Deblurring for the triple-differential yield from Sn+Sn collision @ 270 AMeV

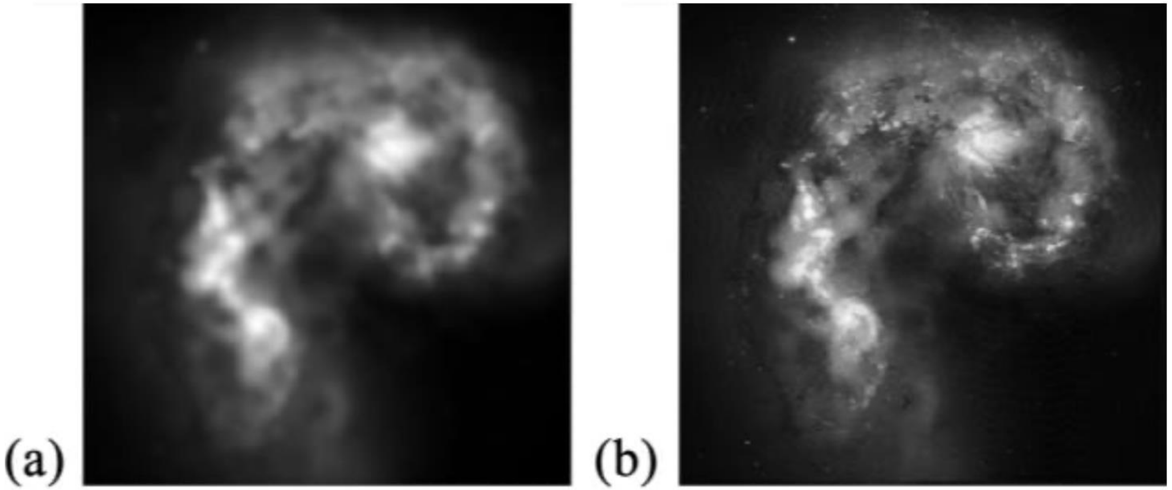
Jeonghyeok Park

Deblurring

Example of blurring and deblurring

Forensic Image deblurring

Original **Blurred** **Restored**



Deblurring in Astronomy (ESA/Hubble)

Deblurring

Richardson-Lucy algorithm

Deblurring process

P. Danielewicz, Phys. Rev. C 105 034608 (2022)
W. H. Richardson, Journal of Optics Society of America 62, 55 (1972)
L. B. Lucy, The Astronomical Journal 79, 745 (1974)

Deblurring

Richardson-Lucy algorithm

$N(X)$ = Original image
$n(Y)$ = Measured image
$P(Y|X)$ = Blurring probability

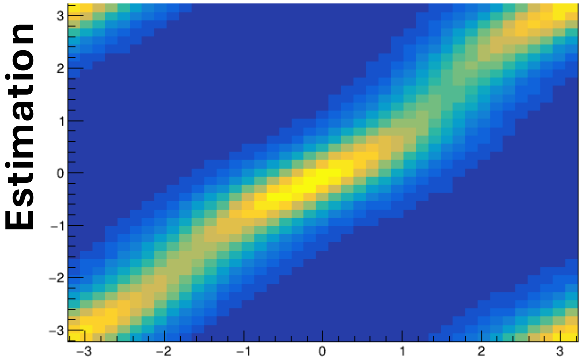
$$n(Y) = \int dX\, P(Y|X)\, N(X)$$

P. Danielewicz, Phys. Rev. C 105 034608 (2022)
W. H. Richardson, Journal of Optics Society of America 62, 55 (1972)
L. B. Lucy, The Astronomical Journal 79, 745 (1974)

Deblurring

Richardson-Lucy algorithm

Construct Transfer matrix



$N(X)$ = Original image

$n(Y)$ = Measured image

$P(Y|X)$ = Blurring probability

$$n(Y) = \int dX\, P(Y|X)\, N(X)$$

P. Danielewicz, Phys. Rev. C 105 034608 (2022)

W. H. Richardson, Journal of Optics Society of America 62, 55 (1972)

L. B. Lucy, The Astronomical Journal 79, 745 (1974)

Deblurring

Richardson-Lucy algorithm

$N^{(r)}(X)$ = Restored image at rth iteration

$$N^{(2)}(\mathrm{X}) = N^{(1)}(X) \int dY \frac{n(Y)}{n^{(1)}(Y)} P(Y|X)$$

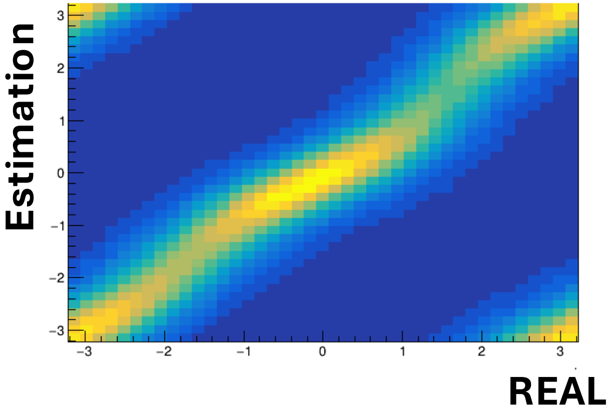
Construct Transfer matrix → **Richardson-Lucy algorithm** →

$N(X)$ = Original image

$n(Y)$ = Measured image

$P(Y|X)$ = Blurring probability

$$n(Y) = \int dX\, P(Y|X)\, N(X)$$



P. Danielewicz, Phys. Rev. C 105 034608 (2022)

W. H. Richardson, Journal of Optics Society of America 62, 55 (1972)

L. B. Lucy, The Astronomical Journal 79, 745 (1974)

Deblurring

Richardson-Lucy algorithm

$N^{(r)}(X)$ = Restored image at rth iteration

$$N^{(r+1)}(\mathrm{X}) = N^{(r)}(X) \int dY \frac{n(Y)}{n^{(r)}(Y)} P(Y|X)$$

Construct Transfer matrix

Richardson-Lucy algorithm

$N(X)$ = Original image

$n(Y)$ = Measured image

$P(Y|X)$ = Blurring probability

$$n(Y) = \int dX\, P(Y|X)\, N(X)$$

P. Danielewicz, Phys. Rev. C 105 034608 (2022)
W. H. Richardson, Journal of Optics Society of America 62, 55 (1972)
L. B. Lucy, The Astronomical Journal 79, 745 (1974)

Deblurring

Richardson-Lucy algorithm

$N^{(r)}(X)$ = Restored image at rth iteration

$$N^{(r+1)}(\mathrm{X}) = N^{(r)}(X) \int dY \, \frac{n(Y)}{n^{(r)}(Y)} P(Y|X)$$

Construct Transfer matrix

Richardson-Lucy algorithm

$N(X)$ = Original image

$n(Y)$ = Measured image

$P(Y|X)$ = Blurring probability

$$n(Y) = \int dX \, P(Y|X) \, N(X)$$

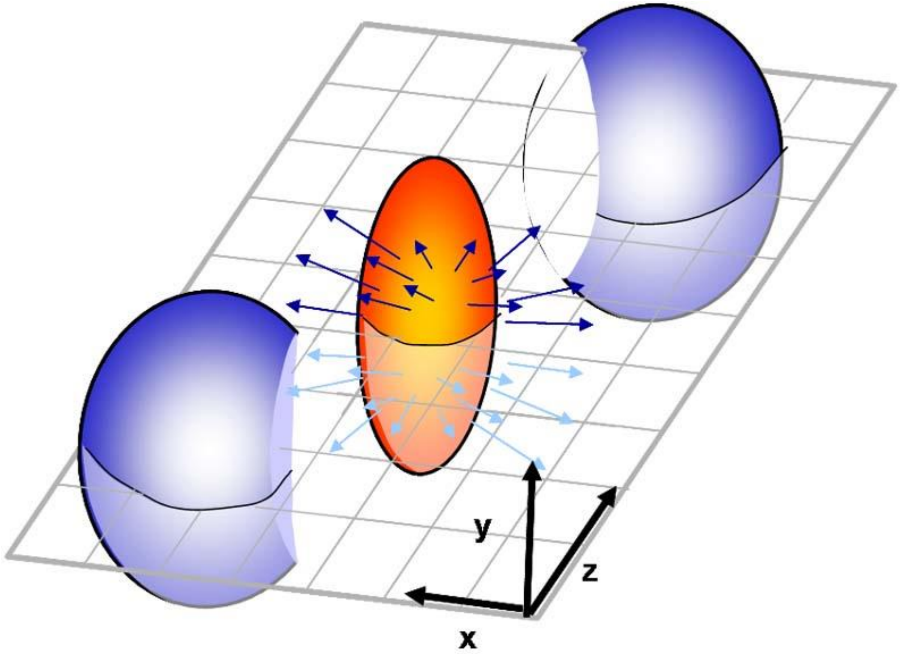
P. Danielewicz, Phys. Rev. C 105 034608 (2022)
W. H. Richardson, Journal of Optics Society of America 62, 55 (1972)
L. B. Lucy, The Astronomical Journal 79, 745 (1974)

Triple differential yield

- Triple differential yield can be written as functions of momentum, rapidity, and angle relative to reaction plane.

- Reaction plane consists of impact parameter axis (x) and beam axis (z) in heavy-ion collision.
  - p^x: Momentum parallel to plane
  - p^y: Momentum perpendicular to plane

e.g.) Collective flow can be extracted by triple differential yield.



$$\phi \equiv \phi_p - \Psi_{RP}$$

$$\frac{d^3N}{p_T dp_T dy d\phi} \propto (1 + 2v_1\cos\phi + 2v_2\cos 2\phi + ..)$$

$$y \leftarrow y_0 \equiv \frac{y_{LAB}}{y_{NN}^{CM}} - 1$$

Reaction plane dispersion

Reaction plane angle blurring in methodology

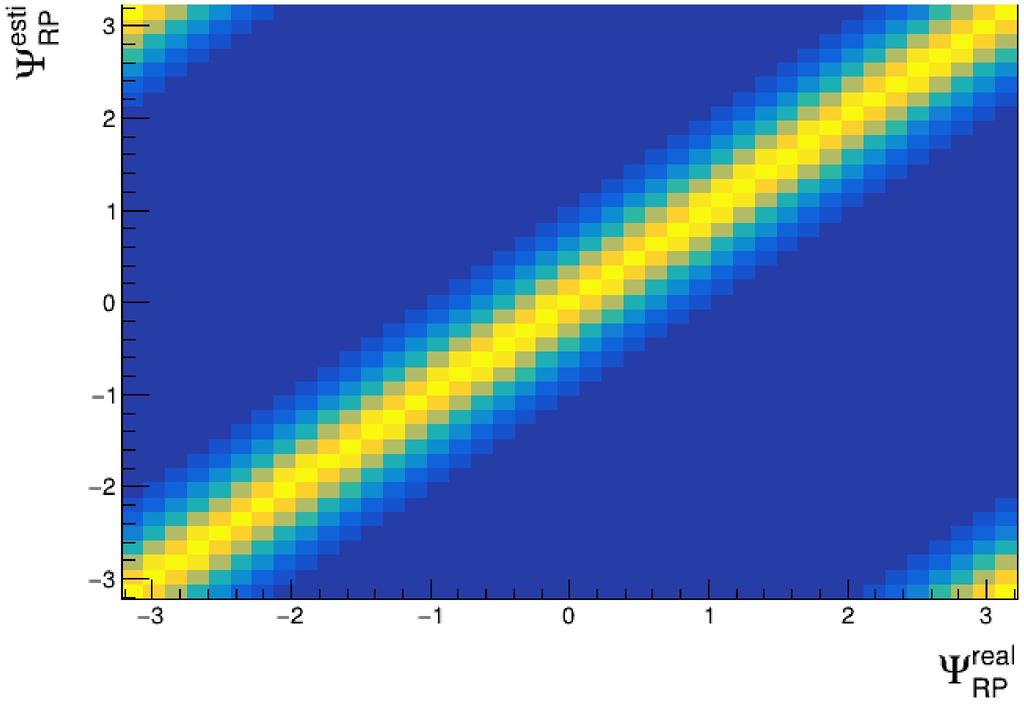
$$\vec{Q} = \sum_{i \neq j} w_i \overrightarrow{p_{Ti}}$$

$$w_i = \begin{cases} +1 & (y_0 > 0.1) \\ -1 & (y_0 < -0.1) \\ 0 & (elsewhere) \end{cases}$$

$$\Psi_{RP} = \mathrm{atan}\left(\frac{Q_y}{Q_x}\right)$$

- Reaction plane (RP) angle can be determined by Q vector from emitted charged particles in heavy-ion collision.
  - Flow model
  - 100 % efficiency assumption
- Reaction plane angle has been "blurred" in determination from Q vector method.

Simulation



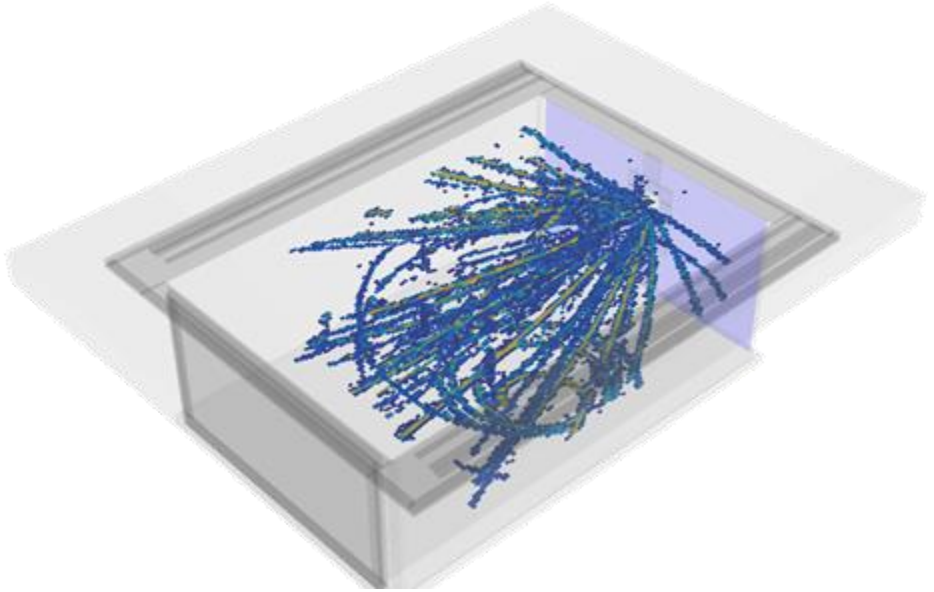
- Ψ_{RP}^{real} = Real RP angle from simulation
- Ψ_{RP}^{esti} = Estimated RP angle from Q

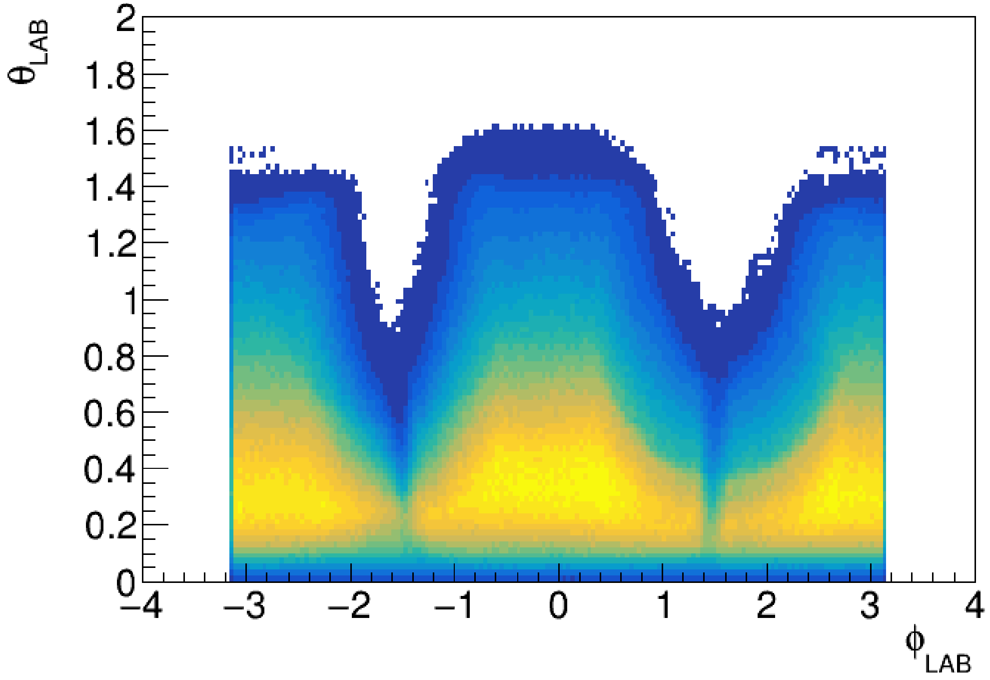
SPiRIT

SAMURAI Pion-Reconstruction and Ion-Tracker

- SPiRIT 2016 experiment @ RIKEN
  - Equation of state
  - TPC (Time Projection Chamber)
  - Placed in SAMURAI magnet (0.5 T)
  - 1344 x 864 mm^2 of pad plane area
  - $^{132}Sn + ^{124}Sn$ @ 270 MeV/u
  - $^{108}Sn + ^{112}Sn$ @ 270 MeV/u

- Stronger blurring problem due to anisotropy in azimuthal angle will be solved via deblurring process.

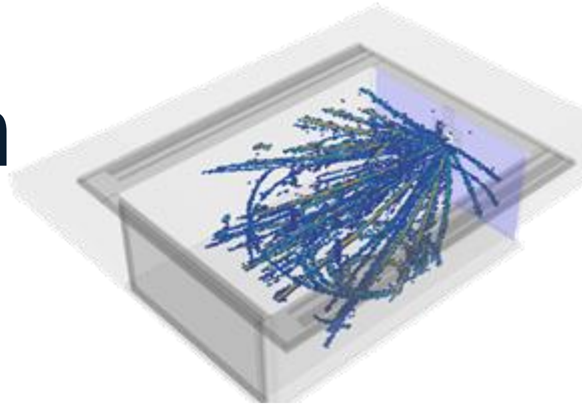




P. Danielewicz, Phys. Lett. B (1985)

Reaction plane dispersion

Reaction plane distortion in SPiRIT



$$\vec{Q} = \sum_{i \neq j} w_i \overrightarrow{p_{Ti}}$$

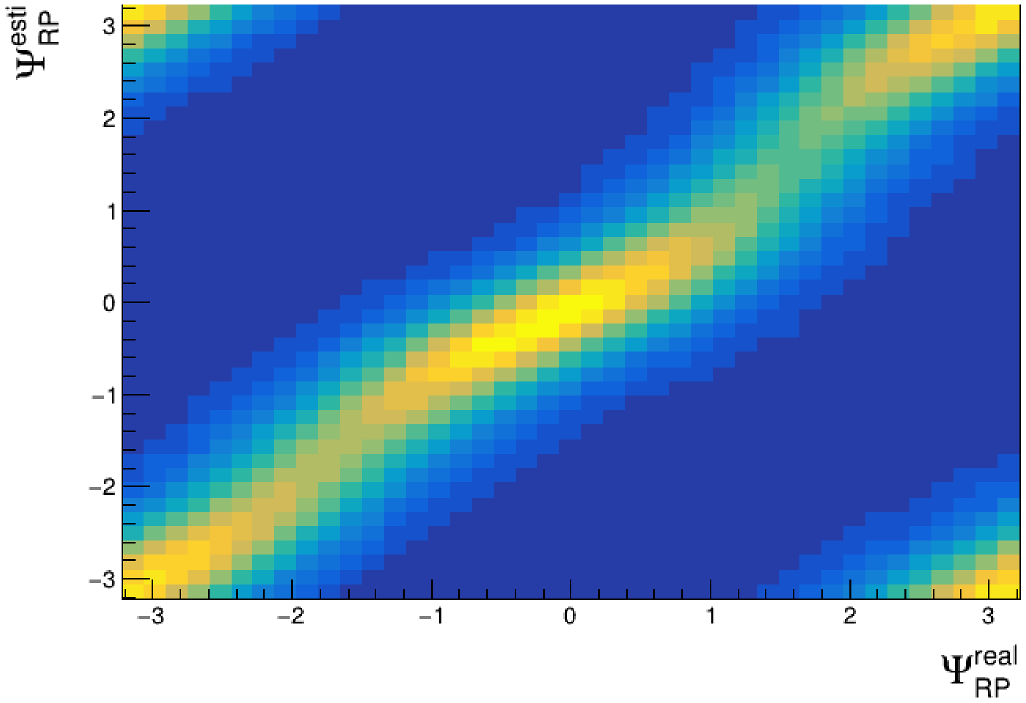
$$\Psi_{RP} = \mathrm{atan}\left(\frac{Q_y}{Q_x}\right)$$

$$w_i = \begin{cases} +1 & (y_0 > 0.1) \\ 0 & (elsewhere) \end{cases}$$

- Reaction plane (RP) angle can be determined by Q vector from emitted charged particles in heavy-ion collision.
  - Considered only forward particle
  - SPiRIT efficiency filtered in flow model
- Due to the detector inefficiency, reaction plane angle distribution has been distorted.

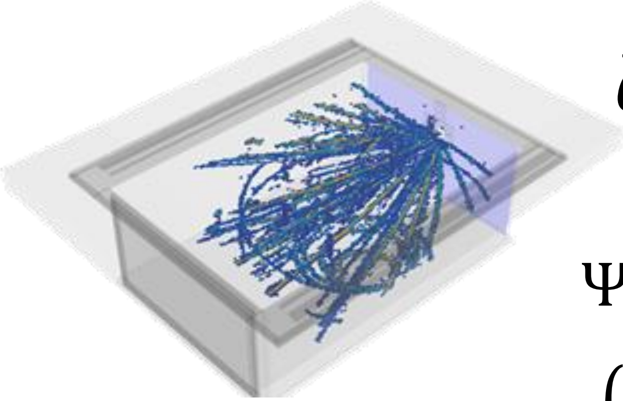
- $\Psi_{\mathrm{RP}}^{\mathrm{real}}$ = Real RP angle from simulation
- $\Psi_{\mathrm{RP}}^{\mathrm{esti}}$ = Estimated RP angle from Q

Simulation



Reaction plane dispersion

Reaction plane distortion in SPiRIT



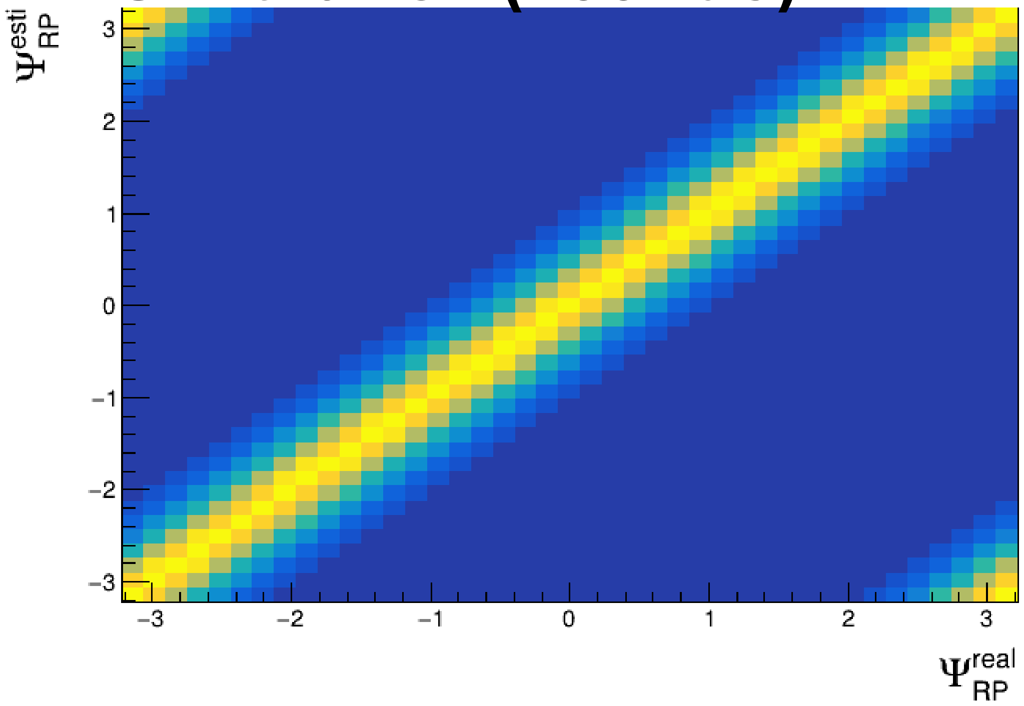
P. Danielewicz, Phys. Lett. B (1985)

$$\vec{Q} = \sum_{i \neq j} w_i \overrightarrow{p_{Ti}}$$

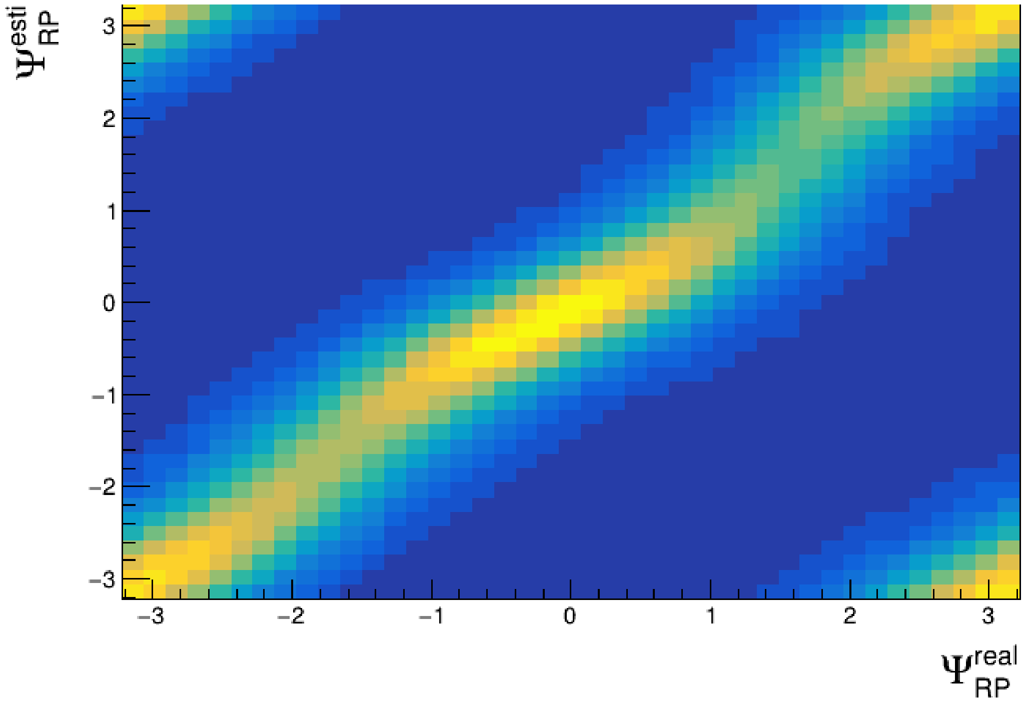
$$\Psi_{RP} = \mathrm{atan}\left(\frac{Q_y}{Q_x}\right)$$

$$w_i = \begin{cases} +1 & (y_0 > 0.1) \\ 0 & (elsewhere) \end{cases}$$

Simulation (100 % ε)

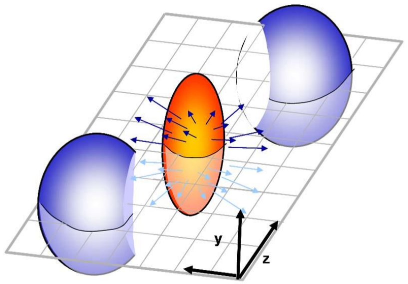


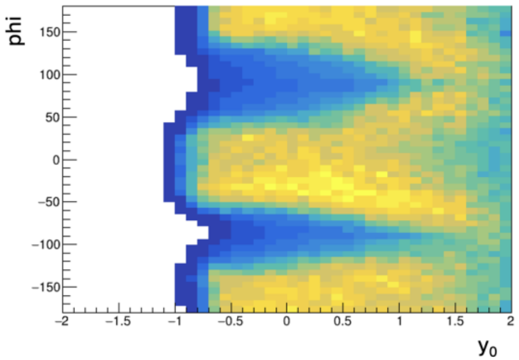
Simulation



Triple differential yield

Deblurring of flow model + SPiRIT efficiency filter

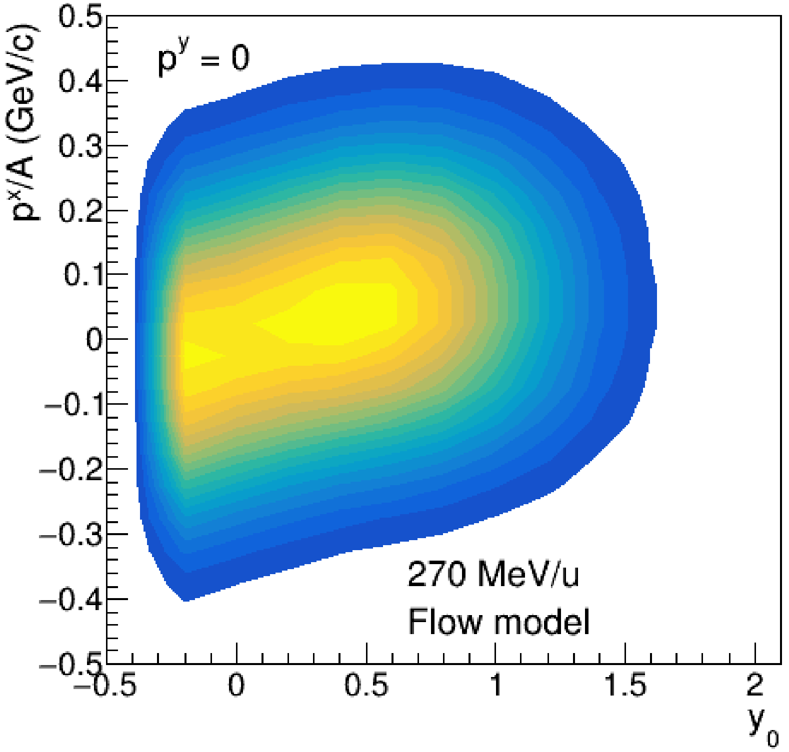


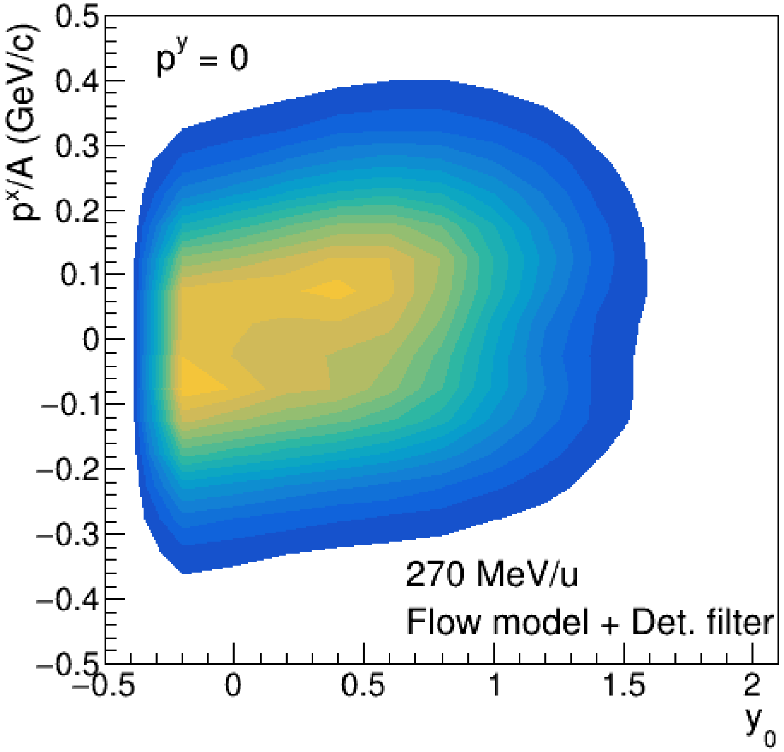


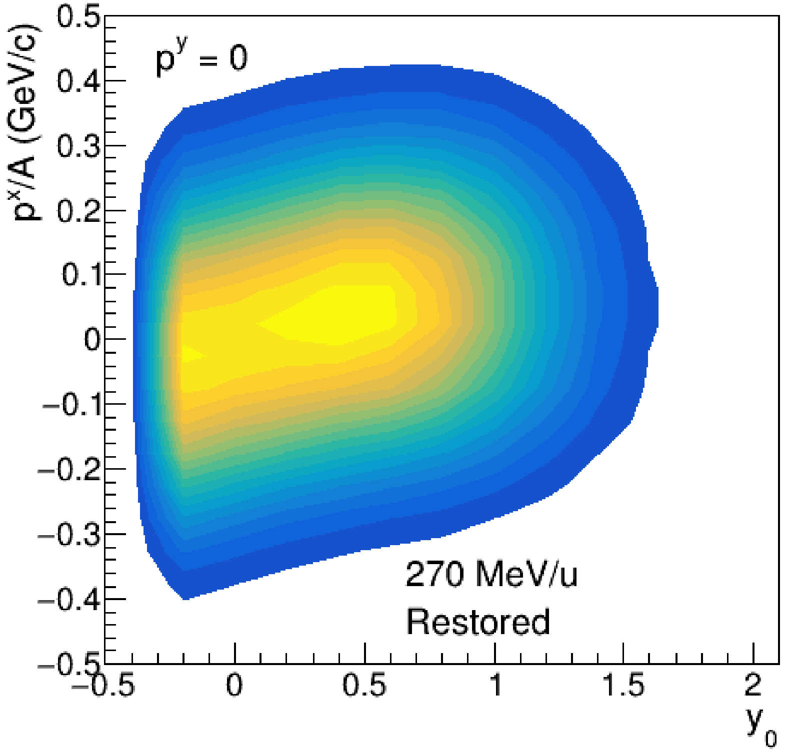
SPiRIT efficiency

Consistent Z-axis scale

Particle= Deuteron







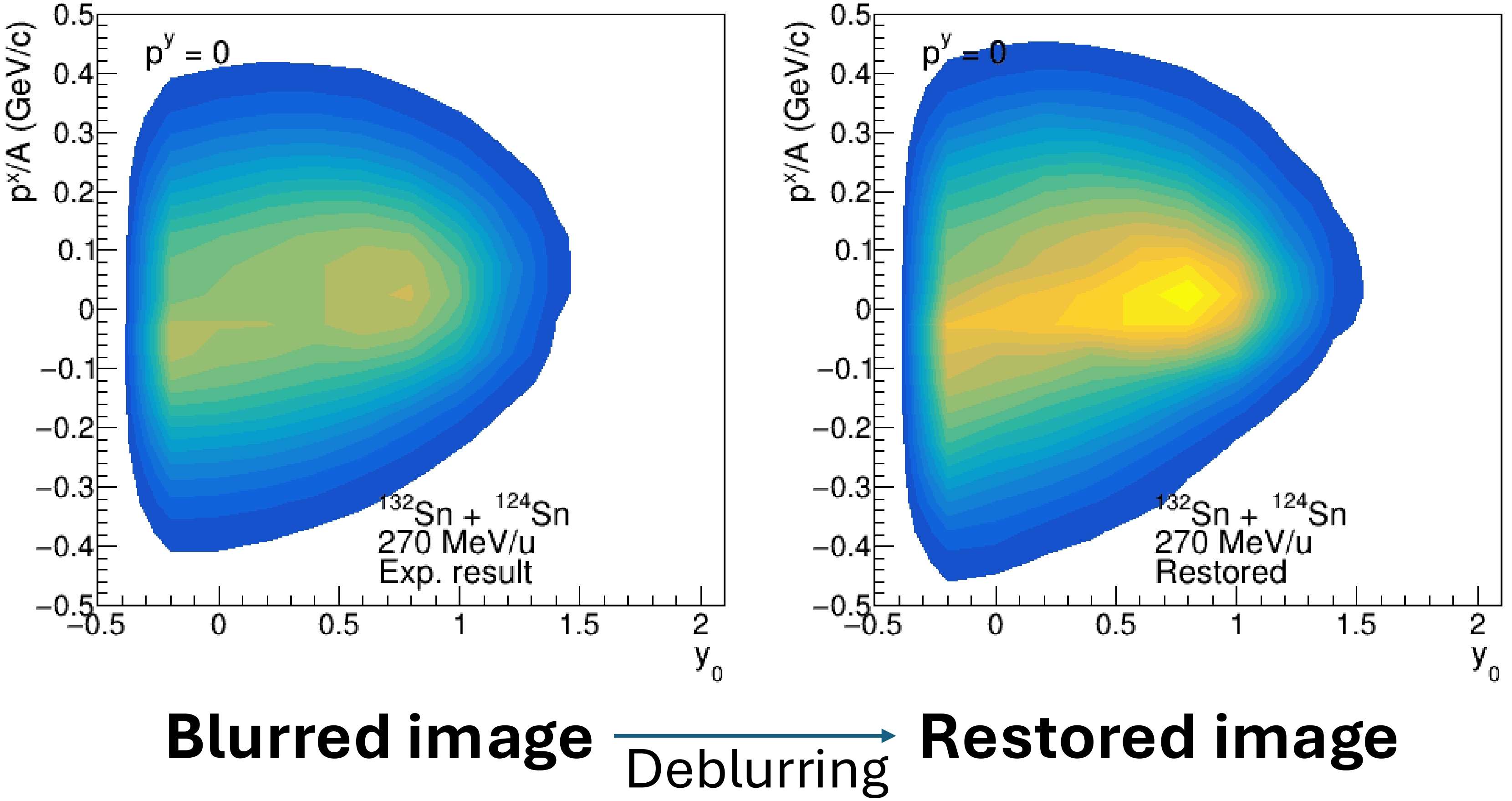
Original image $\xrightarrow{\text{Blurring}}$ **Blurred image** $\xrightarrow{\text{Deblurring}}$ **Restored image**

Triple differential yield

Deblurring of experimental data

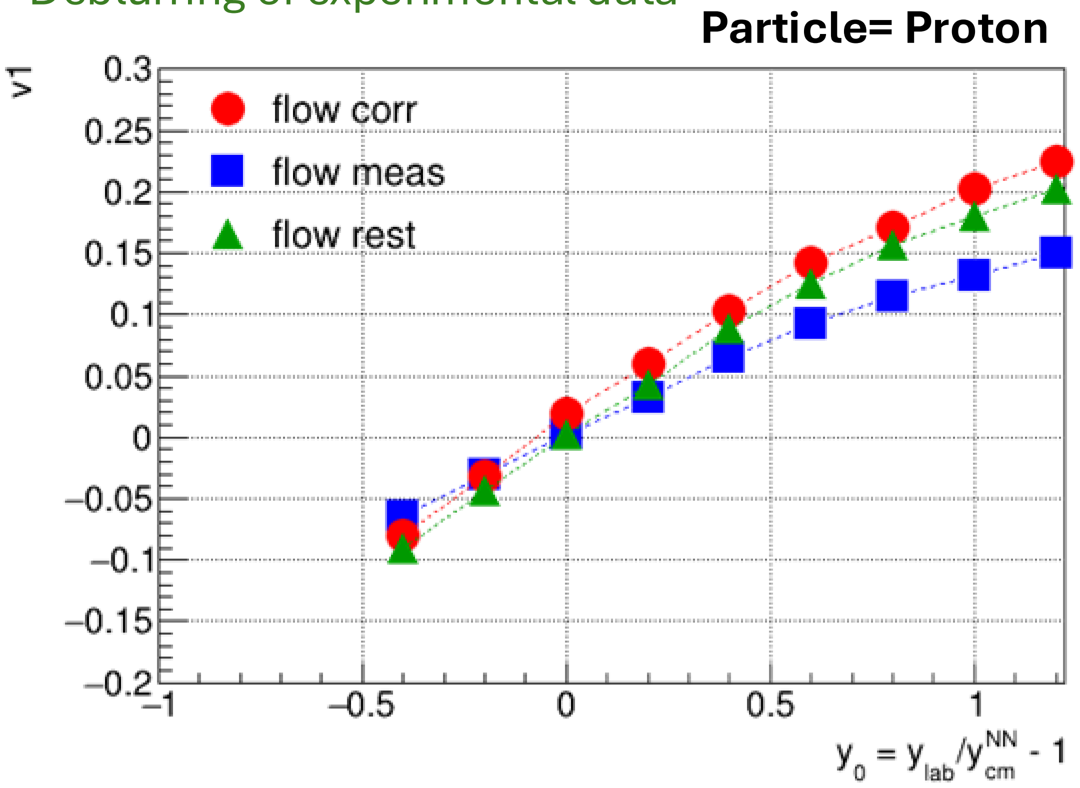
- Clear triple differential yield
  - In-plane momentum vs
    normalized rapidity
- Lower intensity in blurred image due to detection inefficiency

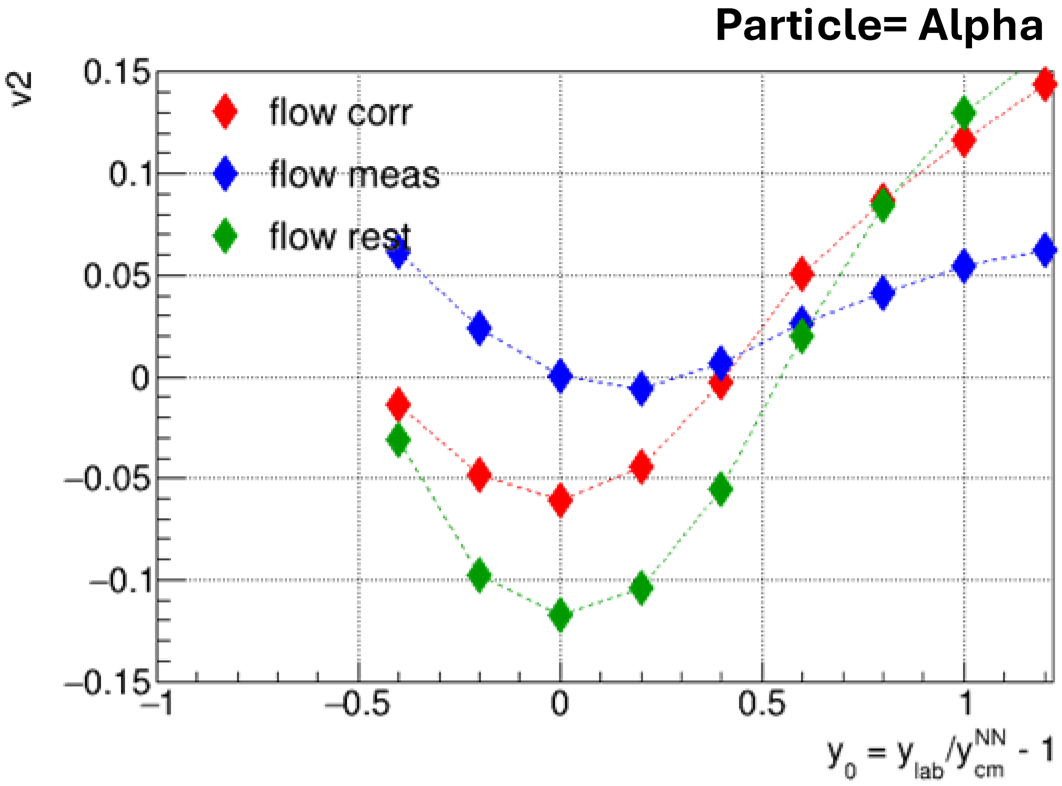
Consistent Z-axis scale **Particle= Deuteron**



Flow parameter

Deblurring of experimental data





- Corrected: Conventional analysis result
- Measured: No corrections (From Q)
- Restored: Deblurring result

$$\frac{d^3N}{p_T dp_T dy d\phi} \propto (1 + 2v_1 \cos\phi + 2v_2 \cos 2\phi + ..)$$

Summary

- Deblurring, the process to remove blurring effect by camera (or detector), has been applied to make clear image of triple differential yield in heavy-ion collision.
- Triple differential yield from SPiRIT data due to the methodology and detection inefficiency has been restored.
- Restored triple differential yield shows nearly consistent results of v1 and stronger v2 flow parameters.

Acknowledgement

Pawel Danielewicz

Professor of Physics

Theoretical Nuclear Physics
Joined the laboratory in 1988

FRIB

Backups

Richardson-Lucy algorithm

- Acting blurring to the original image causes measured image.
  - $n(Y) = \int dX\, P(Y|X)\, N(X)$

- Yielding from Bayesian consideration, Richardson-Lucy handles deconvolution iteratively.

$N(X)$ = Original image (distribution)
$n(Y)$ = Measured image (distribution)
$P(Y|X)$ = Blurring probability
$Q(X|Y)$ = Complementary probability to $P(Y|X)$

$$n(Y) = \int dX\, P(Y|X)\, N(X)$$

$$Q(X|Y)\, n(Y)\, dY dX = P(Y|X)\, N(X)\, dX dY$$

$$Q(X|Y) = \frac{P(Y|X) N(X)}{\int dX'\, P(Y|X')\, N(X')}$$

$$\mathrm{N}(\mathrm{X}) = \frac{\int dY\, Q(X|Y)\, n(Y)}{\int dY\, P(Y|X)}$$

Rough assumption
$n = \varepsilon N$
$N = n/\varepsilon$
ε = efficiency

$$Q^{(r)}(X|Y) = \frac{P(Y|X) N^{(r)}(X)}{\int dX'\, P(Y|X')\, N^{(r)}(X')}$$

$$N^{(r+1)}(\mathrm{X}) = \frac{\int dY\, Q^{(r)}(X|Y)\, n(Y)}{\int dY\, P(Y|X)}$$

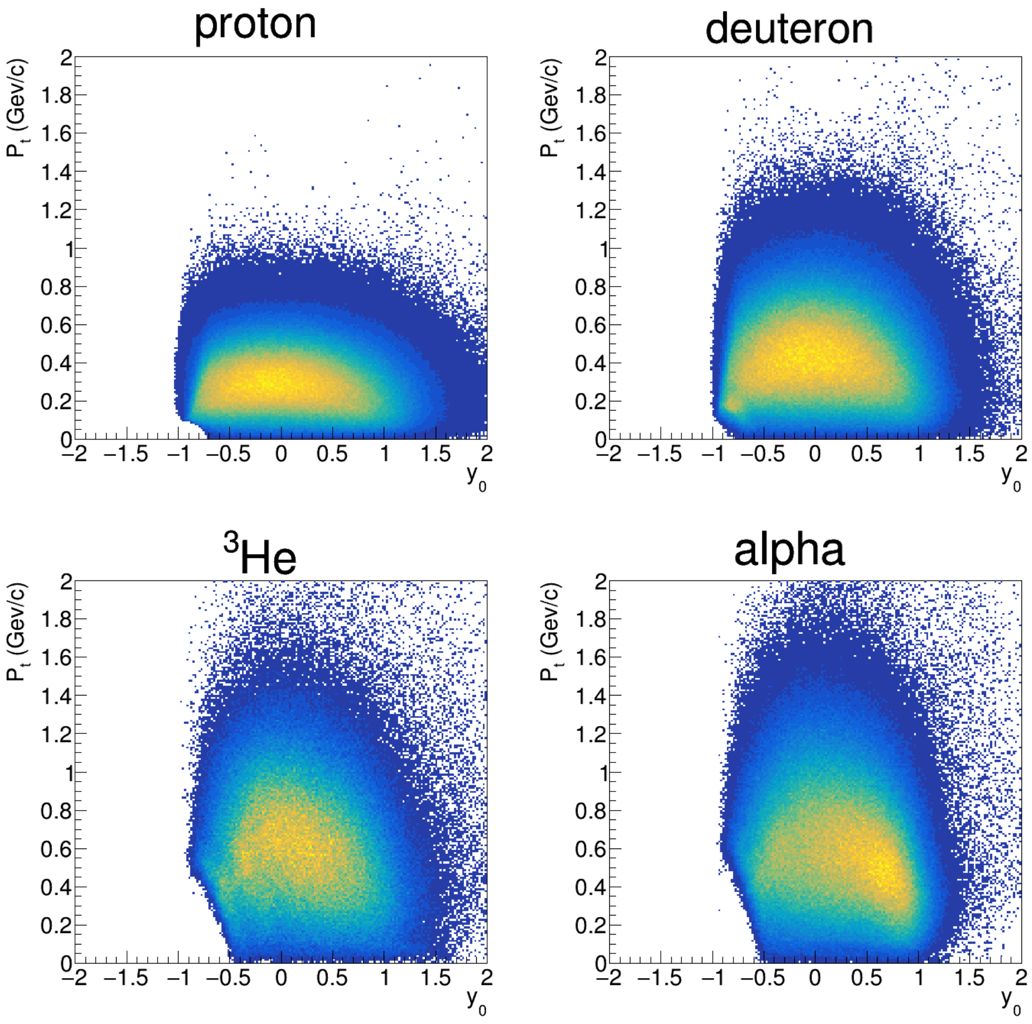
P. Danielewicz, Phys. Rev. C 105 034608 (2022)
W. H. Richardson, Journal of Optics Society of America 62, 55 (1972)
L. B. Lucy, The Astronomical Journal 79, 745 (1974)

Backups

$^{132}Sn + {}^{124}Sn$ @ 270 MeV/u

SPiRIT experimental data

- Phase space distribution of SPiRIT experimental data.
  - Transverse momentum
  - Rapidity
- Impact parameter ($0.03 < b_0 < 0.2$)
  - $b_0 = b/b_{max}$
  - mid-central collision
- Backward rapidity cut in analysis
  - $y_0 < -0.3$ (for 3He)
  - $y_0 < -0.5$ (for others)



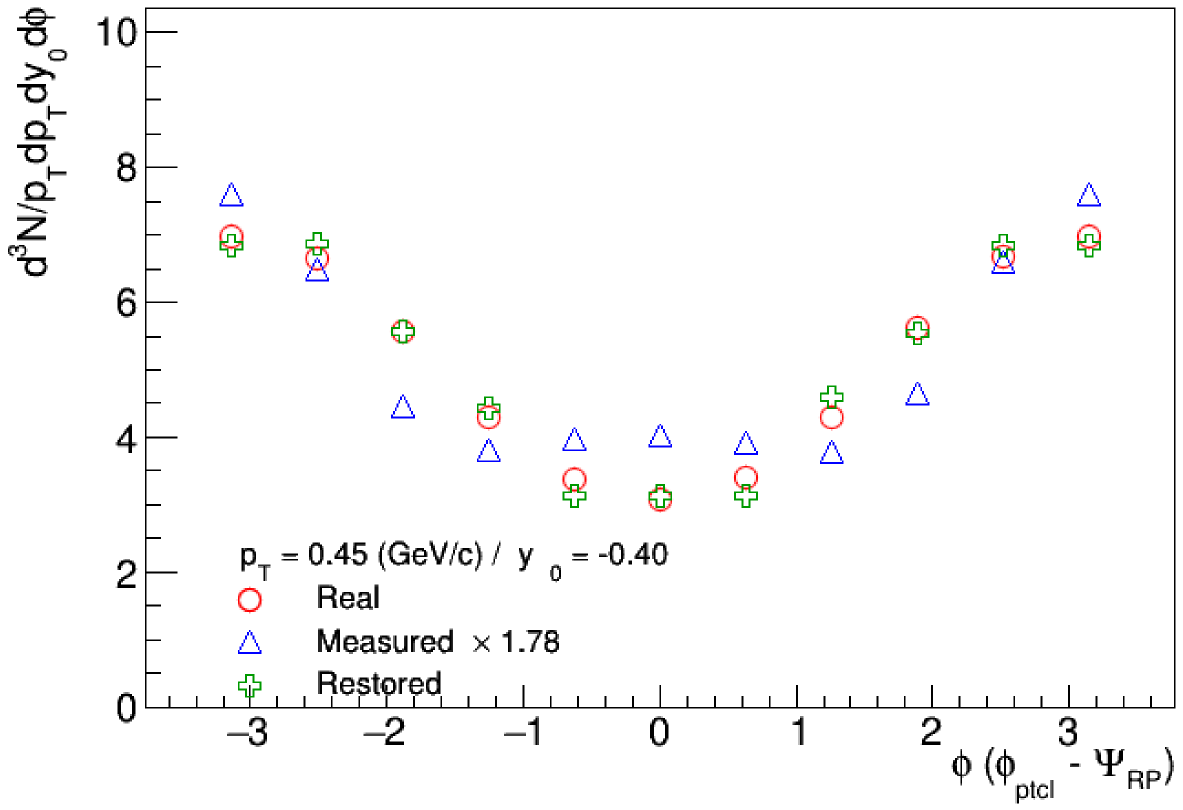
$$\vec{Q} = \sum_{i \neq j} w_i \overrightarrow{p_{Ti}}$$

$$\Psi_{RP} = \mathrm{atan}\left(\frac{Q_y}{Q_x}\right)$$

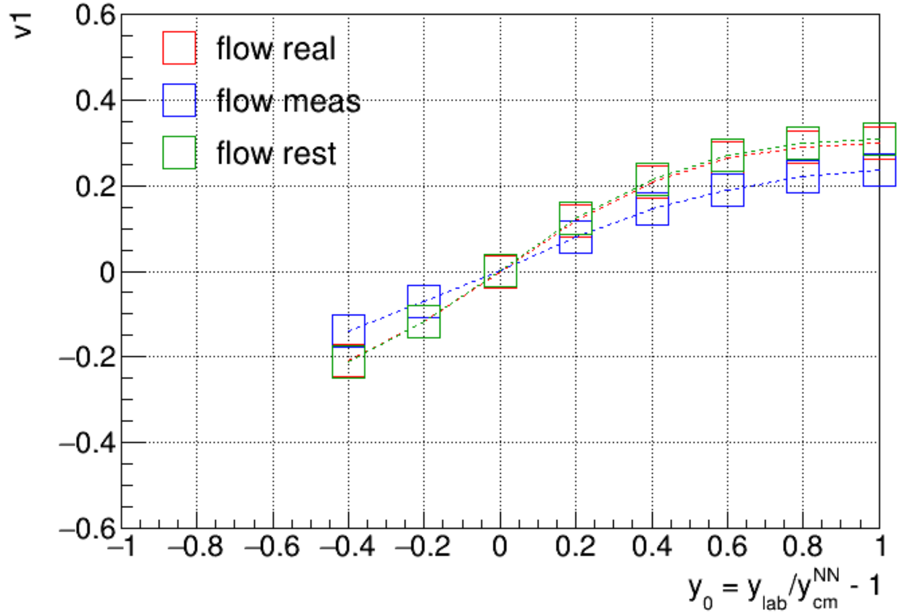
$$w_i = \begin{cases} +1 & (y_0 > 0.1) \\ 0 & (elsewhere) \end{cases}$$

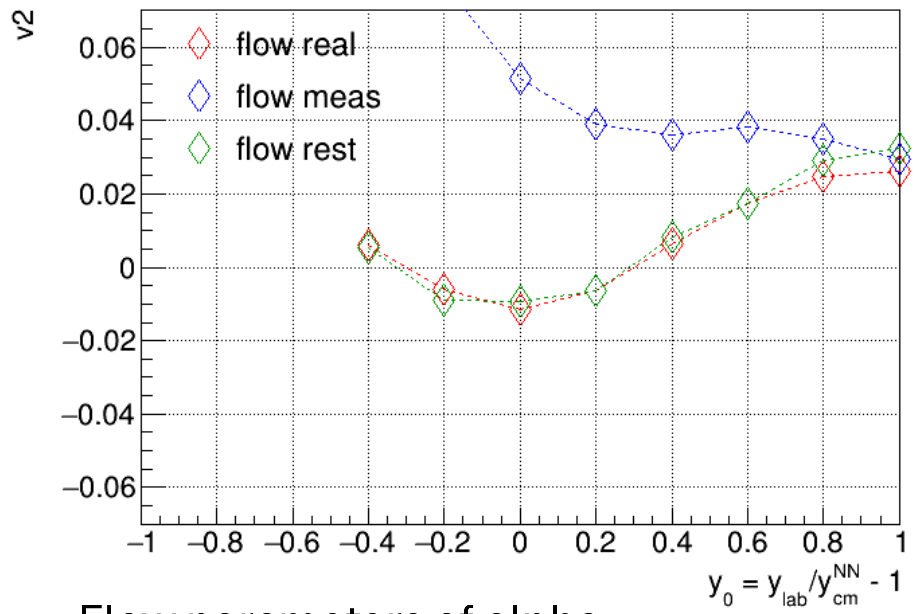
Backups

Deblurring of flow model + SPiRIT efficiency filter



Deuteron angle distribution (at backward)

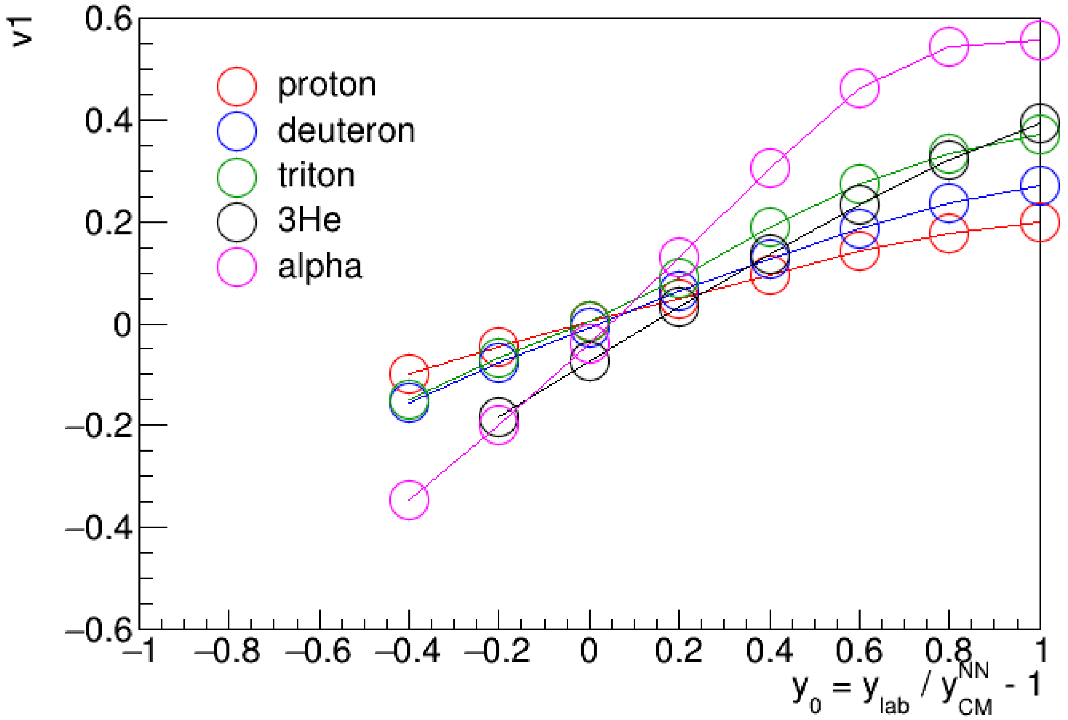


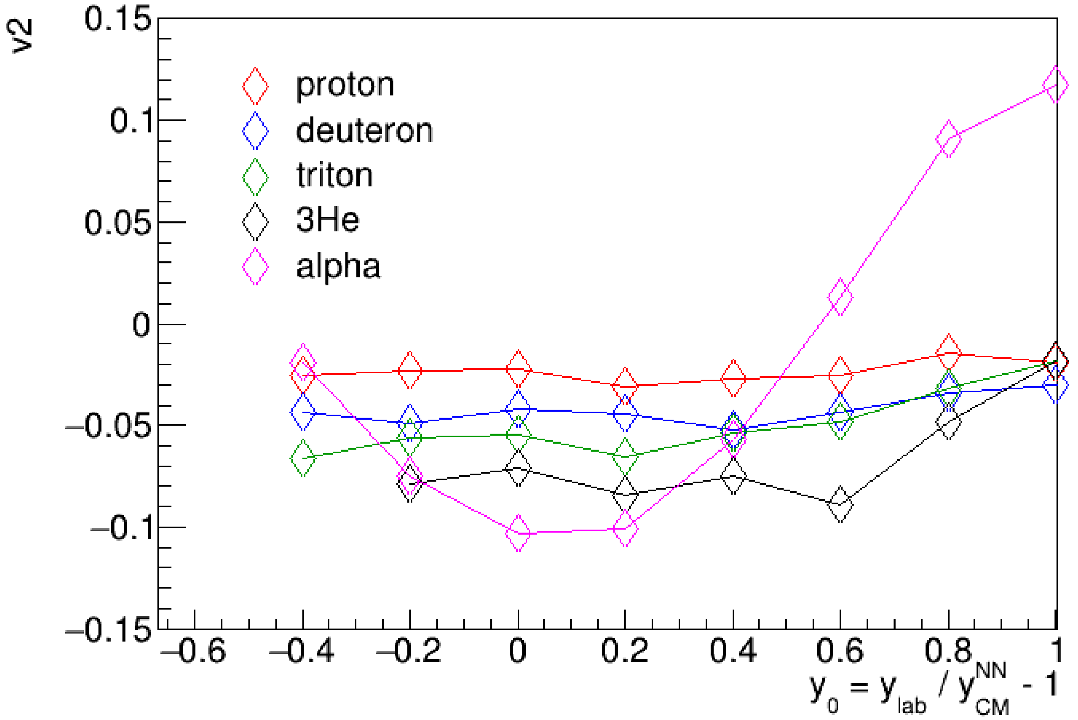


Flow parameters of alpha

Backups

Flow parameter from the deblurring of experimental data

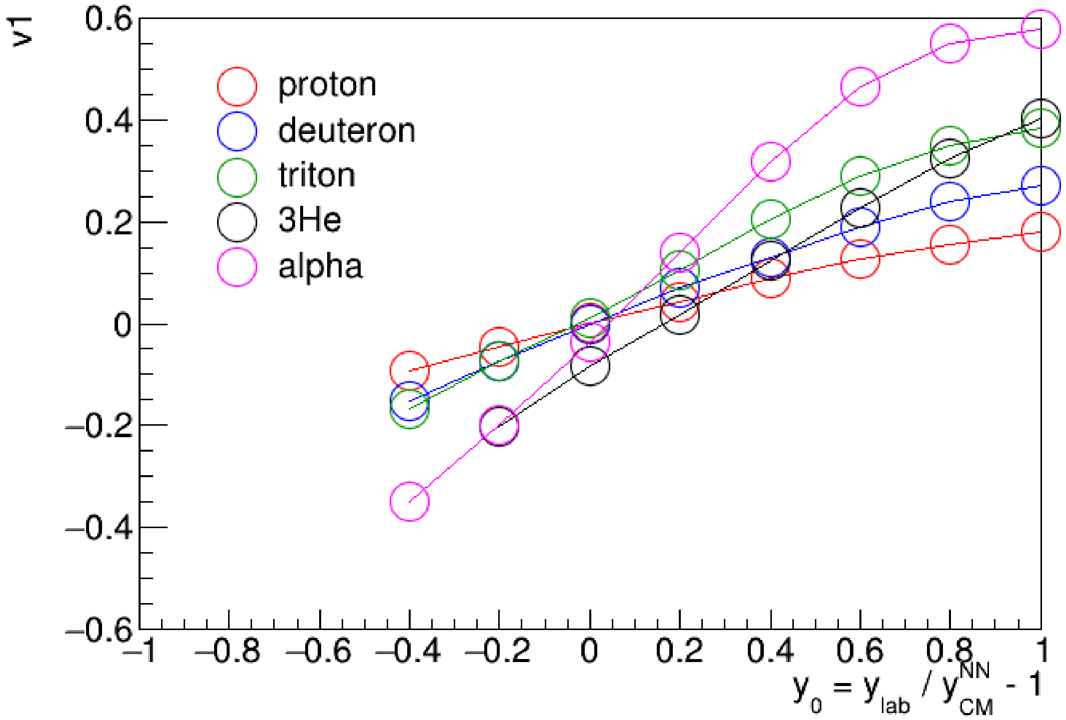


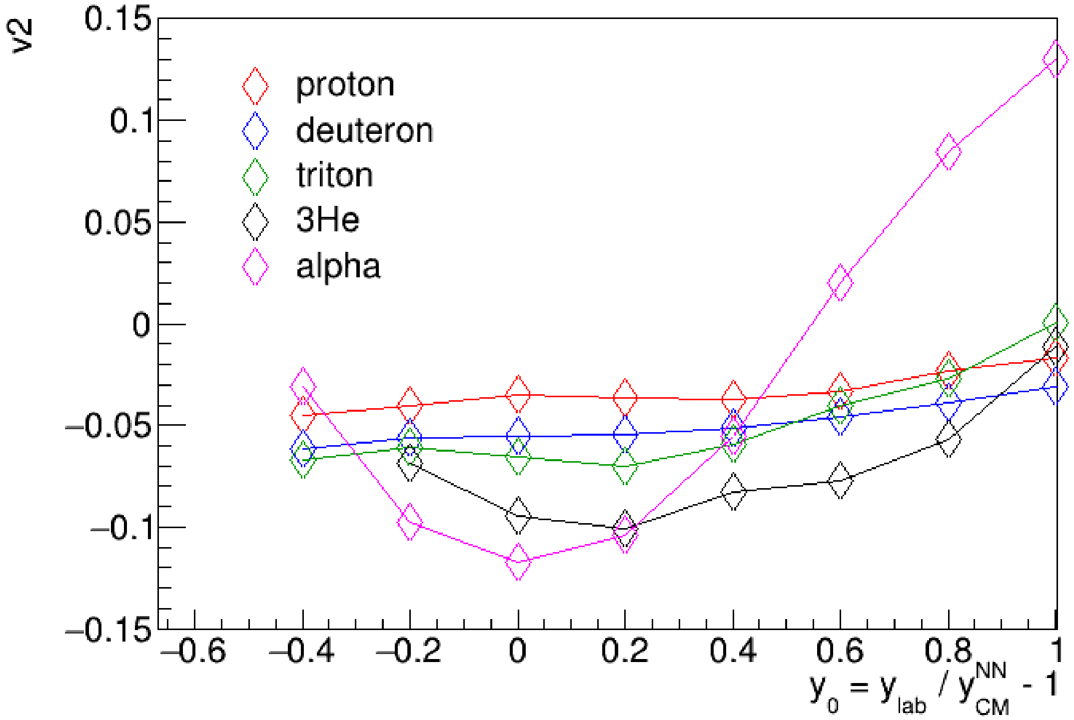


^{108}Sn + ^{112}Sn @ 270 AMeV (0.03 < b_0 < 0.2)

Backups

Flow parameter from the deblurring of experimental data

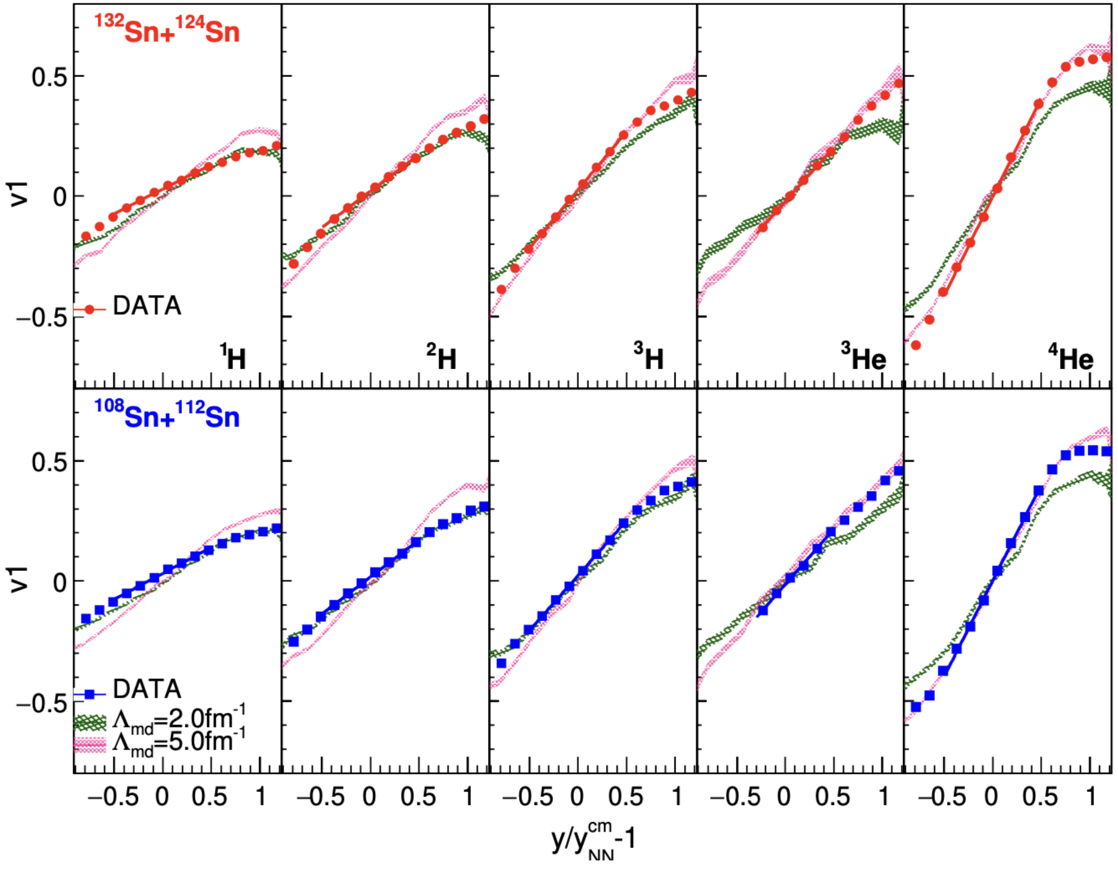


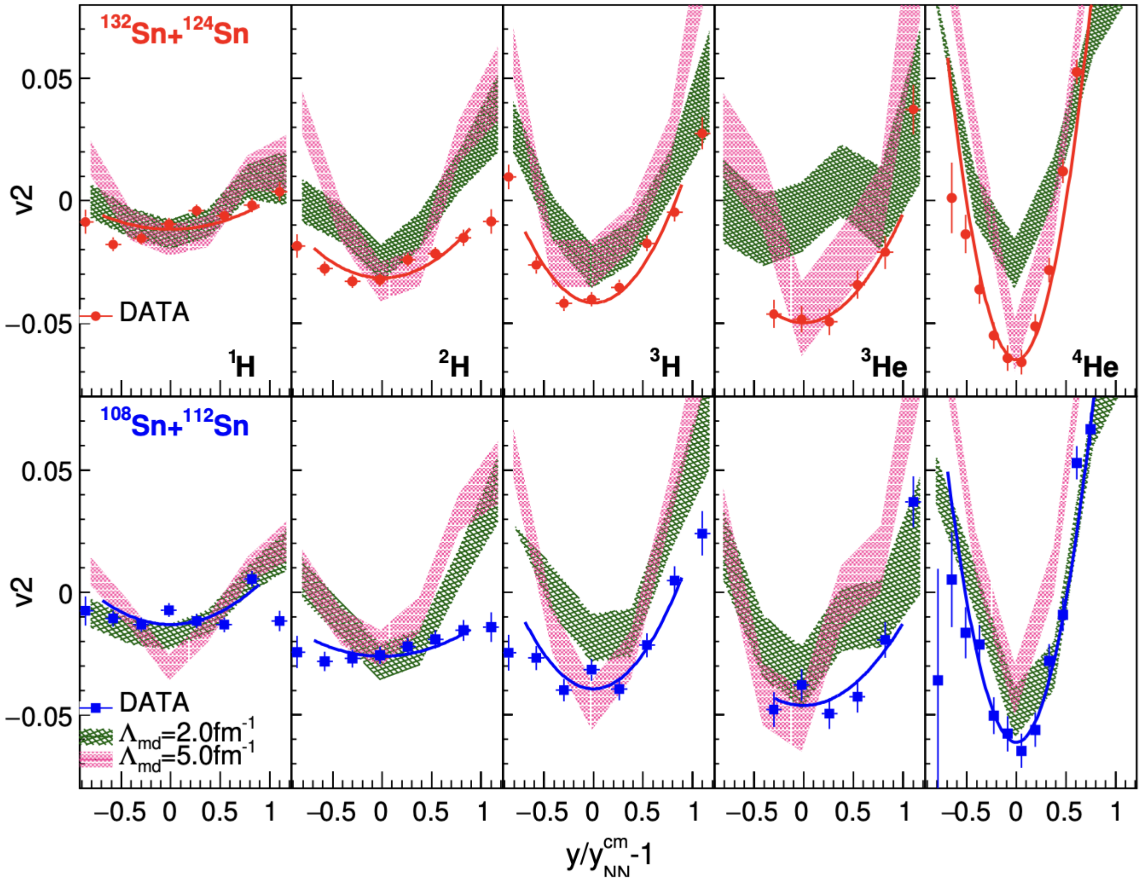


^{132}Sn + ^{124}Sn @ 270 AMeV (0.03 < b_0 < 0.2)

Backups

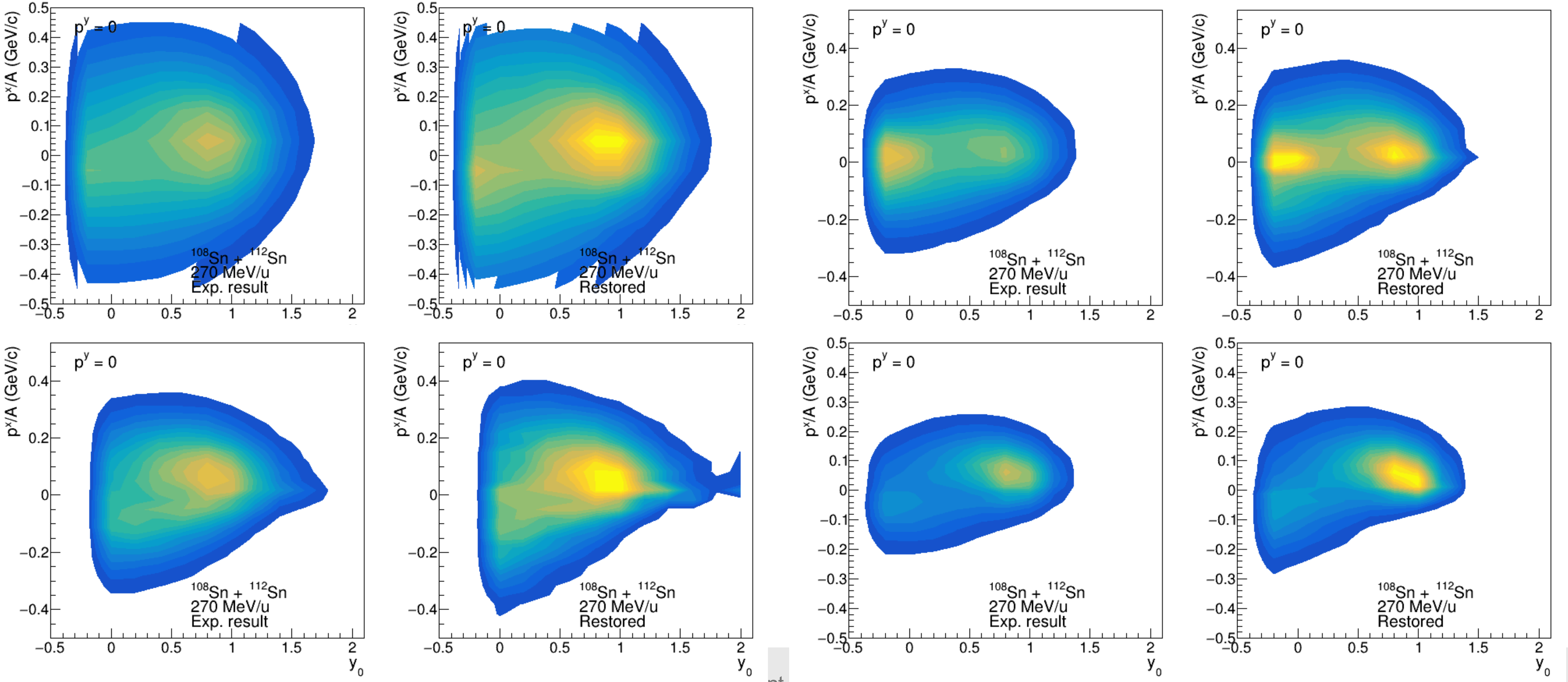
Flow parameter of conventional method





Backups

Deblurring of experimental data



Backups

Deblurring of experimental data