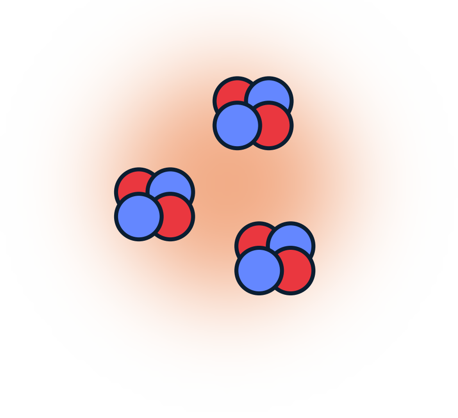
Korea-China joint workshop for rare isotope physics

Cluster Models in the Study of Light Nuclei



Bo Zhou (周波)

Fudan University

July. 08, 2025

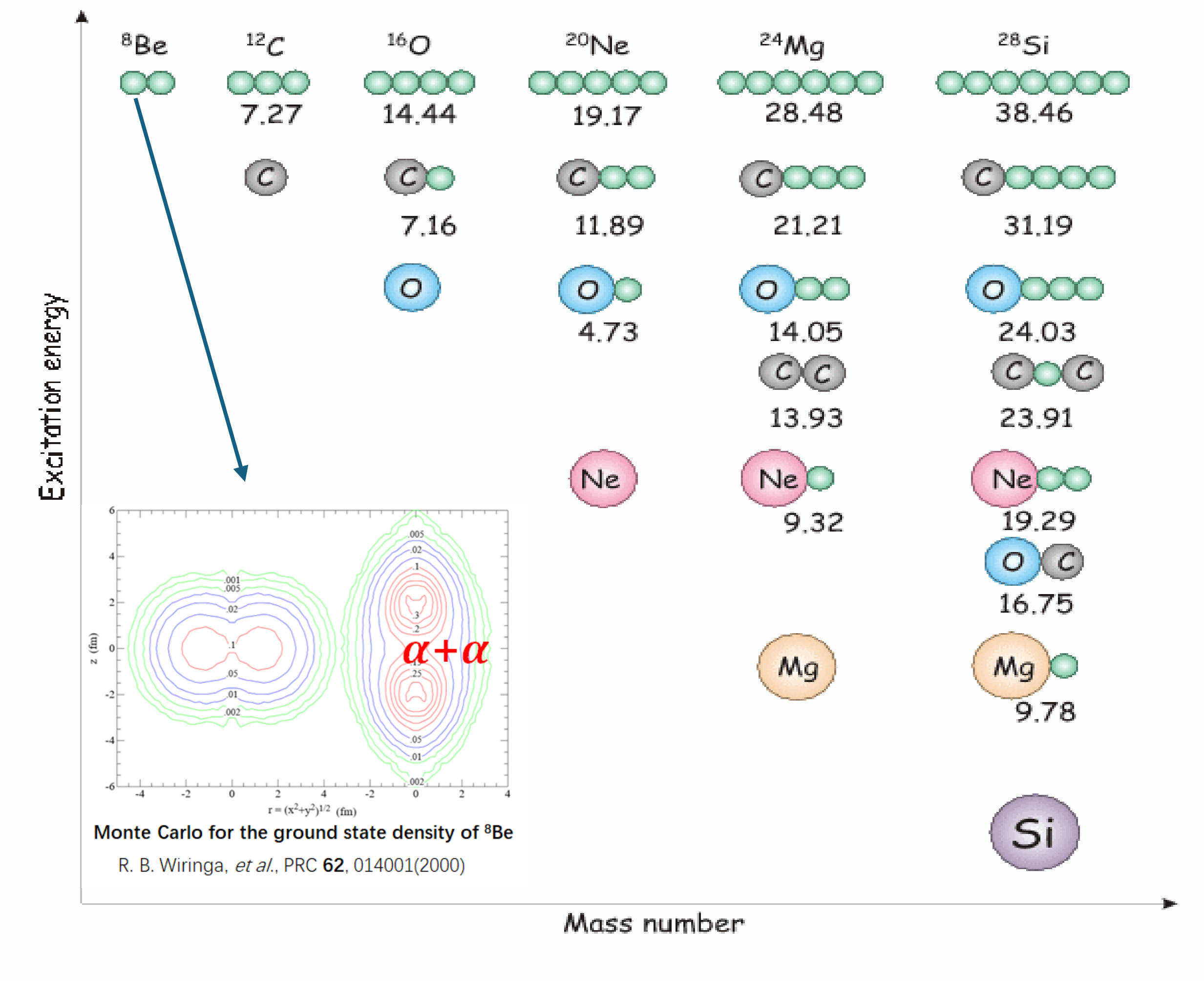
Benisiam Hall, Benikea Hotel, Jeju island

Outline

- From ^{12}C to condensate state
- $3\alpha+p$ clustering in ^{13}N
- $\alpha+\alpha+n$ three-body reaction
- Summary and Prospect

Nuclear Cluster Physics

Ikeda diagram of light nuclei



Monte Carlo for the ground state density of ^{8}Be

R. B. Wiringa, *et al.*, PRC **62**, 014001(2000)

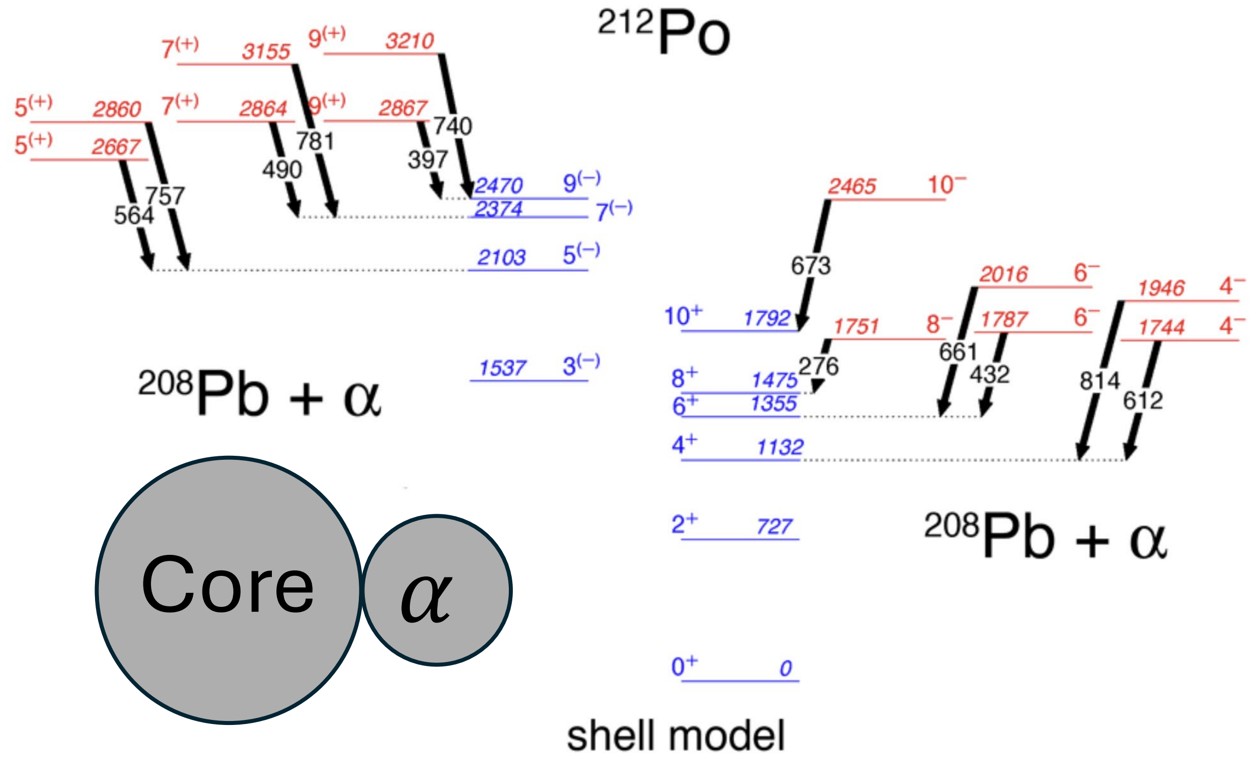
Clustering in heavy nuclei ?

Selected for a Viewpoint in *Physics*

104, 042701 (2010) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2010

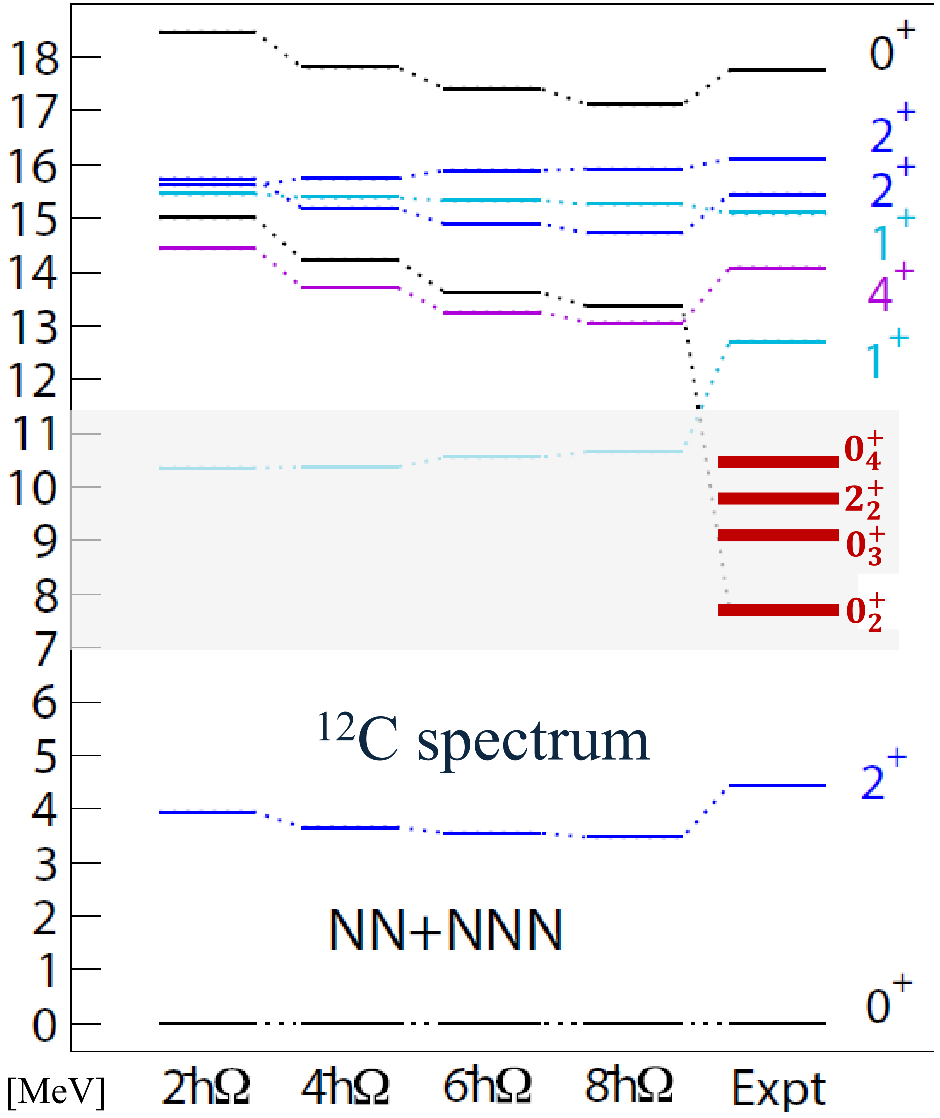
Novel Manifestation of α-Clustering Structures: New "α + ^{208}Pb" States in ^{212}Po Revealed by Their Enhanced $E1$ Decays

A. Astier,[1] P. Petkov,[1,2] M.-G. Porquet,[1] D. S. Delion,[3,4] and P. Schuck[5]



https://physics.aps.org/articles/v3/8

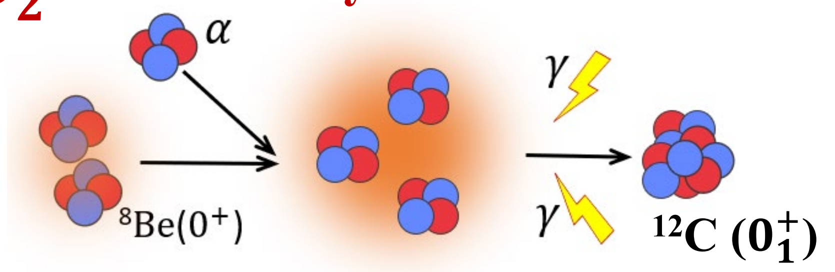
Cluster states of ^{12}C



Recent No-Core-Shell-Model calculations

V.Somà,P.Navrátil, *et al.* PRC,101,014318 (2020)

0_2^+ Hoyle State



Hoyle state & Bose-Einstein Condensate.

Rev. Mod. Phys. **89**, 011002 (2017)

$0_{3,4}^+$

Two broad resonance states with large decay width

Phys. Rev. C **84**, 054308 (2011)

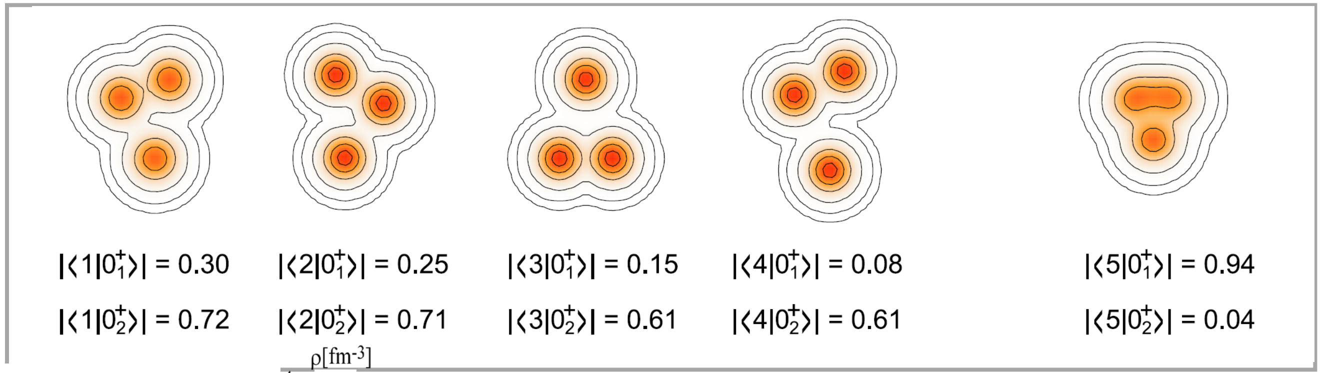
2_2^+

Long puzzle and it now has been confirmed for its existence.

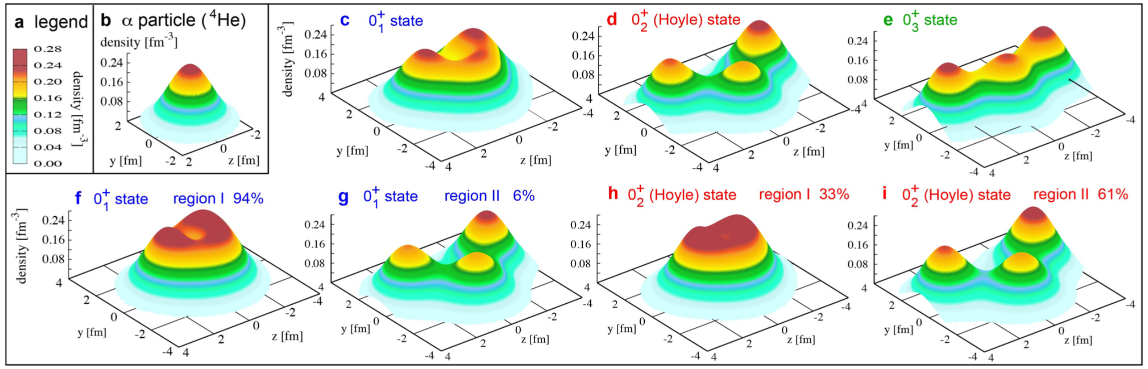
Phys. Rev. Lett. **110**, 152502 (2013)

well-developed clustering states

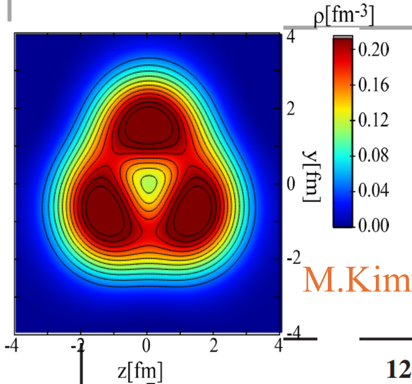
Shape/Structure of the ^{12}C



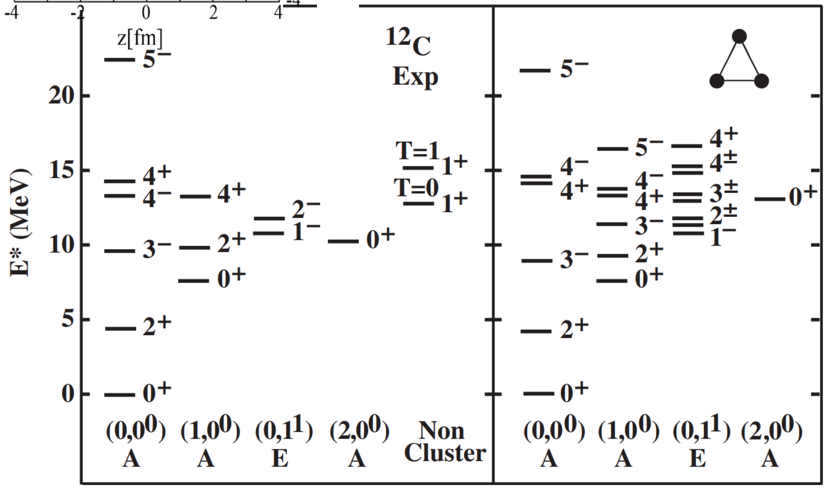
M. Chernykh, et al., PRL 98, 032501 (2007)



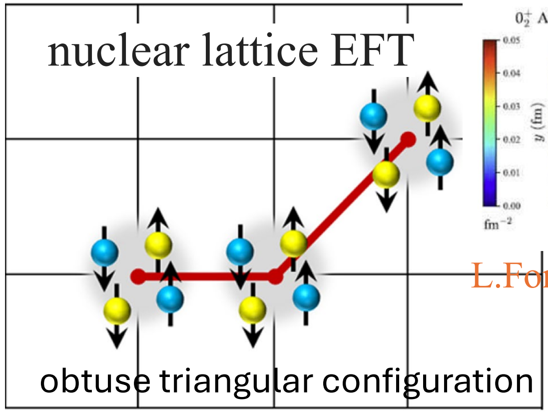
Otsuka, et al., Nat Commun 13, 2234 (2022)



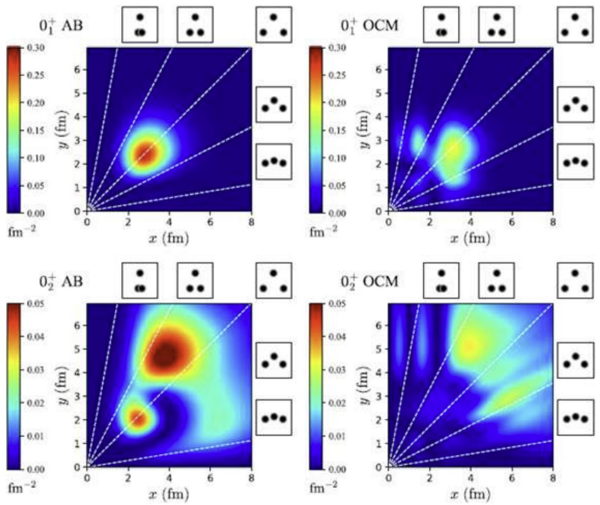
M.Kimura, Eur. Phys. J. A 60, 77 (2024)



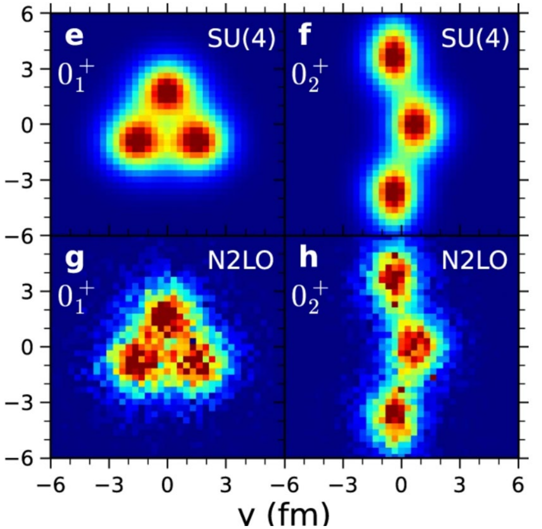
D J Marín-Lámbarri,et al., PRL 113, 012502 (2014)



E.Epelbaum,et al,,PRL 109,252501 (2012)

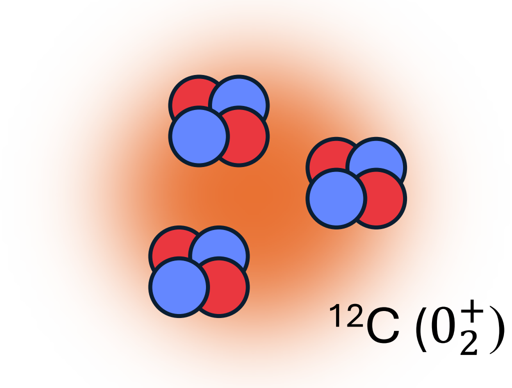


L.Fortunato, *Few-Body Syst* **65**, 1 (2024).



S.Shen, et al., *Nat Commun* **14**, 2777 (2023)

Hoyle state of ^{12}C



3α Bose-Einstein state

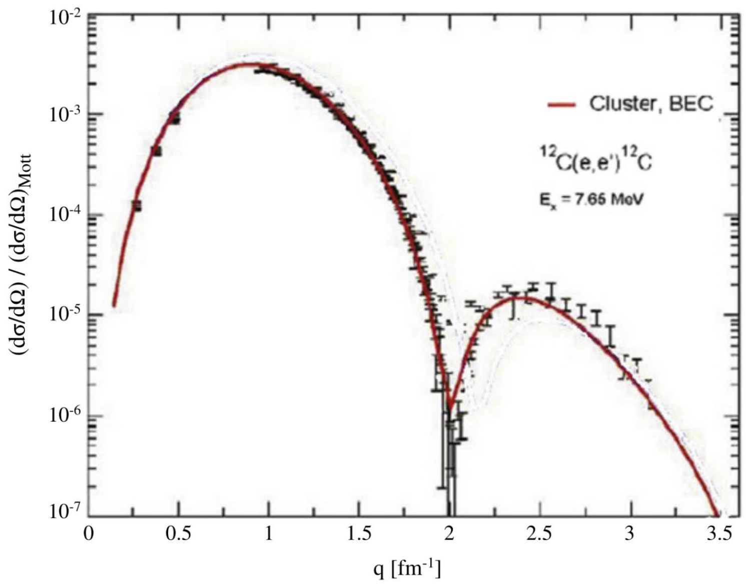
$$\Psi_{3\alpha}^{\mathrm{THSR}} = \mathcal{A}\left\{\exp\left[-\frac{2}{B^2}(\boldsymbol{X}_1^2 + \boldsymbol{X}_2^2 + \boldsymbol{X}_3^2)\right]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\}$$

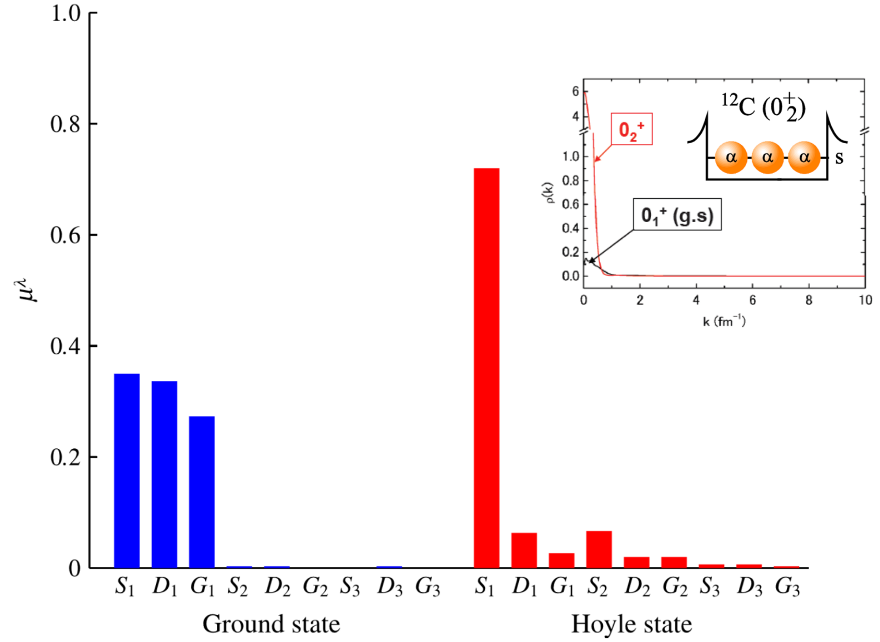
$$= \exp\left(-\frac{6}{B^2}\boldsymbol{\xi}_3^2\right)\mathcal{A}\left\{\exp\left(-\frac{4}{3B^2}\boldsymbol{\xi}_1^2 - \frac{1}{B^2}\boldsymbol{\xi}_2^2\right)\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\},$$

$$\boldsymbol{\xi}_1 = \boldsymbol{X}_1 - \frac{1}{2}(\boldsymbol{X}_2 + \boldsymbol{X}_3), \qquad \boldsymbol{\xi}_2 = \boldsymbol{X}_2 - \boldsymbol{X}_3, \qquad \boldsymbol{\xi}_3 = \frac{1}{3}(\boldsymbol{X}_1 + \boldsymbol{X}_2 + \boldsymbol{X}_3)$$

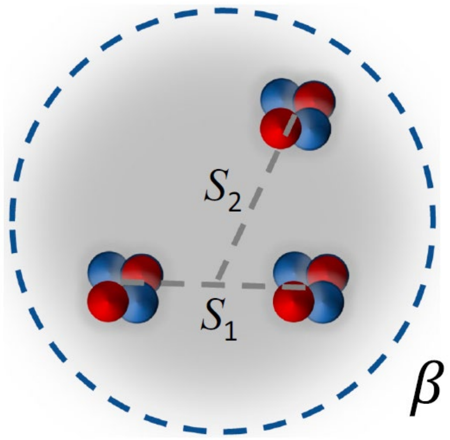
THSR, PRL 87, 192501 (2001)

Y. Funaki et al. / Progress in Particle and Nuclear Physics 82 (2015) 78–132





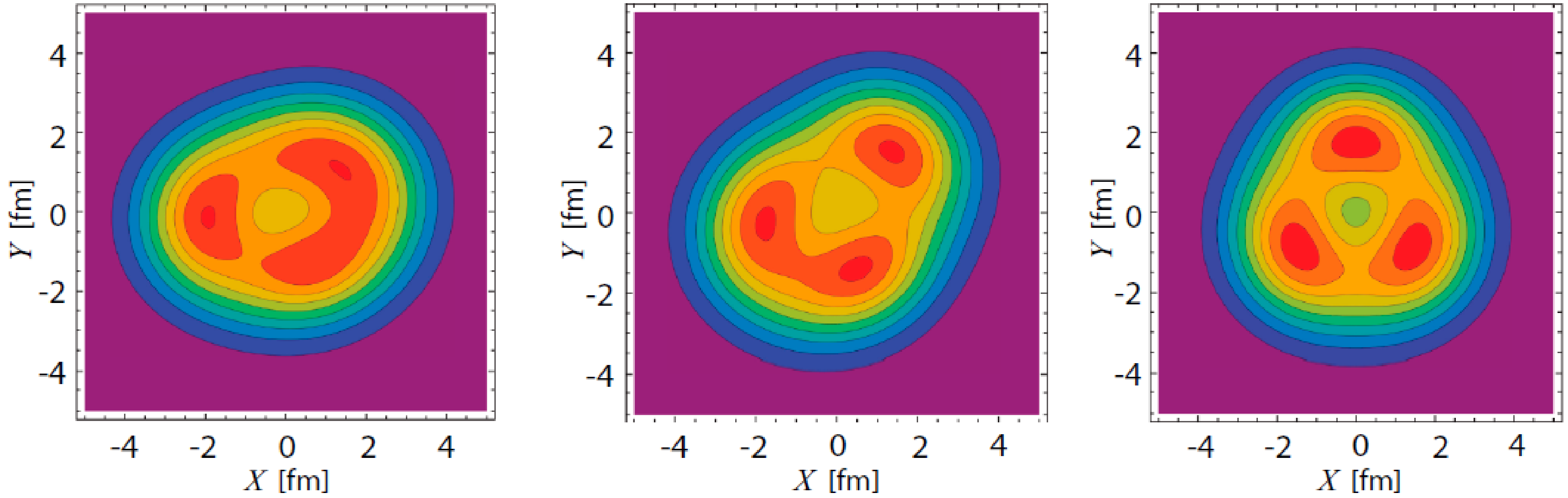
Nonlocalized cluster motion of 3α clusters in ^{12}C



We really obtained the single high-accuracy THSR-type wave functions for 3^- and 4^- states,

$$\propto \mathcal{A}\{\exp[-\frac{(\boldsymbol{\xi}_1-\boldsymbol{S}_1)^2}{b^2+2\boldsymbol{\beta}^2}-\frac{(\boldsymbol{\xi}_2-\boldsymbol{S}_2)^2}{3/4\ (b^2+2\boldsymbol{\beta}^2)}]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\}$$

Size parameters β obtained by variational calculations.



$|\langle\Phi^{3^-}(3/2,3/2,1/2)|\Phi^{3^-}_{\rm GCM}\rangle|^2$=0.94 $|\langle\Phi^{3^-}(1,3/2,3/2)|\Phi^{3^-}_{\rm GCM}\rangle|^2$=0.93 $|\langle\Phi^{3^-}(3/2,0,3/2)|\Phi^{3^-}_{\rm GCM}\rangle|^2$=0.94

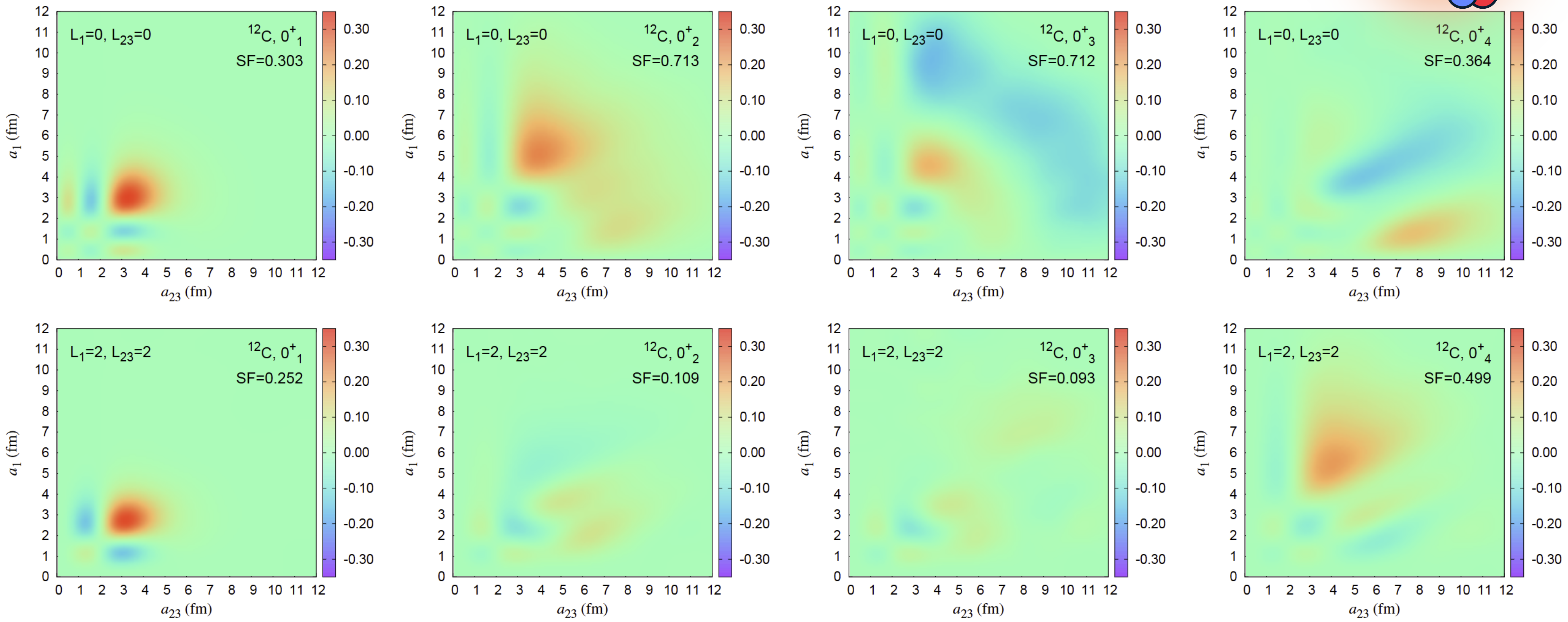
$|\langle\Phi^{4^-}(3/2,3/2,1/2)|\Phi^{4^-}_{\rm GCM}\rangle|^2$=0.92 $|\langle\Phi^{4^-}(1,3/2,3/2)|\Phi^{4^-}_{\rm GCM}\rangle|^2$=0.92 $|\langle\Phi^{4^-}(3/2,0,3/2)|\Phi^{4^-}_{\rm GCM}\rangle|^2$=0.92

B. Zhou, et al., Phys. Rev. C **99**, 051303(R) (2019).

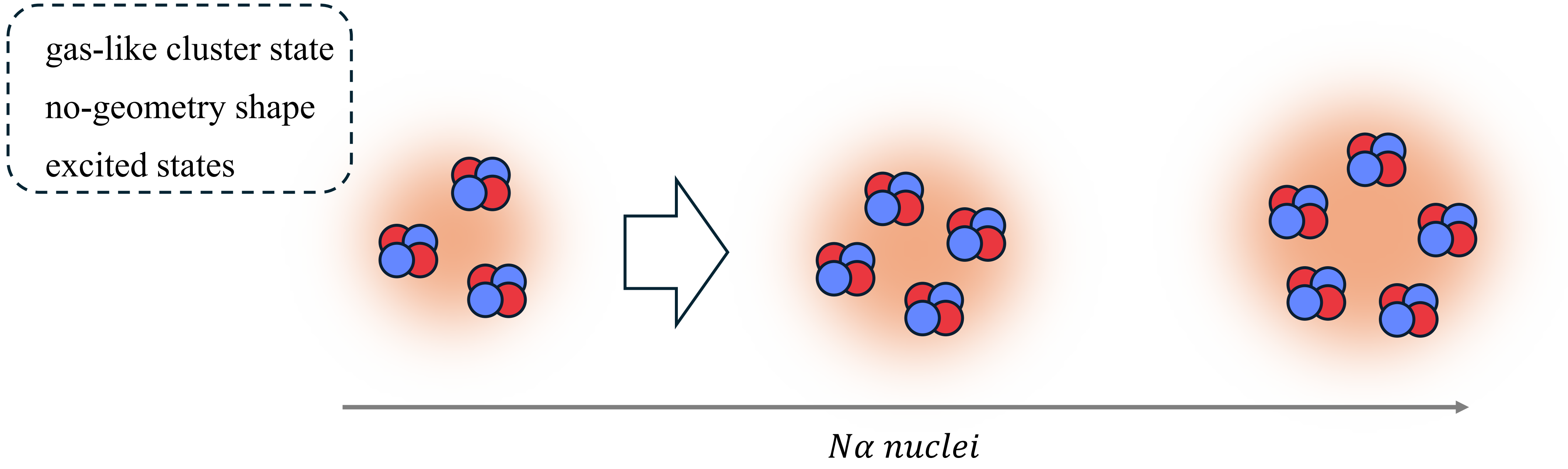
Two-body overlap function (Two-body RWA)

$[\alpha \otimes [\alpha \otimes \alpha]_0]_0 \otimes [0 \otimes 0]_0$

arXiv:2501.10664



$$\mathcal{Y}_c^{J\pi}(a_1,a_2) = \sqrt{\frac{A!}{C_1!C_2!C_3!}} \left\langle \frac{\delta(r_1-a_1)\delta(r_2-a_2)}{r_1^2 r_2^2} \left[[Y_{l_1}(\hat{r}_1)\otimes Y_{l_2}(\hat{r}_2)]_L \otimes \left[\Phi_{C_1}^{j_1\pi_1}\otimes\left[\Phi_{C_2}^{j_2\pi_2}\otimes\Phi_{C_3}^{j_3\pi_3}\right]_{j_{23}}\right]_{j_{123}}\right]_{JM} \middle| \Psi_M^{J\pi}\right\rangle$$



Search for the Nα condensate state

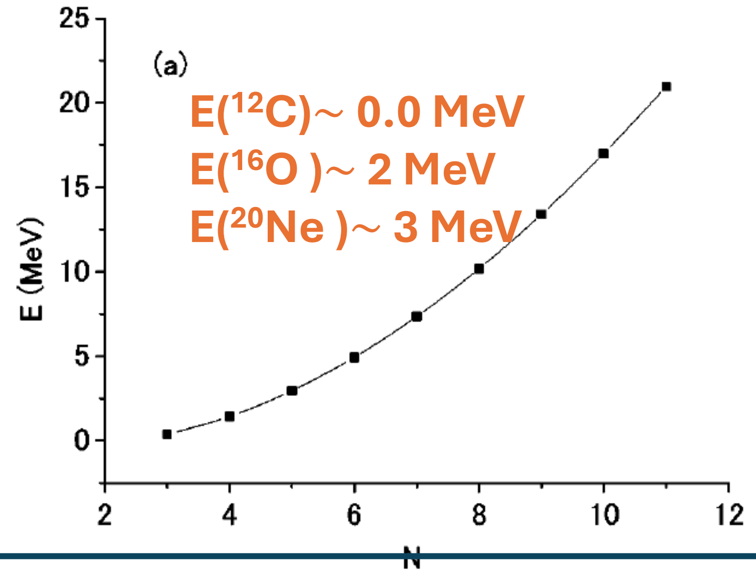
3α condensate	4α condensate	5α condensate
(Hoyle state)	(0_6^+ state)	(?)
2001 (THSR)	2008~ (OCM,THSR)	2019~

study of alpha condensate in finite nuclei

Multi-α condensation

Dilute multi-α cluster condensed states with spherical and axially deformed shapes are studied with the Gross-Pitaevskii equation and Hill-Wheeler equation where the α cluster is treated as a structureless boson, **it is predicted to exist in heavier self-conjugate 4*N* nuclei up to *N*=10.**

T. Yamada and P. Schuck, Phys. Rev. C 69, 024309 (2004).



Some candidates for α condensate were found from experiments for ^{12}C and ^{16}O.

Rev. Mod. Phys. **89**, 011002 (2017).

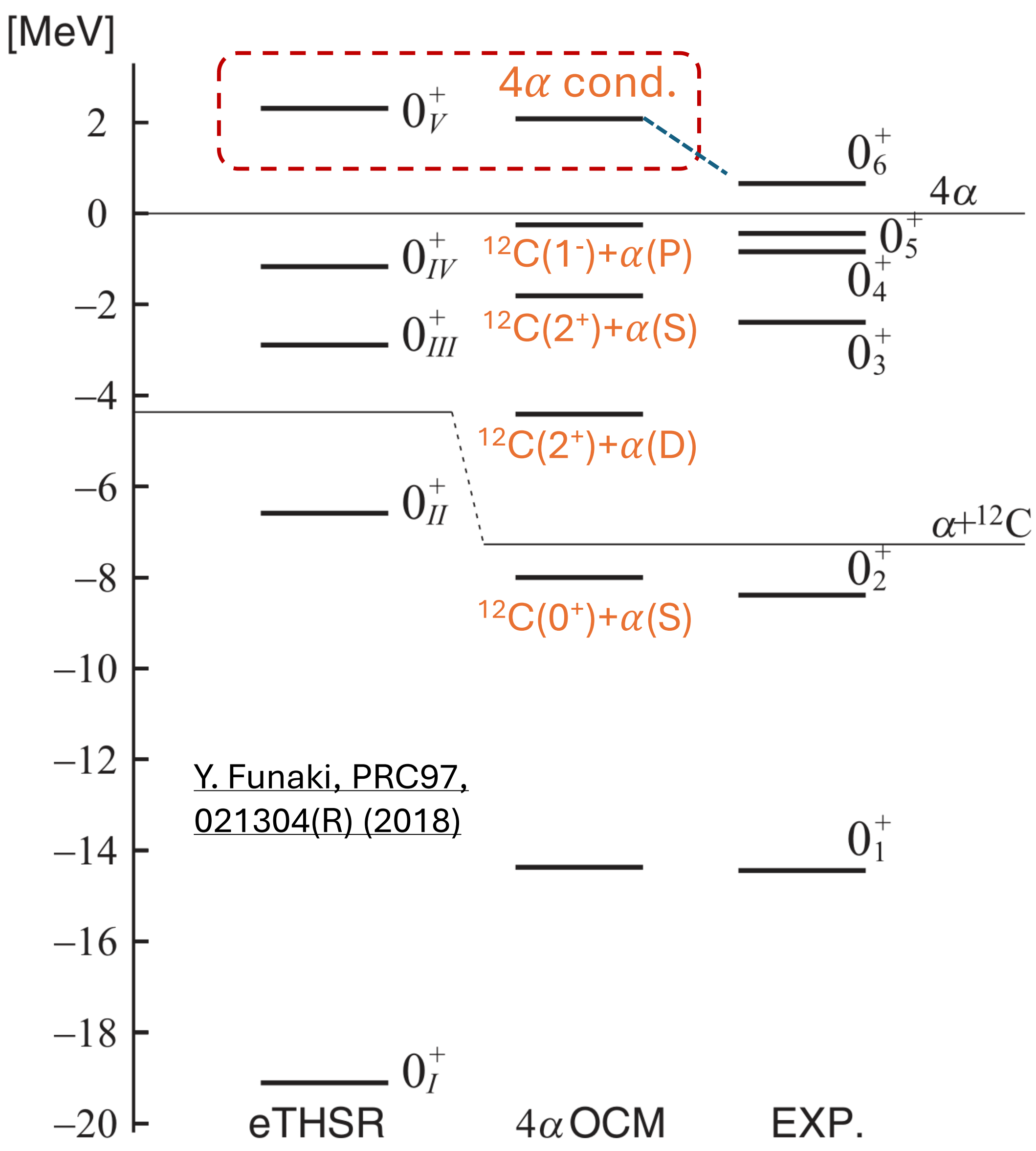
No experimental signatures for α condensation were observed

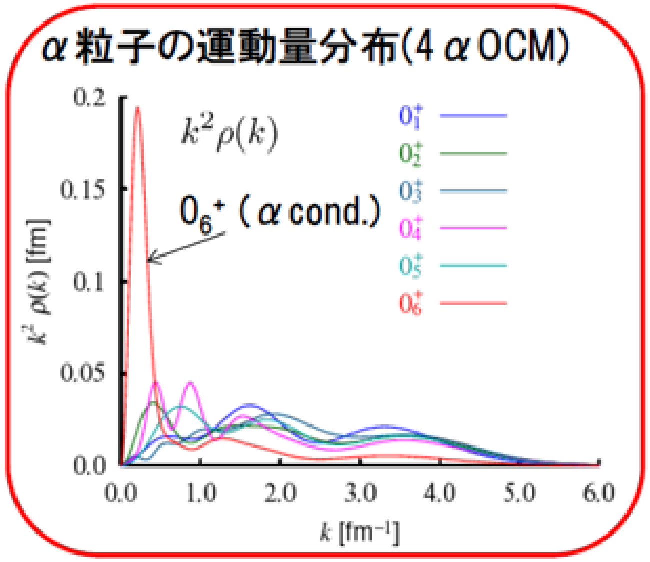
Phys. Rev. C **100**, 034320 (2019)

An experimental way of testing Bose-Einstein condensation of clusters in the atomic nucleus is reported. The enhancement of cluster emission and the multiplicity partition of possible emitted clusters could be direct signatures for the condensed states.

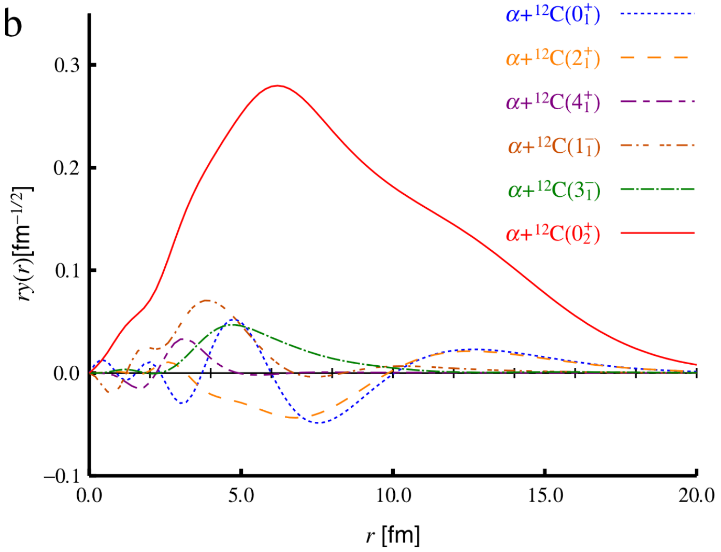
PRL **96**, 192502 (2006)

Alpha condensate in ^{16}O

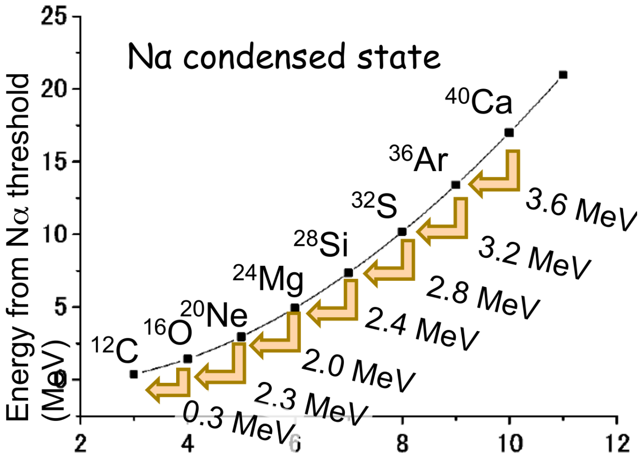




4αOCM
Y. F. et al., PRL **101**, 082502 (2008).
4αTHSR
Y. F. et al., PRC **82**, 024312 (2010).

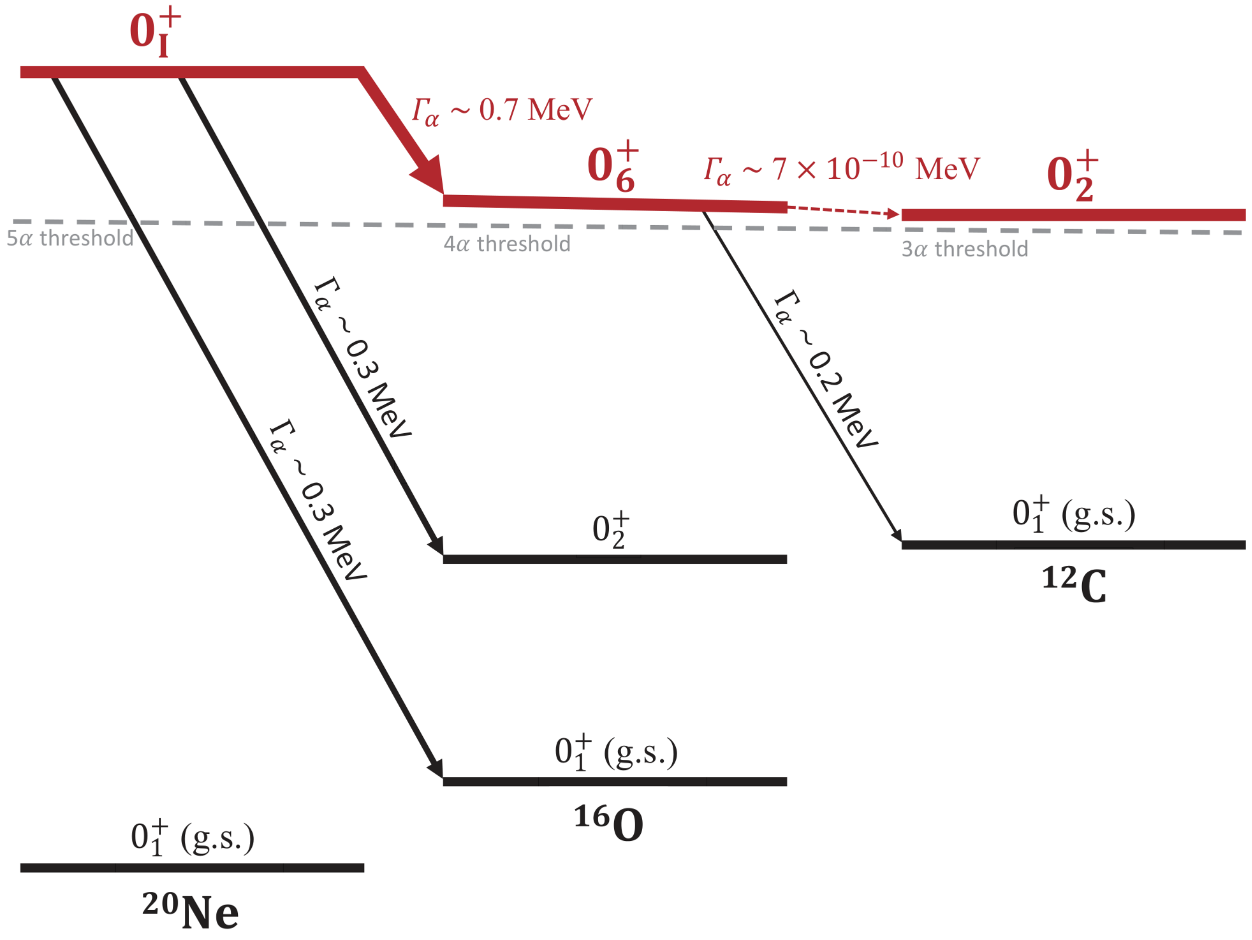


The decay scheme and connections



T. Yamada and P. Schuck, PRC 69, 024309 (2004).

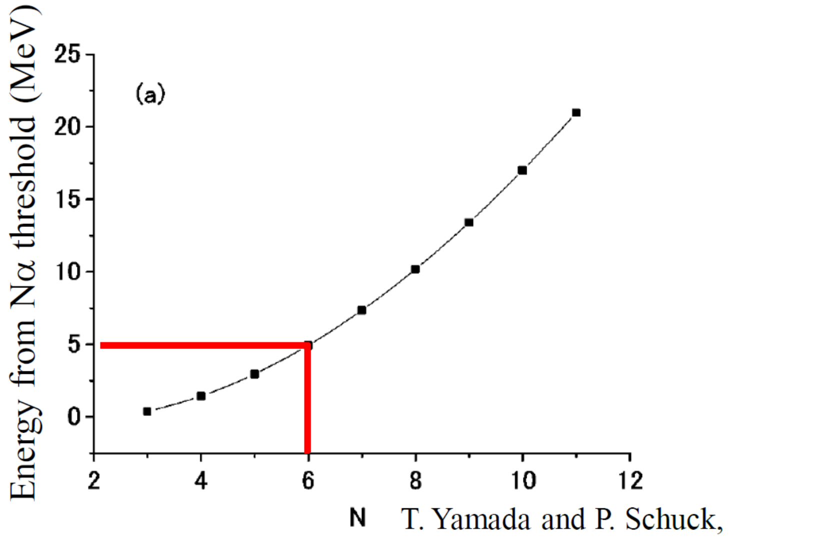
Exotic clustering structure ?



B. Zhou, et al., *Nat. Commun.* **14**, 8206 (2023).

The 6α clustering structure probed by Inelastic Scattering

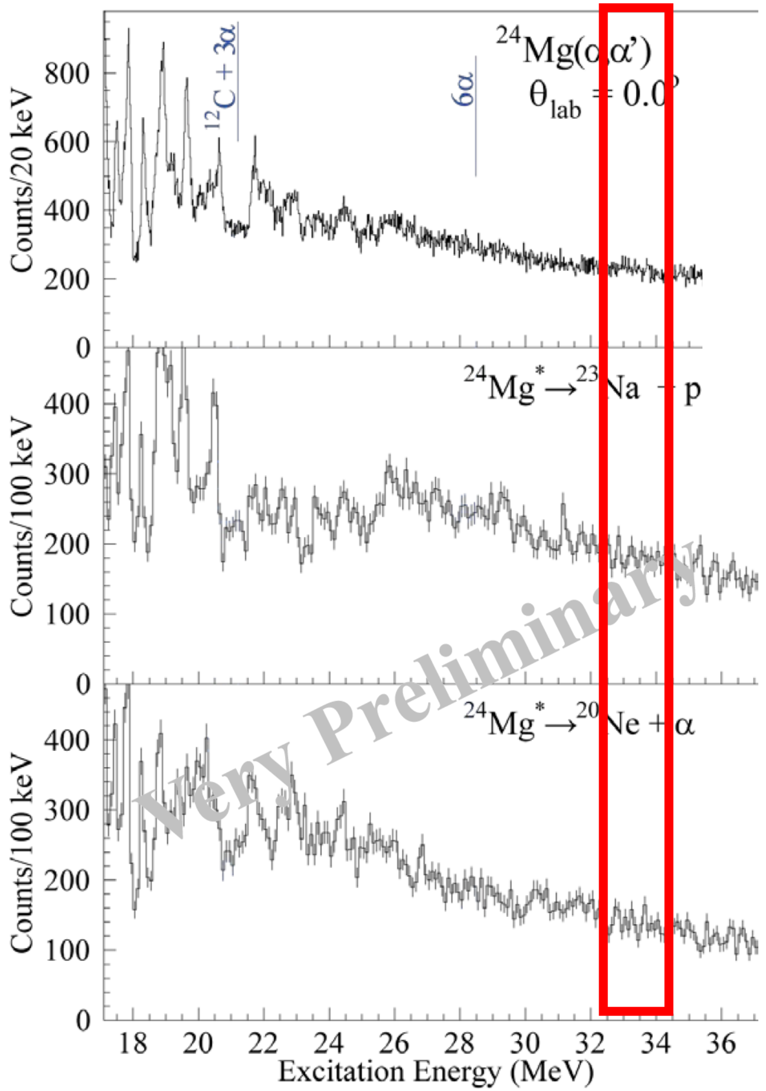
6α condensed state was searched for in the highly excited region.



T. Yamada and P. Schuck, Phys. Rev. C **69**, 024309 (2004).

- 6α condensed state is expected at 5 MeV above the 6α threshold.
  - $E_x \sim 28.5 + 5 = 33.5$ MeV
- No significant structure suggesting the 6α condensed state.
  - Several small structures indistinguishable from the statistical fluctuation. → Need more statistics.

by measuring the 12C+12C scattering



PLB,848 (2024)

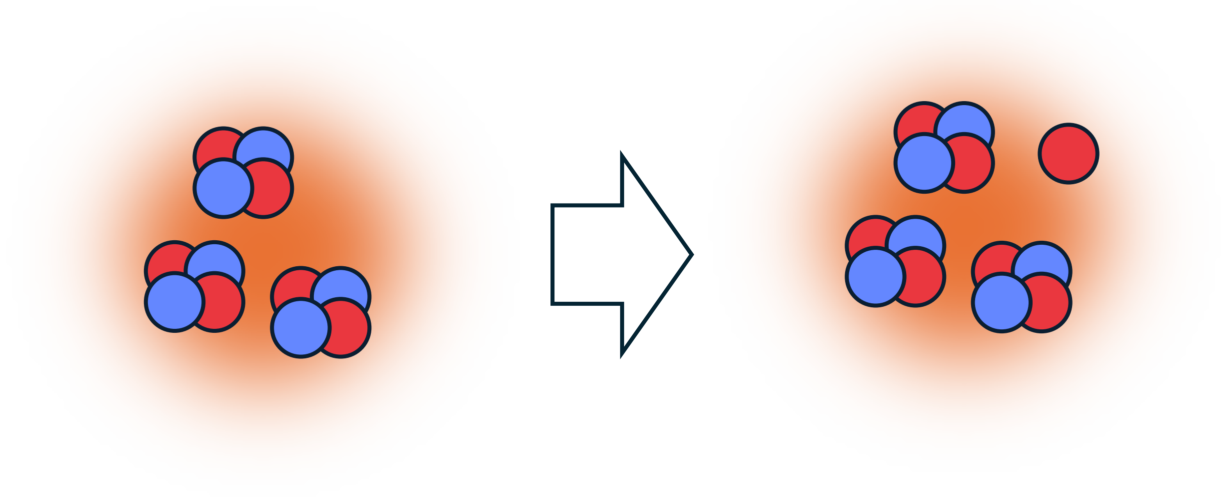
A. Tohsaki et al. / Nuclear Physics A738 (2004) 259–263 261

Table 1
The independent number of permutations for each kernel. Here, the case of the norm kernel for ^{24}Mg is added. The final row shows a full number of permuations without any reduction for the norm kernel.

	^{8}Be(2α)	^{12}C(3α)	^{16}O(4α)	^{20}Ne(5α)	^{24}Mg(6α)
norm	3	9	35	185	1614
kinetic	7	34	242	2546	
two-body	9	58	669	10912	
three-body	40	366	6773	156617	
$(n!)^4$	16	1296	3.32×10^5	2.07×10^8	2.79×10^{11}

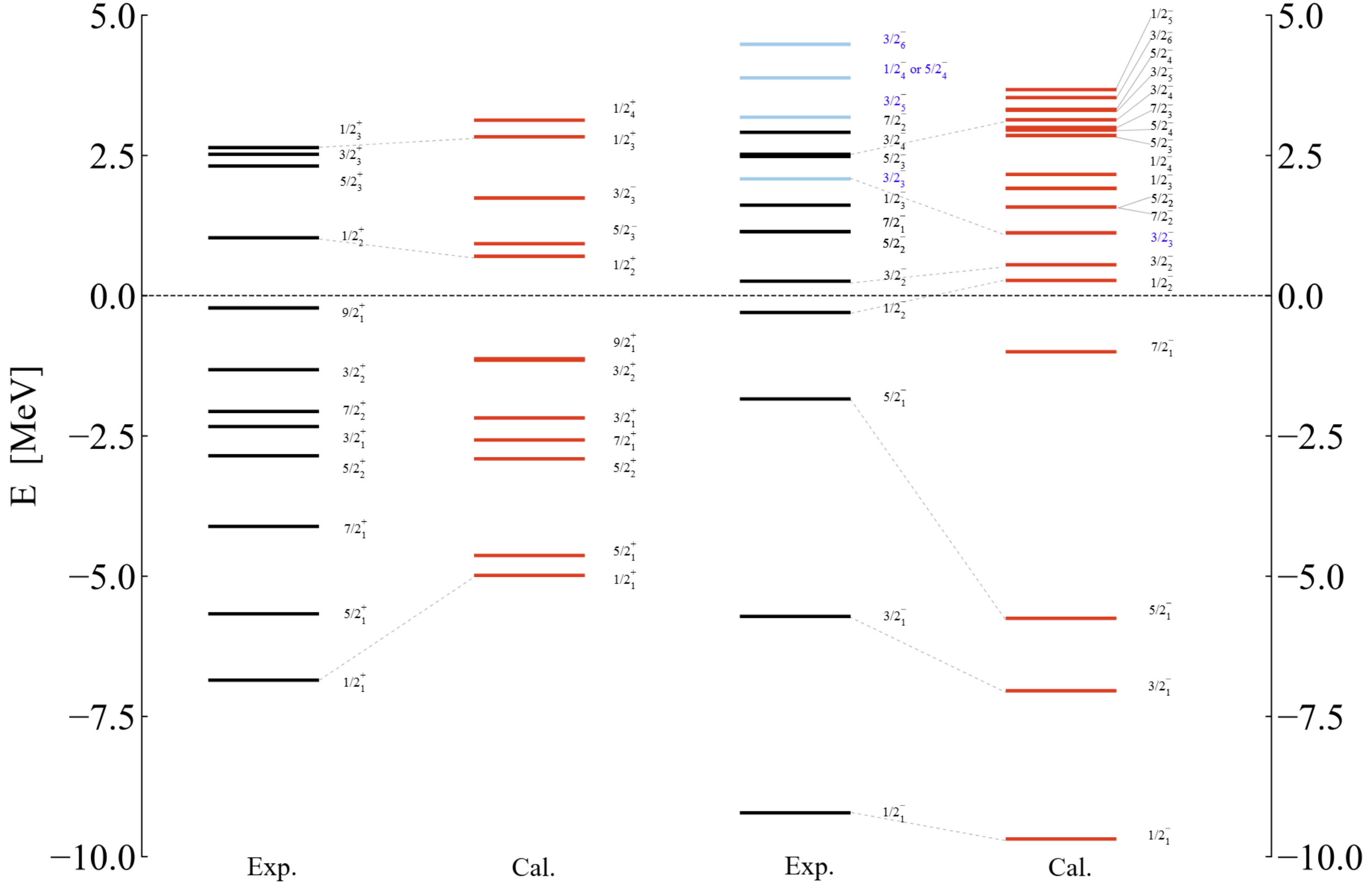
$$\langle \Psi_{n\alpha}^{\mathrm{THSR}}(\beta)|\mathcal{O}|\Psi_{n\alpha}^{\mathrm{THSR}}(\beta')\rangle = \sum_{p=0}^{m_p^{(1)}-1} W_p^{(1)} I_p^{(1)}$$

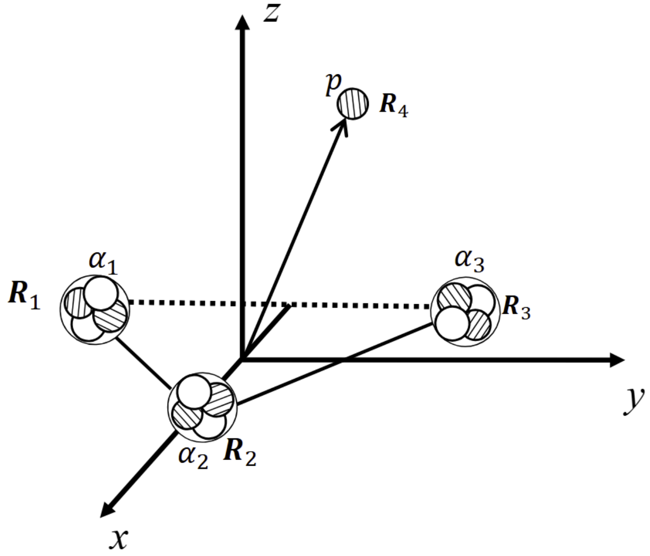
Remains challenging in theoretical calculations



Clustering structure of $3\alpha+p$ in ^{13}N

Hoyle-analog state in ^{13}N





Transition	Present	Experiment
$B(E1, 3/2^-_1 \to 1/2^+_1)$	0.016	0.036
$B(E1, 1/2^+_1 \to 1/2^-_1)$	0.0007	0.036 ± 0.004
$B(E2, 3/2^-_1 \to 1/2^-_1)$	4.87	
$B(E2, 5/2^-_1 \to 1/2^-_1)$	3.71	
$B(E2, 1/2^-_1 \to 3/2^+_1)$	21.58	

arXiv:2501.18303

Hoyle-analog state in ^{13}N

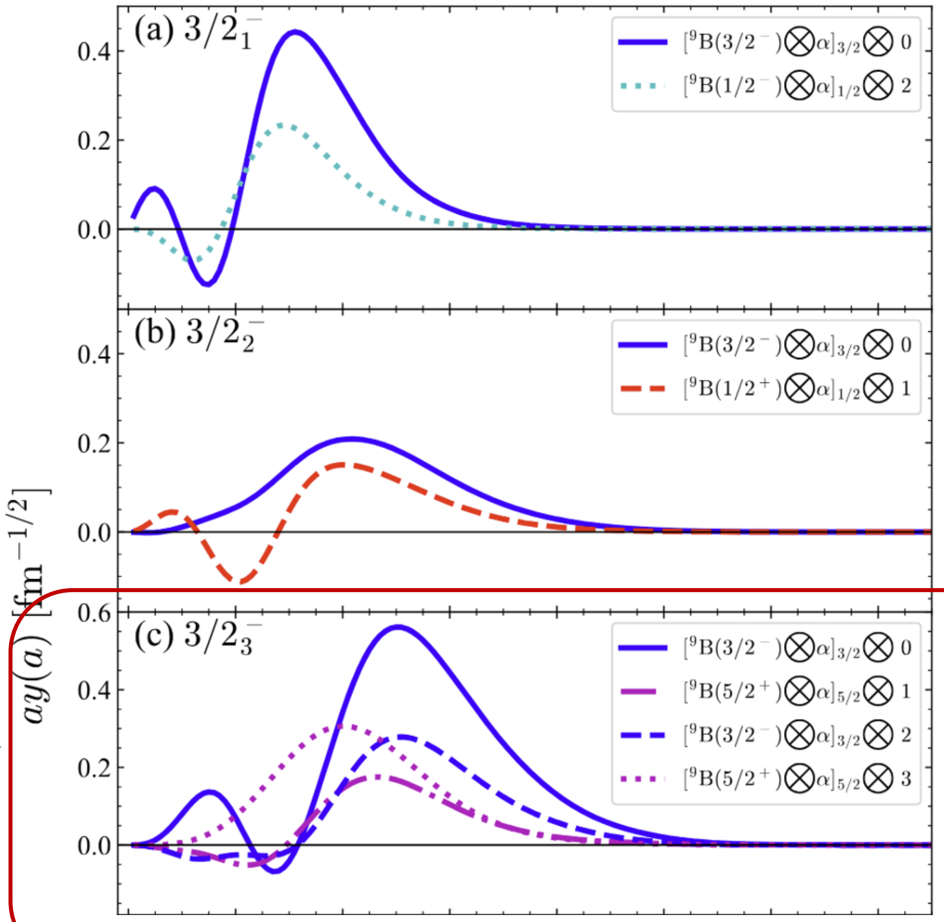
PHYSICAL REVIEW C **109**, 054308 (2024)

Cluster structure of $3\alpha + p$ states in ^{13}N

J. Bishop,[1,2] G. V. Rogachev,[1,3,4] S. Ahn,[5] M. Barbui,[1] S. M. Cha,[5] E. Harris,[1,3] C. Hunt,[1,3] C. H. Kim,[6] D. Kim,[5] S. H. Kim,[6] E. Koshchiy,[1] Z. Luo,[1,3] C. Park,[5] C. E. Parker,[1] E. C. Pollacco,[7] B. T. Roeder,[1] M. Roosa,[1,3] A. Saastamoinen,[1] and D. P. Scriven[1,3]

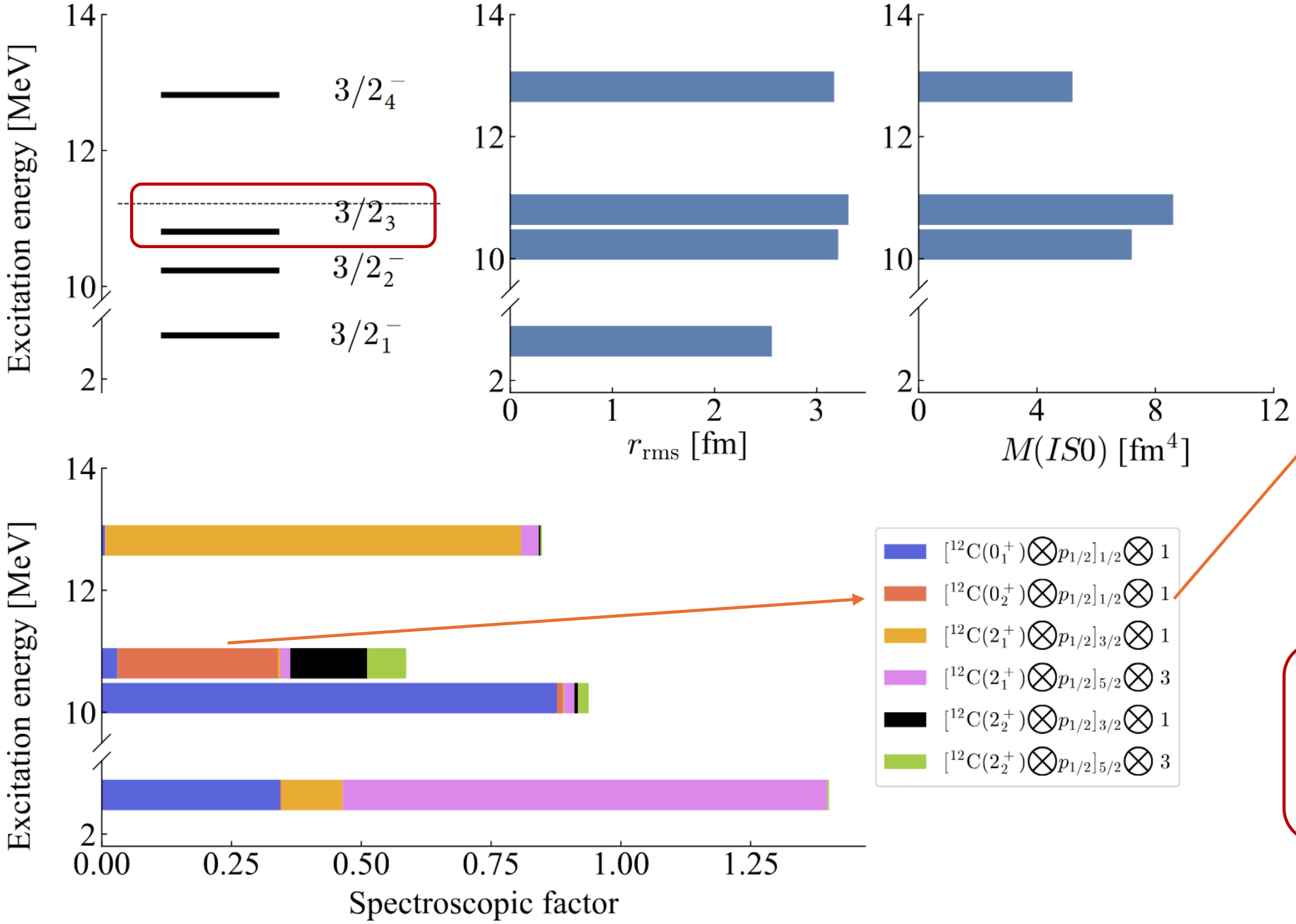
[1]*Cyclotron Institute, Texas A&M University, College Station, Texas 77843, USA*
[2]*School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom*
[3]*Department of Physics & Astronomy, Texas A&M University, College Station, Texas 77843, USA*
[4]*Nuclear Solutions Institute, Texas A&M University, College Station, Texas 77843, USA*
[5]*Center for Exotic Nuclear Studies, Institute for Basic Science, 34126 Daejeon, Republic of Korea*
[6]*Department of Physics, Sungkyunkwan University, Suwon 16419, Republic of Korea*
[7]*IRFU, CEA, Université Paris-Saclay, F-91191 Gif-Sur-Yvette, France*

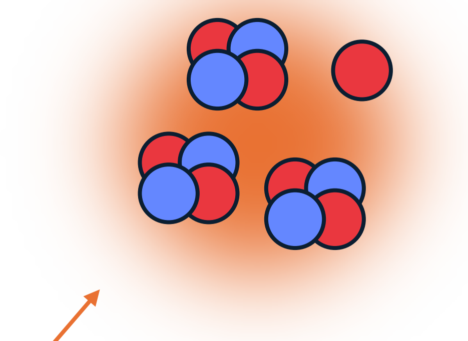
Background: Cluster states in ^{13}N are extremely difficult to measure due to the unavailability of $^{9}B + \alpha$ elastic-scattering data.
Purpose: Using β-delayed charged-particle spectroscopy of ^{13}O, clustered states in ^{13}N can be populated and measured in the $3\alpha + p$ decay channel.
Methods: One-at-a-time implantation and decay of ^{13}O was performed with the Texas Active Target Time Projection Chamber. 149 $\beta 3\alpha p$ decay events were observed and the excitation function in ^{13}N reconstructed.
Results: Four previously unknown α-decaying excited states were observed in ^{13}N at an excitation energy of 11.3, 12.4, 13.1, and 13.7 MeV decaying via the $3\alpha + p$ channel.
Conclusions: These states are seen to have a $[^{9}B(g.s) \otimes \alpha / p + {}^{12}C(0_2^+)]$, $[^{9}B(\frac{1}{2}^+) \otimes \alpha]$, $[^{9}B(\frac{5}{2}^+) \otimes \alpha]$, and $[^{9}B(\frac{5}{2}^+) \otimes \alpha]$ structure, respectively. A previously seen state at 11.8 MeV was also determined to have a $[p + {}^{12}C(g.s.) / p + {}^{12}C(0_2^+)]$ structure. The overall magnitude of the clustering is not able to be extracted, however, due to the lack of a total width measurement. Clustered states in ^{13}N (with unknown magnitude) seem to persist from the addition of a proton to the highly α-clustered ^{12}C. Evidence of the $\frac{1}{2}^+$ state in ^{9}B was also seen to be populated by decays from $^{13}N^{\star}$.



This obtained state corresponds to the state observed at 11.3 MeV

Hoyle-analog state in ^{13}N



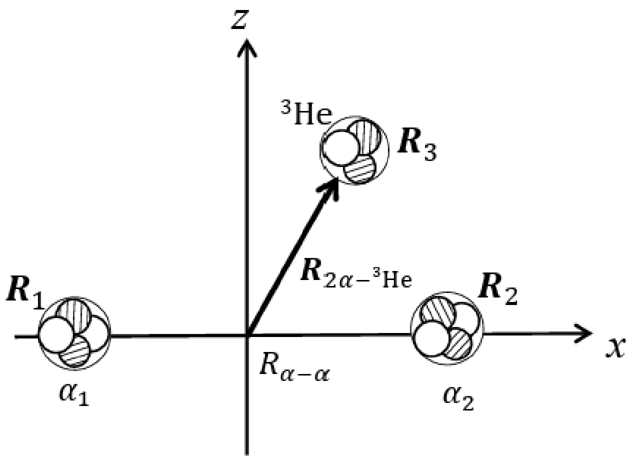


$\langle \mathrm{Hoyle} + p | ^{13}\mathrm{N\ state} \rangle$

$$y^{J\pi}_{j_1\pi_1 j_2\pi_2 j_{12} l}(a) = \sqrt{\frac{A!}{(1+\delta_{C_1C_2})\,C_1!C_2!}} \times \left\langle \frac{\delta(r-a)}{r^2} \left[Y_l(\hat{r}) \left[\Phi^{j_1\pi_1}_{C_1} \Phi^{j_2\pi_2}_{C_2} \right]_{j_{12}} \right]_{JM} \middle| \Psi^{J\pi}_M \right\rangle$$

arXiv:2501.18303

Gas-like states in ^{11}C

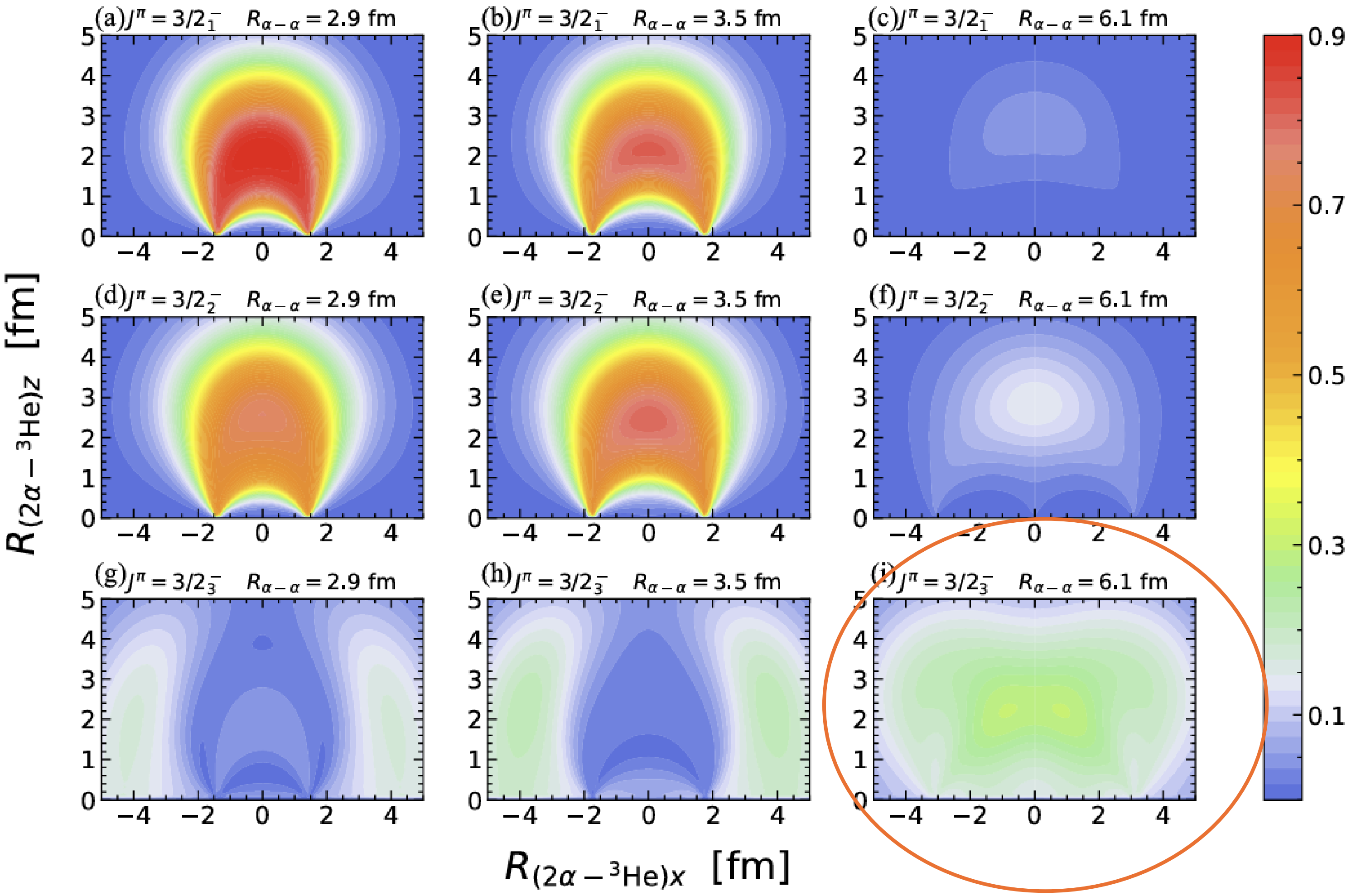


$$\Phi(\boldsymbol{R}_1, \boldsymbol{R}_2, \boldsymbol{R}_3) = \mathcal{A}\{\Phi_\alpha(\boldsymbol{R}_1)\Phi_\alpha(\boldsymbol{R}_2)\Phi_{^3\mathrm{He}}(\boldsymbol{R}_3)\},$$

$$\Phi_\alpha(\boldsymbol{R}) = \mathcal{A}\{\prod_{i=1}^{4} \phi(\boldsymbol{R}, \boldsymbol{r}_i)\chi_i\tau_i\},$$

$$\Phi_{^3\mathrm{He}}(\boldsymbol{R}) = \mathcal{A}\{\prod_{i=1}^{3} \phi(\boldsymbol{R}, \boldsymbol{r}_i)\chi_i\tau_i\},$$

$$\phi(\boldsymbol{R}, \boldsymbol{r}_i) = (\frac{1}{\pi b^2})^{3/4} e^{-\frac{(\boldsymbol{r}_i - \boldsymbol{R})^2}{2b^2}},$$



Y.Y Cao, et al., Phys. Rev. C 111, 024309 (2025)

PHYSICAL REVIEW LETTERS **131**, 242501 (2023)

Editors' Suggestion | Featured in Physics

Observation of the Exotic 0_2^+ Cluster State in ^{8}He

Z. H. Yang,[1,2,*,†] Y. L. Ye,[1,*,‡] B. Zhou,[3,4,5] H. Baba,[2] R. J. Chen,[6] Y. C. Ge,[1] B. S. Hu,[1] H. Hua,[1] D. X. Jiang,[1] M. Kimura,[2,5,7] C. Li,[2] K. A. Li,[6] J. G. Li,[1] Q. T. Li,[1] X. Q. Li,[1] Z. H. Li,[1] J. L. Lou,[1] M. Nishimura,[2] H. Otsu,[2] D. Y. Pang,[8] W. L. Pu,[1] R. Qiao,[1] S. Sakaguchi,[2,9] H. Sakurai,[2] Y. Satou,[10] Y. Togano,[2] K. Tshoo,[10] H. Wang,[2,11] S. Wang,[2] K. Wei,[1] J. Xiao,[1] F. R. Xu,[1] X. F. Yang,[1] K. Yoneda,[2] H. B. You,[1] and T. Zheng[1]

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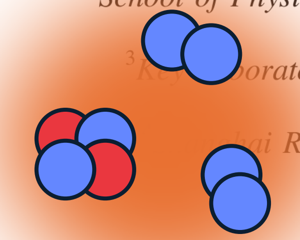
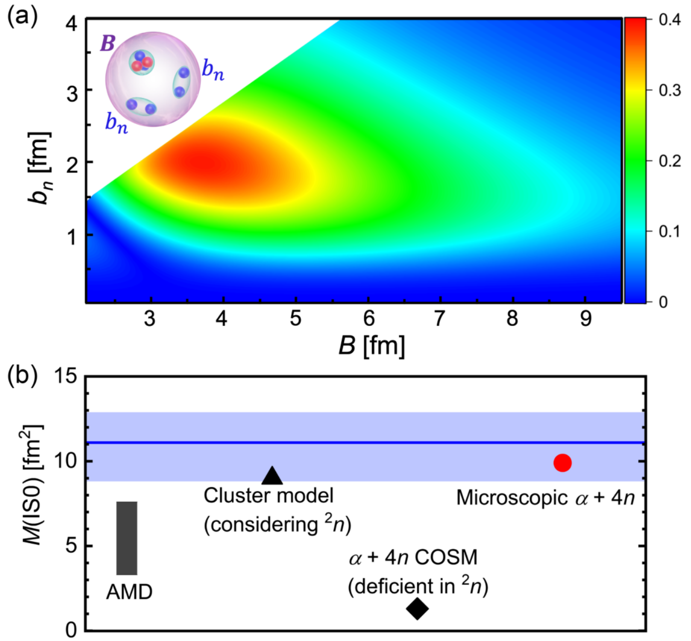
[3]Key Laboratory of Nuclear Physics and Ion-beam Application (MOE), Institute of Modern Physics, Fudan University, Shanghai 200433, China
...ai Research Center for ...
[5]Departm...
[6]Institute of Mo...



$$\Phi(\boldsymbol{B}, b_n) \propto \mathcal{A}\left\{\exp\left[-\frac{4\xi_1^2}{3B^2}-\frac{3\xi_2^2}{2B^2}\right]\times\phi_\alpha(b_\alpha)\phi_{{}^2_1n}(b_n)\phi_{{}^2_2n}(b_n)\right\},$$



PTEP

Prog. Theor. Exp. Phys. **2018**, 041D01 (10 pages)
DOI: 10.1093/ptep/pty034

Letter

New trial wave function for the nuclear cluster structure of nuclei

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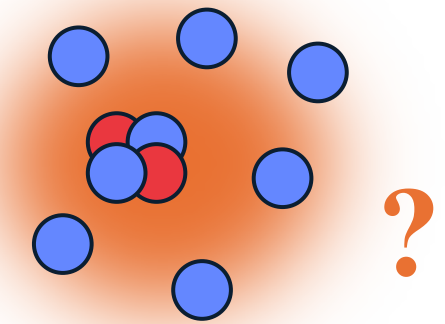
A new trial wave function is proposed for nuclear cluster physics, in which an exact solution to the long-standing center-of-mass problem is given. In the new approach, the widths of the

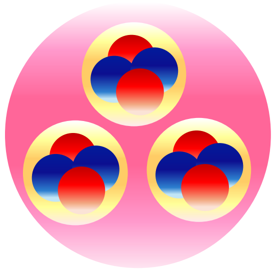
$$\Psi(\mathbf{r}) = \Phi_{\rm g}(\mathbf{r}_g)\Phi_{\rm int}(\mathbf{r}_i - \mathbf{r}_j)$$

$$\begin{aligned}\Psi_{\rm new} &= \widehat{L}_{n-1}(\beta)\widehat{G}_n(\beta_0)\widehat{D}(\boldsymbol{Z})\Phi_0(\boldsymbol{r})\\ &= \int d^3\widetilde{T}_1\cdots d^3\widetilde{T}_{n-1}\exp[-\sum_{i=1}^{n-1}\frac{\widetilde{T}_i^2}{\beta_i^2}]\int d^3R_1\cdots d^3R_n\exp[-\sum_{i=1}^{n}(\frac{A_i}{\beta_0^2-2b_i^2})(\boldsymbol{R}_i-\boldsymbol{Z}_i-\boldsymbol{T}_i)^2]\Phi_0(\boldsymbol{r}-\boldsymbol{R})\\ &= n_0\exp[-\frac{A}{\beta_0^2}X_g^2]\,\mathcal{A}\{\prod_{i=1}^{n-1}\exp[-\frac{1}{2B_i^2}(\boldsymbol{\xi}_i-\boldsymbol{S}_i)^2]\prod_{i=1}^{n}\phi_i^{\rm int}(b_i)\}.\end{aligned}$$

a tool for studying the cluster correlations

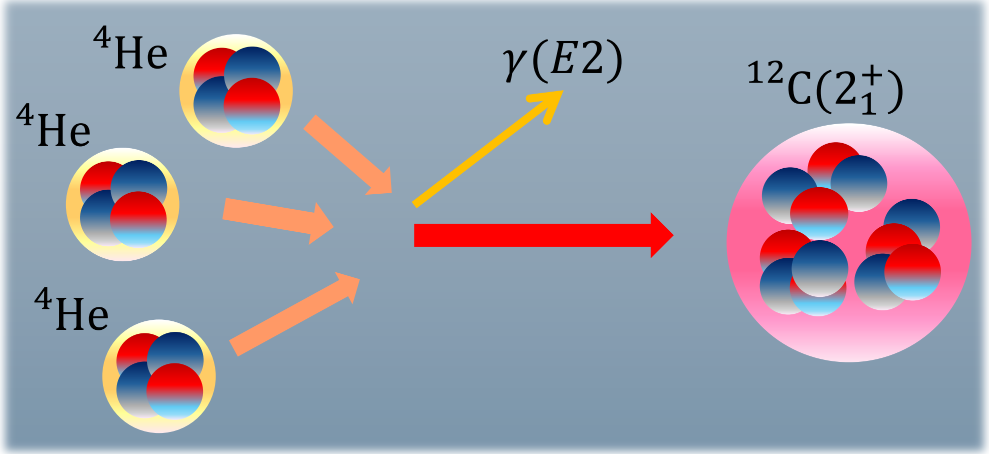


Triple α process

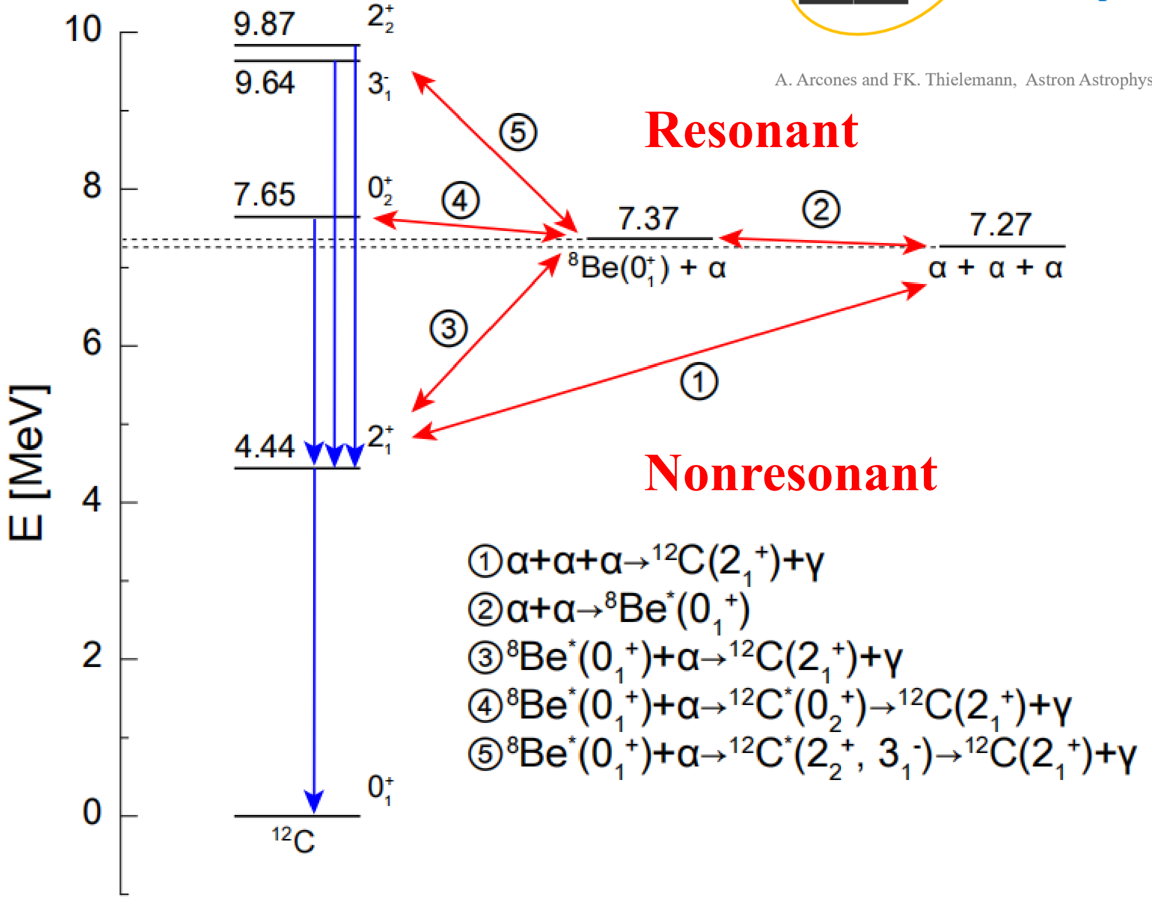


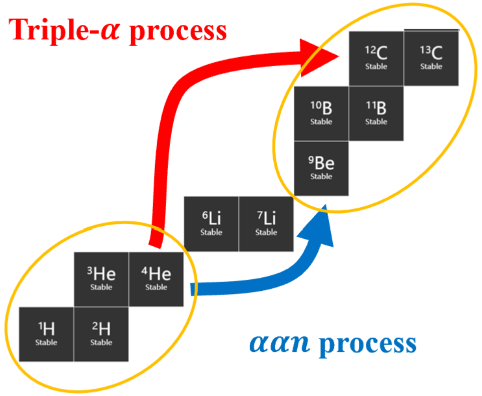
Nuclear astrophysics
- Bridging the gaps at A = 5 and A = 8
- Generation of elements with A > 8

Triple-α process



Hoyle state
$^{12}C(0_2^+, 7.65MeV)$

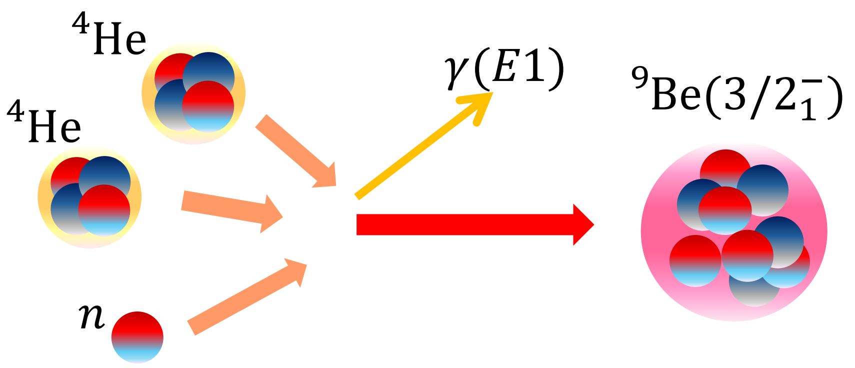




A. Arcones and FK. Thielemann, Astron Astrophys Rev 31, 1 (2023)

Pei Luanhong, et al., SCIENTIA SINICA Physica, Mechanica & Astronomica, 2024, ISSN 1674-7275

Sequential picture and Direct picture



Sequential picture (级联)

Intermediate state: $\boldsymbol{X}$

$$a + b \to \boldsymbol{X}$$

$$\boldsymbol{X} + c(\to A^*) \to A + \gamma$$

Characteristics:

1. **Specific**: assumed the reaction mechanism
2. **Integrated with experiments**: two-body cross-section, resonance

$$N_A^2\langle abc\rangle \sim N_A^2 \frac{\hbar}{\Gamma_\alpha(\boldsymbol{X})}\langle ab\rangle\langle \boldsymbol{X}c\rangle$$

- NACRE

Direct picture

$$a + b + c \to A + \gamma$$

Characteristics:

1. **General**
2. **Three-body non-resonant process**
   (Important at low temperatures)
3. **Complicated calculation**
   (Three-body scattering state)

$$\langle abc\rangle = (1+\Delta_{abc})\frac{1}{\omega_i}\left(\frac{4\pi^2\beta^2\hbar^4}{\mu}\right)^{3/2}\frac{8\pi(\lambda+1)}{\hbar\lambda[(2\lambda+1)!!]^2} \times \sum_{M_f\mu}\sum_i e^{-\beta E_i}\left(\frac{E_i-E_f}{\hbar c}\right)^{2\lambda+1}\left|\langle\Phi_f|M_{\lambda\mu}|\Phi_i\rangle\right|^2,$$

- Transformed harmonic oscillator method
- Three-body Breit-Wigner
- **Imaginary-time method**

Imaginary-time method

Introduce inverse temperature

$$\beta = 1/(k_B T)$$

Reaction rate (from **direct picture**)

$$\langle abc\rangle = (1+\Delta_{abc})\frac{1}{\omega_i}\left(\frac{4\pi^2\beta^2\hbar^4}{\mu}\right)^{3/2}\frac{8\pi(\lambda+1)}{\hbar\lambda[(2\lambda+1)!!]^2} \times \sum_{M_f\mu}\sum_i e^{-\beta E_i}\left(\frac{E_i-E_f}{\hbar c}\right)^{2\lambda+1}\left|\langle\Phi_f|M_{\lambda\mu}|\Phi_i\rangle\right|^2,$$

$$\sim \sum_i e^{-\beta E_i}\left(\frac{E_i-E_f}{\hbar c}\right)^{2\lambda+1}\left\langle\Phi_f\left|M_{\lambda\mu}|\Phi_i\rangle\langle\Phi_i|M_{\lambda\mu}^{+}\right|\Phi_f\right\rangle$$

Scattering state

Purpose: remove initial state (Scattering state)

The spectral representation of H

$$f(\hat{H}) = \sum_{n\in\text{bound}} f(E_n)|\Phi_n\rangle\langle\Phi_n| + \sum_{i\in\text{scattering}} f(E_i)|\Phi_i\rangle\langle\Phi_i|,$$

T. Akahori, et al, Phys. Rev. C 92, 022801(R) (2015)
K. Yabana et al., Phys. Rev. C 85, 055803 (2012)

Take $f(x)$ as the following form

$$f(x) = e^{-\beta x}\left(\frac{x-E_f}{\hbar c}\right)^{2\lambda+1}$$

Therefore

$$\sum_i e^{-\beta E_i}\left(\frac{E_i-E_f}{\hbar c}\right)^{2\lambda+1}|\Phi_i\rangle\langle\Phi_i|$$

$$= f(\hat{H})\left(\mathbf{1} - \sum_{n\in\text{bound}}|\Phi_n\rangle\langle\Phi_n|\right)$$

Introduce projection operator

$$\hat{P} = \mathbf{1} - \sum_{n\in bound}|\Phi_n\rangle\langle\Phi_n|$$

Imaginary-time reaction rate

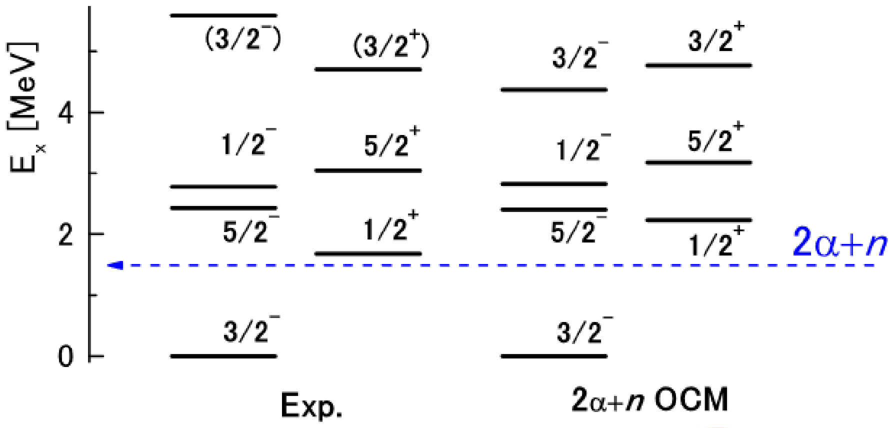
$$\langle abc\rangle = (1+\Delta_{abc})\frac{1}{\omega_i}\left(\frac{4\pi^2\beta^2\hbar^4}{\mu}\right)^{3/2}\frac{8\pi(\lambda+1)}{\hbar\lambda[(2\lambda+1)!!]^2} \times \langle\Phi_f|M_{\lambda\mu}e^{-\beta H}\left(\frac{H-E_f}{\hbar c}\right)^{2\lambda+1}\hat{P}M^{\dagger}_{\lambda\mu}|\Phi_f\rangle, \quad (62)$$

Scattering state + Bound state → Bound state

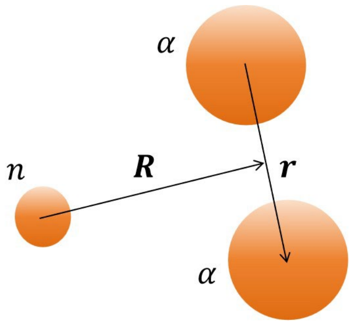
$\alpha\alpha n$ process

$$\boldsymbol{\alpha + \alpha + n \rightarrow {}^{9}\text{Be}(3/2_1^-) + \gamma}$$

Spectrum of ^{9}Be (OCM)



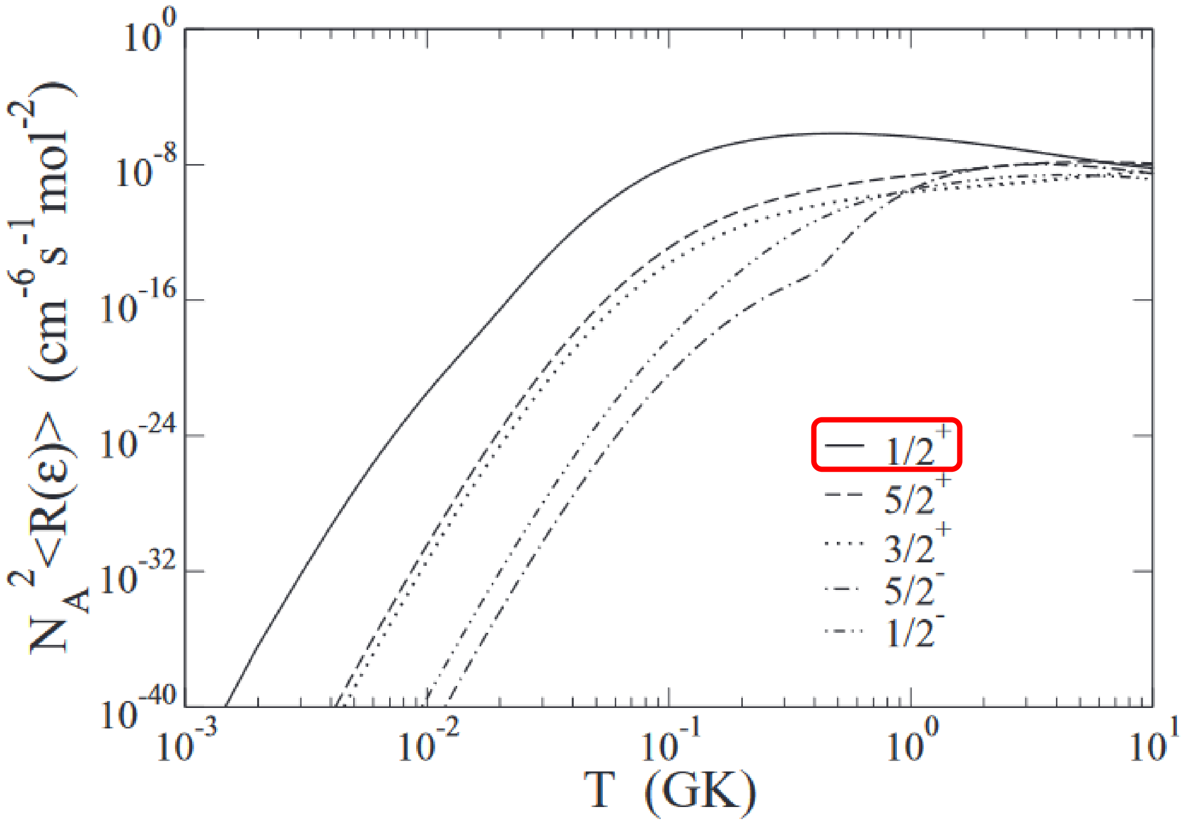
T-type Jacobi



$$< \mathbf{Rr}|\Phi_f > = \sum_{JLl} \frac{u^J_{Ll}(R,r)}{Rr} \{[Y_L(\hat{R})Y_l(\hat{r})]_J \otimes \chi_{1/2}\}_{J_{total},M_f}$$

T. Yamada and Y. Funaki, Phys. Rev. C 92, 034326 (2015)

The contributions from the different states to the reaction rate



E1 transition: $\lambda = 1$

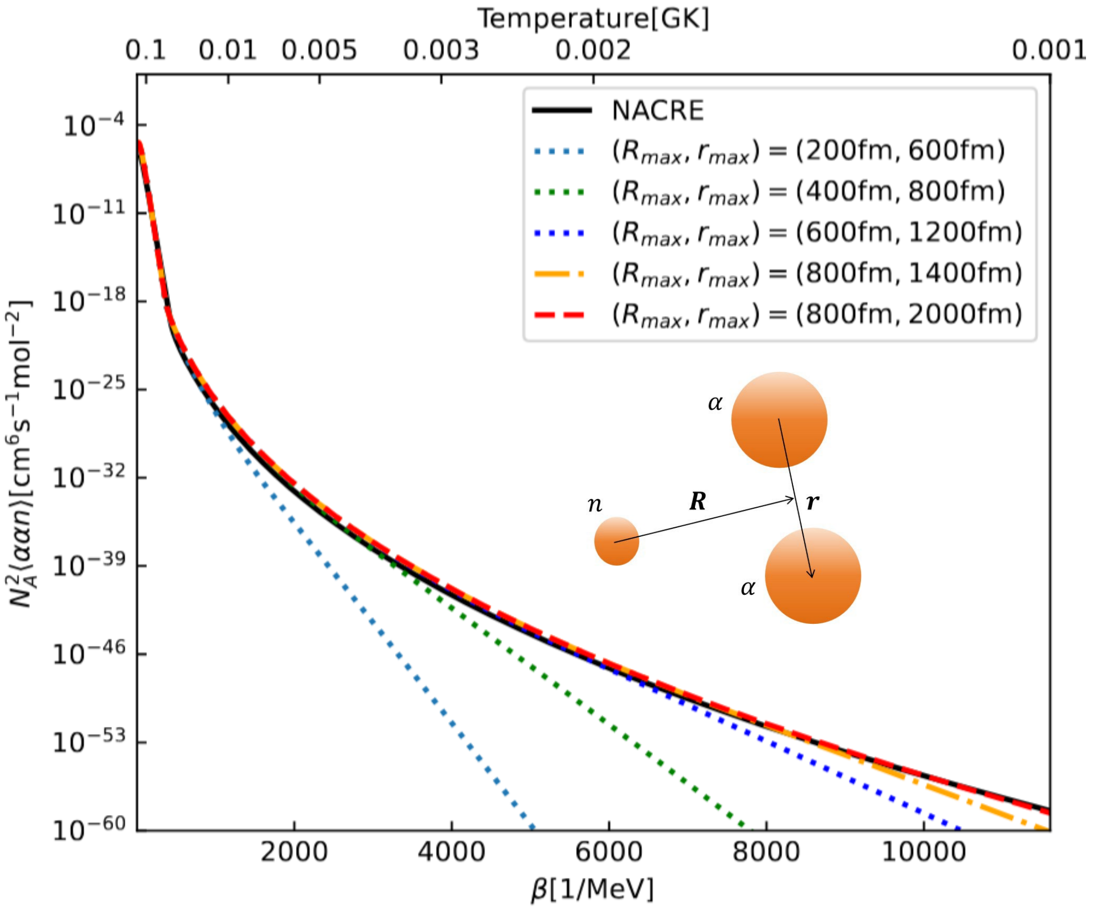
J. Casal, et al., Phys. Rev. C 90, 044304 (2014)

$\alpha\alpha n$ process

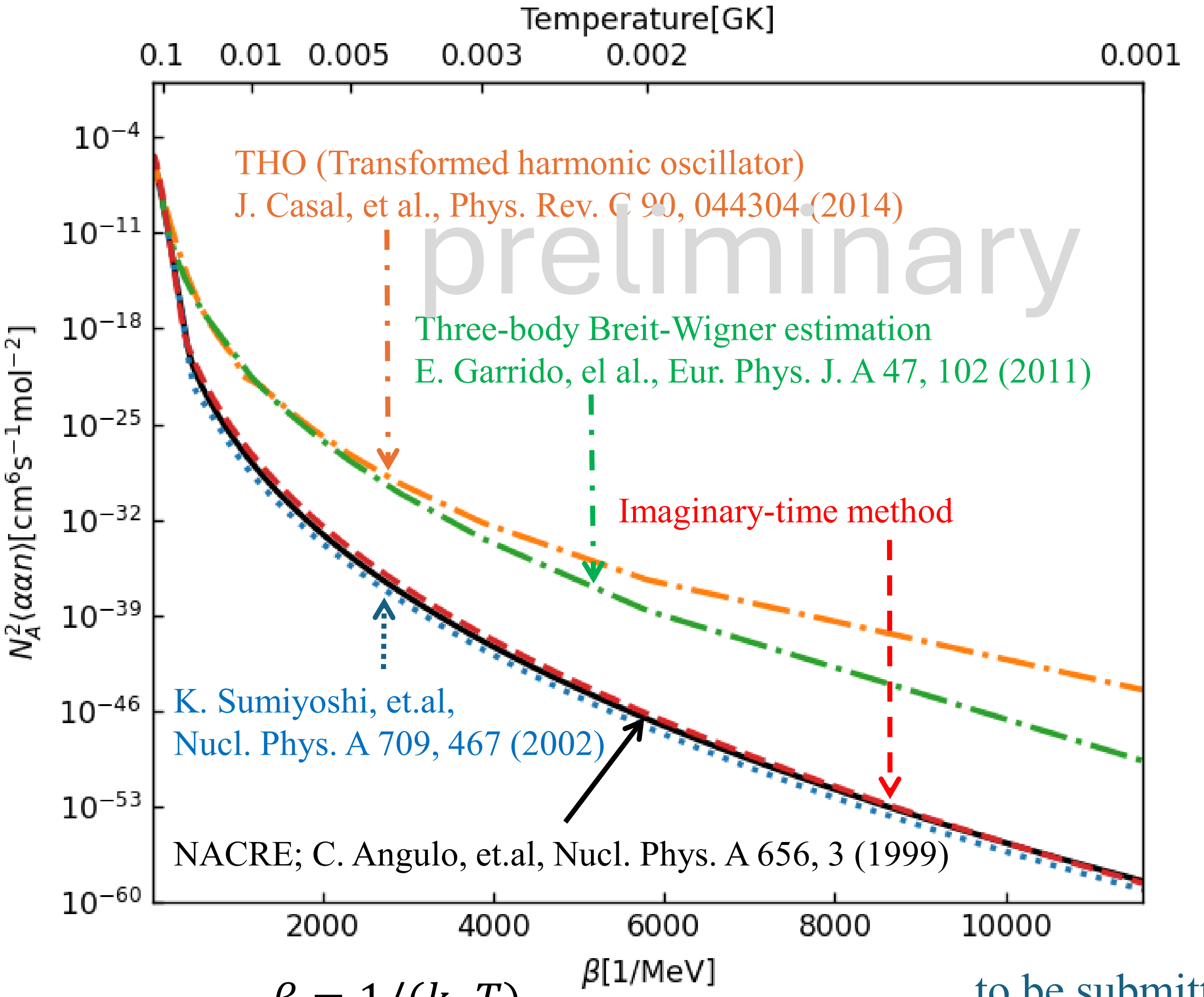
$$\boldsymbol{\alpha + \alpha + n \rightarrow {}^9\mathrm{Be}(3/2_1^-) + \gamma}$$

Convergence

Reaction rates under different truncations



Reaction rates compared with other methods

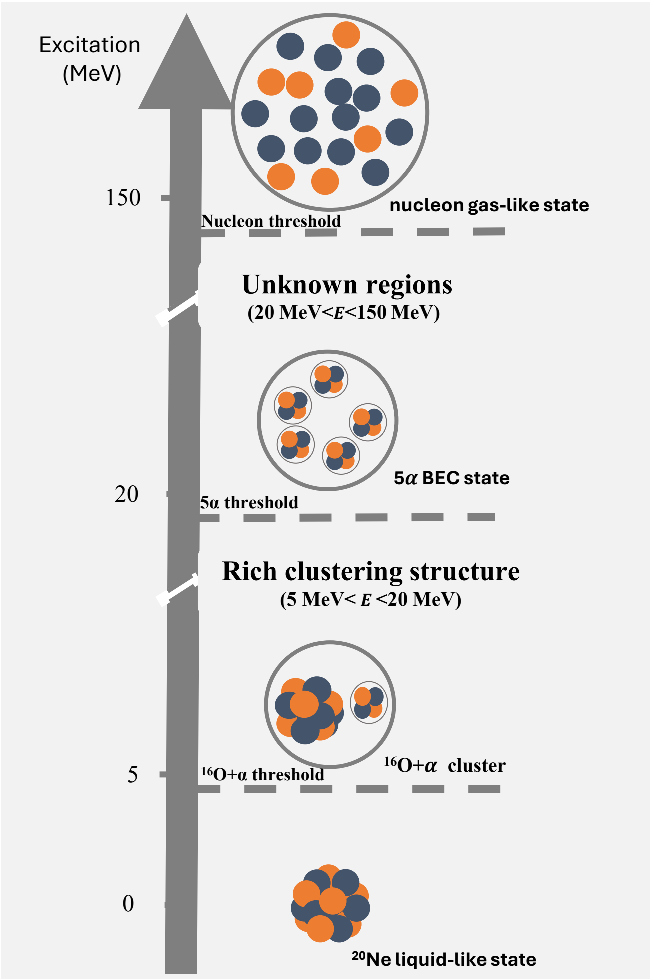


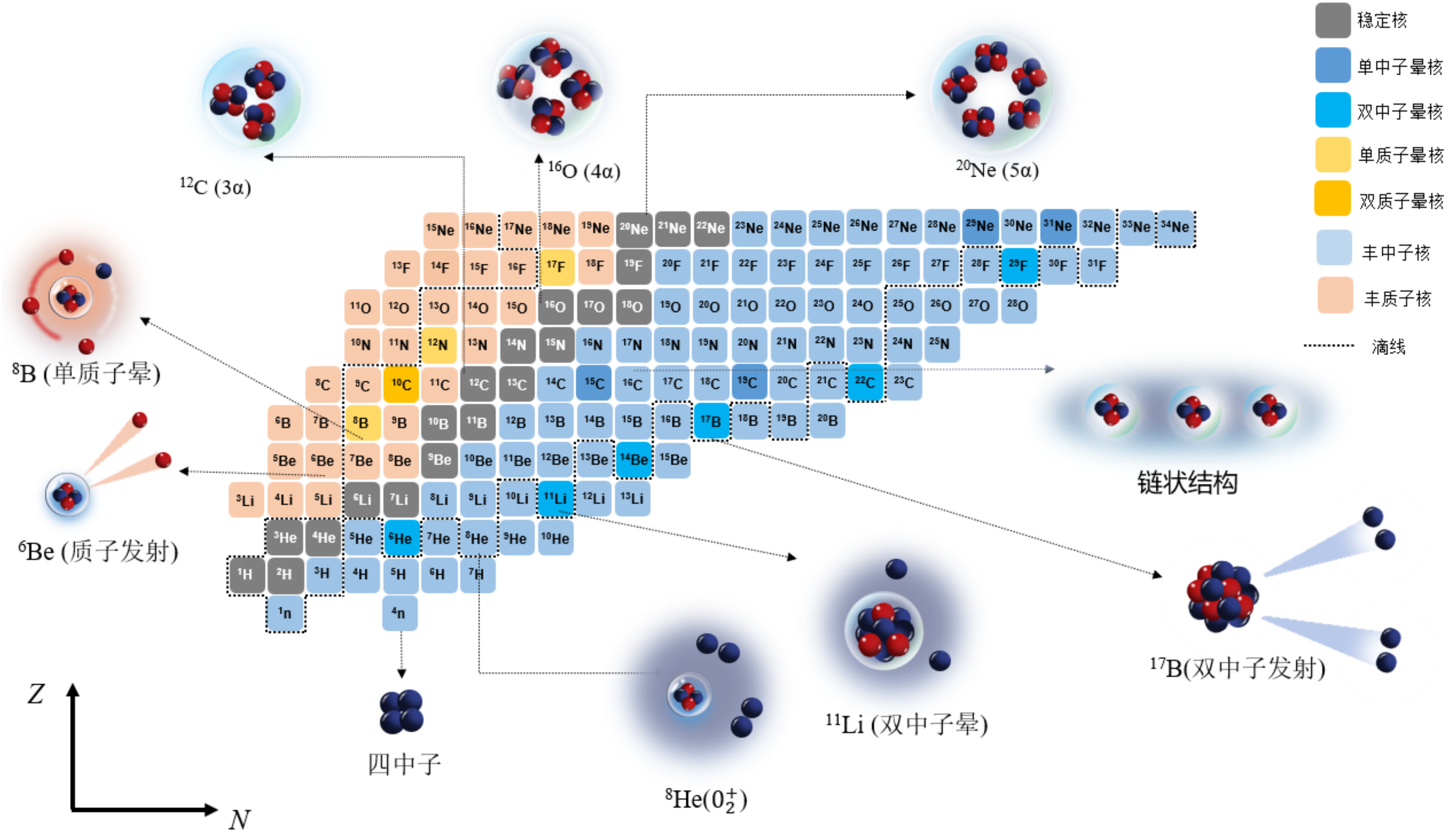
$\beta = 1/(k_B T)$

to be submitted

Summary and Prospect

rich clustering structure





Evolution of structure of ^{20}Ne

explore novel clustering structure of light nuclei

Thanks for my collaborators

and your attentions.