華南理工大學
SOUTH CHINA UNIVERSITY OF TECHNOLOGY

Preequilibrium Cluster Emission and Systematics on the Formation Cross Sections of Superheavy Nuclei (SHN)

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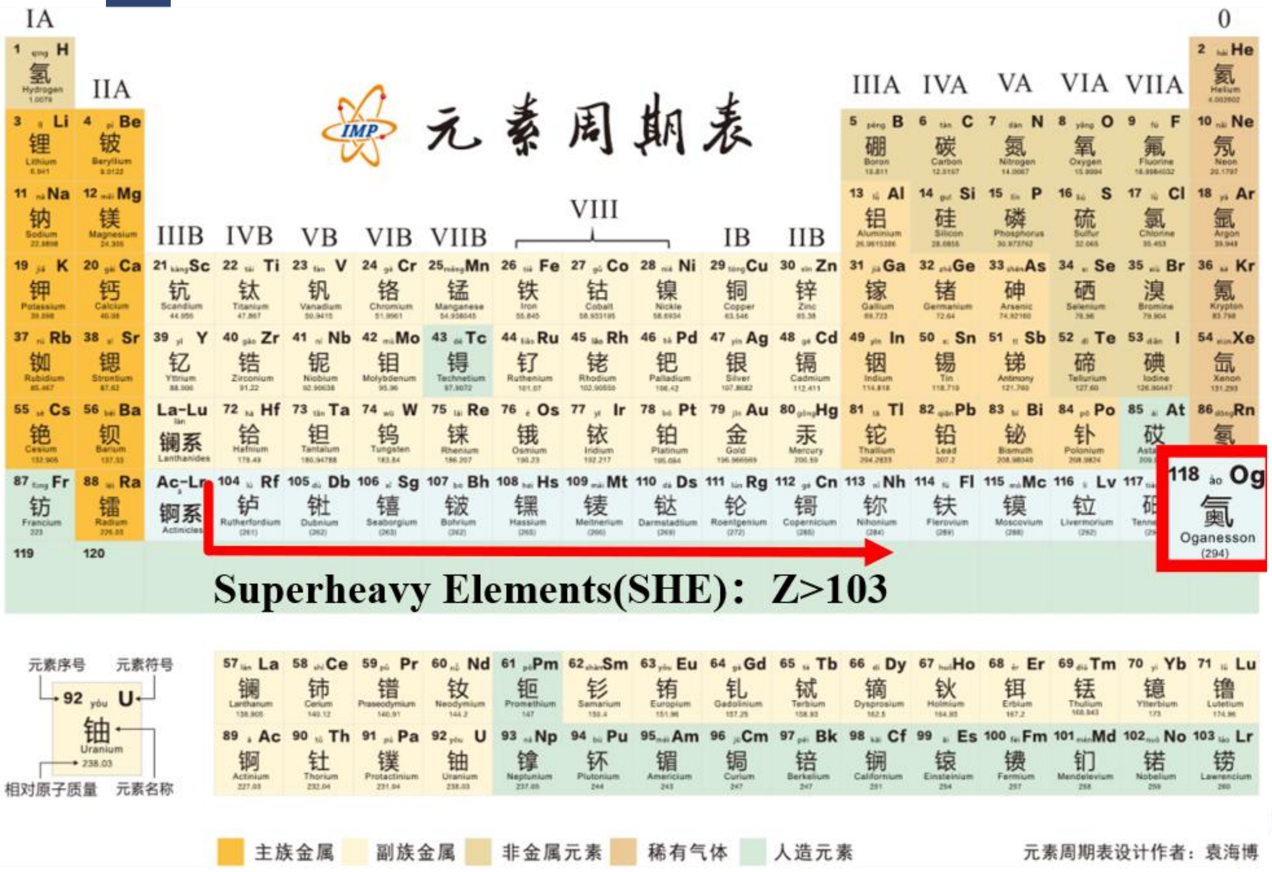
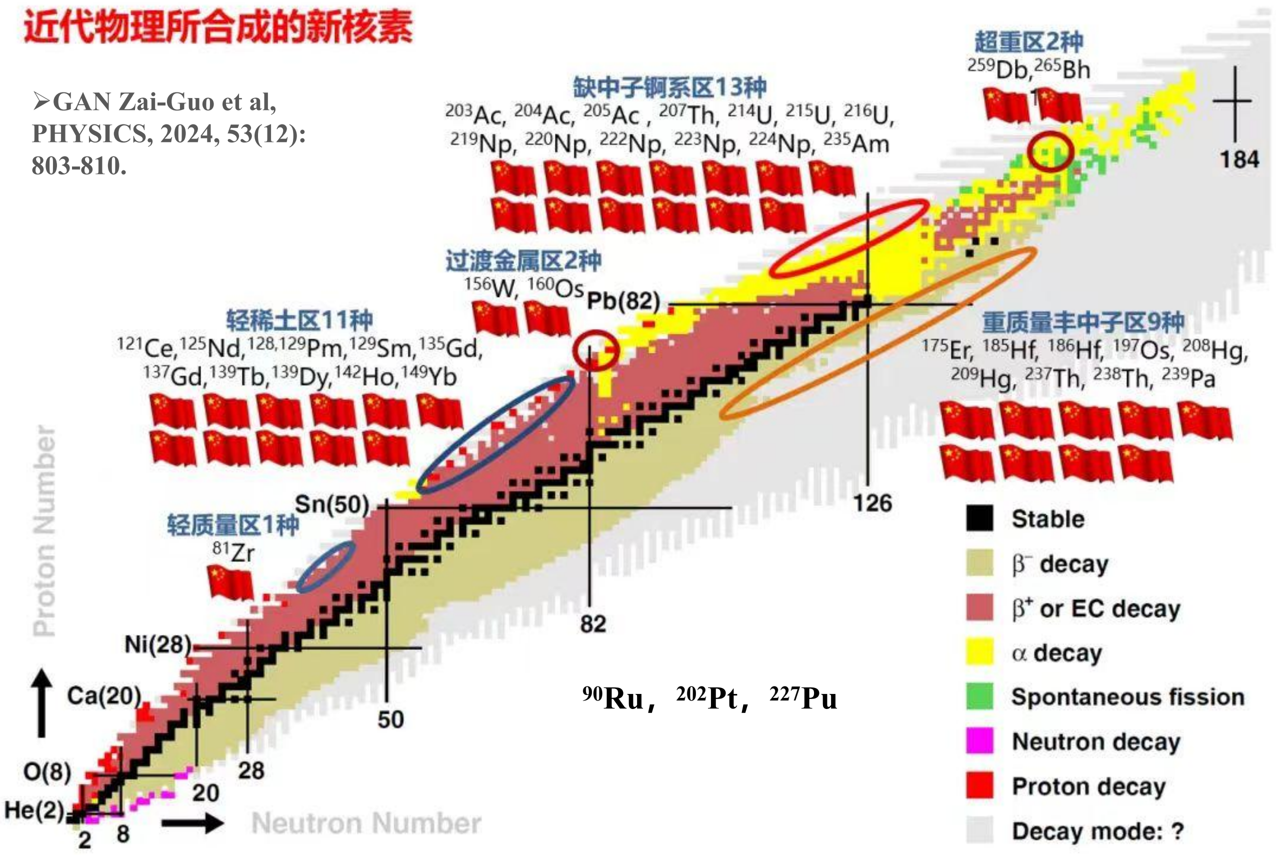
Supervisor: Zhao-Qing Feng (冯兆庆)

Outline

1. Research Motivation
2. Preequilibrium Cluster Emission
3. Formation Cross Sections of SHN

1.Research Motivation：superheavy nuclei





Significance of SHN research：

- Expand the periodic table of elements
- Explore the limits of atomic nucleus mass and charge
- Study atomic nucleus decay and structural properties
- Test the shell model theory of atomic nuclei
- Reveal the nuclear synthesis process in nuclear astrophysics

Difficulties in the synthesis of new SHN：

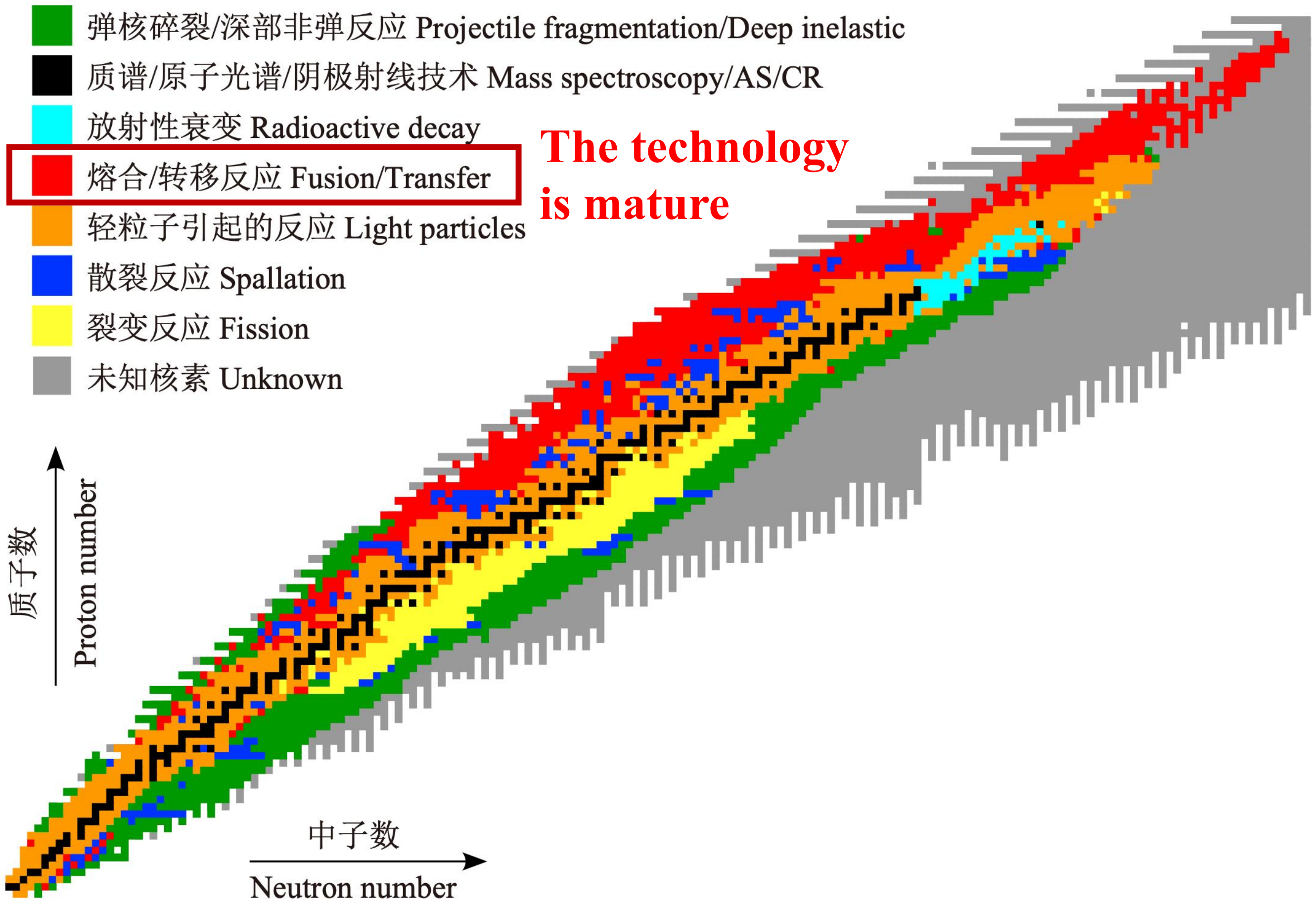
- Extremely low production cross-section and fleeting lifetime
- The products are difficult to separate and identify
- The experimental cost is high and the cycle is long

Accurate Theoretical Predictions Are Needed！

1.Research Motivation：MNT reactions

In the multi-nucleon transfer (MNT) reactions:

- Generate broad isotope distribution depending on transfer channels.
- Angular and excitation energy of compound nucleus distribute widely.
- Producing neutron-rich heavy and superheavy nuclei.



ELSEVIER

Nuclear Physics A
Volume 176, Issue 2, 29 November 1971, Pages 284-288

New isotopes ^{29,30}Mg, ^{31,32,33}Al, ^{33,34,35,36}Si, ^{35,36,37,38}P, ^{39,40}S and ^{41,42}Cl produced in bombardment of a ^{232}Th target with 290 MeV ^{40}Ar ions

A.G. Artukh, V.V. Avdeichikov †, G.F. Gridnev, V.L. Mikheev, V.V. Volkov, J. Wilczyński

Multi-nucleon transfer reactions (MNT) experiment progress

Lab.	Reaction system	References
Dubna	^{136}Xe + ^{208}Pb ^{156,160}Gd + ^{186}W	Phys. Rev. C 86, 044611 (2012) Phys. Rev. C 96, 064621 (2017)
GSI	^{238}U+^{238}U ^{48}Ca+^{248}Cm ^{48}Ca+^{238}U	Phys. Rev. Lett. 39, 385 (1977) Phys. Rev. Lett. 41, 469 (1978) Phys. Lett. B 748, 199 (2015) Eur. Phys. J. A 56, 224 (2020)
GANIL	^{238}U+^{238}U ^{136}Xe+^{198}Pt	IJMPE 17, 2235-2239 (2008) Phys. Rev. Lett. 115, 172503 (2015)
Argonne	^{136}Xe+^{208}Pb ^{204}Hg+^{198}Pt	Phys. Rev. C 91, 064615 (2015) Physics Letters B 771, 119-124 (2017)
RIKEN	^{238}U+^{198}Pt→^{241}U	Phys. Rev. Lett. 130, 132502 (2023)

1.Research Motivation: pre-equilibrium clusters

☐ Theoretically

➢ The Exciton Model

- ➢ M. Blann, Rev. Nucl. Sci. 25, 123 (1975).
- ➢ E. Gadioli et al, Phys. Rev. C 16, 1404 (1977).
- ➢ OV Fotina et al, Int. J. Mod. Phys. E 19, 1134 (2010).

➢ The Dinuclear System (DNS) Model

- ➢ Z. Q. Feng, Phys. Rev. C 107, 054613 (2023).

➢ The significance of studying pre-equilibrium clusters

✓the correlation of spatial configuration of nucleons

✓the cluster structure of the stable or unstable nuclide

✓the cluster formation mechanism in nuclear reactions

✓the kinetic mechanism of MNT reactions

✓… …

- ➢ P. E. Hodgson and E. Běták, Phys. Rep. 374, 1-89 (2003).

☐ Experimentally

➢ 1977, IMP:

第 1 卷 第 1 期　高能物理与核物理　Vol. 1, No. 1
1977 年 11 月　PHYSICA ENERGIAE FORTIS ET PHYSICA NUCLEARIS　November, 1977

$^{12}C+^{209}Bi$

^{12}C 轰击 ^{209}Bi 时发射的 α 粒子

沈文庆　徐树威　王大延　谢元祥　郭中言　李祖玉

（中国科学院近代物理研究所）

➢ 1980, IMP:

Nuclear Physics A
Volume 349, Issues 1–2, 3–10 November 1980, Pages 285-300

$^{12}C+^{209}Bi$

Product cross sections for the reaction of ^{12}C with ^{209}Bi ☆

Jin Gen-Ming [1, 2], Xie Yuan-Xiang, Zhu Yong-Tai, Shen Wen-Ging, Sun Xi-Jun, Guo Jun-Sheng, Liu Guo-Xing, Yu Ju-Sheng, Sun Chi-Chang, J.D. Garrett

➢ 1980, RIKEN:

Nuclear Physics A
Volume 334, Issue 1, 28 January 1980, Pages 127-143

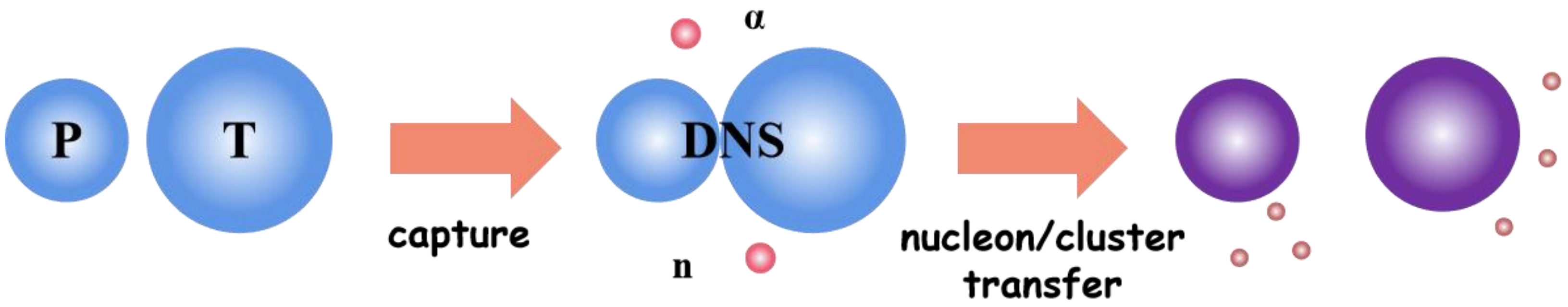
$^{14}N+^{159}Tb$, ^{169}Tm, ^{181}Ta, ^{197}Au, ^{209}Bi

Preequilibrium α-particle emission in heavy-ion reactions

H. Utsunomiya †, T. Nomura, T. Inamura, T. Sugitate ††, T. Motobayashi ‡

2.Preequilibrium Cluster Emission

□ The Dinuclear System Model (DNS)



Schematic diagram of multinucleon transfer reactions in the DNS model

- 冯兆庆. 近垒重离子熔合反应和超重核合成机制研究[D]. 甘肃:中国科学院近代物理研究所,2007.

- Emission cross-section of pre-equilibrium cluster:

$$\sigma_\nu(E_k,\theta,t)=\sum_{J=0}^{J_{max}}\sum_{Z_1=Z_\nu}^{Z_{max}}\sum_{N_1=N_\nu}^{N_{max}}\sigma_{cap}(E_{c.m.},J)\times\int f(B)\times P(Z_1,N_1,E_1(E_{c.m.},J),t,\beta,B)\times P_\nu(Z_\nu,N_\nu,E_k)dB$$

Emission probability of the cluster:

$$P_\nu(Z_\nu,N_\nu,E_k)=\Delta t\frac{\Gamma_\nu}{\hbar}$$

$$\nu=n,p,d,t,{}^3He,\alpha,{}^{6,7}Li,{}^{8,9}Be$$

arXiv:2505.02337

The Dinuclear System Model (DNS)—Capture process:

Capture cross section:

$$\sigma_{cap}\left(E_{c.m.},J\right)=\frac{\pi\hbar^2(2J+1)T\left(E_{c.m.},J\right)}{2\mu E_{c.m.}} \qquad \mu=m_n\frac{A_P A_T}{A_P+A_T}$$

Penetration probability:

$$T\left(E_{c.m.},J\right)=\int f(B)\,T\left(E_{c.m.},J,B\right)dB$$

1. for the light and medium systems: the Hill-Wheeler formula

$$T(E_{\mathrm{c.m.}},J,B)=\left\{1+\exp\left\{-\frac{2\pi}{\hbar\omega(J)}\left[E_{\mathrm{c.m.}}-B-\frac{\hbar^2 J(J+1)}{2\mu R_{\mathrm{B}}^2(\mathrm{J})}\right]\right\}\right\}^{-1}$$

- D. L. Hill and J. A. Wheeler, Phys. Rep. 89, 1102-1145 (1953).

Barrier distribution function:

Gaussian form $f(B)=\frac{1}{N}\exp\left(-\left(\frac{B-B_m}{\Delta}\right)^2\right)$

$\Delta=(B_C-B_S)/2 \quad B_{\mathrm{m}}=(B_C+B_{\mathrm{S}})/2 \quad \int f(B)dB=1$

2. for the heavy systems: the classical trajectory approach

$$T\left(E_{c.m.},J,B\right)=0,\ E_{c.m.}<B+\frac{J(J+1)\hbar^2}{2\mu R_C^2};\quad T\left(E_{c.m.},J,B\right)=1,\ E_{c.m.}>B+\frac{J(J+1)\hbar^2}{2\mu R_C^2};\quad R_C=r_{0c}\left(A_P^{1/3}+A_T^{1/3}\right)$$

□ The Dinuclear System Model (DNS)—Transfer dynamics:

Master equation:

Only the transfer of Nucleons:

$$\frac{dP(Z_1,N_1,E_1,t)}{dt} = \sum_{Z_1'} W_{Z_1,N_1;Z_1',N_1}(t)\left[d_{Z_1,N_1}P(Z_1',N_1,E_1',t) - d_{Z_1',N_1}P(Z_1,N_1,E_1,t)\right] + \sum_{N_1'} W_{Z_1,N_1;Z_1,N_1'}(t) \times \left[d_{Z_1,N_1}P(Z_1,N_1',E_1',t) - d_{Z_1,N_1'}P(Z_1,N_1,E_1,t)\right].$$

$$W_{Z_1,N_1,Z_1',N_1} = \frac{\tau_{mem}(Z_1,N_1,E_1;Z_1',N_1,E_1')}{d_{Z_1,N_1}d_{Z_1',N_1}\hbar^2}\sum_{i,i'}|\langle i'|V|i\rangle|^2$$

- Z. Q. Feng, Phys. Rev. C 108, L051601 (2023).

Nucleons (n, p)+Clusters (d, t, ^{3}He, ^{4}He):

$$\begin{aligned}\frac{dP(Z_1,N_1,E_1,\beta_1,B,t)}{dt} &= \sum_{Z_1'=Z_1\pm1} W_{Z_1,N_1,\beta_1;Z_1',N_1,\beta_1'}(t)[d_{Z_1,N_1}P(Z_1',N_1,E_1',\beta_1',B,t) - d_{Z_1',N_1}P(Z_1,N_1,E_1,\beta_1,B,t)] \\ &+ \sum_{N_1'=N_1\pm1} W_{Z_1,N_1,\beta_1;Z_1,N_1',\beta_1'}(t)[d_{Z_1,N_1}P(Z_1,N_1',E_1',\beta_1',B,t) - d_{Z_1,N_1'}P(Z_1,N_1,E_1,\beta_1,B,t)] \\ &+ \sum_{Z_1'=\pm1,N_1'=N_1\pm1} W^{d}_{Z_1,N_1,\beta_1;Z_1',N_1',\beta_1'}(t)[d_{Z_1,N_1}P(Z_1',N_1',E_1',\beta_1',B,t) - d_{Z_1',N_1'}P(Z_1,N_1,E_1,\beta_1,B,t)] \\ &+ \sum_{Z_1'=\pm1,N_1'=N_1\pm2} W^{t}_{Z_1,N_1,\beta_1;Z_1',N_1',\beta_1'}(t)[d_{Z_1,N_1}P(Z_1',N_1',E_1',\beta_1',B,t) - d_{Z_1',N_1'}P(Z_1,N_1,E_1,\beta_1,B,t)] \\ &+ \sum_{Z_1'=\pm2,N_1'=N_1\pm1} W^{^3He}_{Z_1,N_1,\beta_1;Z_1',N_1',\beta_1'}(t)[d_{Z_1,N_1}P(Z_1',N_1',E_1',\beta_1',B,t) - d_{Z_1',N_1'}P(Z_1,N_1,E_1,\beta_1,B,t)] \\ &+ \sum_{Z_1'=\pm2,N_1'=N_1\pm2} W^{\alpha}_{Z_1,N_1,\beta_1;Z_1',N_1',\beta_1'}(t)[d_{Z_1,N_1}P(Z_1',N_1',E_1',\beta_1',B,t) - d_{Z_1',N_1'}P(Z_1,N_1,E_1,\beta_1,B,t)].\end{aligned}$$

$$W^{s}_{Z_1,N_1;Z_1',N_1'} = G_s\frac{\tau_{mem}(Z_1,N_1,E_1;Z_1',N_1',E_1')}{d_{Z_1,N_1}d_{Z_1',N_1'}\hbar^2}\sum_{ii'}\left|\langle i'|V|i\rangle\right|^2$$

- R. Mattiello et al, Phys. Rev. C 55, 1443 (1997).
- Z. Q. Feng, Phys. Rev. C 102, 044604 (2020).

□ The Dinuclear System Model (DNS)—Preequilibrium cluster emission:

Emission probability : $P_\nu(Z_\nu, N_\nu, E_k) = \Delta t \frac{\Gamma_\nu}{\hbar}$

- **the time step:** $\Delta t = 0.5(0.25)\times 10^{-22}s$
- **the particle decay widths Γ_ν:**

the Weisskopf evaporation theory

$$\Gamma_\nu(E^*, J) = (2s_\nu + 1)\frac{m_\nu}{\pi^2\hbar^2\rho(E^*, J)} \times \int_0^{E^*-B_\nu-E_{rot}-V_c} \varepsilon \times \rho(E^* - B_\nu - E_{rot} - V_c - \varepsilon, J)\sigma_{inv}(\varepsilon)d\varepsilon$$

- V. Weisskopf, Phys. Rev. 52, 295 (1937).
- P. H. Chen et al, Chin. Phys. C 40, 091002 (2016).

Kinetic energy spectra:

the Monte Carlo method, $\varepsilon_\nu = (0,\ E^* - B_\nu - V_c - E_{rot})$

- **for neutron: Watt Spectrum**

$$\frac{dN_n}{d\epsilon_n} = C_n \frac{\epsilon_n^{1/2}}{T_w^{3/2}} \exp\left(-\frac{\epsilon_n}{T_w}\right) \qquad T_\omega = 1.7 \pm 0.1 MeV$$

- H. Rossner, Phys. Rev. C 45, 719 (1992).

- **for charged particles: Boltzmann Distribution**

$$\frac{dN_\nu}{d\epsilon_\nu} = 8\pi E_k \left(\frac{m}{2\pi T_\nu}\right)^{1/2} \exp\left(-\frac{\epsilon_\nu}{T_\nu}\right) \qquad T_\nu = \sqrt{\frac{E^*}{a}},\quad a = A/12$$

2.Preequilibrium Cluster Emission

□ The Dinuclear System Model (DNS)—Preequilibrium cluster emission:

Angular distributions ： the deflection function method

$$\Theta(J_i) = \Theta_C(J_i) + \Theta_N(J_i)$$

➢ C. Peng and Z. Q. Feng, Eur. Phys. J. A 58, 162 (2022).

➢ **The coulomb deflection**

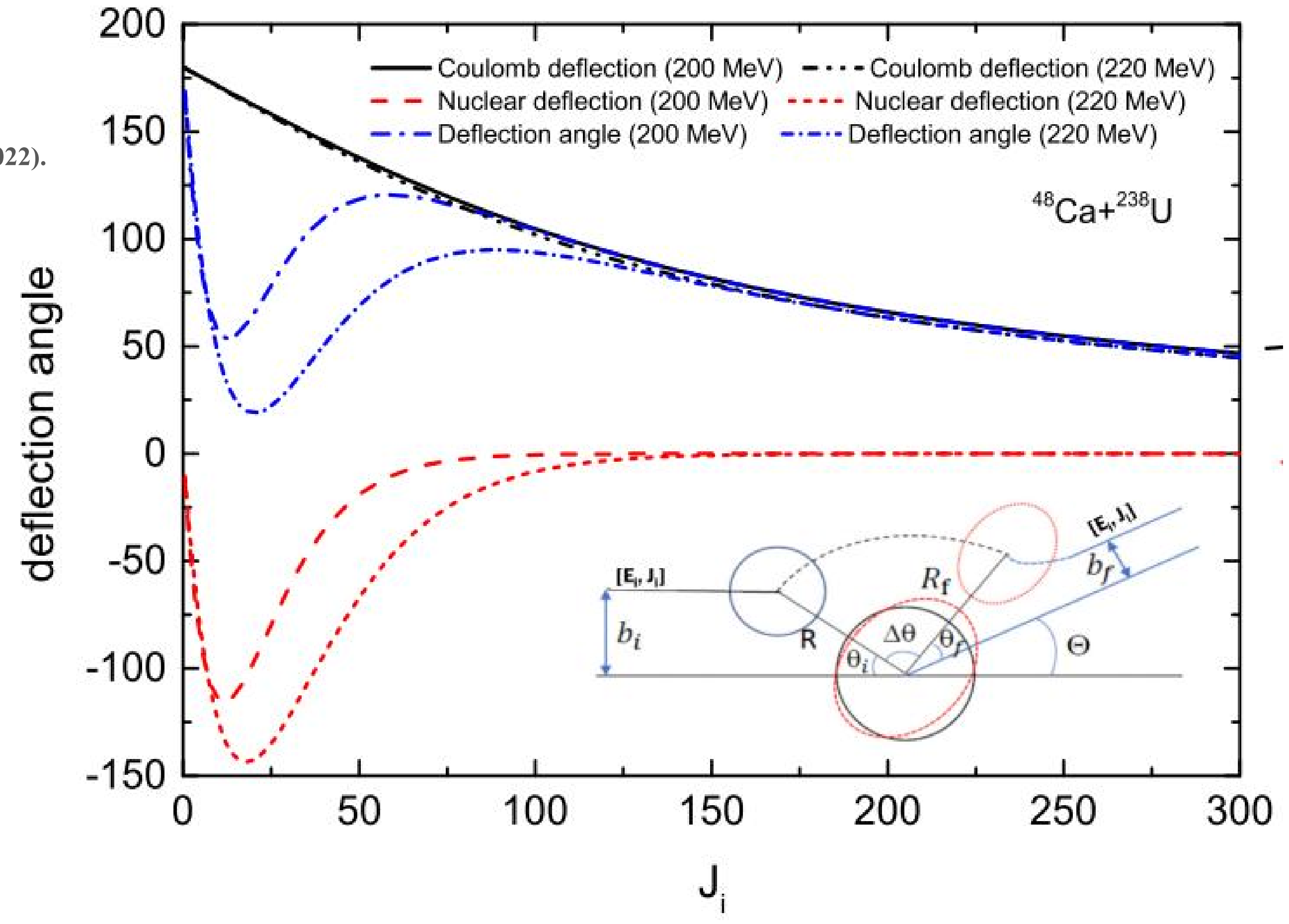
$$\Theta_C(J_i) = 2\tan^{-1}\frac{Z_P Z_T e^2}{2E_{c.m.}b}$$

➢ **The nuclear deflection**

$$\Theta_N(J_i) = -\beta\Theta_C^{gr}(J_i)\frac{J_i}{J_{gr}}\left(\frac{\delta}{\beta}\right)^{\frac{J_i}{J_{gr}}}$$

the grazing angular momentum：

$$J_{gr} = 0.22R_{int}\left[A_{red}\left(E_{c.m.} - V(R_{int})\right)^{1/2}\right]$$



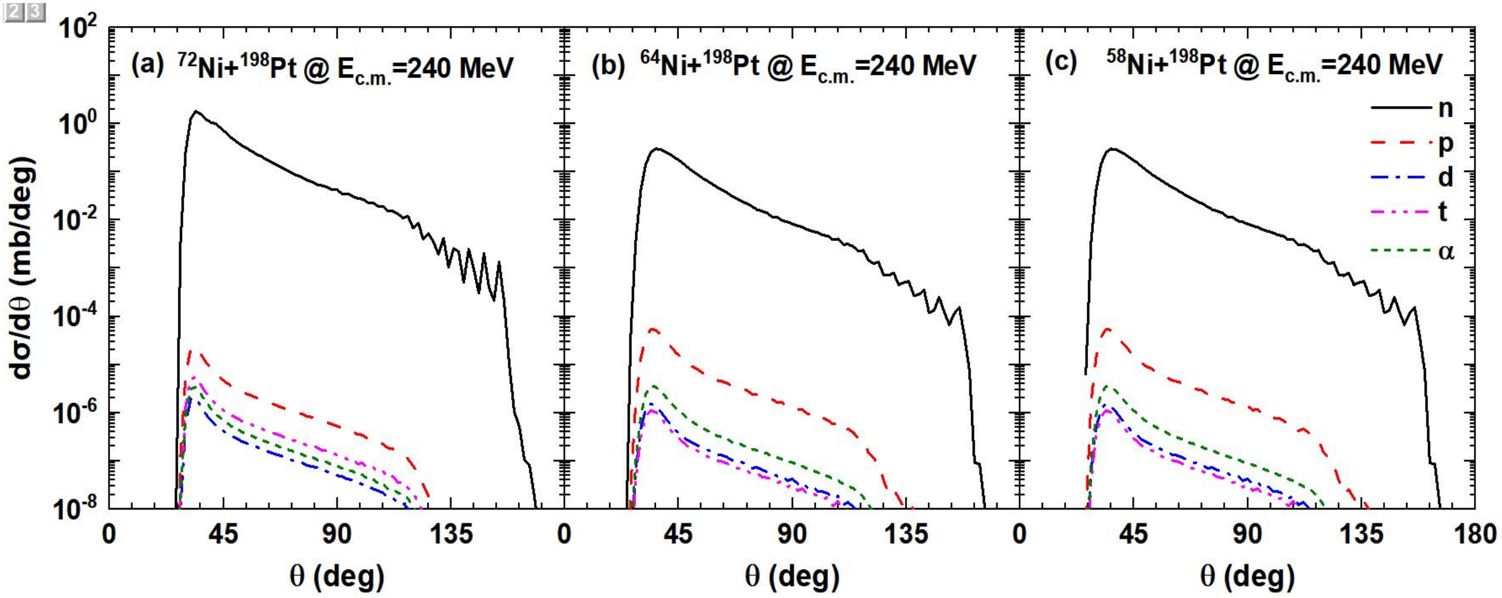
➢ Z. Q. Feng, Phys. Rev. C 107, 054613 (2023).

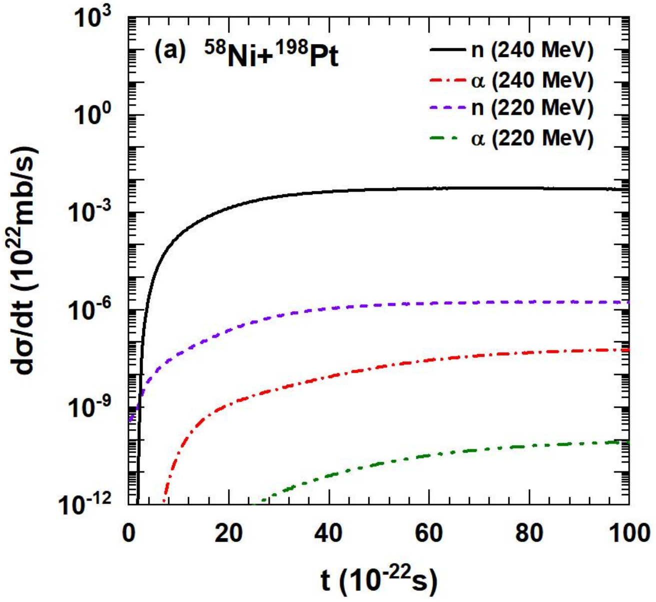
2.Preequilibrium Cluster Emission

arXiv:2505.02337

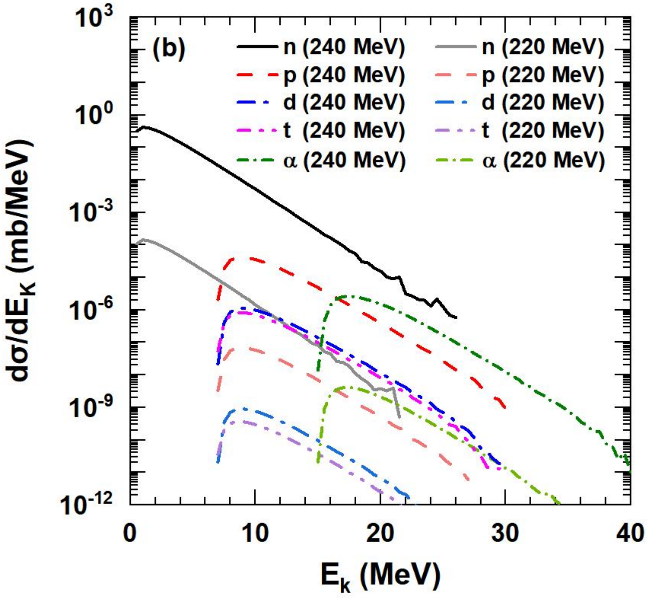
□ Results: $^{58,64,72}Ni+^{198}Pt$

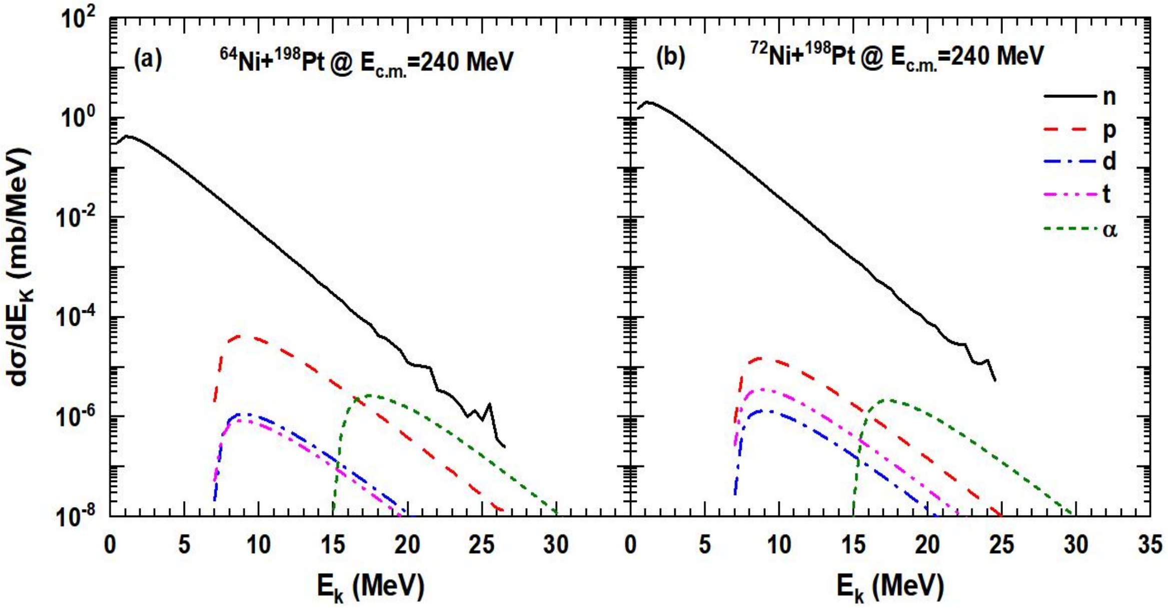
Angular distributions





Temporal evolution





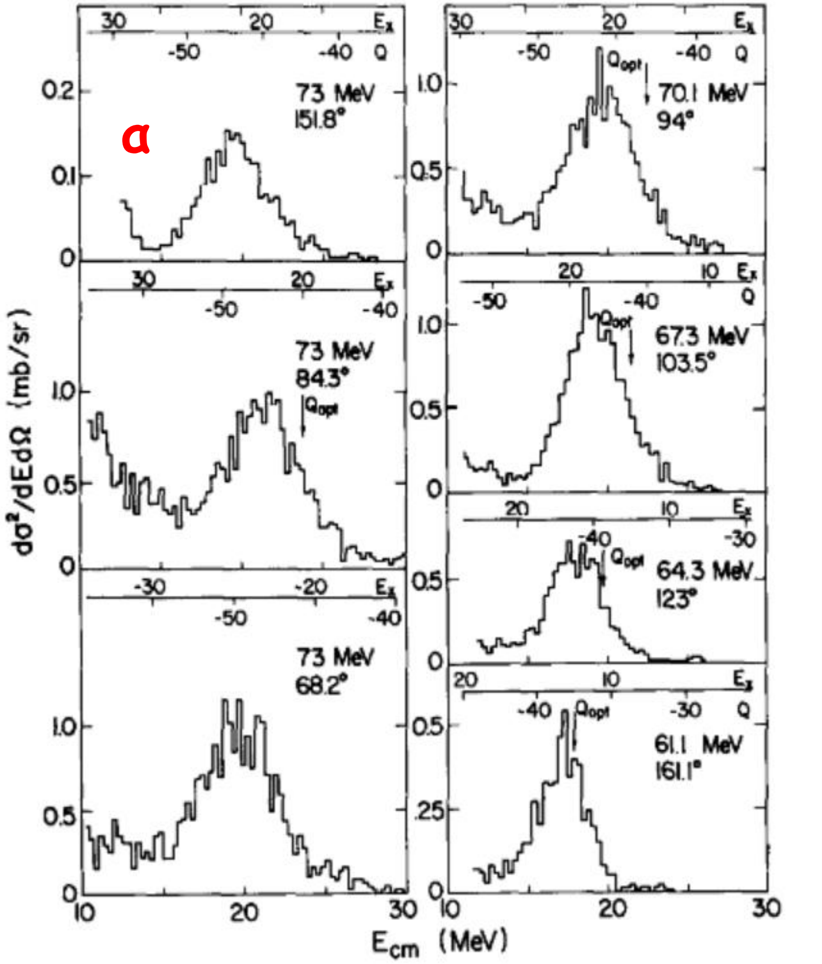
Kinetic energy spectra

2.Preequilibrium Cluster Emission

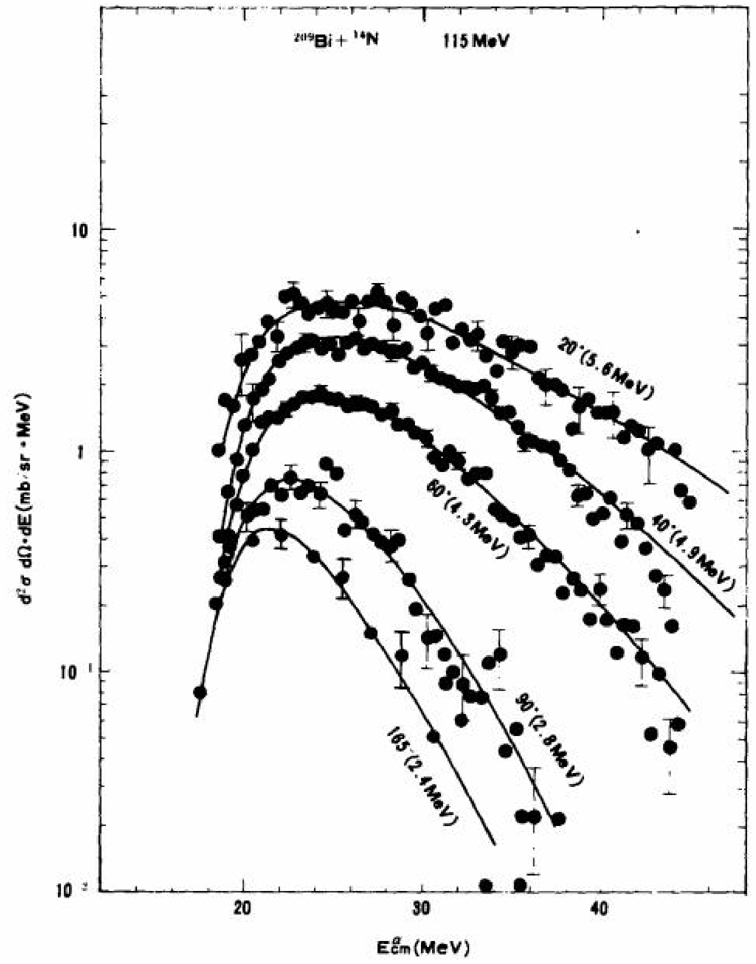
arXiv:2505.02337

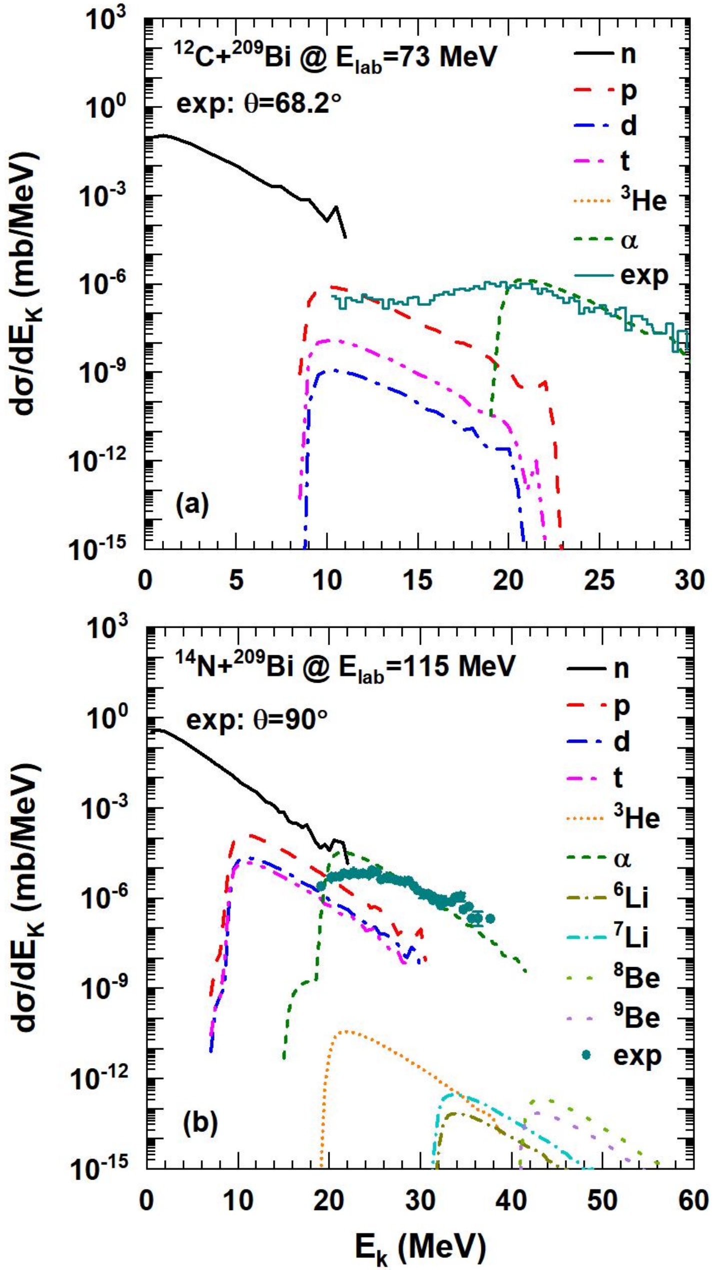
□ Results: ^{12}C, $^{14}N+^{209}Bi$

IMP: $^{12}C+^{209}Bi@E_{lab}=73MeV$



RIKEN:
$^{14}N+^{209}Bi@E_{lab}=115MeV$

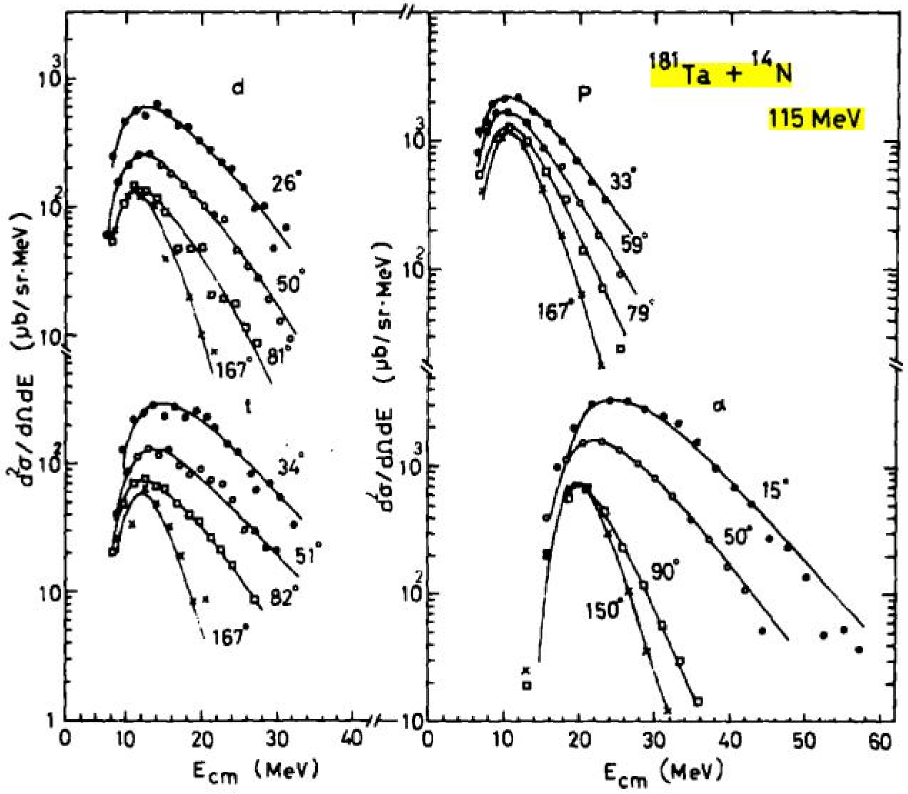


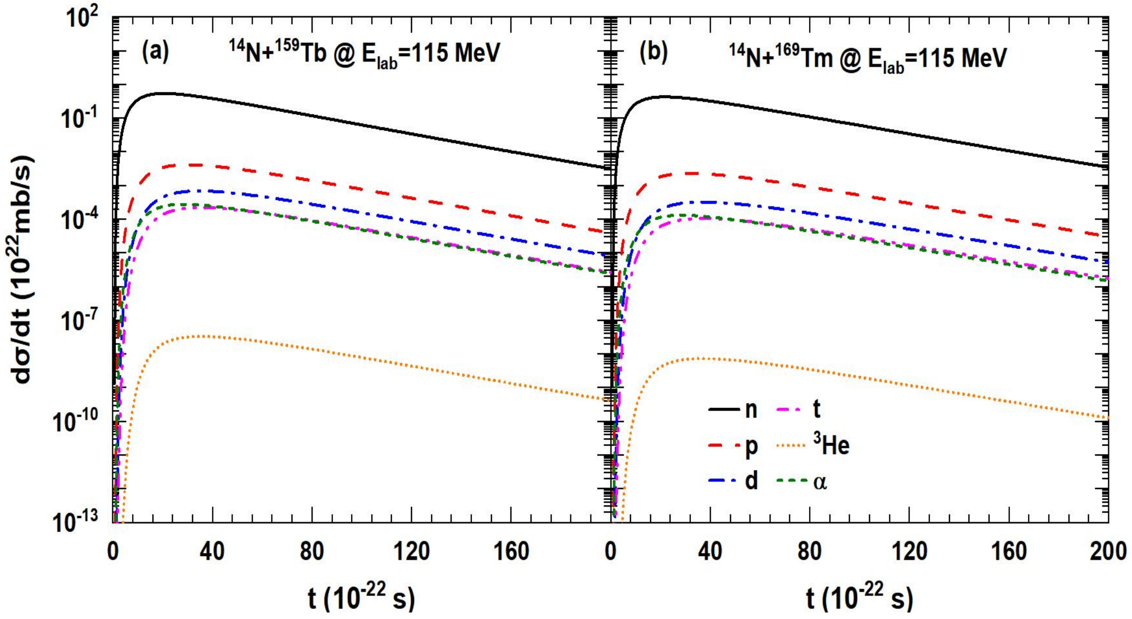


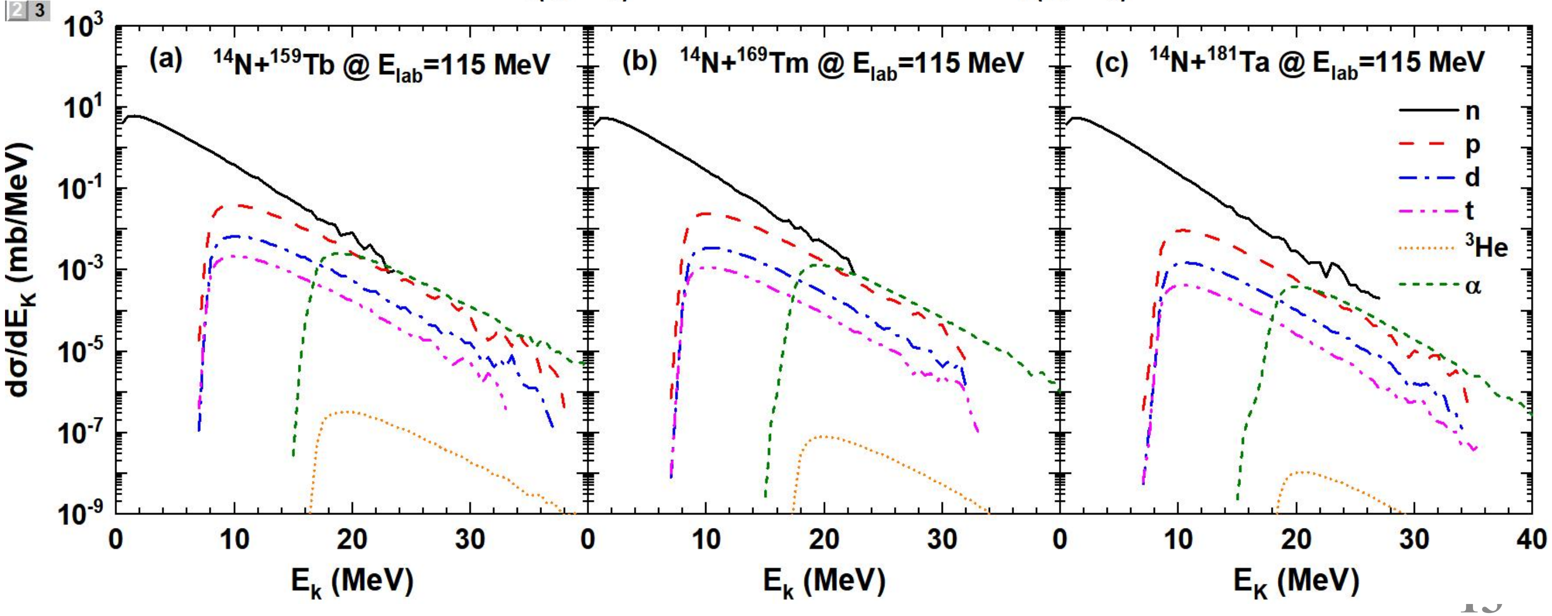
2.Preequilibrium Cluster Emission

arXiv:2505.02337

□ Results: $^{14}N+^{159}Tb, ^{169}Tm, ^{181}Ta$







3.Formation Cross Sections of SHN

The Dinuclear System Model (DNS)

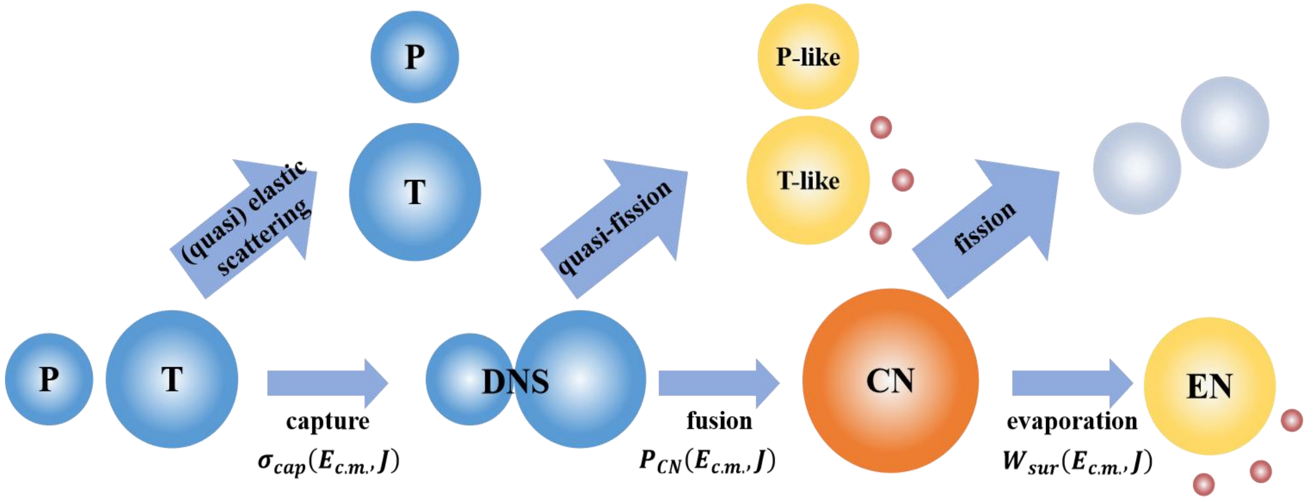
- **Production cross-section of SHN:**

$$\sigma_{ER}(E_{c.m.}) = \sum_{J=0}^{J_{max}} \sigma_{cap}(E_{c.m.},J)P_{CN}(E_{c.m.},J)W_{sur}(E_{c.m.},J)$$

$$\begin{aligned}\frac{dP(Z_1,N_1,E_1,\beta_1,B,t)}{dt} &= \sum_{Z_1'=Z_1\pm1} W^{p}_{Z_1,N_1,\beta_1;Z_1',N_1,\beta_1'}(t)\times[d_{Z_1,N_1}P(Z_1',N_1,E_1',\beta_1',B,t)-d_{Z_1',N_1}P(Z_1,N_1,E_1,\beta_1,B,t)]\\
&+\sum_{N_1'=N_1\pm1} W^{n}_{Z_1,N_1,\beta_1;Z_1,N_1',\beta_1'}(t)\times[d_{Z_1,N_1}P(Z_1,N_1',E_1',\beta_1',B,t)-d_{Z_1,N_1'}P(Z_1,N_1,E_1,\beta_1,B,t)]\\
&+\sum_{Z_1'=Z_1\pm1,N_1'=N_1\pm1} W^{d}_{Z_1,N_1,\beta_1;Z_1,N_1',\beta_1'}(t)\times[d_{Z_1,N_1}P(Z_1',N_1',E_1',\beta_1',B,t)-d_{Z_1',N_1'}P(Z_1,N_1,E_1,\beta_1,B,t)]\\
&+\sum_{Z_1'=Z_1\pm1,N_1'=N_1\pm2} W^{t}_{Z_1,N_1,\beta_1;Z_1,N_1',\beta_1'}(t)\times[d_{Z_1,N_1}P(Z_1',N_1',E_1',\beta_1',B,t)-d_{Z_1',N_1'}P(Z_1,N_1,E_1,\beta_1,B,t)]\\
&+\sum_{Z_1'=Z_1\pm2,N_1'=N_1\pm1} W^{^3He}_{Z_1,N_1,\beta_1;Z_1,N_1',\beta_1'}(t)\times[d_{Z_1,N_1}P(Z_1',N_1',E_1',\beta_1',B,t)-d_{Z_1',N_1'}P(Z_1,N_1,E_1,\beta_1,B,t)]\\
&+\sum_{Z_1'=Z_1\pm2,N_1'=N_1\pm2} W^{\alpha}_{Z_1,N_1,\beta_1;Z_1,N_1',\beta_1'}(t)\times[d_{Z_1,N_1}P(Z_1',N_1',E_1',\beta_1',B,t)-d_{Z_1',N_1'}P(Z_1,N_1,E_1,\beta_1,B,t)]\end{aligned}$$

$$W^{\nu}_{Z_1,N_1;Z_1',N_1'} = G_\nu \frac{\tau_{mem}(Z_1,N_1,E_1;Z_1',N_1',E_1')}{d_{Z_1,N_1}d_{Z_1',N_1'}\hbar^2}\times\sum_{ii'}|\langle Z_1',N_1',E_1',i'|V|Z_1,N_1,E_1,i\rangle|^2.$$

- **σ_{cap}: the probability that the projectile nucleus and the target nucleus overcome the Coulomb barrier and form a DNS.**
  - V.I. Zagrebaev, Phys. Rev. C 64 (2001) 034606.
- **P_{CN}: the formation probability of compound nuclei.**
  - N. Antonenko et al, Phys. Lett. B 319 (1993) 425.
  - N.V. Antonenko et al, Phys. Rev. C 51 (1995) 2635.
- **W_{sur}: the probability of the highly excited compound nuclei surviving by evaporating light particles.**
  - A.S. Zubov et al, Phys. Rev. C 65 (2002) 024308.



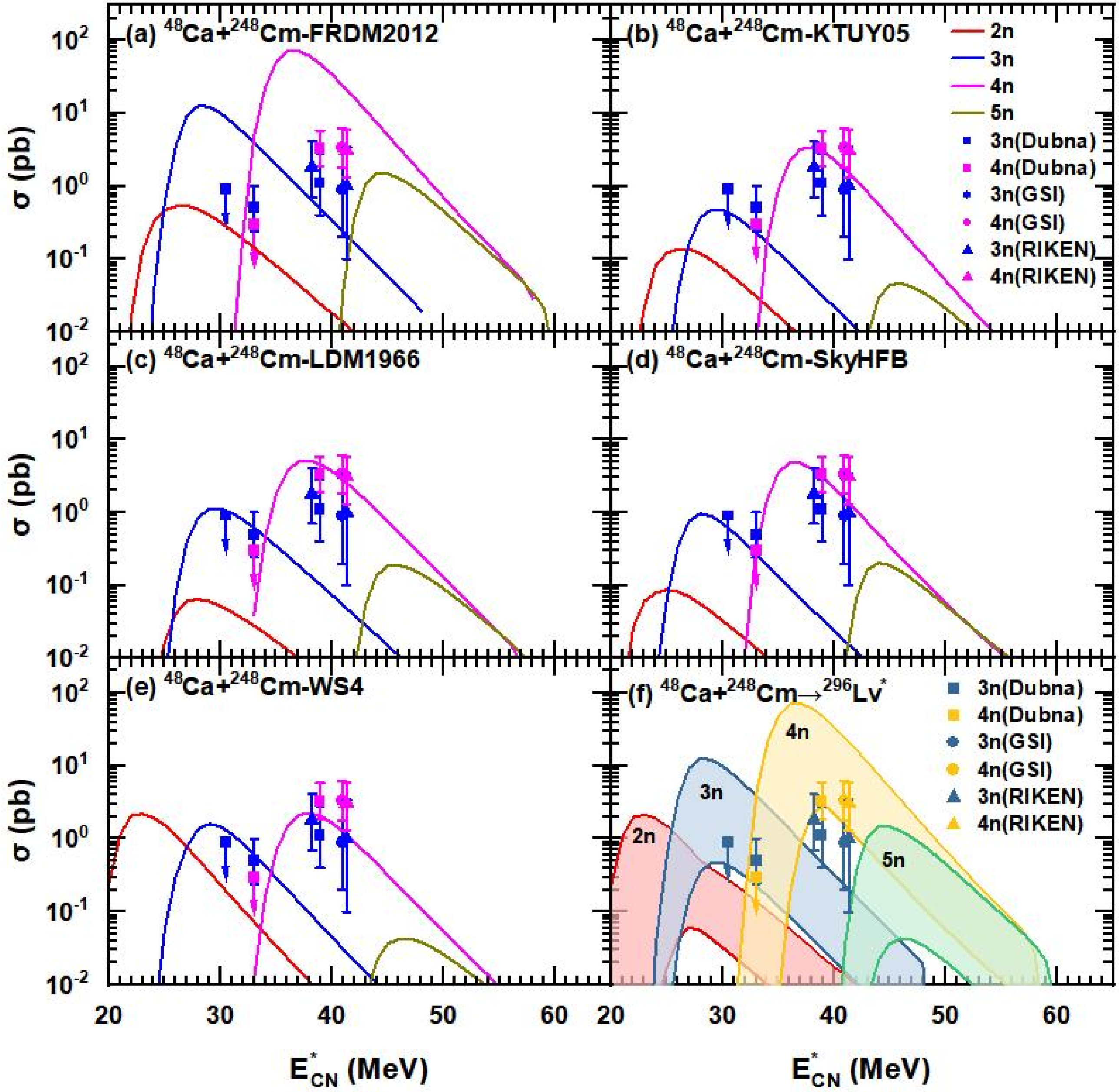
Schematic diagram of fusion evaporation reactions in the DNS model

- 冯兆庆. 近垒重离子熔合反应和超重核合成机制研究[D]. 甘肃:中国科学院近代物理研究所,2007.

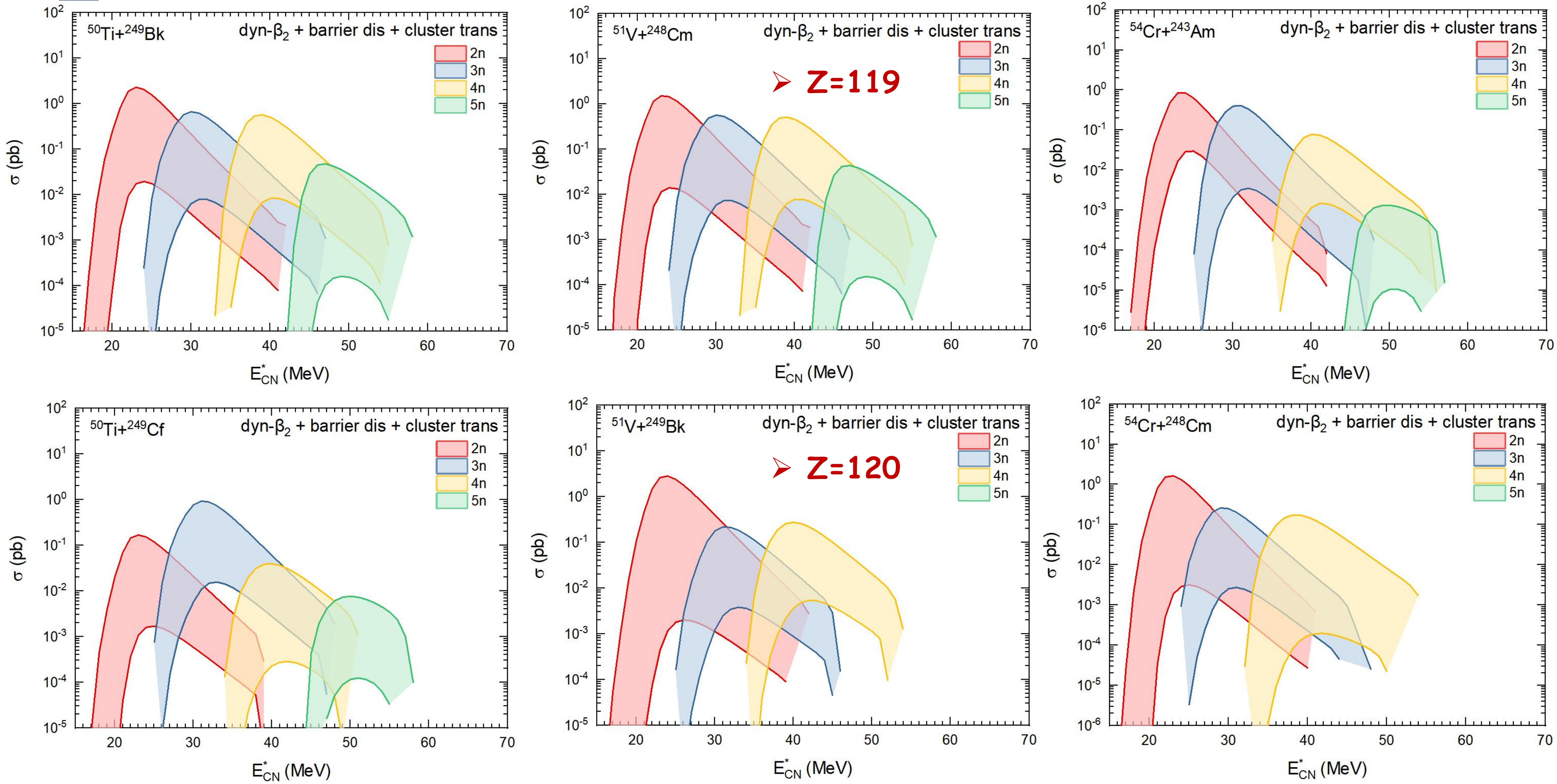
3.Formation Cross Sections of SHN

□ Different mass tables

- **FRDM2012:**
  the finite-range droplet model
  - P. Möller et al, Atomic Data and Nuclear Data Tables 109–110 (2016) 1–204
- **KTUY05:**
  the Koura-Tachibana-Uno-Yamada
  - Hiroyuki Koura et al, Progress of Theoretical Physics 113 (2005) 305–325
- **LDM1966:**
  the little-droplet model
  - William D. Myers et al, Nuclear Physics 81 (1966) 1–60
- **SkyHFB:**
  the Hartree-Fock-Bogoliubov method
  - Y. El Bassem et al, Nuclear Physics A 957 (2017) 22–32
- **WS4:**
  the Weizsäcker-Skyrme mass model
  - Ning Wang et al, Physics Letters B 734 (2014) 215–219



3.Formation Cross Sections of SHN



THANK YOU !

Korea-China Joint Workshop on Rare Isotope Physics

2025.07.10