The 4th Korea-China joint workshop for rare isotope,
Benikea Hotel, Jeju island, Korea, 2025.07.10

Investigations on properties of resonances for multineutron systems

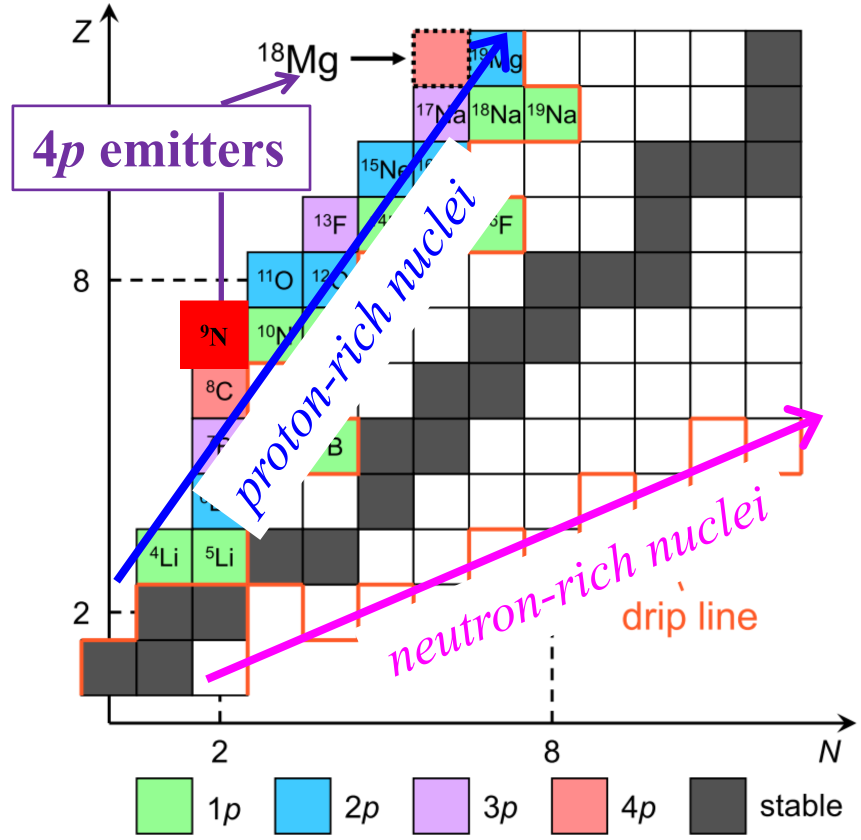
Niu Wan (万牛)
South China University of Technology (华南理工大学)
wanniu@scut.edu.cn

Takayuki Myo(OIT)， Hiroki Takemoto(OMP)， Mengjiao Lyu(NUAA)， Qing Zhao(HZU)，
Hisashi Horiuchi(RCNP)， Masahiro Isaka(Hosei)， Akinobu Doté (KEK, IPNS)

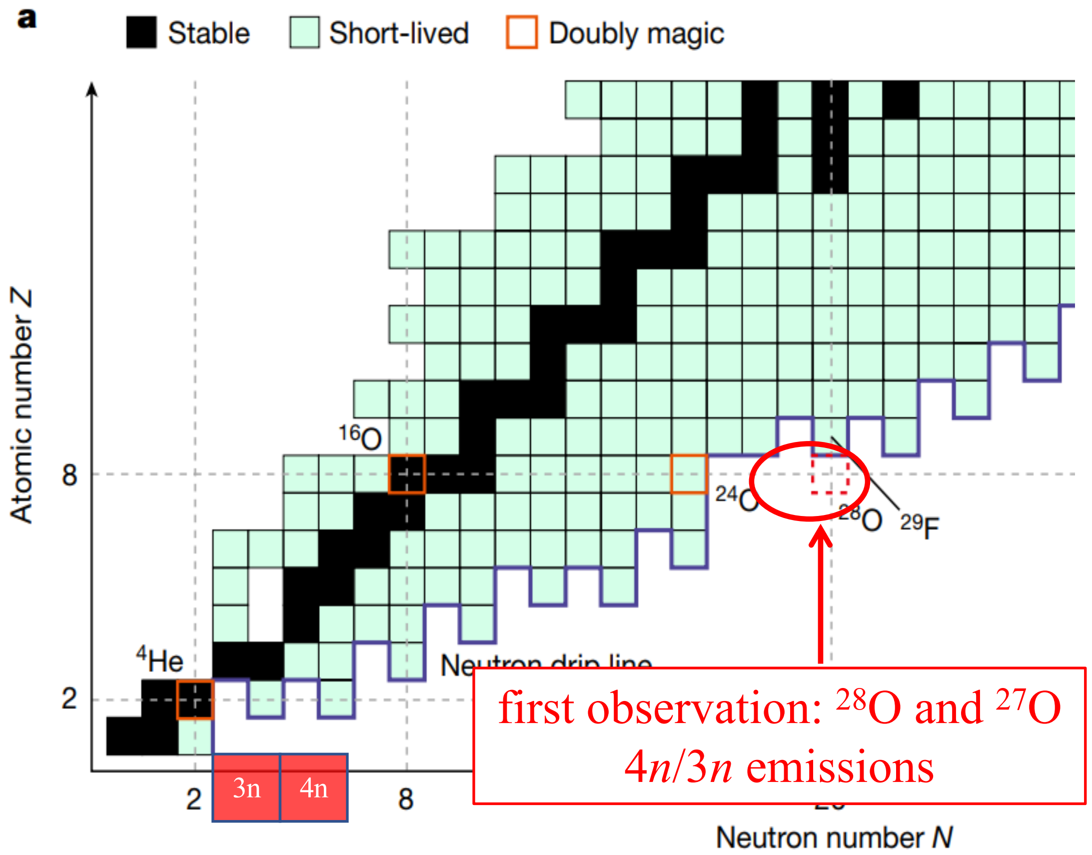
Outline

- Background
- Methods
  - ✓ Tensor-optimized antisymmetrized molecular dynamics (TOAMD)
  - ✓ Inverse analytical continuation in the coupling constant method (IACCC)
  - ✓ Possible resonances of trineutron and tetraneutron
- Summary and Outlook

Background

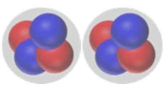


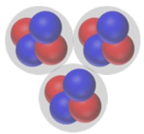
Y. Jin et al., PRL127, 262502 (2021)

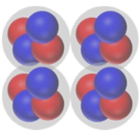


Y. Kondo et al., Nature 620 965 (2023)

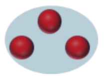
A. A. Ogloblin et al., Treatise on Heavy-Ion Science: Nuclei Far From Stability, ed. by D. A. Bromley, Plenum Press, New York (1989) and references therein

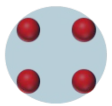

unbound


bound


bound

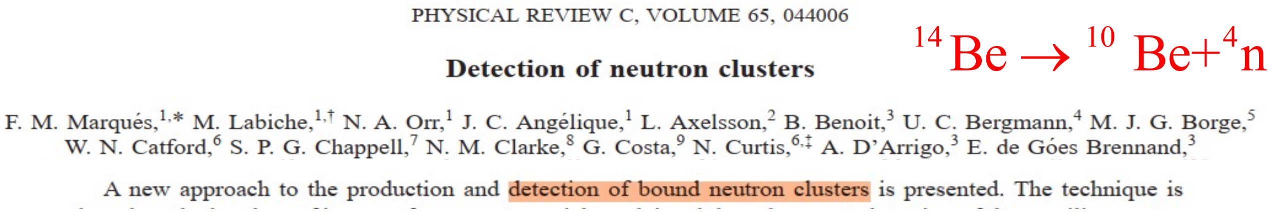

unbound


?


?

Background

F. M. Marques, et al., PRC65, 044006 (2002)


PHYSICAL REVIEW C, VOLUME 65, 044006

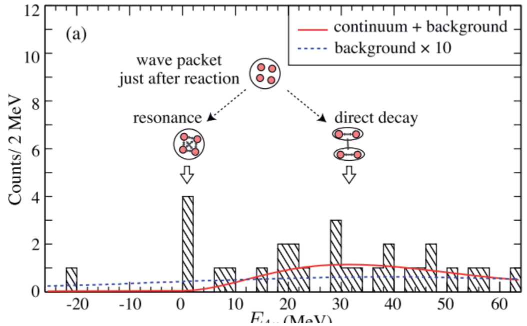
Detection of neutron clusters

F. M. Marqués,[1,*] M. Labiche,[1,†] N. A. Orr,[1] J. C. Angélique,[1] L. Axelsson,[2] B. Benoit,[3] U. C. Bergmann,[4] M. J. G. Borge,[5] W. N. Catford,[6] S. P. G. Chappell,[7] N. M. Clarke,[8] G. Costa,[9] N. Curtis,[6,‡] A. D'Arrigo,[3] E. de Góes Brennand,[3]

A new approach to the production and detection of bound neutron clusters is presented. The technique is

$^{14}\mathrm{Be} \rightarrow {}^{10}\mathrm{Be} + {}^{4}\mathrm{n}$

K. Kisamori et al., PRL116, 052501 (2016)

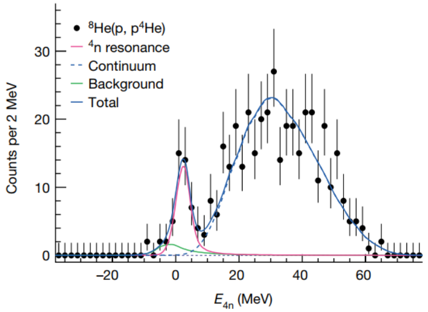


${}^{4}\mathrm{He}\left({}^{8}\mathrm{He},{}^{8}\mathrm{Be}\right)$

E_R=0.83±0.65±1.25 MeV

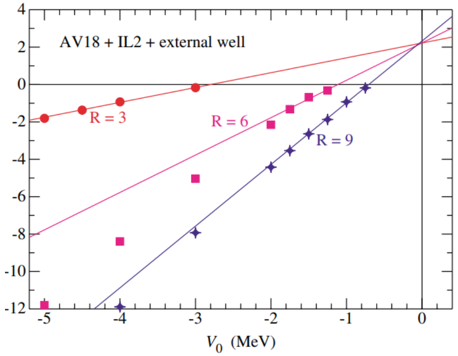
Γ<2.6 MeV

M. Duer et al., Nature 606, 678 (2022)

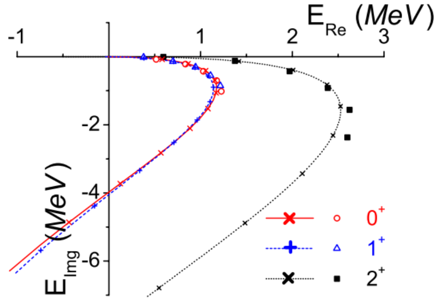


${}^{8}\mathrm{He}\left(p, p\,{}^{4}\mathrm{He}\right)$

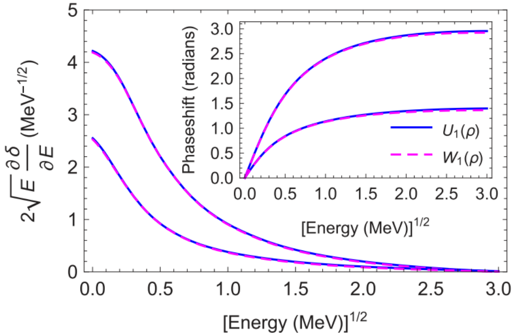
$E_r = 2.37 \pm 0.38(\mathrm{stat.}) \pm 0.44(\mathrm{sys.})$ MeV,
$\Gamma = 1.75 \pm 0.22(\mathrm{stat.}) \pm 0.30(\mathrm{sys.})$ MeV.



*S. C. Pieper, PRL**90**, 252501 (2003)*

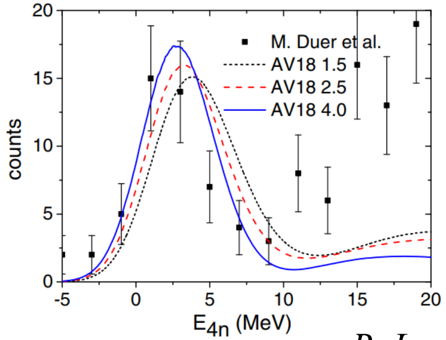


R. Lazauskas et al., PRC72, 034003 (2005); PRC71, 044004 (2005)



Enhanced density of state
No theoretical signal for resonance

M. D. Higgins, et al., PRL125, 052501 (2020); PRC103, 024004 (2021)



4n weakly bound by the core

Final-state interaction between dineutron-dineutron

R. Lazauskas, et al, PRL130, 102501 (2023)

Background

Motivation: correlations among 4n and extrapolation method

Calculations on multineutron systems:

1. Directly search resonant state: Faddeev+CSM et al.

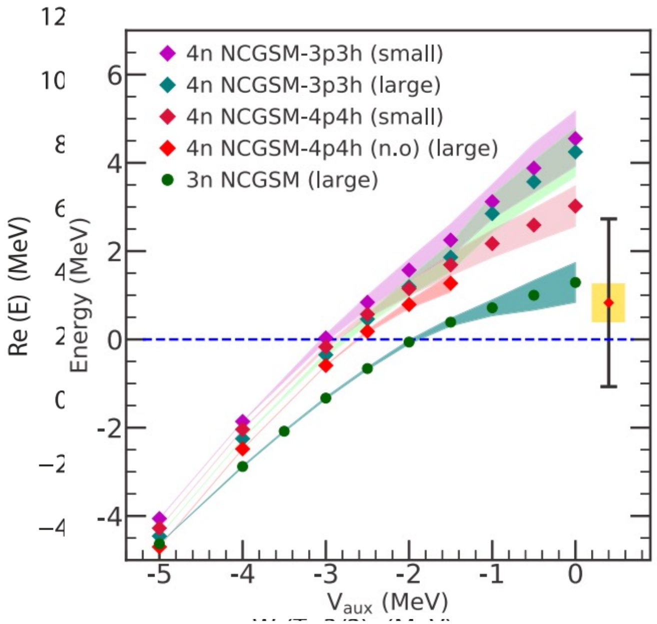
2. Extrapolate from artificial bound state to resonant region

(1) introduce 3BF

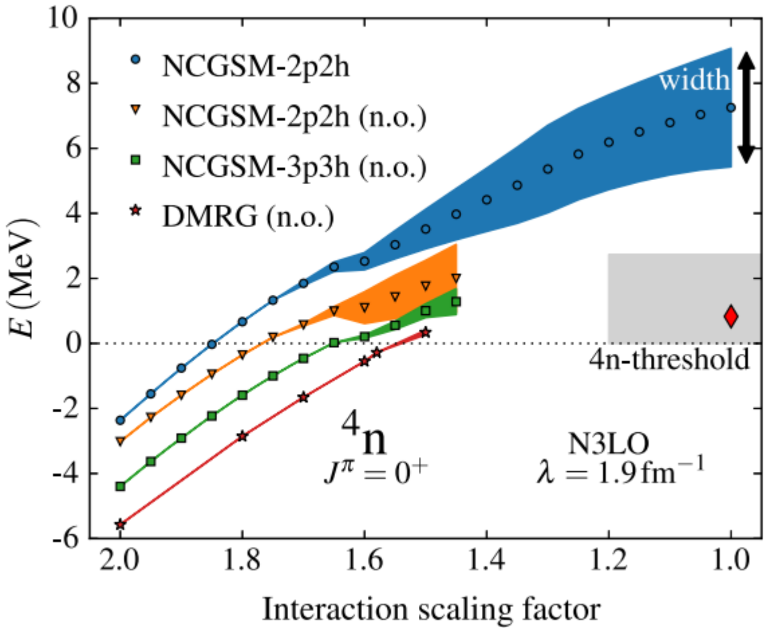
(2) enhance attractive part of NN interaction

(3) introduce external trapped well

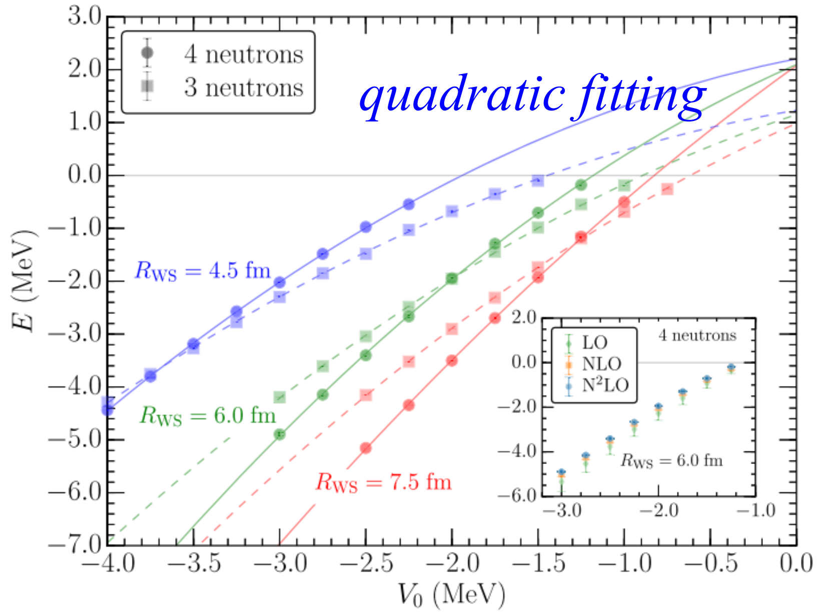
A. M. Shirokov et al., PRL 117, 182502 (2016)
E. Hiyama et al., PRC 93, 044004 (2016)
S. Gandolfi et al., PRL 118, 232501 (2017)
K. Fossez et al., PRL 119, 032501 (2017)
A. Deltuva, PLB 782 238 (2018)
J. G. Li et al., PRC 100, 054313 (2019)
M. D. Higgins et al., PR125, 052501 (2020)
S.Ishikawa, PRC 102, 034002 (2020)



J. G. Li et al., PRC 100, 054313 (2019)



K. Fossez et al., PRL 119, 032501 (2017)



S. Gandolfi et al., PRL 118, 232501 (2017)

Methods

Extrapolate from artificial bound state to resonant region

- Hamiltonian of multineutron systems: realistic AV8' *NN* interaction

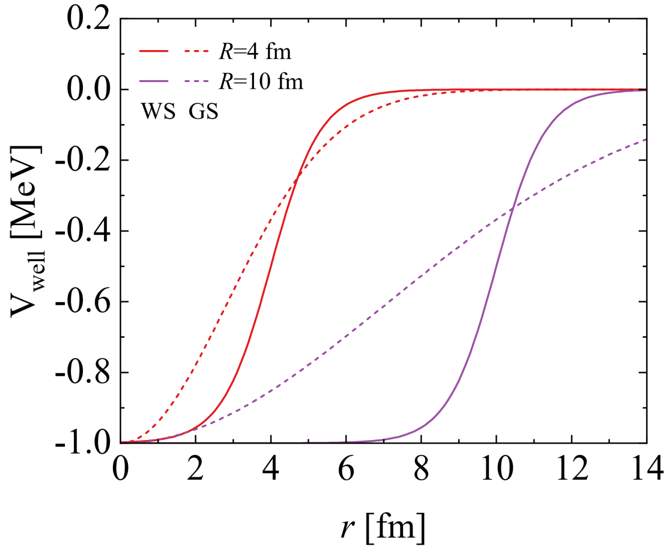
$$H = \sum_{i=1}^{A} t_i - T_{c.m.} + \sum_{i<j}^{A} v_{ij} + \sum_{i=1}^{A} \lambda V_{\rm ex}(\vec{r}_i)$$

$$V_{\rm WS}(\vec{r}) = -\frac{1}{1+exp\left[\frac{(r-R)}{a}\right]}$$

One-body WS/GS external well

$$V_{GS}(\vec{r}) = -\lambda e^{-ar^2}, a = 1/R^2$$

S. C. Pieper, PRL90, 252501 (2003)
S. Gandolfi, et al, PRL118, 232501 (2017)
*J. G. Li, et al, PRC**100**, 054313 (2019)*



- Tensor-optimized antisymmetrized molecular dynamics (TOAMD)

$$\Phi_{\rm AMD} = \frac{1}{\sqrt{A!}} \det\left\{\prod_{i=1}^{A} \phi_i\right\}$$

$$\phi(\vec{r}) = \left(\frac{2\nu}{\pi}\right)^{3/4} e^{-\nu(\vec{r}-\vec{D})^2} \chi_\sigma \chi_\tau$$

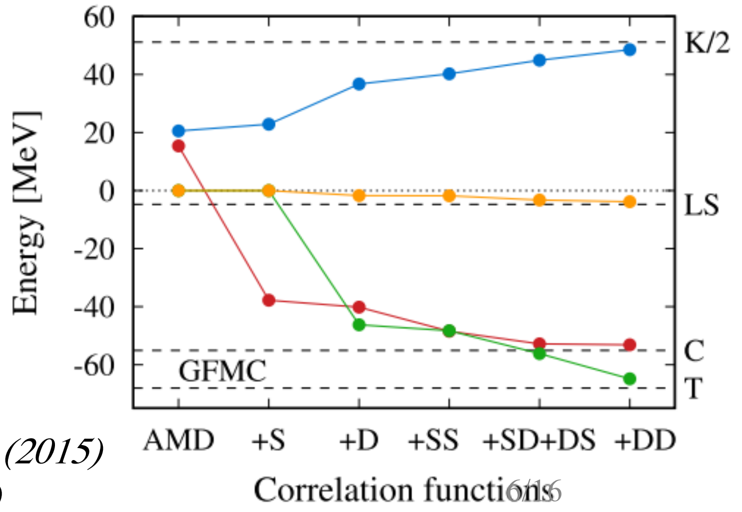
ν: Gaussian range parameter
$\vec{D}$: centroid position of single nucleon

$$\Phi_{\rm TOAMD} = (1 + F_S + F_S F_S)\Phi_{\rm AMD}$$

$$F_S = \sum_{t=0}^{1}\sum_{s=0}^{1}\sum_{i<j}^{A} f_S^{t,s}(\vec{r}_{ij}) O_{ij}^t O_{ij}^s$$

efficient correlation functions

*T. Myo, et al, PTEP**2015**, 073D02 (2015)*
*T. Myo, et al, PLB**769**, 213 (2017)*



Methods

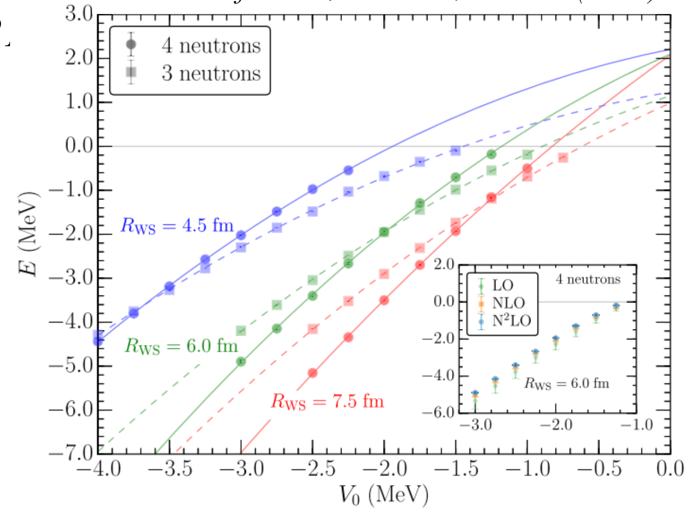
Extrapolate from artificial bound state to resonant region

- Inverse analytical continuation in the coupling constant method (IACCC)

$$H = \sum_{i=1}^{A} t_i - T_{c.m.} + \sum_{i<j}^{A} v_{ij} + \sum_{i=1}^{A} \lambda V_{\text{ex}}(\vec{r}_i)$$

V. I. Kukulin et al., Theory of resonances: Principles and Applications. Kluwer Academic Publishers, Dordrecht/Boston/l
J. Horacek et al., Commun. Comput. Phys. 2.

S. Gandolfi et al., PRL 118, 232501 (2017)



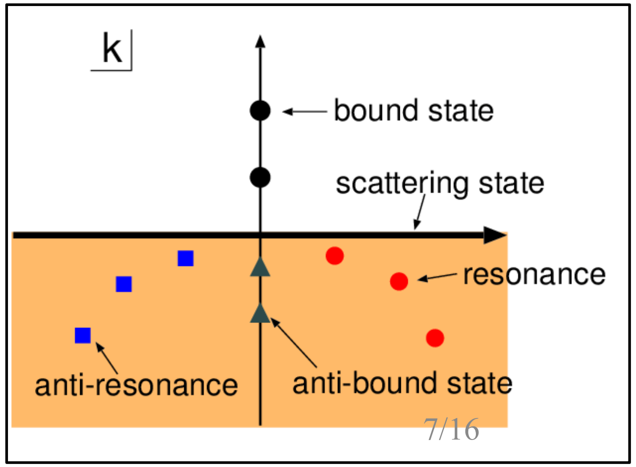
$$E(\lambda) = \frac{\hbar^2 k^2}{2m}, \quad k = i\kappa \qquad \text{ACCC:} \quad \kappa(\lambda) = \kappa\left(\sqrt{\lambda - \lambda_0}\right) = \frac{P_N\left(\sqrt{\lambda - \lambda_0}\right)}{Q_M\left(\sqrt{\lambda - \lambda_0}\right)}$$

IACCC: Padé approximant: $\lambda(\kappa) = \frac{P_N(\kappa)}{Q_M(\kappa)} = \frac{a_0 + a_1\kappa + \cdots + a_N\kappa^N}{1 + b_1\kappa + \cdots + b_M\kappa^M}$

Bound state: $E(\lambda) < 0$: imaginary k and real $\kappa \longrightarrow [a_i, b_j]$

Resonant state: $\lambda = 0 \longrightarrow P_N(\kappa) = 0$

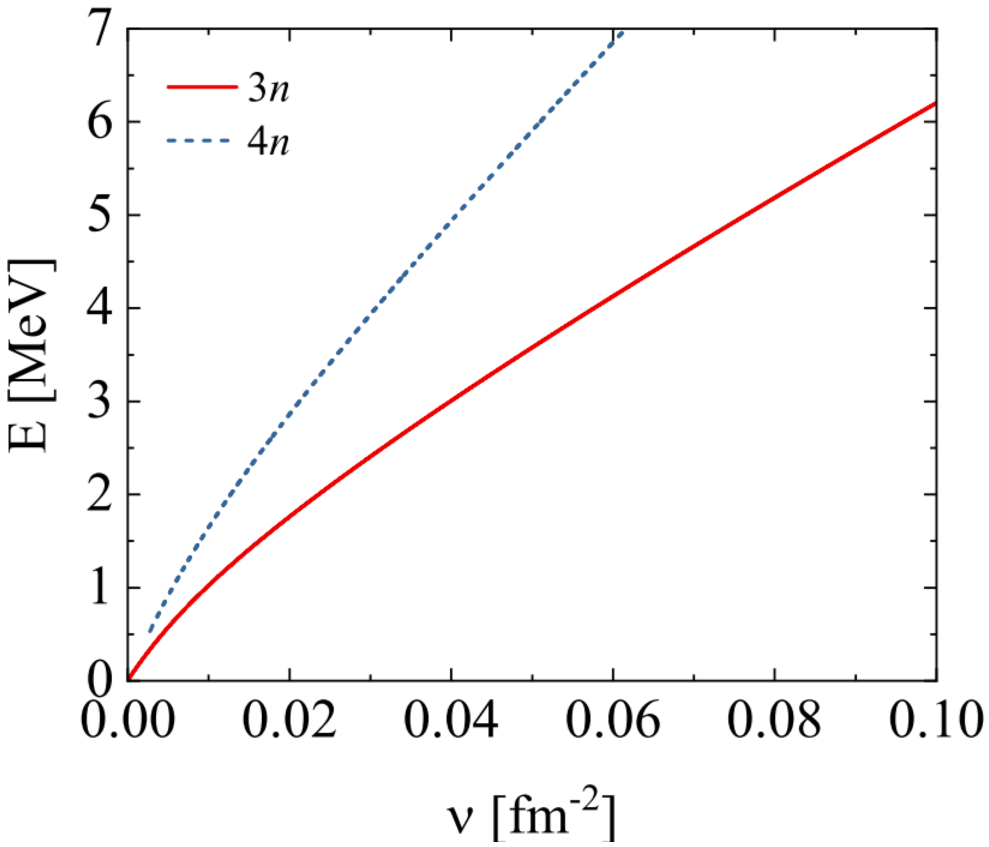
$k = i\kappa$



$$E = \frac{\hbar^2 k^2}{2m} = E_R - \frac{i}{2}\Gamma$$

Results: direct calculations and enhancement of interaction

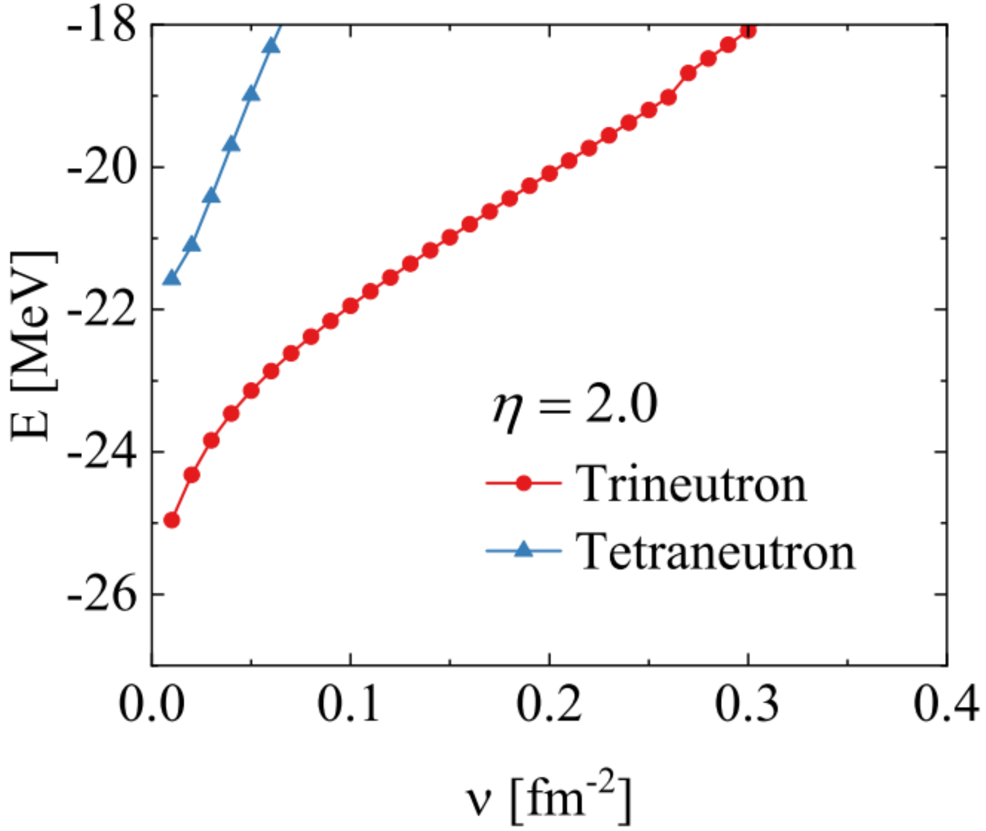
$$H = \sum_{i=1}^{A} t_i - T_{c.m.} + \sum_{i<j}^{A} v_{ij} + \sout{\sum_{i=1}^{A} \lambda V_{\text{ex}}(\vec{r}_i)}$$



No flat region: stabilization method

No further signals for resonances

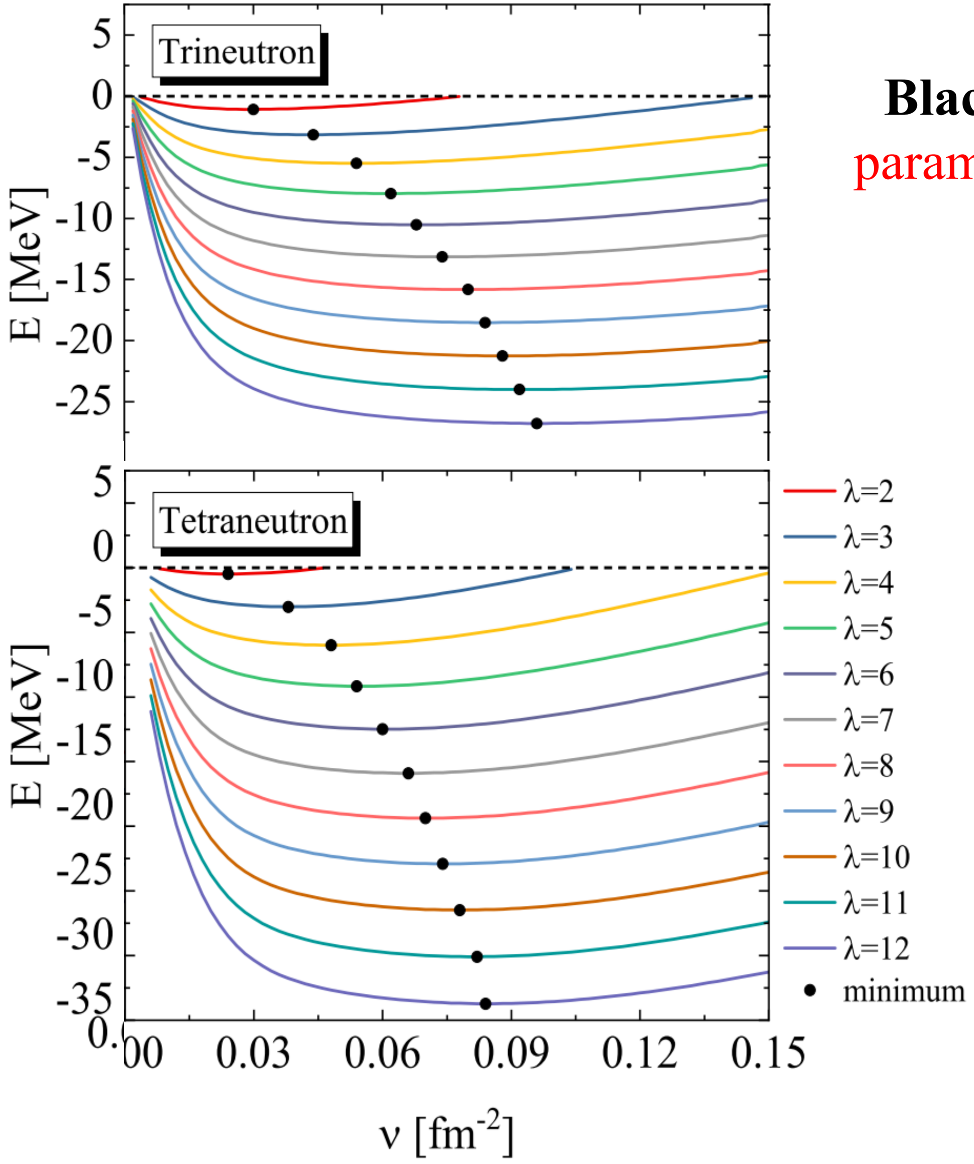
$$H = \sum_{i=1}^{A} t_i - T_{c.m.} + \sum_{i<i}^{A} \boldsymbol{\eta} \cdot v_{ij}$$



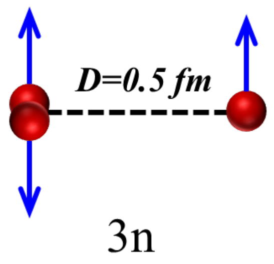
No bound states

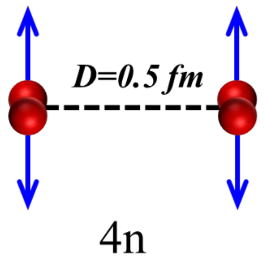
No further signals for resonances

Results: find bound states with WS/GS wells



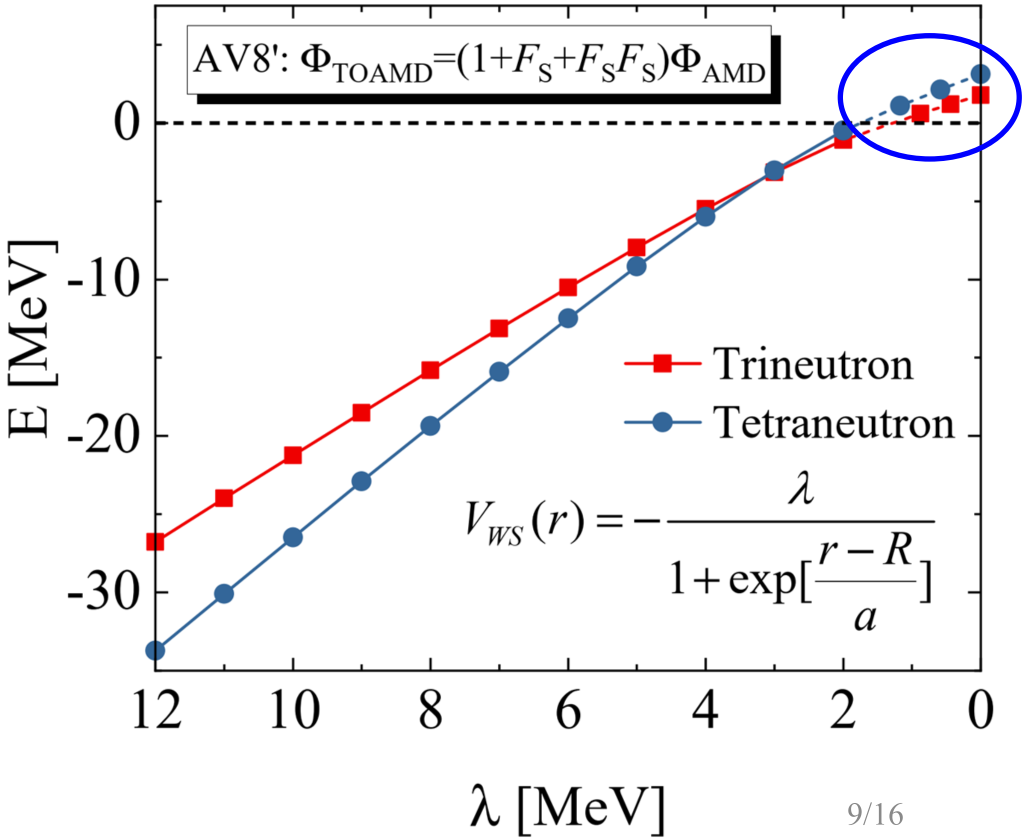
Black points: optimized ν parameter for each strength λ with bound state





$$H = \sum_{i=1}^{A} t_i - T_{c.m.} + \sum_{i<j}^{A} v_{ij} + \sum_{i=1}^{A} \lambda V_{\text{ex}}(\vec{r}_i)$$

IACCC for resonance region above threshold



Results: resonance pole trajectories with WS/GS wells

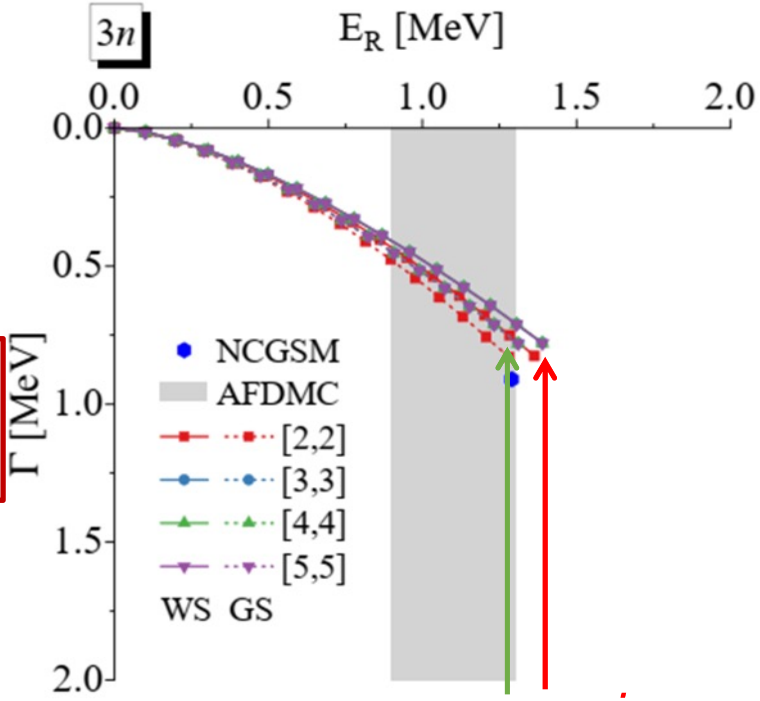
$$\lambda(\kappa)[N,M] = \frac{P_N(\kappa)}{Q_M(\kappa)} = \frac{a_0 + a_1\kappa + \cdots + a_N\kappa^N}{1 + b_1\kappa + \cdots + b_M\kappa^M}$$

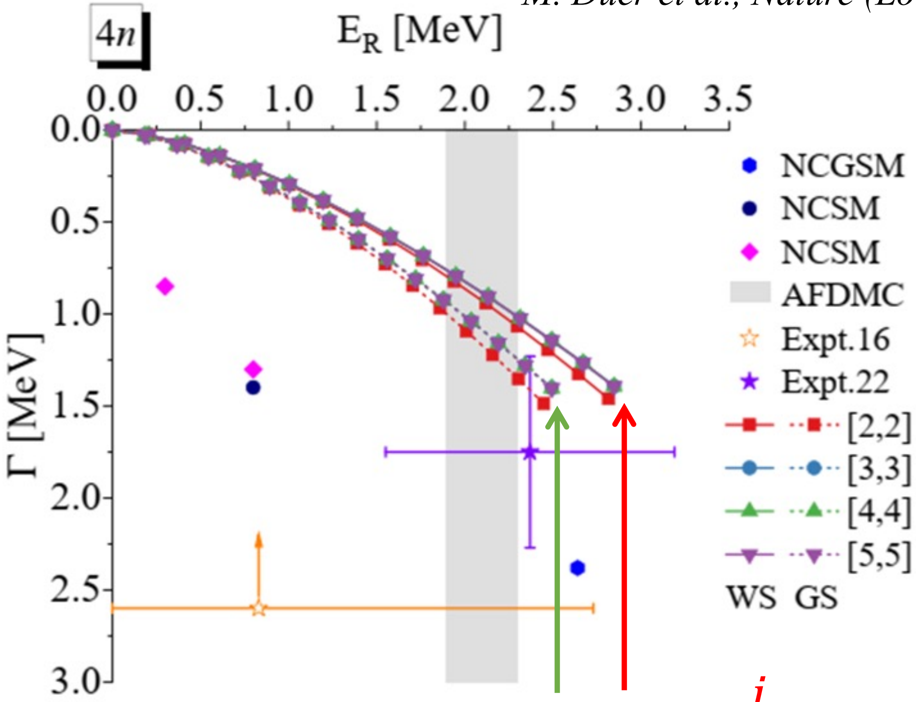
$$E = E_R - \frac{i}{2}\Gamma$$

$E_r = 2.37 \pm 0.38(\text{stat.}) \pm 0.44(\text{sys.})$ MeV,
$\Gamma = 1.75 \pm 0.22(\text{stat.}) \pm 0.30(\text{sys.})$ MeV.

M. Duer et al., Nature (London) 606, 678 (2022)

What Is The Difference?





WS: $E(3n) = 1.39 - \frac{i}{2}0.78\ MeV$ $E(4n) = 2.85 - \frac{i}{2}1.39\ MeV$ R=6 fm

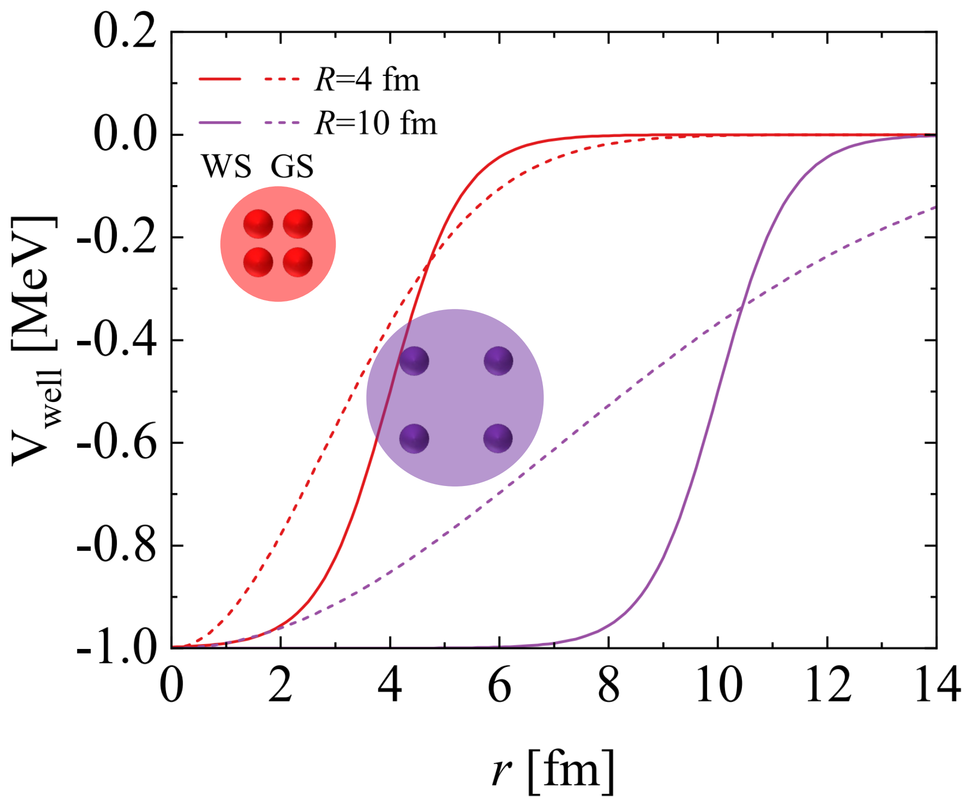
GS: $E(3n) = 1.31 - \frac{i}{2}0.78\ MeV$ $E(4n) = 2.49 - \frac{i}{2}1.40\ MeV$ R=5 fm

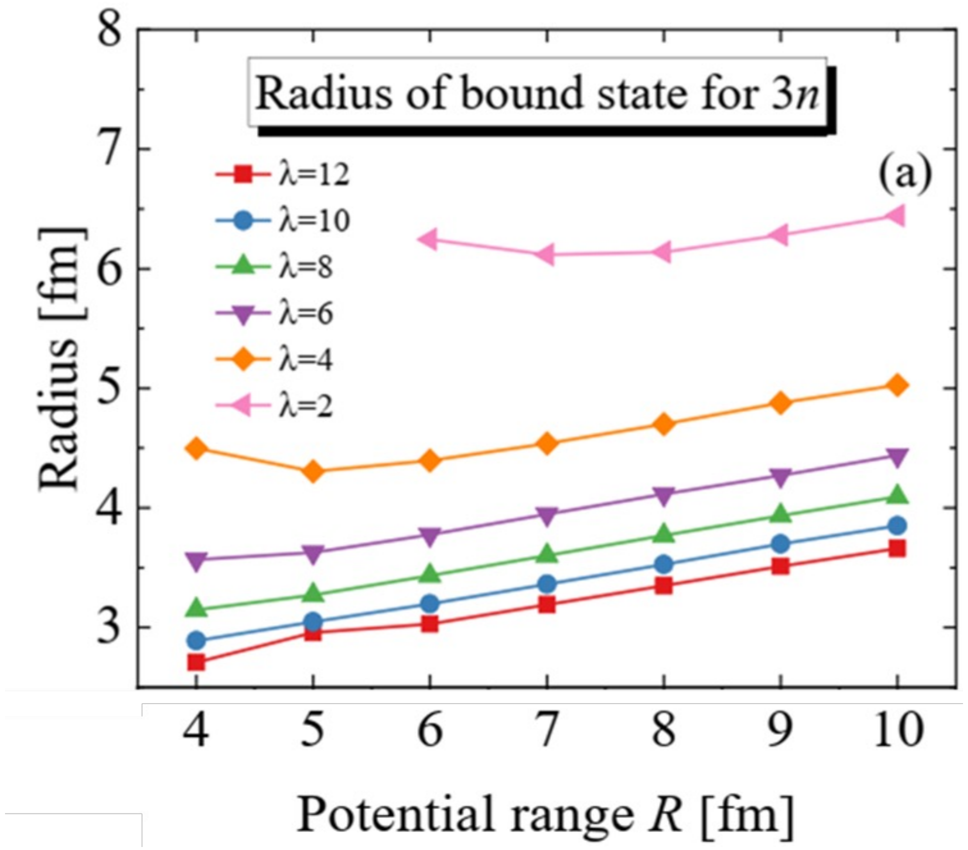
NCGSM: $E(3n) = 1.29 - \frac{i}{2}0.90\ MeV$ $E(4n) = 2.64 - \frac{i}{2}2.38\ MeV$ R=4 fm

J. G. Li, et al, PRC ***100****, 054313 (2019)*

Results: potential range R dependence

Dependence on the potential range R of external well $V_{GS}(\vec{r}) = -\lambda e^{-ar^2}, a = 1/R^2$

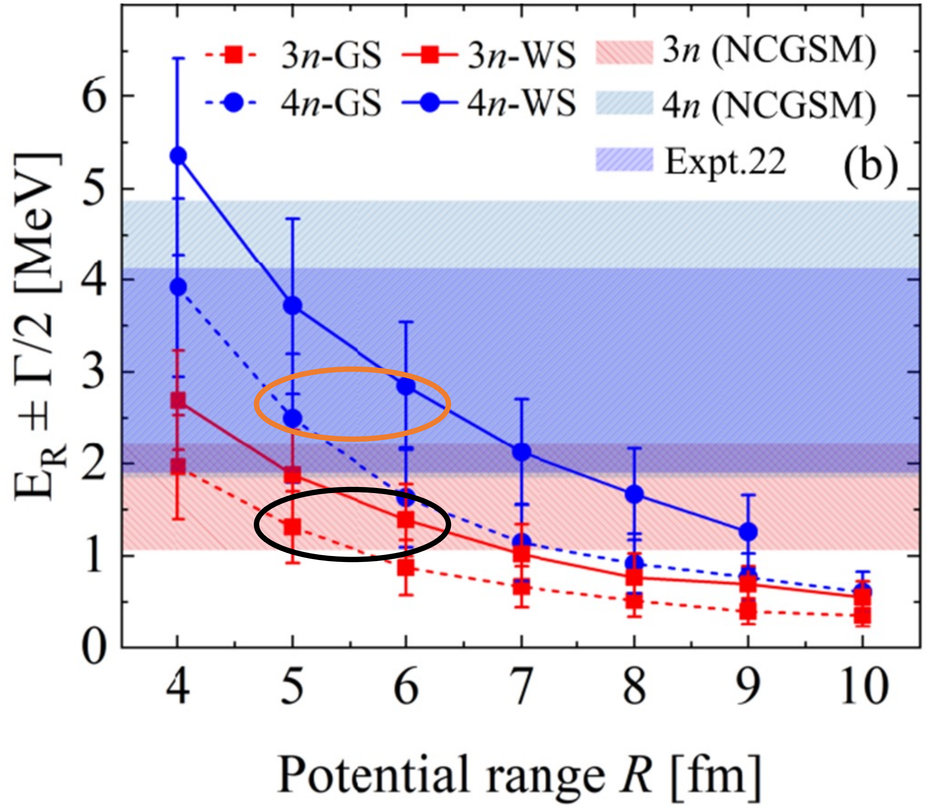


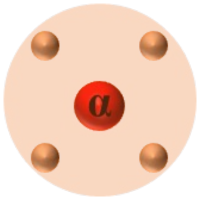


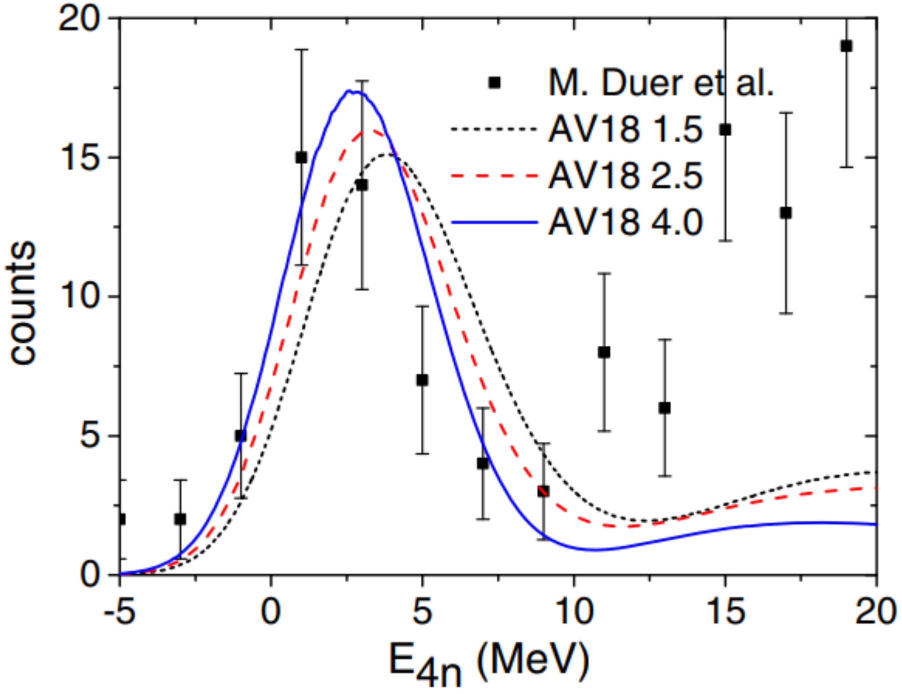
additional compression from narrow potential range

Results: potential range R dependence

Dependence on the potential range R of external w







R. Lazauskas, et al, PRL130, 102501 (2023)

Reaction model and response function

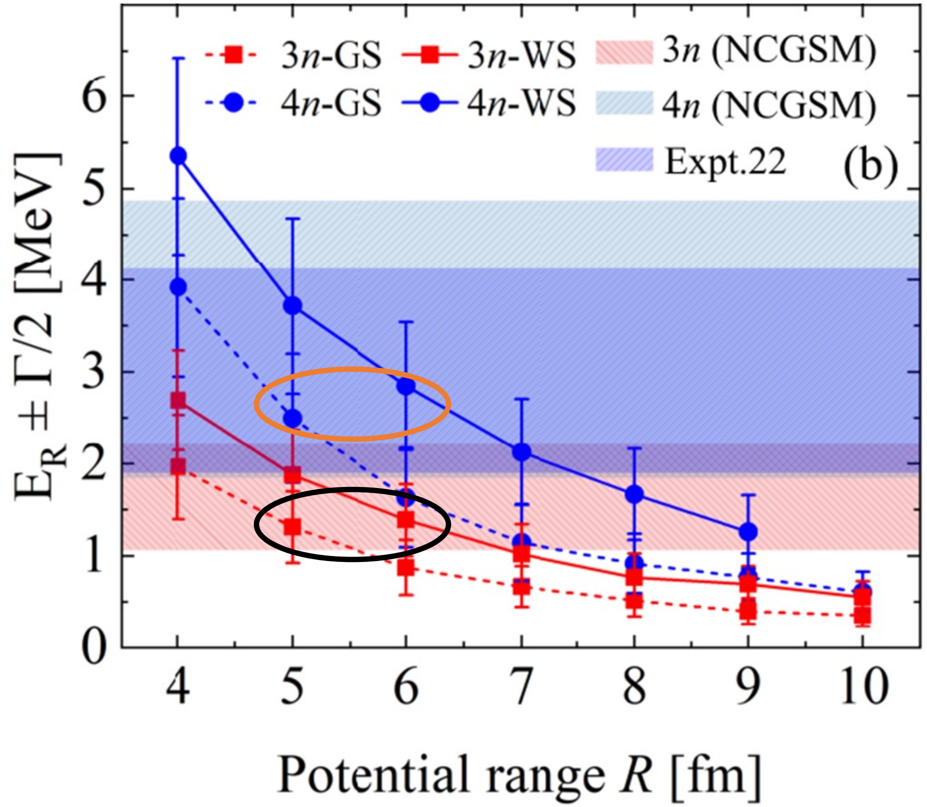
Our calculations were constrained only by the requirement of four valence neutrons to be weakly bound by a nuclear core. Thus, our study addresses a class of reactions

emergence of a sharp low energy peak in the missing mass spectrum of a $4n$ decay. Such phenomena might also be seen in some systems of cold atoms.

Specific confinement from external attraction for pure neutron systems

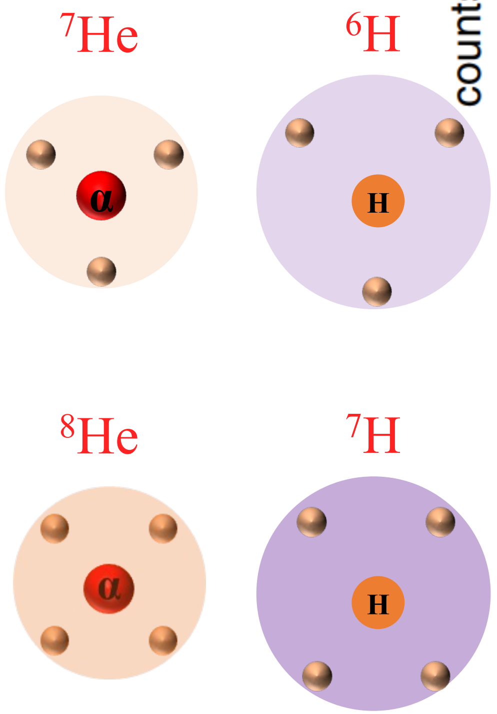
Results: potential range R dependence

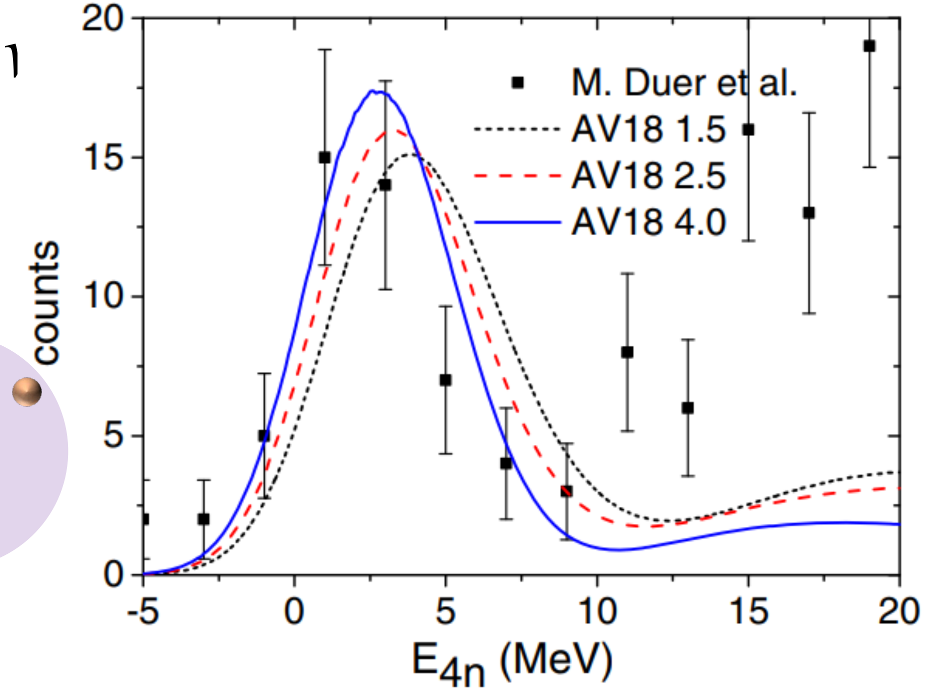
Dependence on the potential range R of external



Can this tendency be measured?

Isotopes? Different cores?





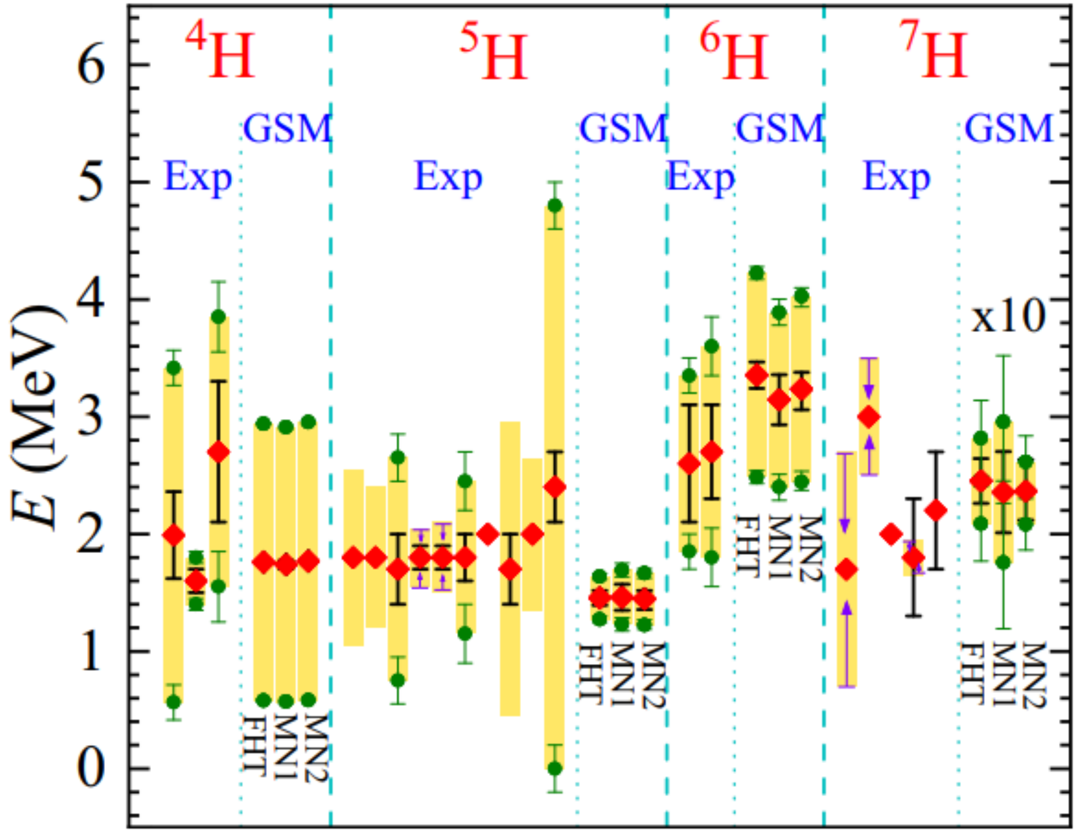
R. Lazauskas, et al, PRL130, 102501 (2023)

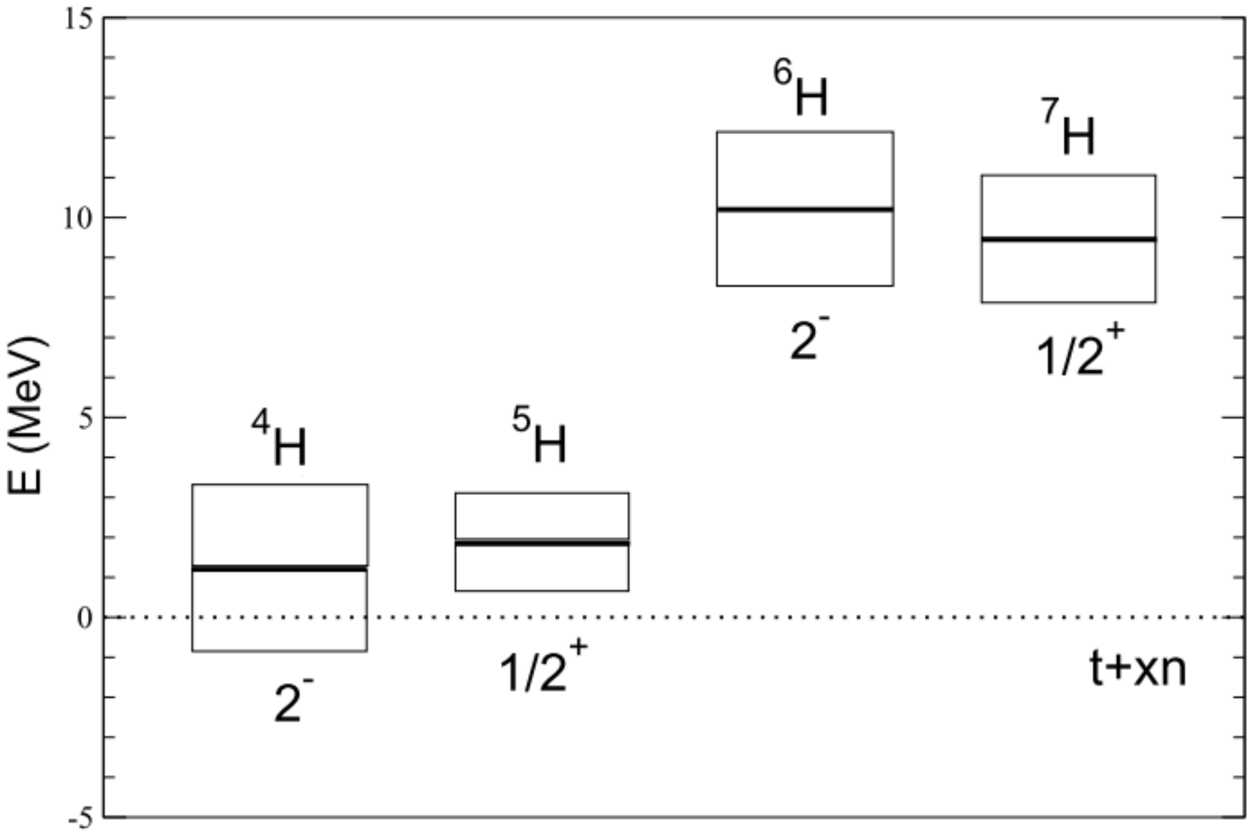
Reaction model and response function

Our calculations were constrained only by the requirement of four valence neutrons to be weakly bound by a nuclear core. Thus, our study addresses a class of reactions

emergence of a sharp low energy peak in the missing mass spectrum of a 4n decay. Such phenomena might also be seen in some systems of cold atoms.

Existence limit for Hydrogen isotopes

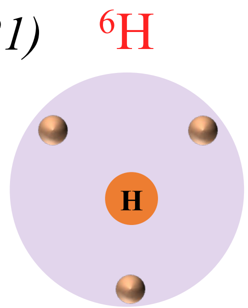


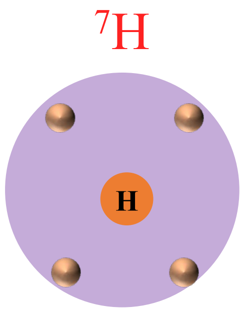


H. H. Li, et al., PRC 104, L061306 (2021)

E. Hiyama, PLB 833, 137367 (2022)

GSM





GEM+ stabilization method

Summary

- Under WS/GS external potential wells, bound states are optimized and resonance pole trajectories are obtained
- Theoretical resonances of both $3n$ and $4n$ are obtained, and the results are consistent with previous predictions and experimental data
- Dependence of the resultant resonances on the potential range of external well are obtained
- **Nonexistence** of resonances for $3n$ and $4n$ are supported by present TOAMD+IACCC using realistic AV8' *NN* interaction
- Possibility for measurements on resonances with different nuclear cores
- Existence limit for hydrogen isotopes

Thanks for your attention