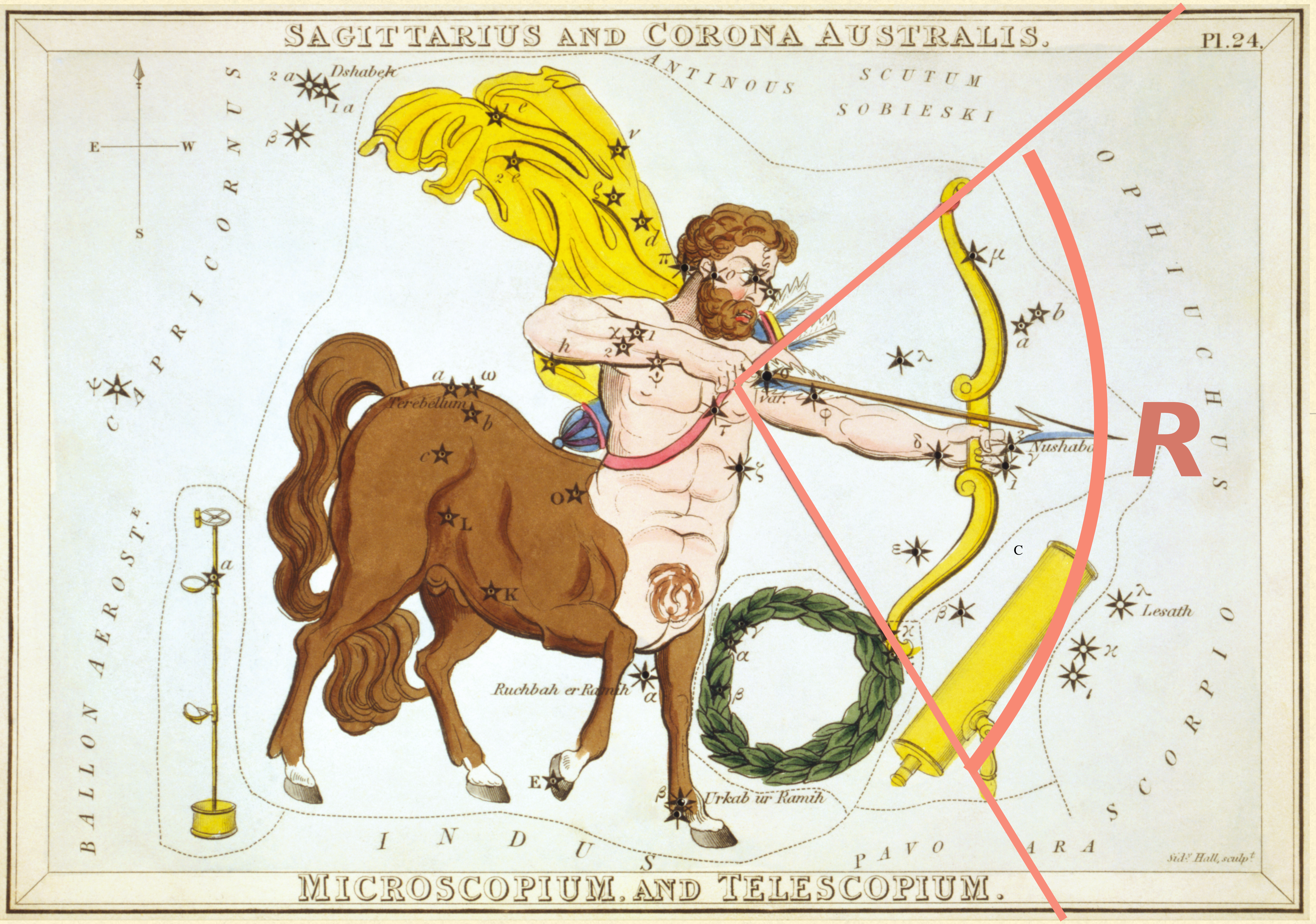
CENTAURIC EVENT SHAPES IN DIS



Urania's Mirror (1824)

Work in progress with:

Andrew Dotson (NMSU)

June-Haak Ee (LANL, MIT)

Yiannis Makris (LANL, INFN)

John Terry (LANL, ANL)

Christopher Lee

SCET 2026

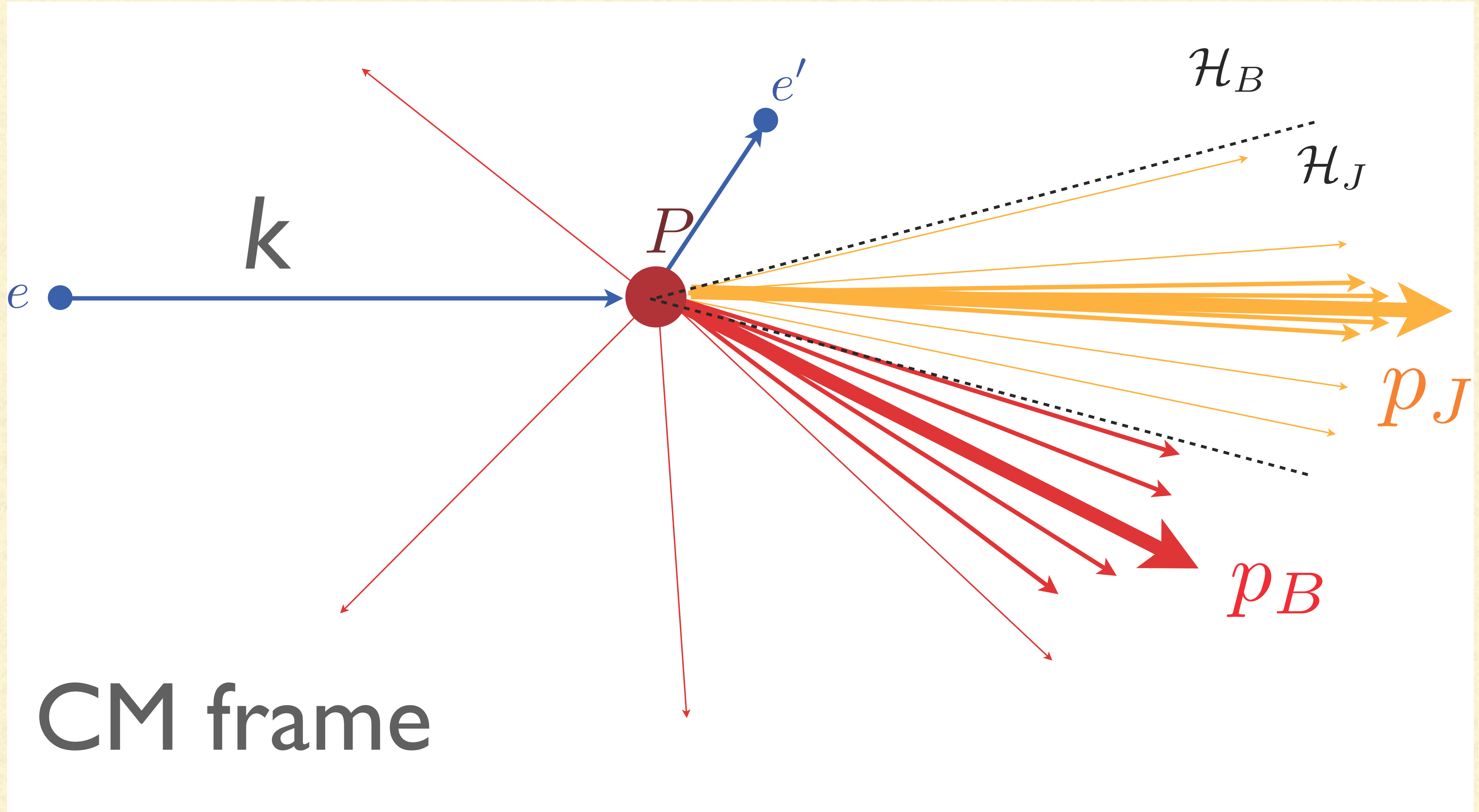
March 4, 2026

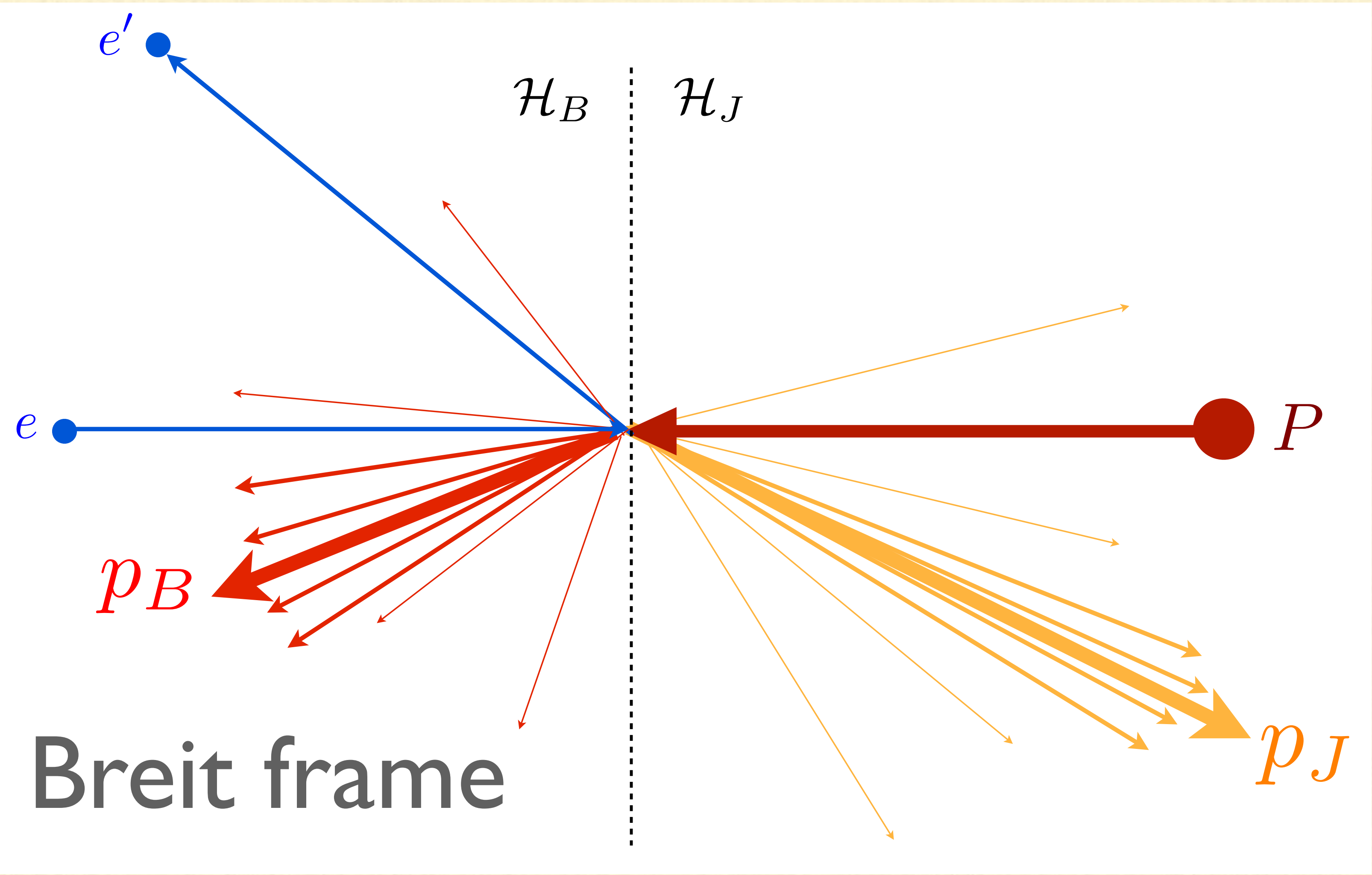
LA-UR-26-21579

GLOBAL EVENT SHAPES IN DIS

- 1-jettiness in DIS

$$\tau_1 = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B \cdot p_i, q_J \cdot p_i\}$$





- Choices of axes:

[1303.6952]

$\tau_1^a : \quad q_B = xP\,, \quad q_J = \text{jet axis } P_J$ *aligned with jet axis*

$\tau_1^b : \quad q_B = xP\,, \quad q_J = q + xP$ *hemispheres in Breit frame*

$\tau_1^c : \quad q_B = P\,, \quad q_J = k$ *hemispheres in CM frame*

PREDICTIONS FOR DIS THRUST

- DIS thrust
  [hep-ph/9912488]

$$\tau_1^b = \tau_Q = 1 - \frac{2}{Q} \sum_{i \in \mathcal{H}_J} p_z^i$$

- Factorization for τ_1^b:
  [1303.6952]

$$\frac{d\sigma}{dx\, dQ^2\, d\tau_1^b} = \frac{d\sigma_0}{dx\, dQ^2\, d\tau_1^b} \int dt_J\, dt_B\, dk_S\, \delta\left(\tau_1^b - \frac{t_J + t_B}{Q^2} - \frac{k_S}{Q}\right) \sum_q H_q(y, Q^2, \mu) S_{\rm hemi}(k_S, \mu)$$
$$\times \int d^2\mathbf{p}_\perp J_q(t_J - \mathbf{p}_\perp^2, \mu) \mathscr{B}_q(t_B, x, \mathbf{p}_\perp^2, \mu)$$

- Factorization for τ_1^a:
  [1204.5469]
  [1303.6952]

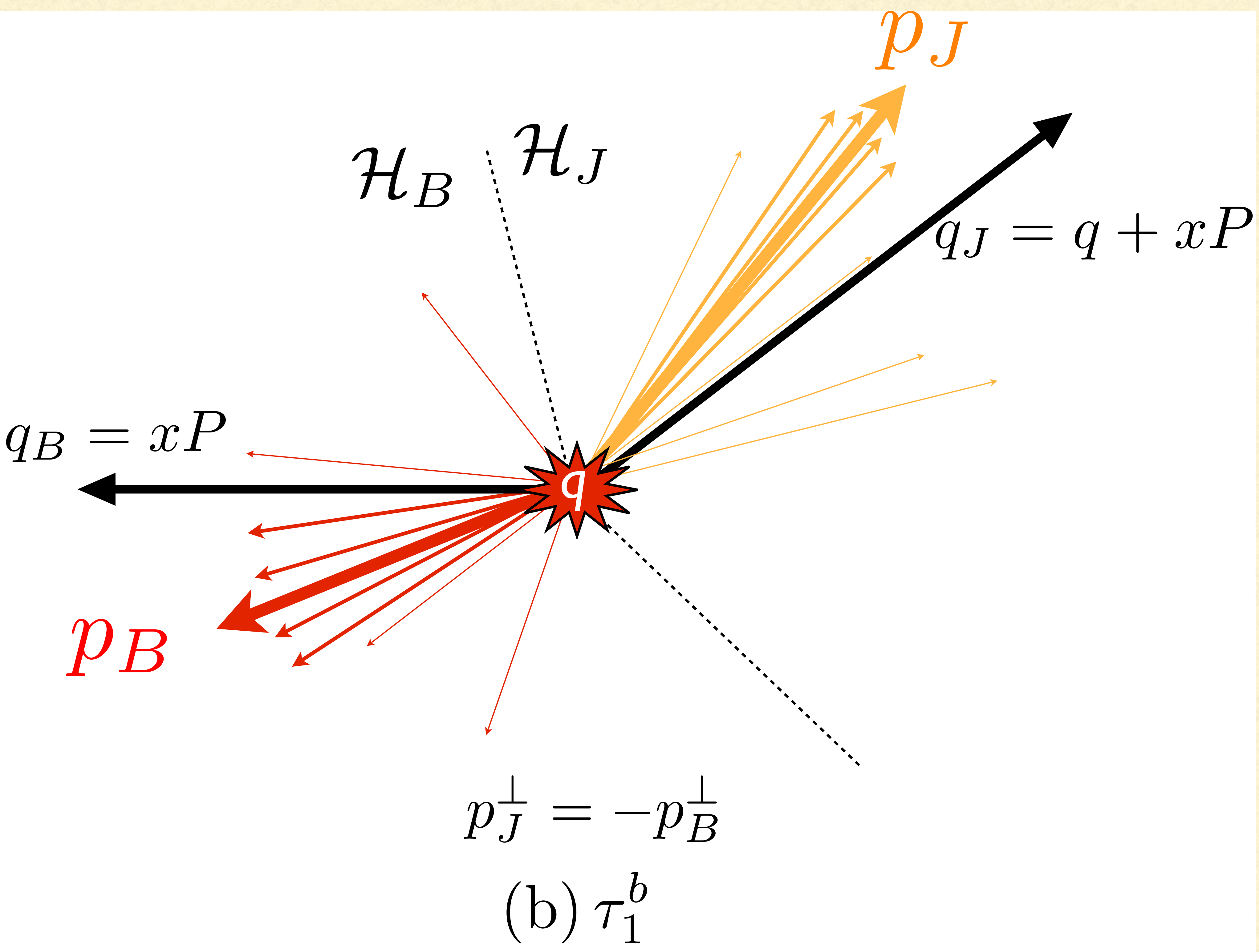
$$\frac{d\sigma}{dx\, dQ^2\, d\tau_1^a} = \frac{d\sigma_0}{dx\, dQ^2\, d\tau_1^a} \int dt_J\, dt_B\, dk_S\, \delta\left(\tau_1^b - \frac{t_J + t_B}{Q^2} - \frac{k_S}{Q}\right) \sum_q H_q(y, Q^2, \mu) S_{\rm hemi}(k_S, \mu)$$
$$\times J_q(t_J, \mu) B_q(t_B, x, , \mu)$$

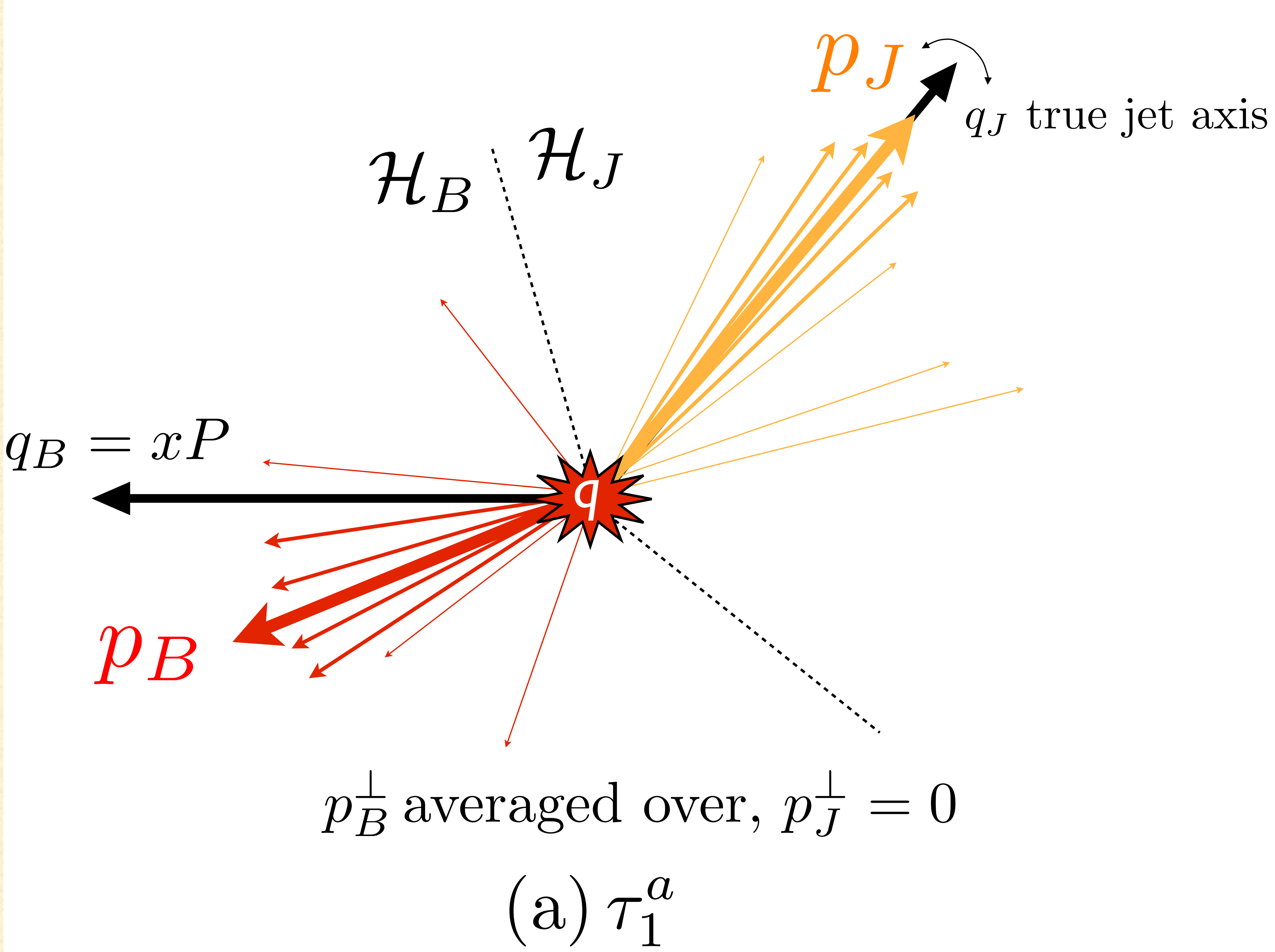
PREDICTIONS FOR DIS THRUST

- DIS thrust
  [hep-ph/9912488]

$$\tau_1^b = \tau_Q = 1 - \frac{2}{Q} \sum_{i \in \mathscr{H}_J} p_z^i$$

[1303.6952]

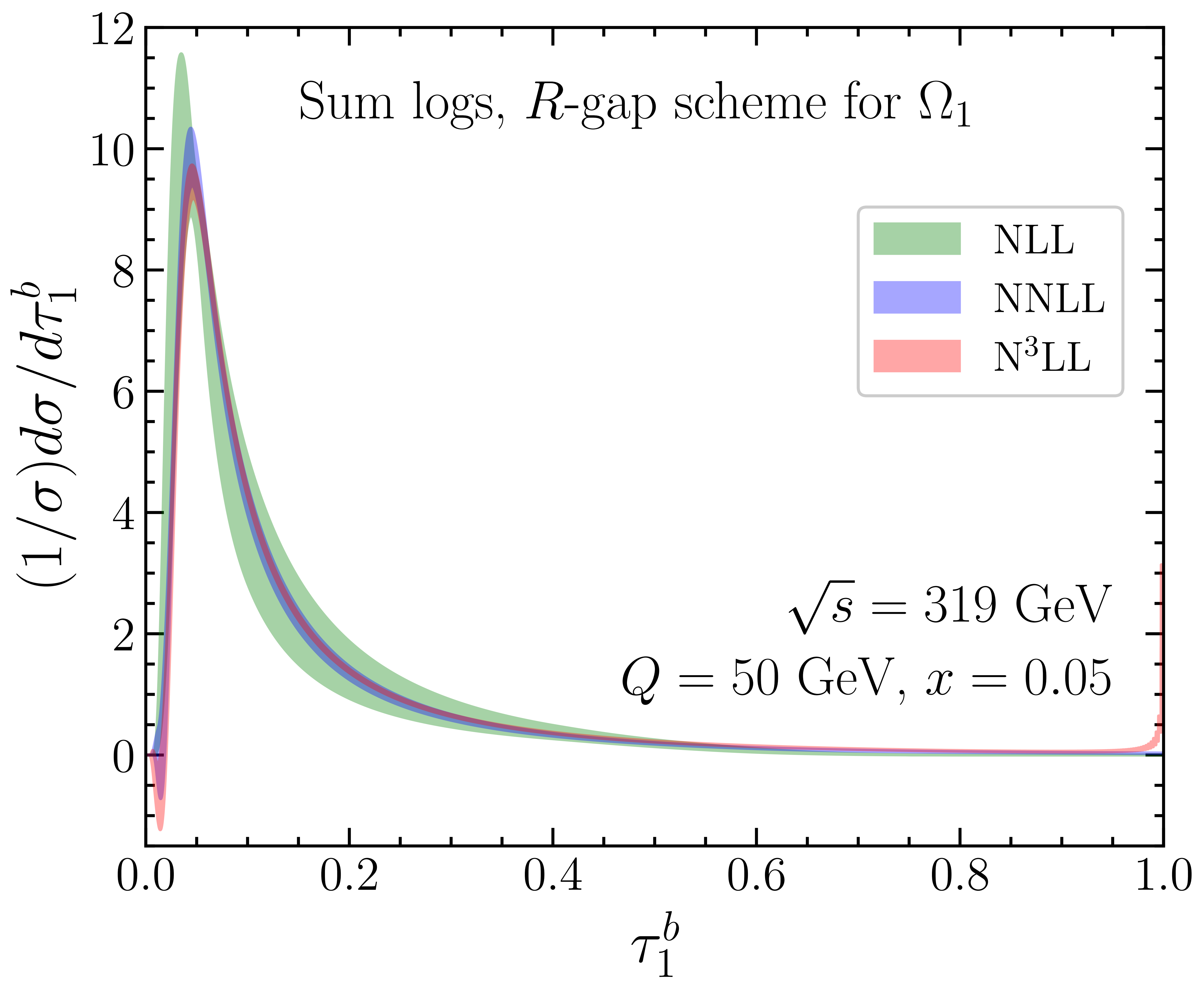


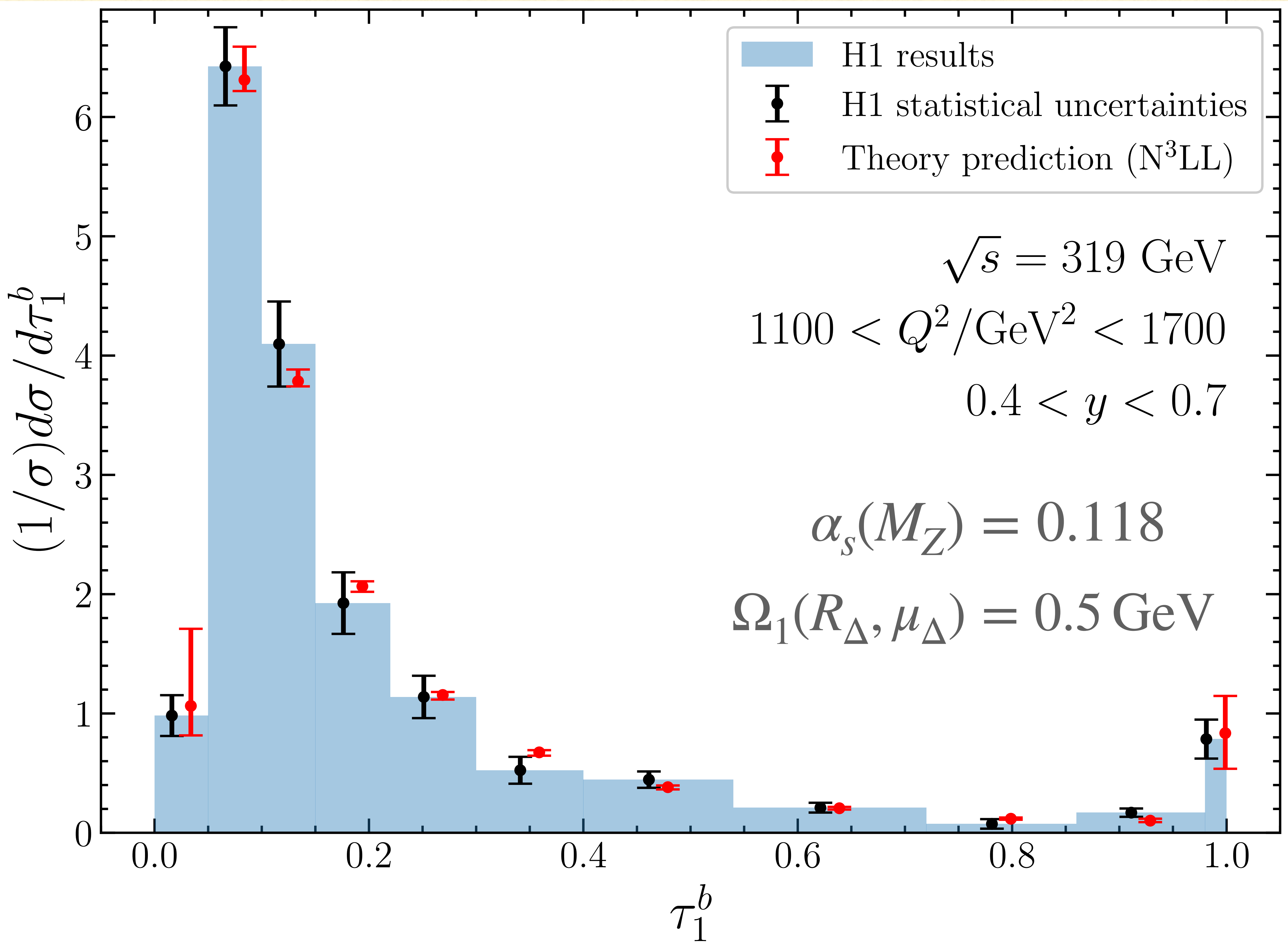


PREDICTIONS FOR DIS THRUST

[2504.05234]

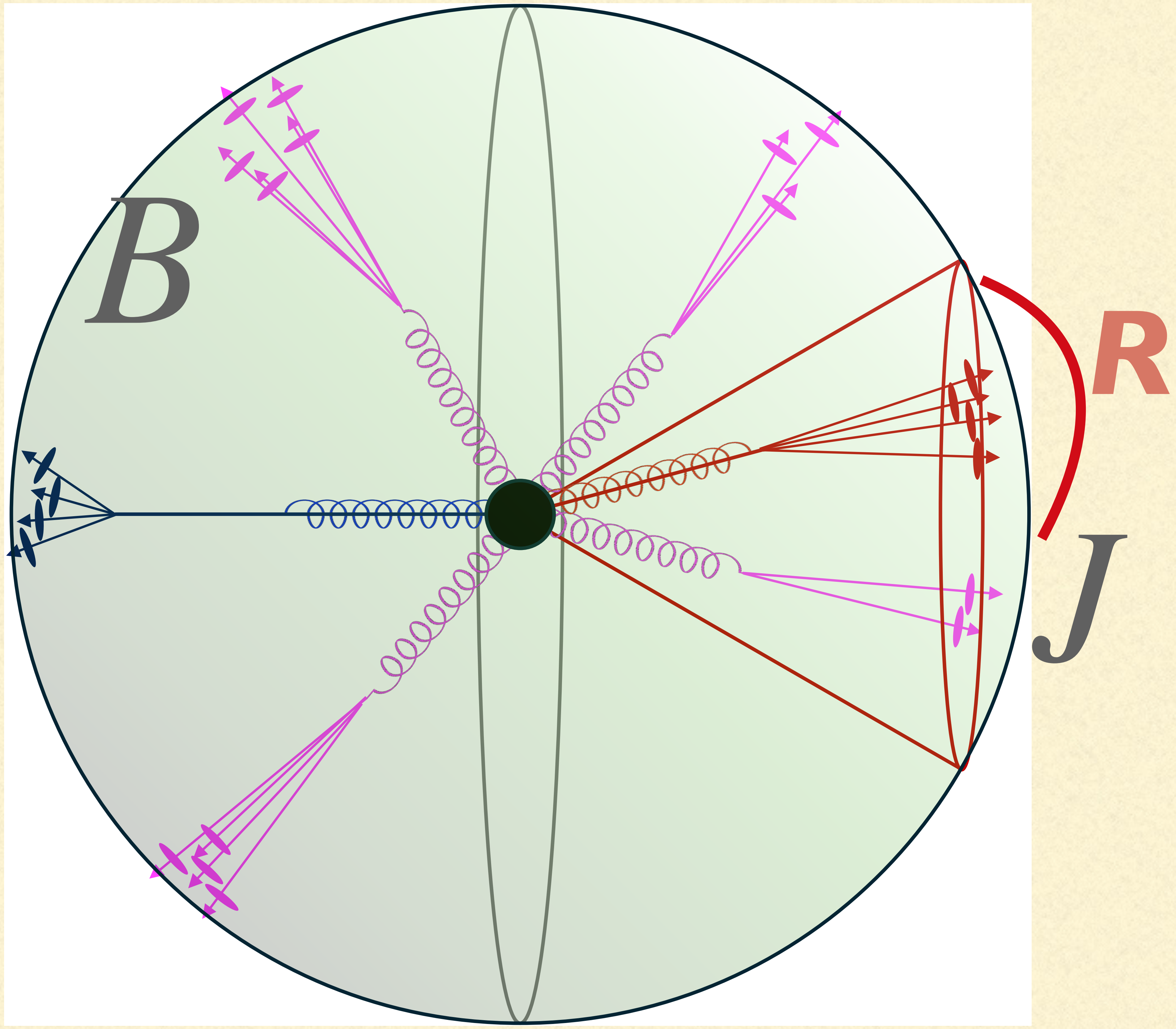
- Resummed, matched predictions with nonperturbative shape function





GENERALIZED DIS 1-JETTINESS WITH JET REGIONS

- One way: Change by hand the definition of jet & beam regions to be determined by jet algorithm



$$\tau_1^J = \frac{2}{Q^2}\left\{ \sum_{i\in J} q_J \cdot p_i + \sum_{i\notin J} q_B \cdot p_i \right\}$$

- cf. fixed-order calculation of τ_1^a with jet axis determined by algorithms [2202.08040]

- Our motivation: explore scaling of nonperturbative corrections with size of jet region R

k_T JET ALGORITHMS FOR DIS

[2006.10751]

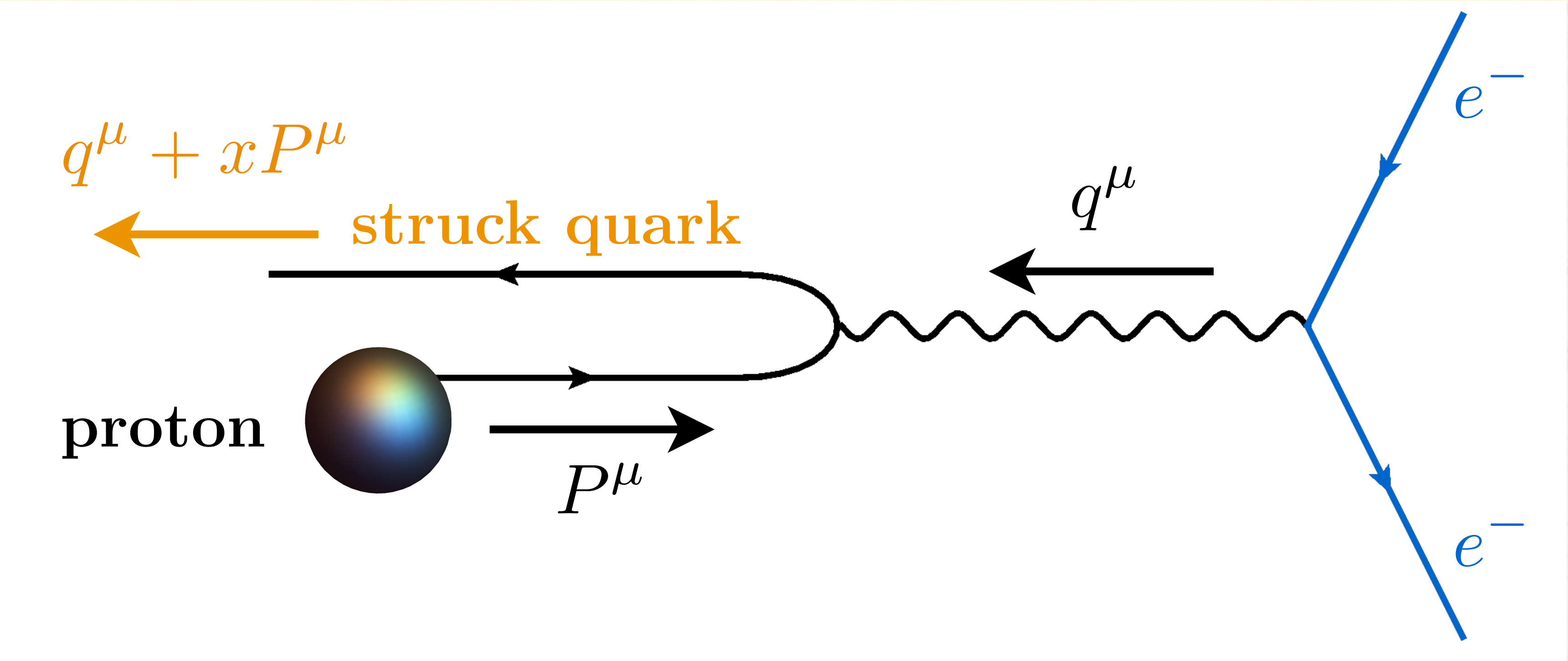
- k_T class of jet clustering algorithms: $d_{ij} = \min(p_{Ti}^{2p}, p_{Tk}^{2p}) f_{ij}(R)\,, \quad d_{iB} = p_{Ti}^{2p}$
  - *pp* longitudinally invariant [LI] measure:

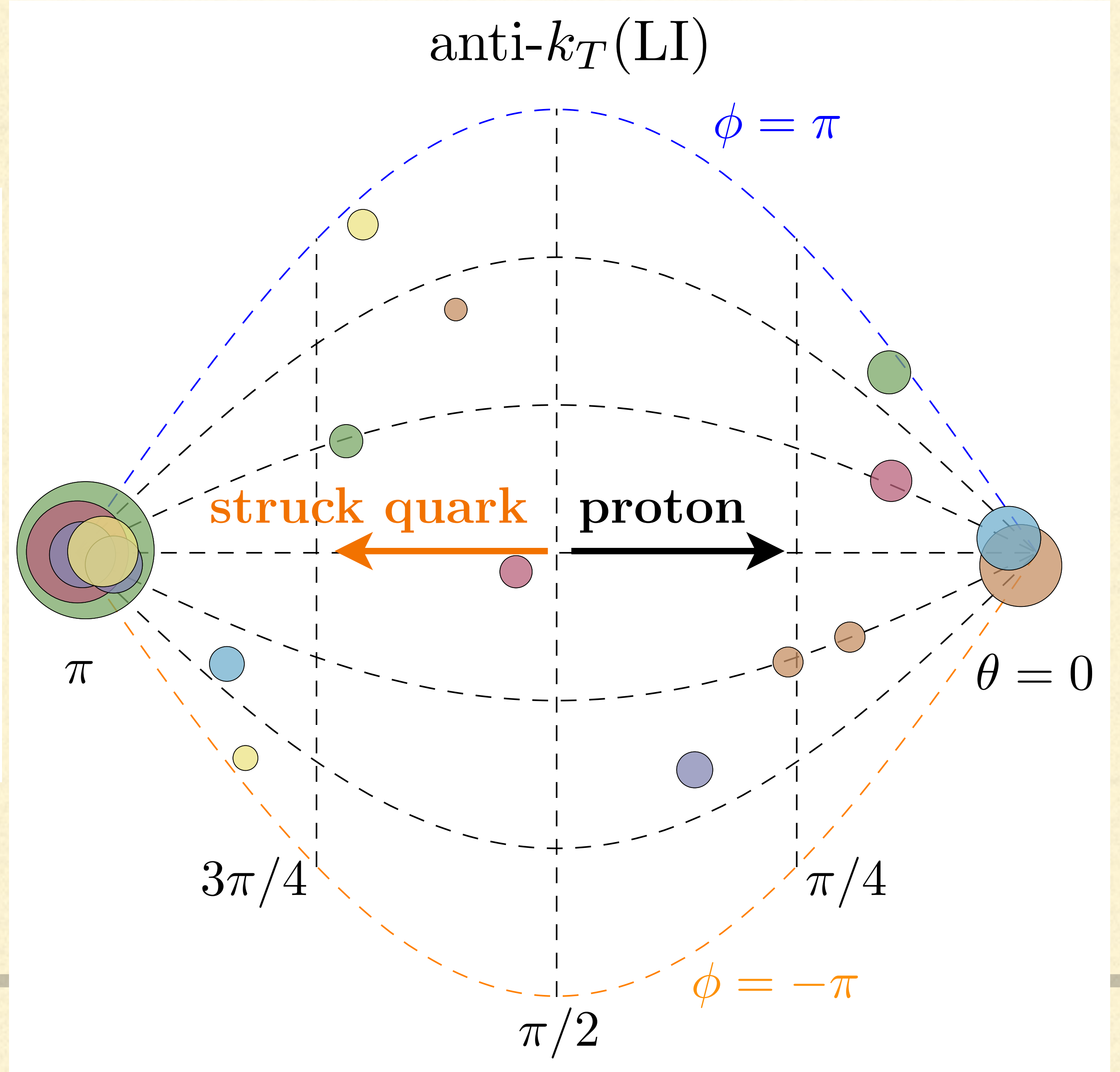
$$f_{ij}^{LI}(R) = \frac{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}{R^2}$$

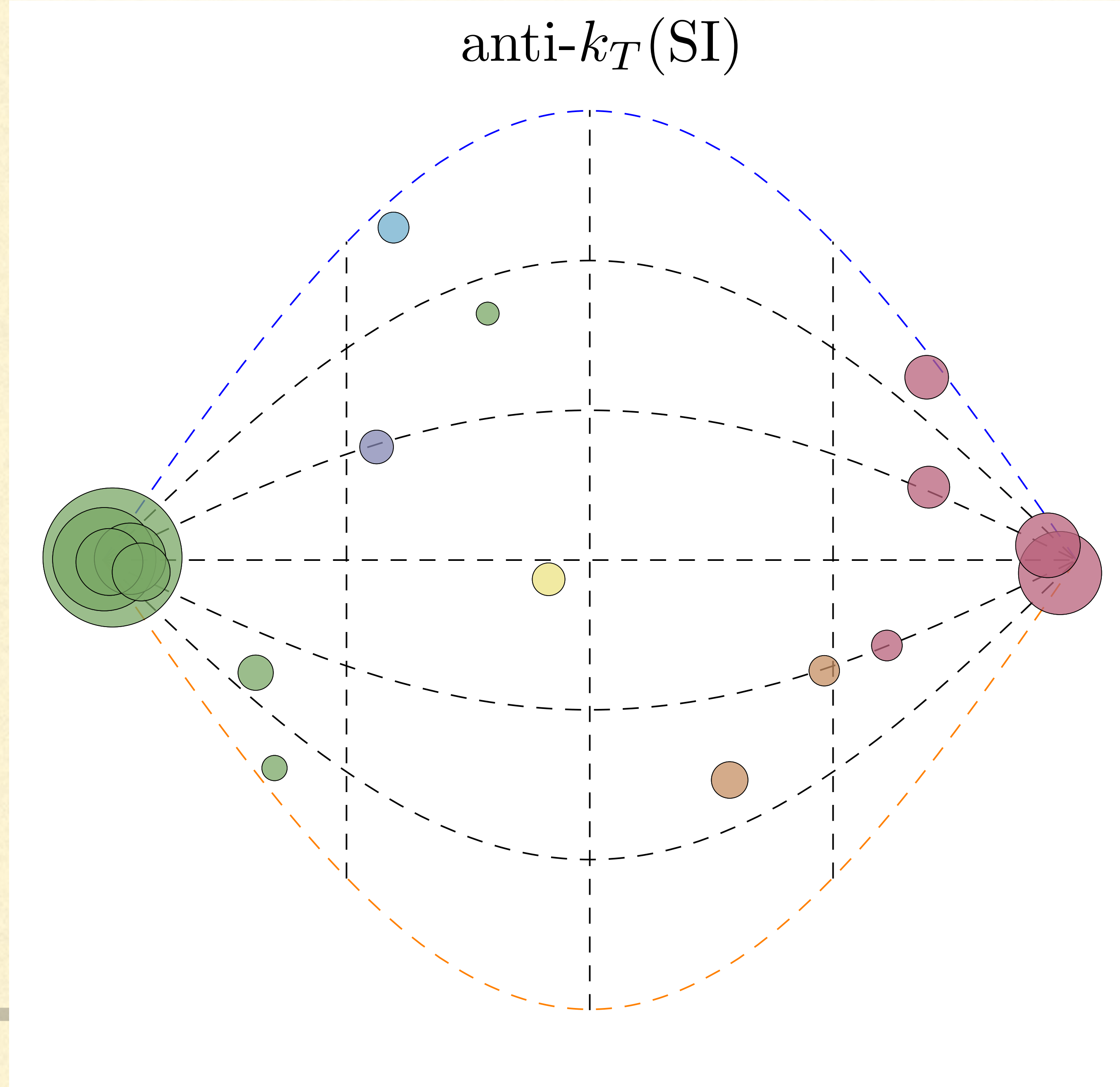
  - Cannot reconstruct jet in struck quark direction $y \to -\infty$
  - e+e- spherically invariant [SI] measure:

$$f_{ij}^{SI}(R) = \frac{1 - \cos\theta_{ij}}{1 - \cos R}$$

  - Not invariant along boosts



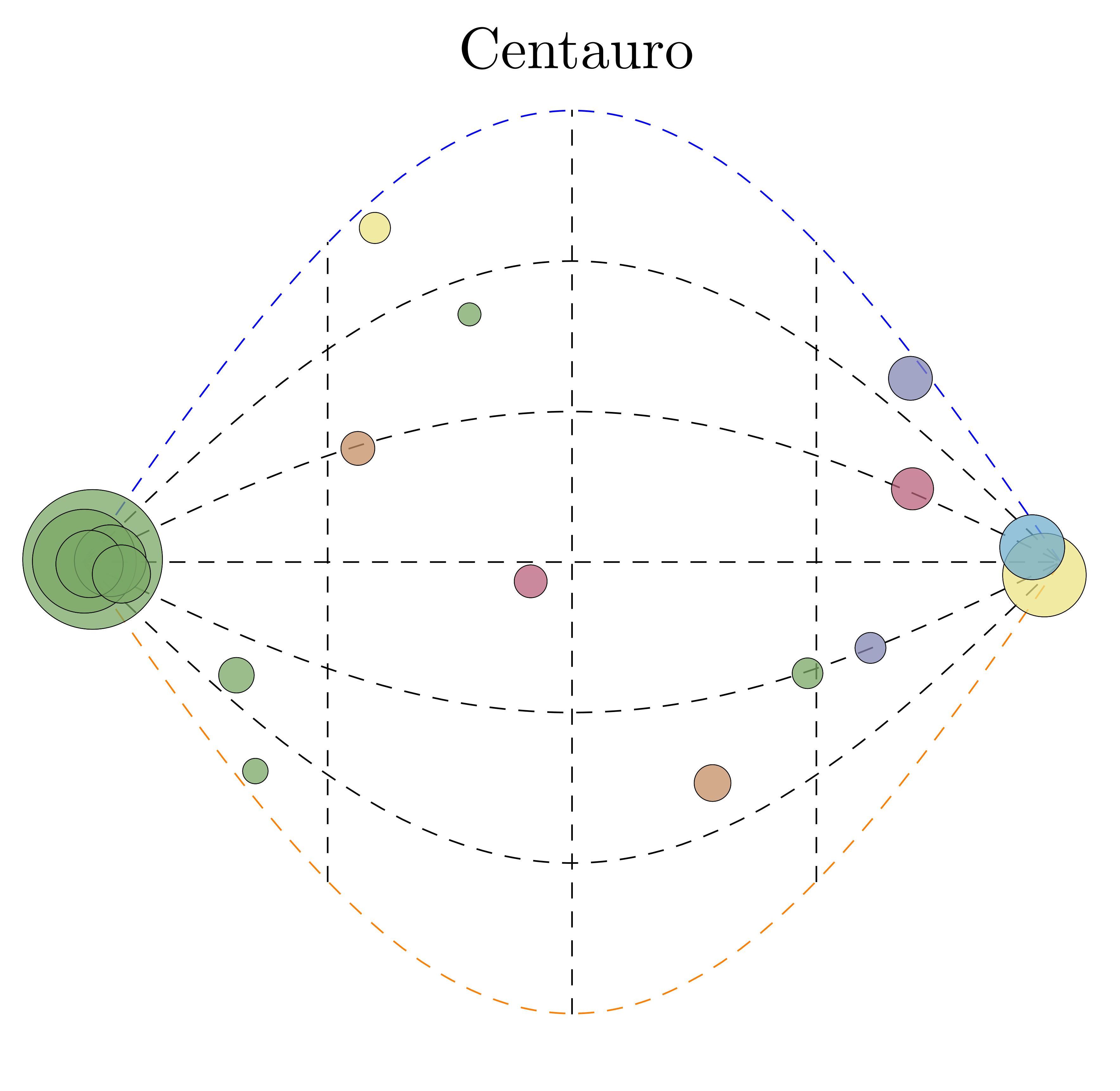




CENTAURO JET ALGORITHMS

[2006.10751]

- Suited for DIS: Measure that mimics *pp* measure in *p* direction, e^+e^- measure in the e^- direction, maintains boost invariance



$$f_{ij}^{\text{Cent}}(R) = \frac{(\bar{\eta}_i - \bar{\eta}_j)^2 + 2\bar{\eta}_i\bar{\eta}_j(1 - \cos\Delta\phi_{ij})}{R^2}$$

$$\bar{\eta}_i \equiv \frac{2p_\perp^i}{\bar{n}\cdot p_i}$$

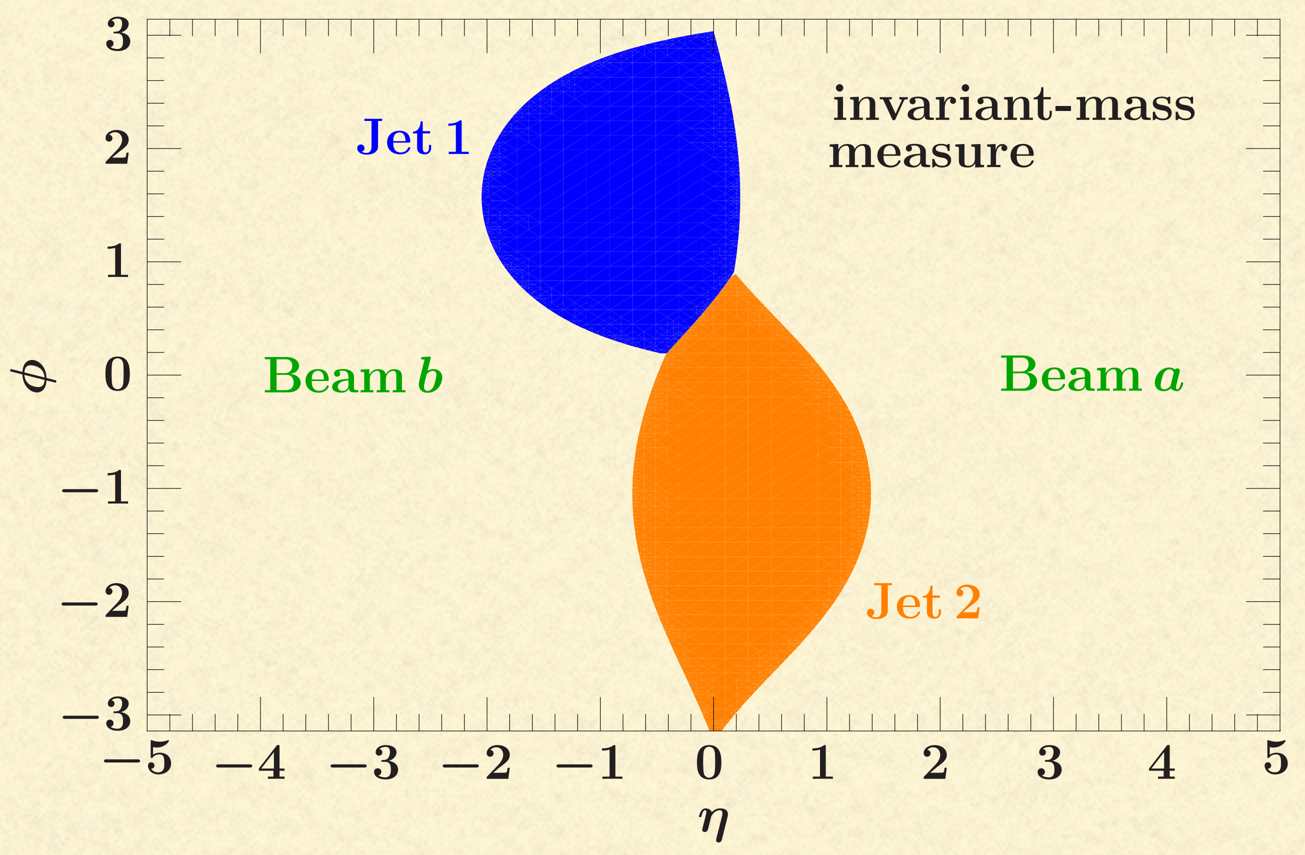
- In backward limit:

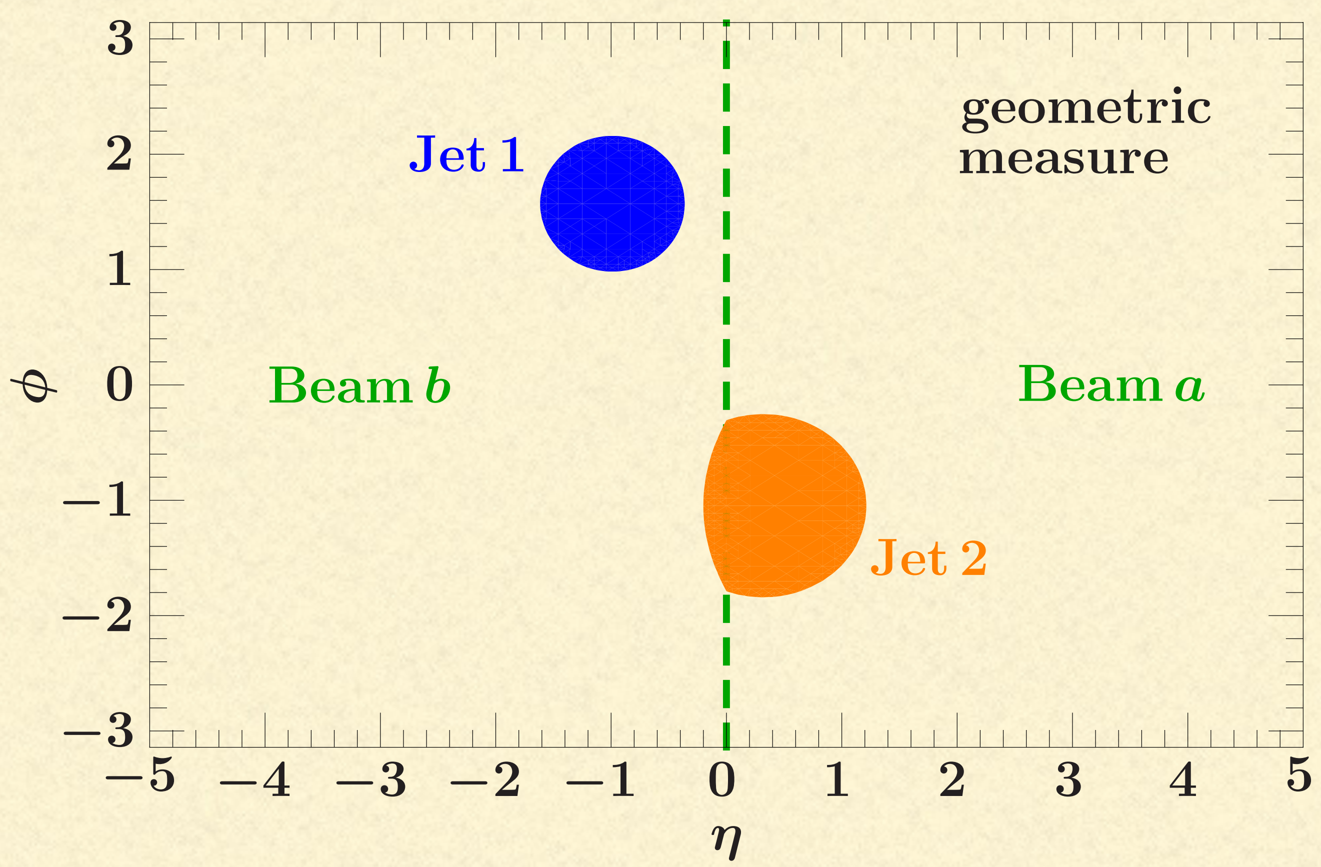
$$1 - \cos\theta_{ij} \simeq \frac{1}{2}(\theta_i - \theta_j)^2 + \bar{\theta}_i\bar{\theta}_j(1 - \cos\Delta\phi_{ij})$$

$$\bar{\theta} \equiv \pi - \theta$$

JET REGIONS BY REWEIGHTING

- Second way, reweight 1-jettiness definition to make jettiness regions mimic jet regions:



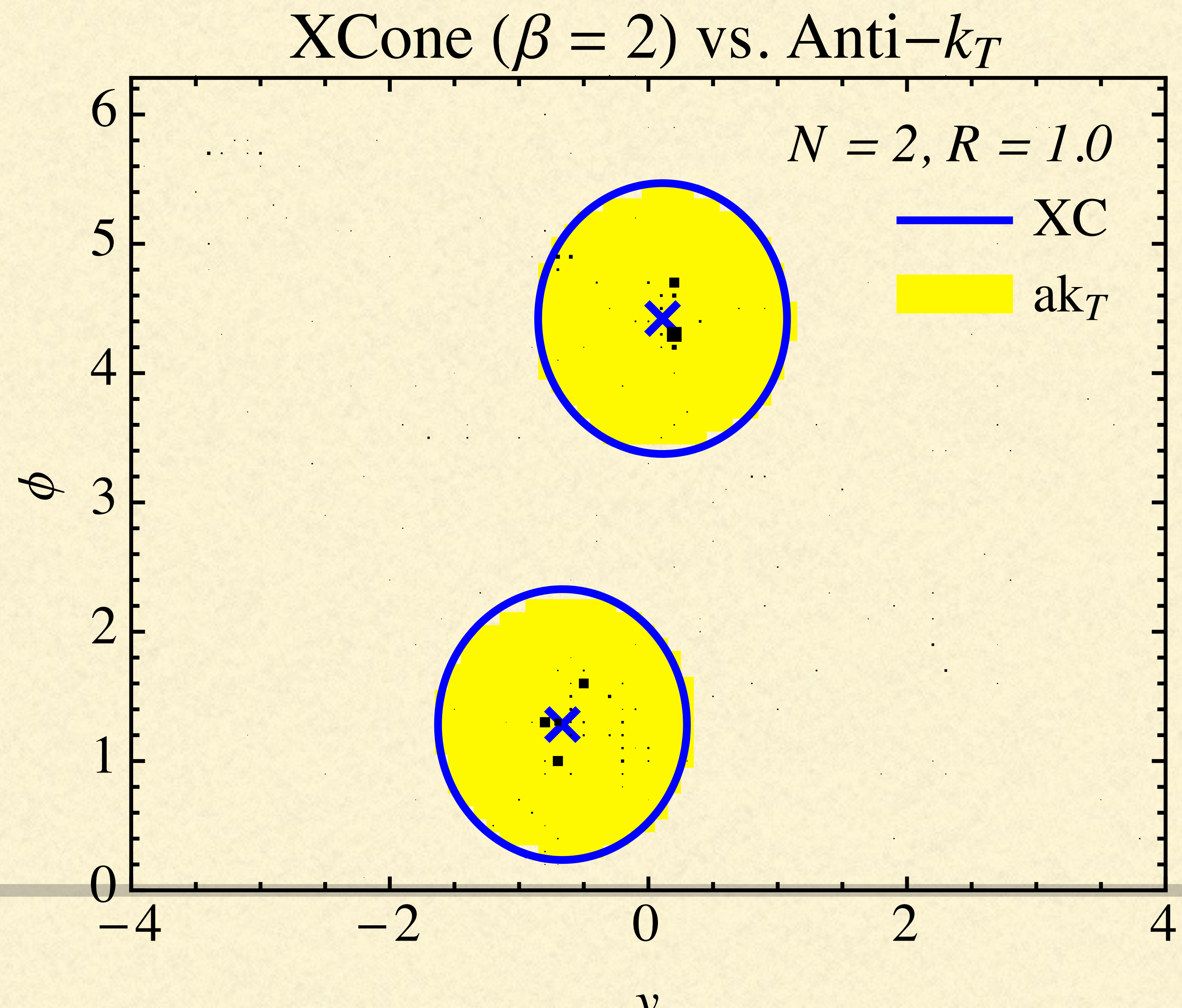


- Has been explored:
    - Soft functions for generalized *N*-jettiness: [1102.4344] [1704.08262]
    - XCone jet algorithms: [1508.01516]

$$\mathcal{T}_N = \sum_i \min\{\rho_{\text{jet}}(p_i, n_1), \ldots, \rho_{\text{jet}}(p_i, n_N), \rho_{\text{beam}}(p_i)\}$$

- XCone default measure:

$$\rho_{\text{jet}}(p_i, n_A) = \frac{2\cosh y_A}{R^2} n_A \cdot p_i\,, \quad \rho_{\text{beam}}(p_i) = p_{Ti}$$



CENTAURIC 1-JETTINESS

- 1-jettiness weights:

$$\tau_1^J = \frac{2}{Q^2}\left\{\sum_{i\in J} q_J \cdot p_i + \sum_{i\notin J} q_B \cdot p_i\right\} \qquad q_B^\mu = \omega_B \frac{n_B^\mu}{2}, \quad q_J^\mu = \omega_J \frac{n_J^\mu}{2}$$

$$= \sum_{i\in X} \min\left\{\frac{n_B \cdot p_i}{Q_B}, \frac{n_J \cdot p_i}{Q_J}\right\} \qquad Q_{B,J} = \frac{Q^2}{\omega_{B,J}}$$

for some $Q_{B,J}$?

- Condition for Centauro jet boundary:

$$\frac{\omega_J}{\omega_B} = \frac{Q_B}{Q_J} = \frac{4}{R^2 \cos^2(\theta_J/2)}$$

HYBRID CENTAURIC 1-JETTINESS

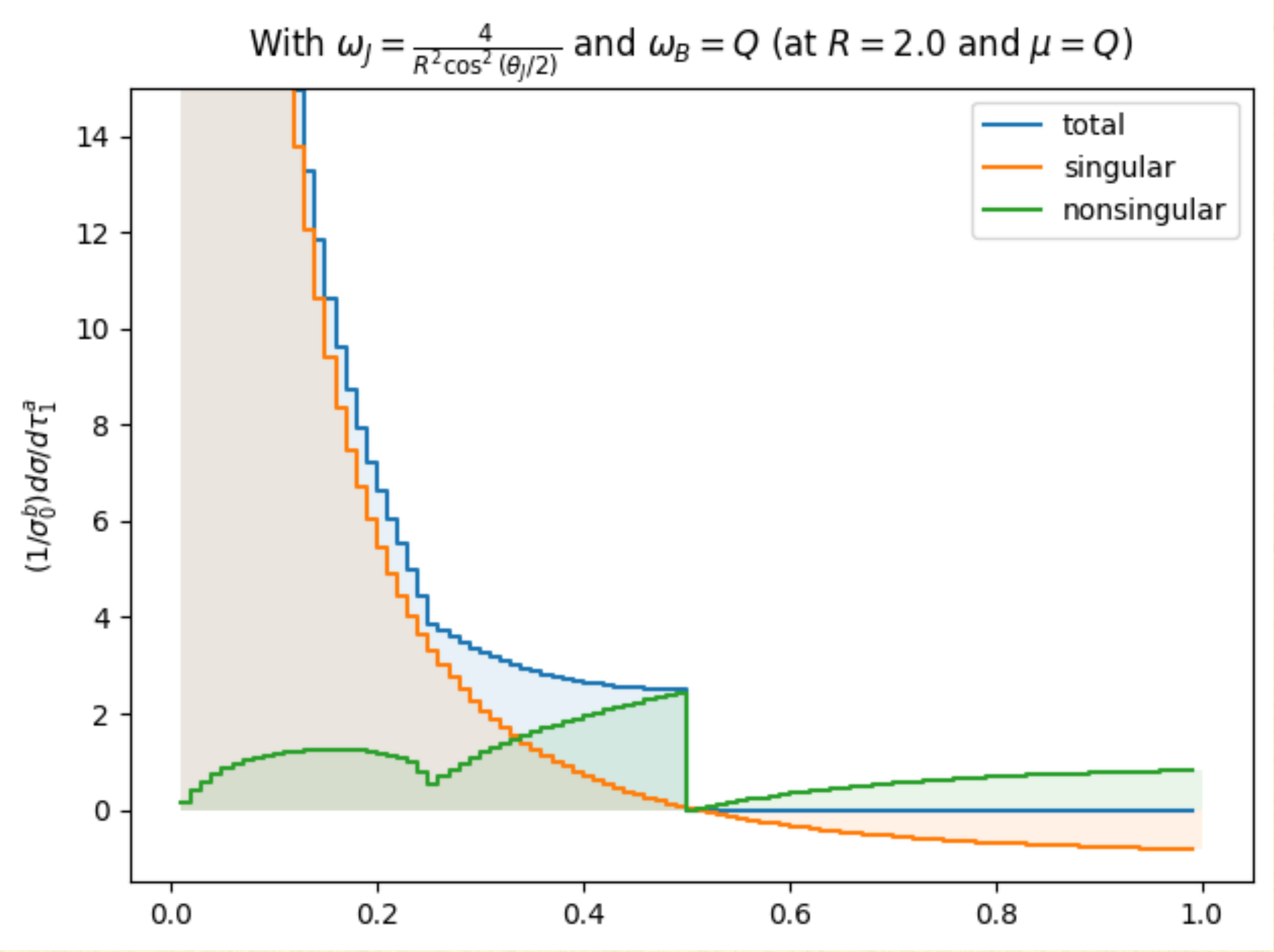
- I: For jet selection, use the weights:
  (assign particles to jets minimizing this τ_1)

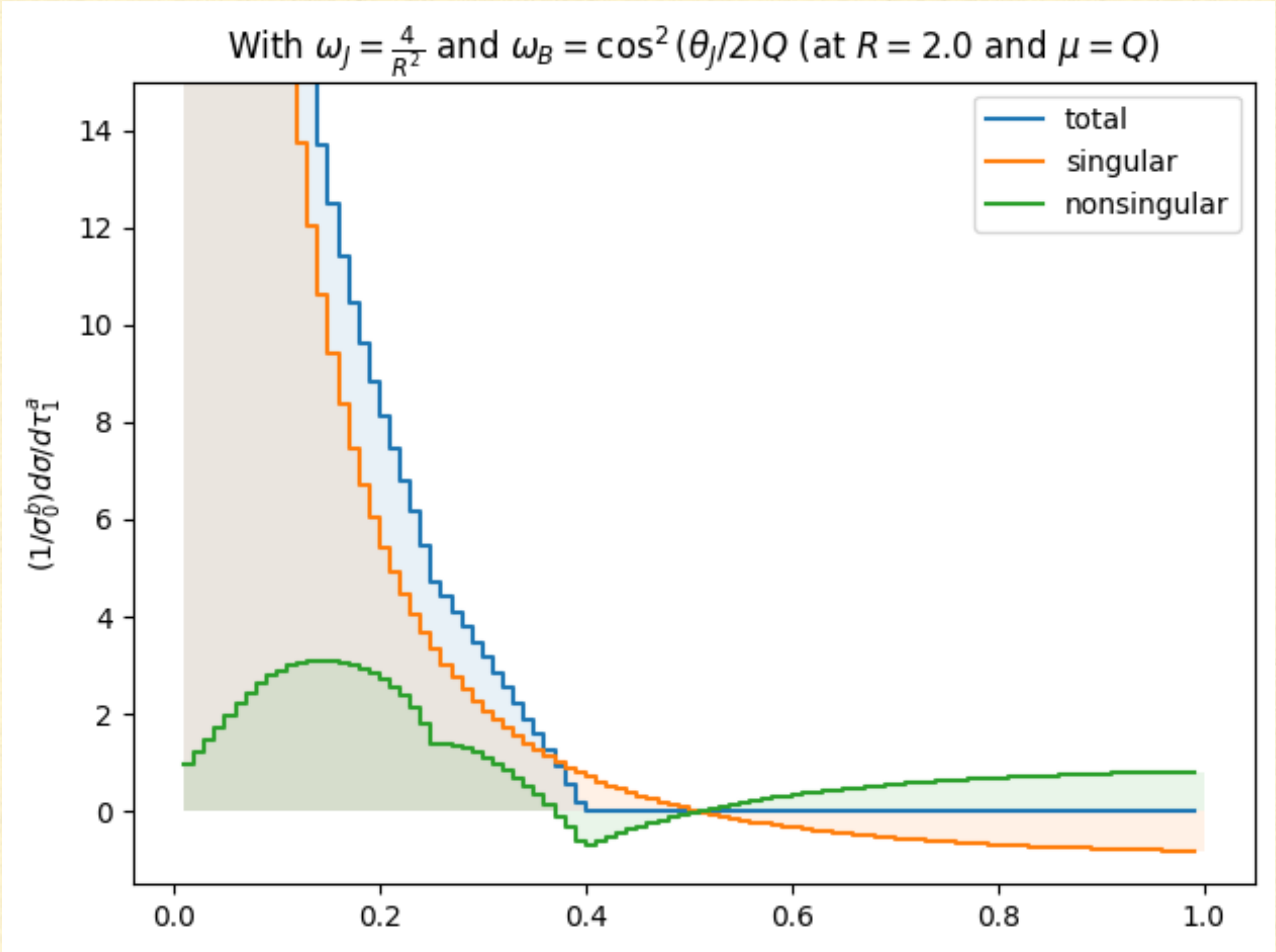
$$Q_B = Q \,,\, Q_J = \frac{QR^2 \cos^2(\theta_J/2)}{4}$$

- II: For computing the actual value of the 1-jettiness:

$$Q_B = \frac{Q}{\cos^2(\theta_J/2)} \,,\, Q_J = \frac{QR^2}{4}$$

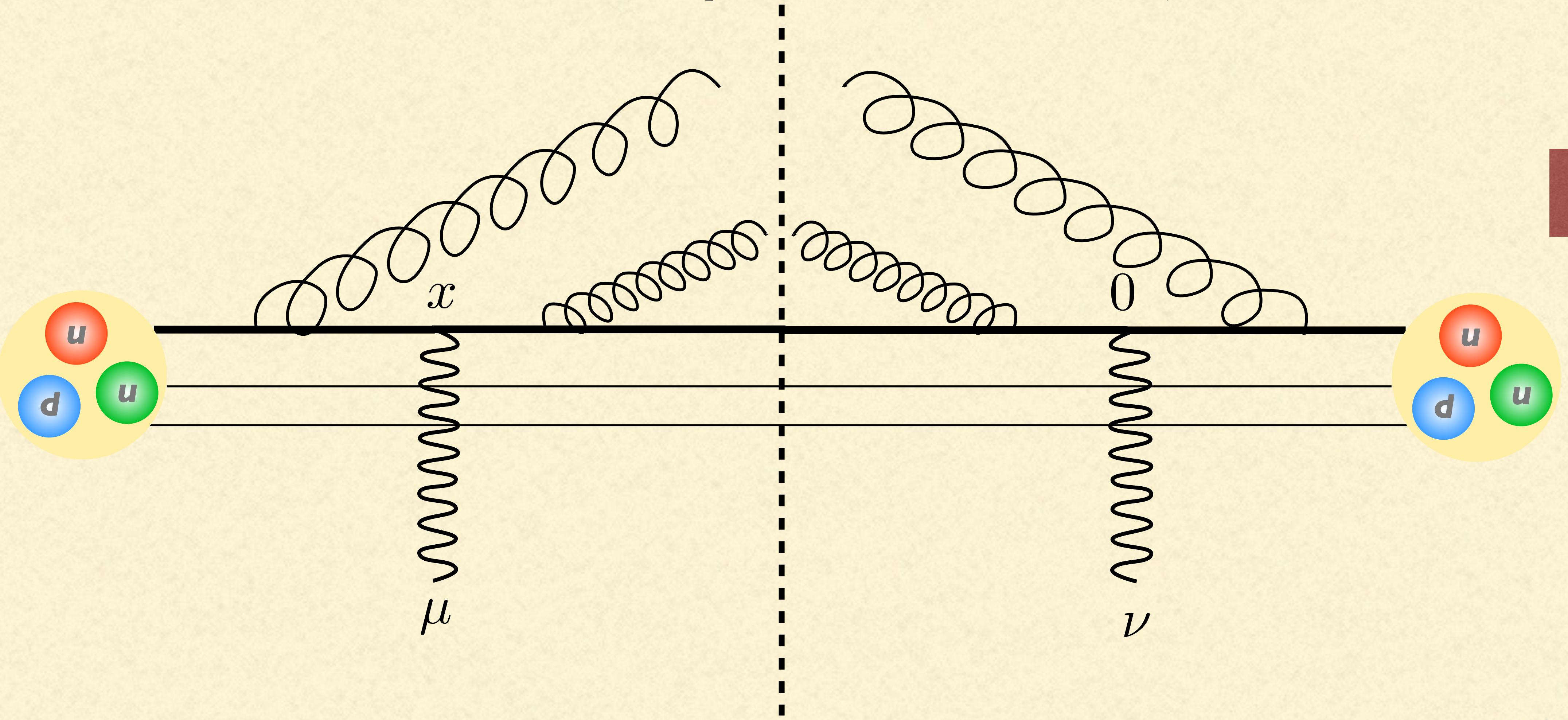
- Doesn't matter for small τ,
  but we wanted to make nice plots for you for all τ

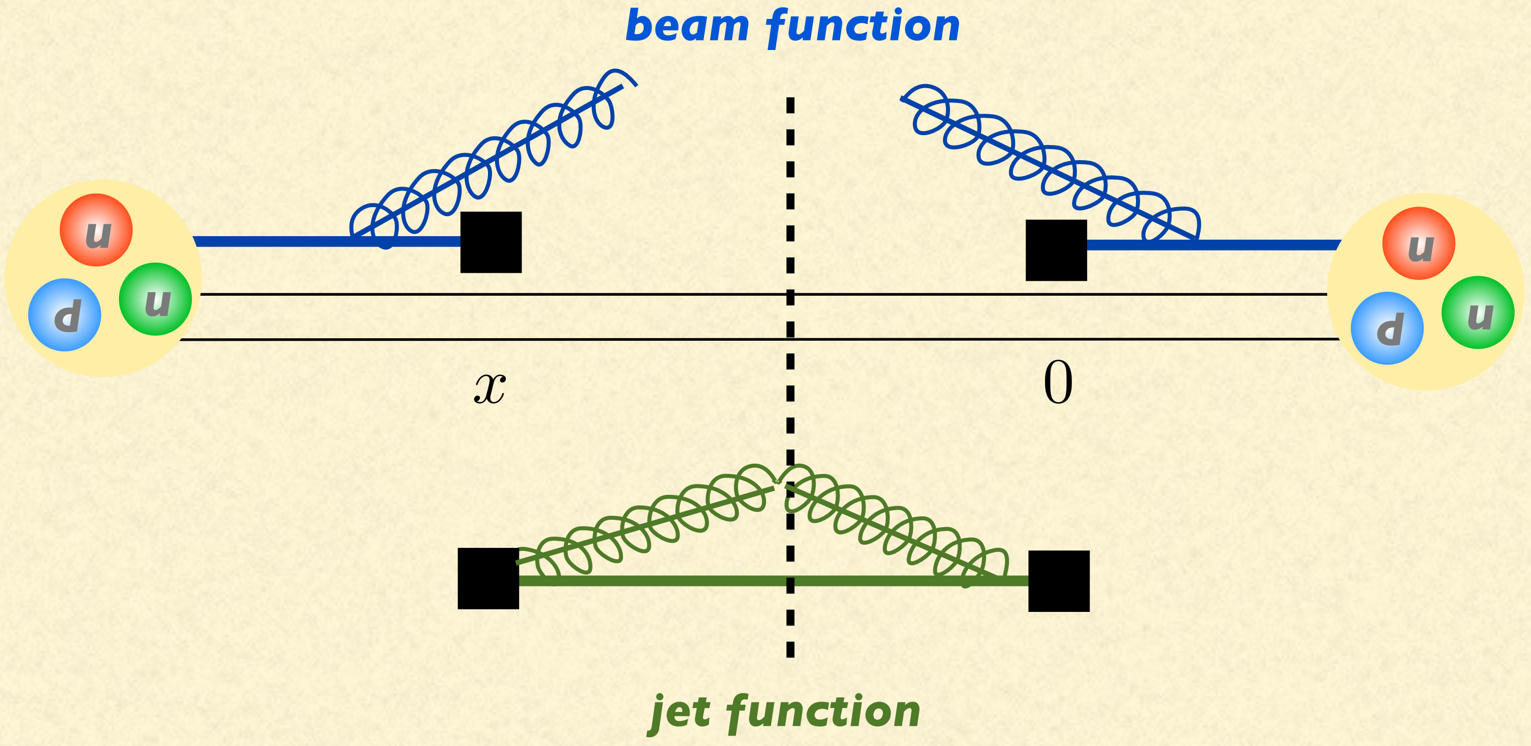


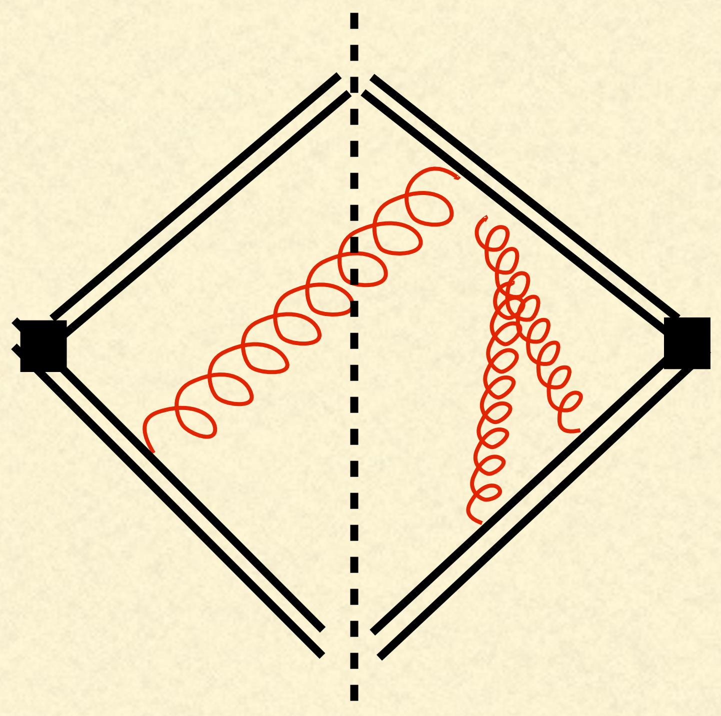


FACTORIZATION FOR τ_1^C

- Similar to hemisphere DIS 1-jettiness, valid for large enough R [1303.6952]







$$\sigma(\tau_1^C) = \sigma^{\text{sing}}(\tau_1^C) + \sigma^{\text{ns}}(\tau_1^C)$$

$$\frac{d\sigma^{\text{sing}}}{d\tau_1^C} = \sigma_0 \sum_q H_q(y, Q^2, \mu) \int dt_J\, dt_B\, dk_S\, \delta\Big(\tau_1^C - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big) S_{\text{hemi}}(k_S, \mu) J_q(t_J, \mu) B_q(t_B, x, \mu)$$

$$(\theta_J \ll 1)$$

σ^{ns} get from NLOJet++

FACTORIZATION FOR τ_1^C

[1303.6952]

$$\frac{d\sigma^{\text{sing}}}{d\tau_1^C} = \sigma_0 \sum_q H_q(y, Q^2, \mu) \int dt_J\, dt_B\, dk_S\, \delta\Big(\tau_1^C - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big) S_{\text{hemi}}(k_S, \mu) J_q(t_J, \mu) B_q(t_B, x, \mu)$$

- Relate to usual beam, jet and hemisphere soft functions by rescaling with Lorentz invariants:

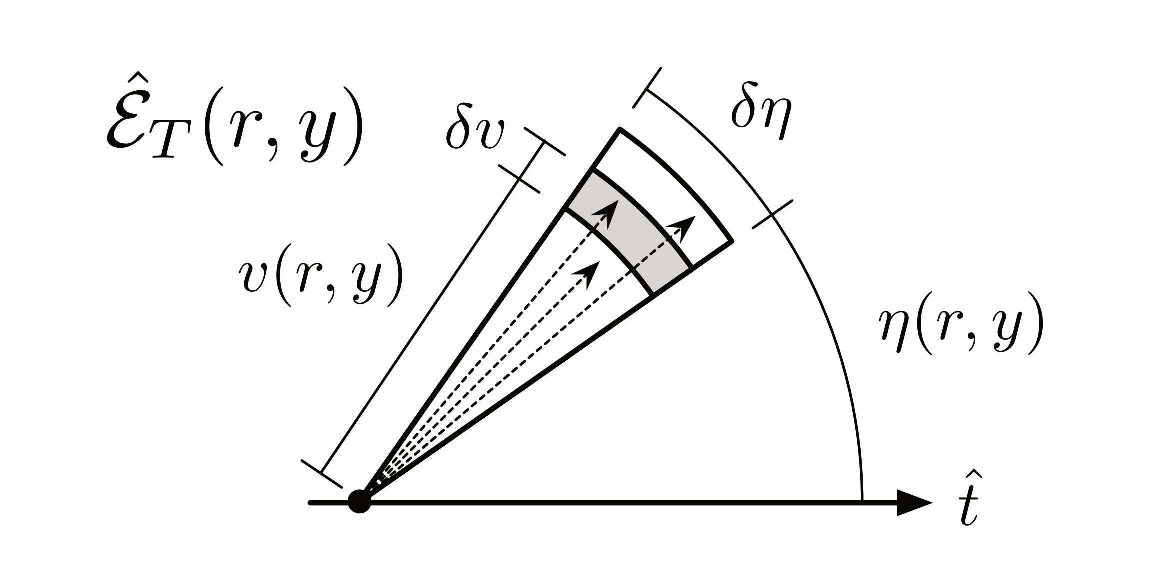
	Hemisphere $\tau_1^{a,b}$	Centauro τ_1^C
$s_B = -\frac{q_J \cdot q}{q_J \cdot q_B} Q^2$	$s_B = Q^2$	$s_B = \frac{Q^2}{\cos^2(\theta_J/2)} \to Q^2$
$s_J = \frac{q_B \cdot q}{q_J \cdot q_B} Q^2$	$s_J = Q^2$	$s_J = \frac{Q^2 R^2}{4}$
$Q_R = \frac{Q^2}{\sqrt{2 q_J \cdot q_B}}$	$Q_R = Q$	$Q_R = \frac{QR}{2\cos(\theta_J/2)} \to \frac{QR}{2}$

NONPERTURBATIVE CORRECTIONS TO τ_1^C

- Soft function for *J*, *B* regions:

$$S(k_J, k_B, \mu) = \frac{1}{N_C}\mathrm{Tr}\langle 0|Y_n^\dagger Y_{\bar{n}}\delta\Big(k_J - \int drdyd\phi\, f_{\mathrm{alg}}^J(r,y,\phi)\hat{\mathscr{E}}_T(r,y,\phi)\Big)\delta\Big(k_B - \int drdyd\phi\, f_{\mathrm{alg}}^B(r,y,\phi)\hat{\mathscr{E}}_T(r,y,\phi)\Big)Y_{\bar{n}}^\dagger Y_n|0\rangle$$

[1209.3781]



$$\hat{\mathscr{E}}_T(r,y,\phi)|X_s\rangle = \sum_{i\in X_s} m_{Ti}\delta(r-r_i)\delta(y-y_i)\delta(\phi-\phi_i)|X_s\rangle$$

$$m_{Ti} = \sqrt{p_\perp^2 + m^2}\,, \quad r = p_\perp/m_\perp$$

- Weighting factor for algorithms:

$$f_{\mathrm{alg}}^J(r,y,\phi) = \Theta_{\mathrm{alg}}(r,y,\phi)\tilde{f}_J(r,y,\phi)$$

$$f_{\mathrm{alg}}^B(r,y,\phi) = [1-\Theta_{\mathrm{alg}}(r,y,\phi)]\tilde{f}_B(r,y,\phi)$$

- For measurement of the 1-jettiness / thrust:

$$\tilde{f}_J(r,y,\phi) = \tilde{f}_J(y) = e^y$$

$$\tilde{f}_B(r,y,\phi) = \tilde{f}_B(y) = e^{-y}$$

- For soft phase space with Centauro alg.:

$$\Theta_{\mathrm{alg}}(r,y,\phi) = \Theta_{\mathrm{Cent}}(y) = \Theta\Big(\ln\frac{R}{2} - y\Big)$$

NONPERTURBATIVE CORRECTIONS TO τ_1^C

- Soft function for *J*, *B* regions:

$$S(k_J, k_B, \mu) = \delta(k_J)\delta(k_B) + \delta'(k_J)\delta(k_B)\Omega^J_{\rm alg} + \delta(k_J)\delta'(k_B)\Omega^B_{\rm alg} + \cdots$$

$$\Omega^{J,B}_{\rm alg} = \int dr dy d\phi f^{J,B}_{\rm alg}(r,y,\phi)\frac{1}{N_C}\mathrm{Tr}\langle 0|Y_n^\dagger Y_{\bar n}\hat{\mathcal{E}}_T(r,y,\phi)Y_{\bar n}^\dagger Y_n|0\rangle$$

- Centauro:

$$\Omega^{J,B}_{\rm Cent} = C^{J,B}_{\rm Cent}(R)\Omega_1^{\rm J-scheme} \qquad \Omega_1^{\rm J-scheme} = \frac{1}{N_C}\mathrm{Tr}\langle 0|Y_n^\dagger Y_{\bar n}\hat{\mathcal{E}}_T(y=0)Y_{\bar n}^\dagger Y_n|0\rangle$$

$$. \; C^J_{\rm Cent}(R) = \frac{R}{2}, C^B_{\rm Cent}(R) = \frac{2}{R}$$

Exact in *R*!

- “Type II” τ_1^C :

$$. \; \Delta\tau_1^c = \frac{1}{Q_J}\Omega^J_{\rm Cent} + \frac{1}{Q_B}\Omega^B_{\rm Cent} = \frac{4}{QR}\Omega_1^{\rm J-scheme}$$

$$Q_J = \frac{QR^2}{4}, Q_B = Q$$

(also follows directly from reweighted def of τ_1^C)

THEORETICAL PREDICTIONS

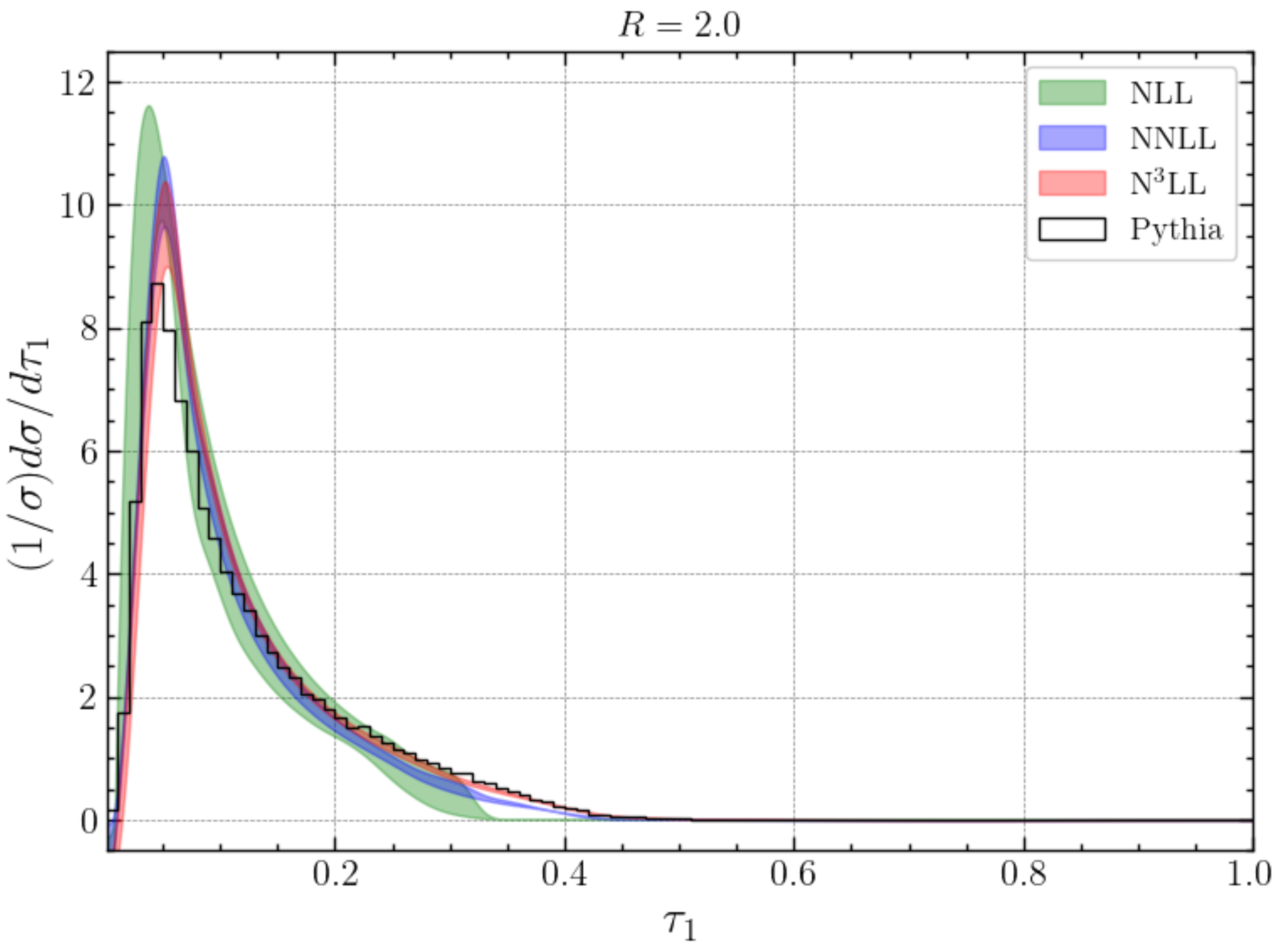
- N^{3}LL resummed + $\mathcal{O}(\alpha_s^2)$ fixed-order matched, convolved with shape function with R_{gap} renormalon subtractions

- Plots for kinematics at HERA: $\sqrt{s} = 319\,\mathrm{GeV}\,, Q = 50\,\mathrm{GeV}\,, x = 0.05$

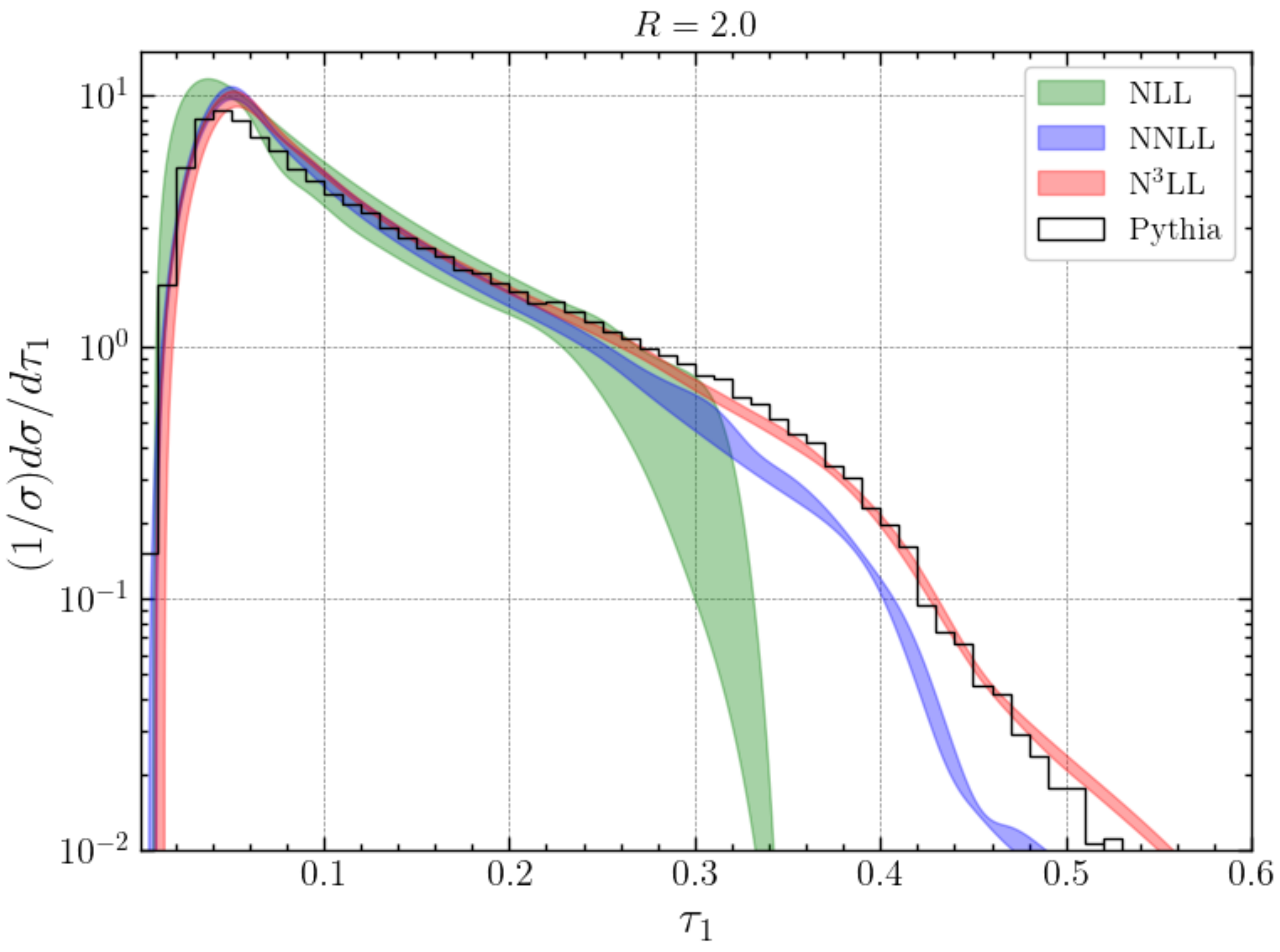
- Default values $\alpha_s(M_Z) = 0.118\,, \quad \Omega_1(R_\Delta, \mu_\Delta) - \Delta_0 = 500\,\mathrm{MeV}$

$$R_\Delta = \mu_\Delta = 2\,\mathrm{GeV}\,, \Delta_0 = 50\,\mathrm{MeV}$$

R=2.0

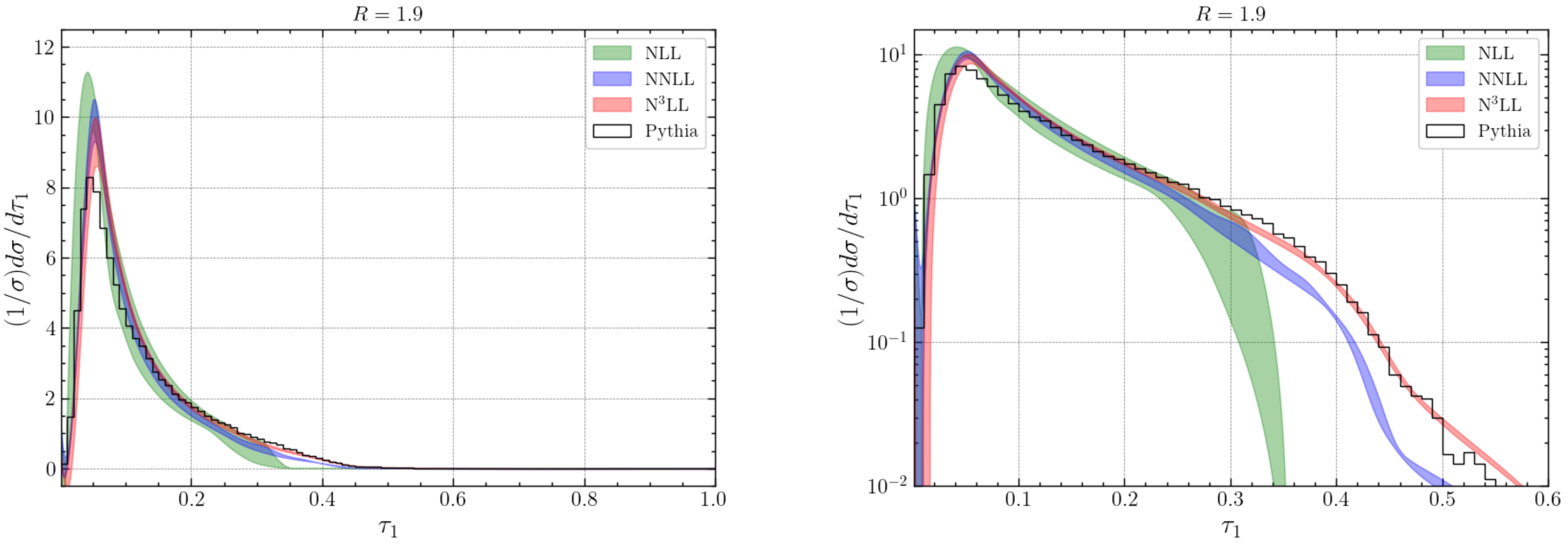


$R = 2.0$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0



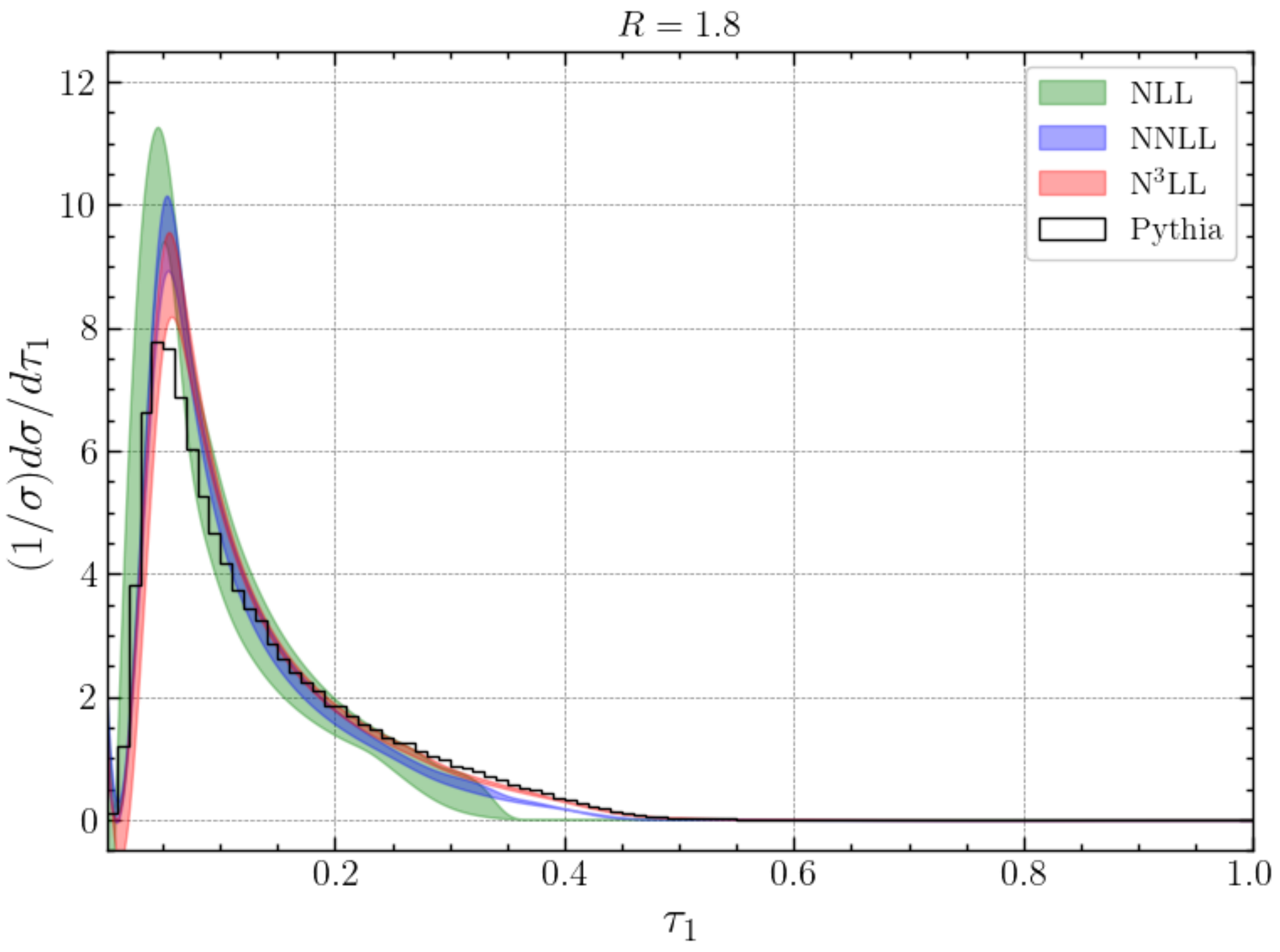
$R = 2.0$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
10^{-2}
10^{-1}
10^0
10^1
0.1
0.2
0.3
0.4
0.5
0.6

R=1.9

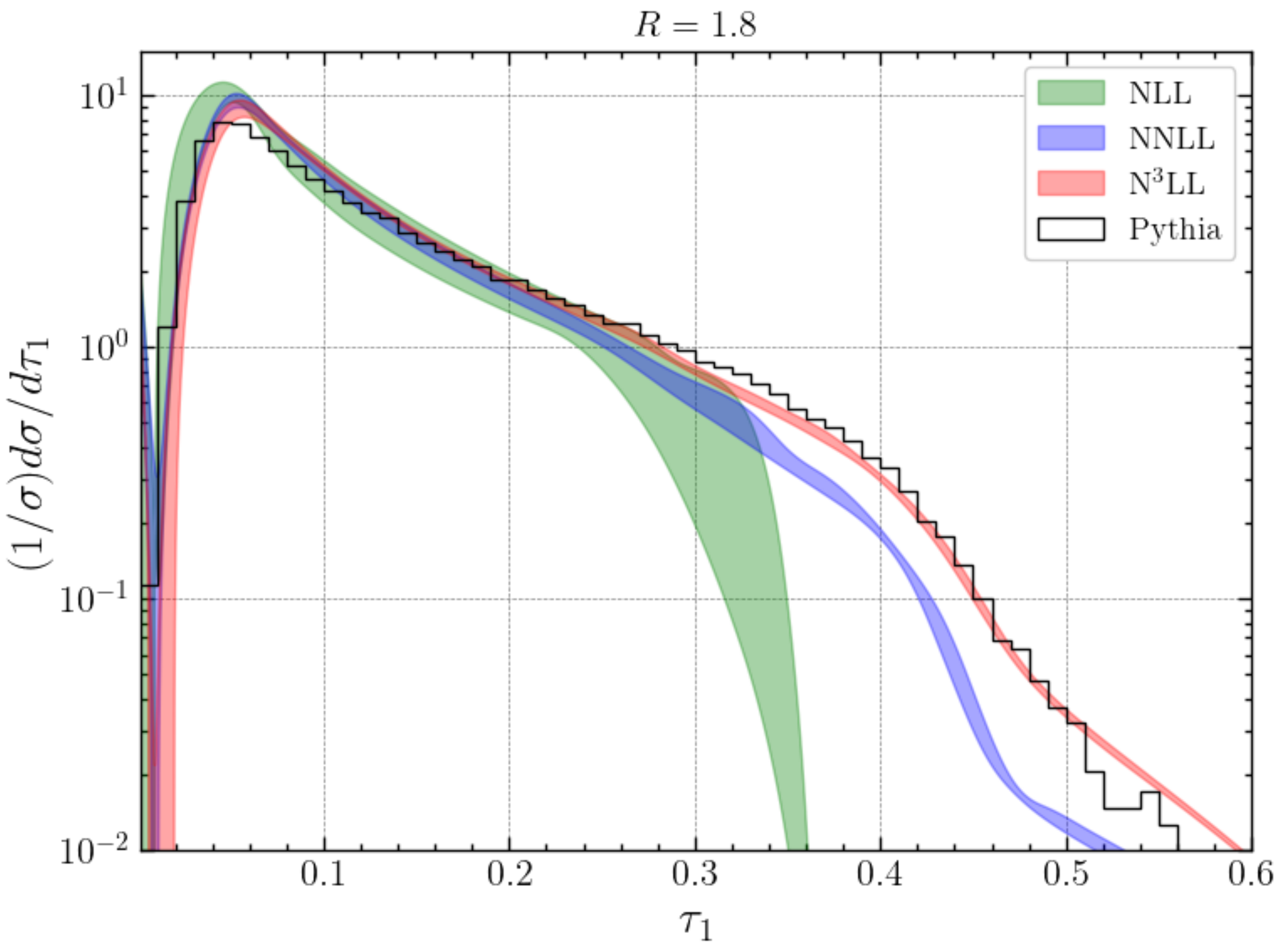


$R = 1.9$
$(1/\sigma)d\sigma/d\tau_1$
τ_1
NLL
NNLL
N^{3}LL
Pythia
$R = 1.9$
$(1/\sigma)d\sigma/d\tau_1$
τ_1
NLL
NNLL
N^{3}LL
Pythia

R=1.8

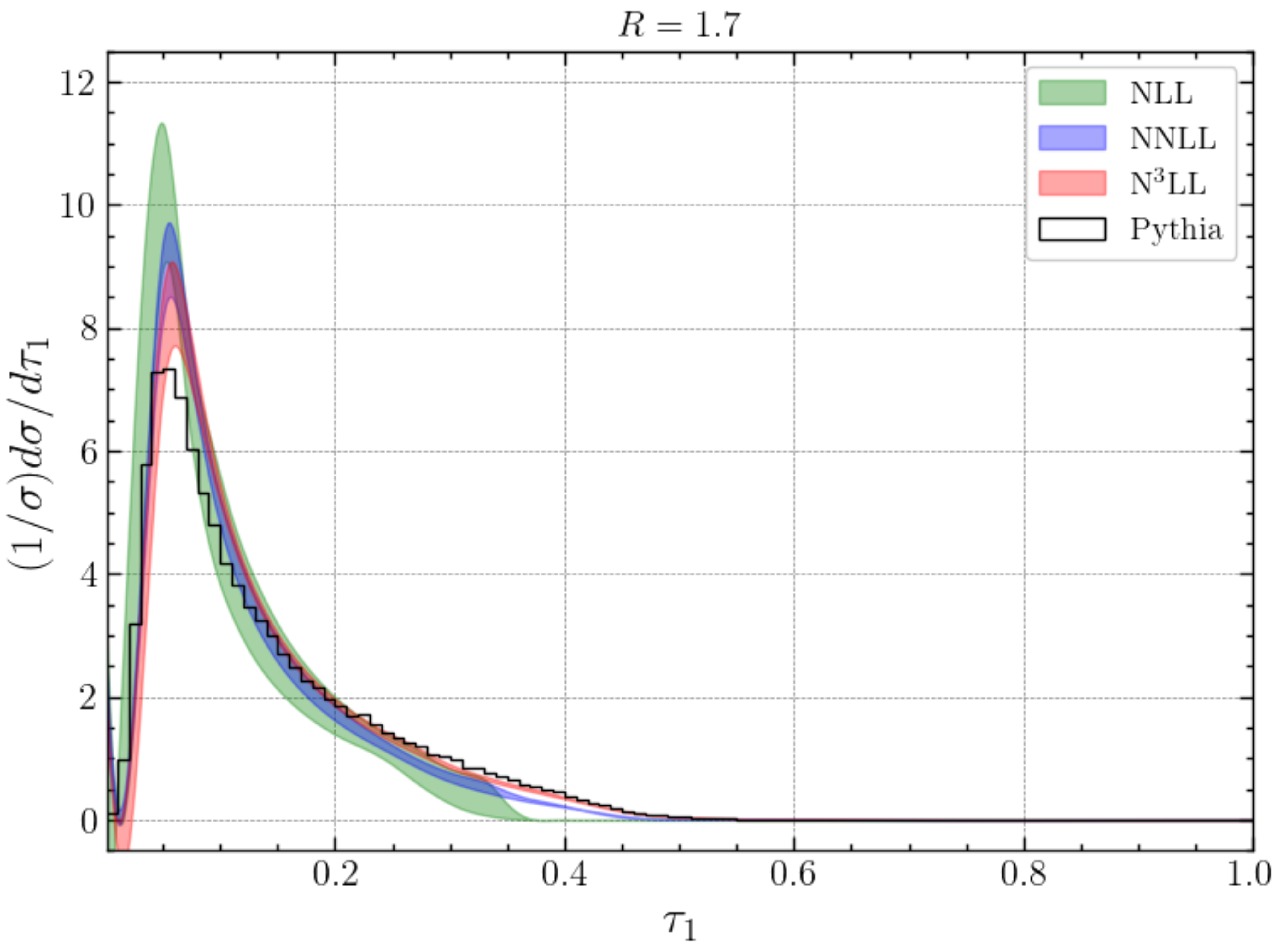


$R = 1.8$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1

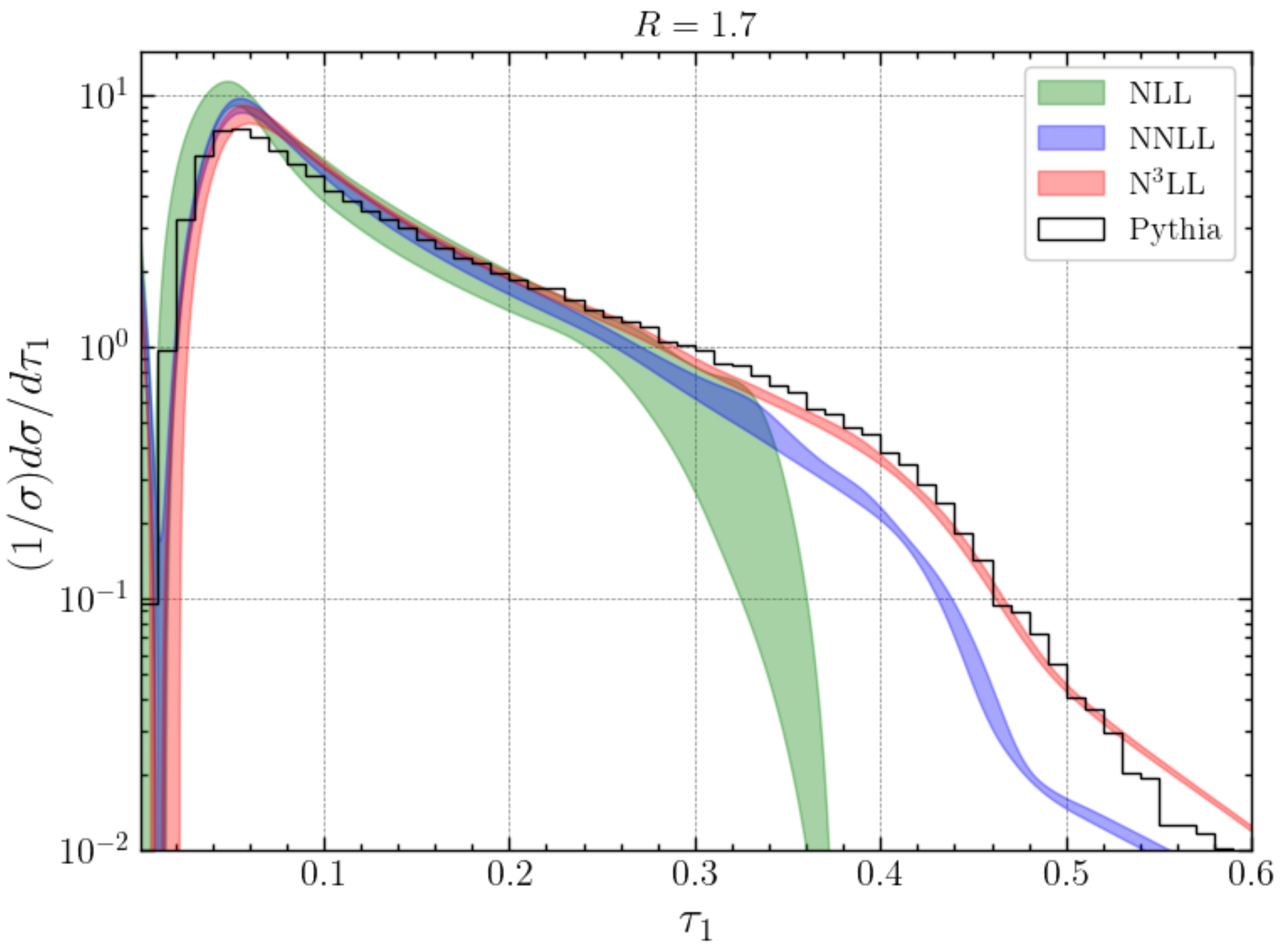


$R = 1.8$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1

R=1.7

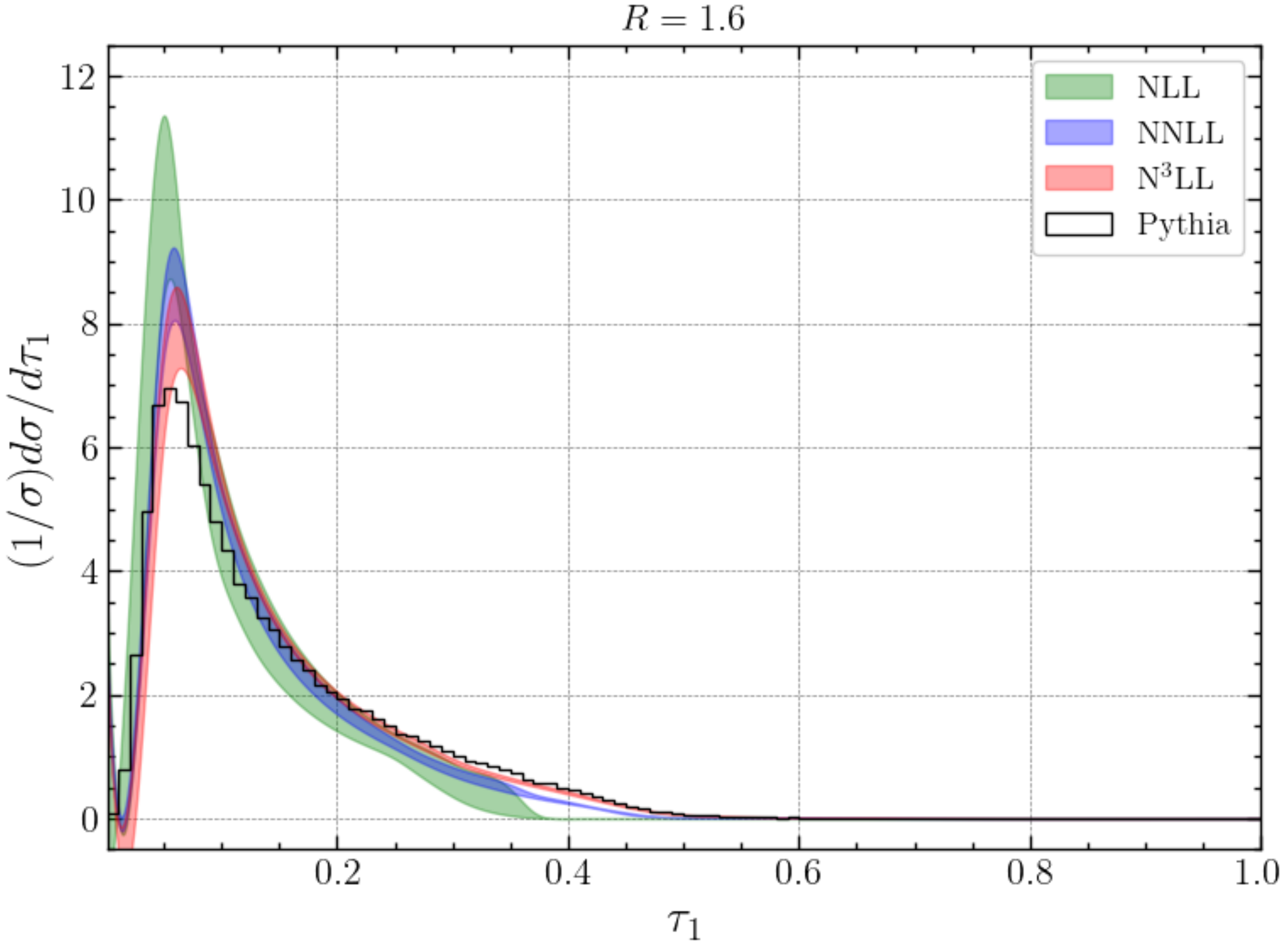


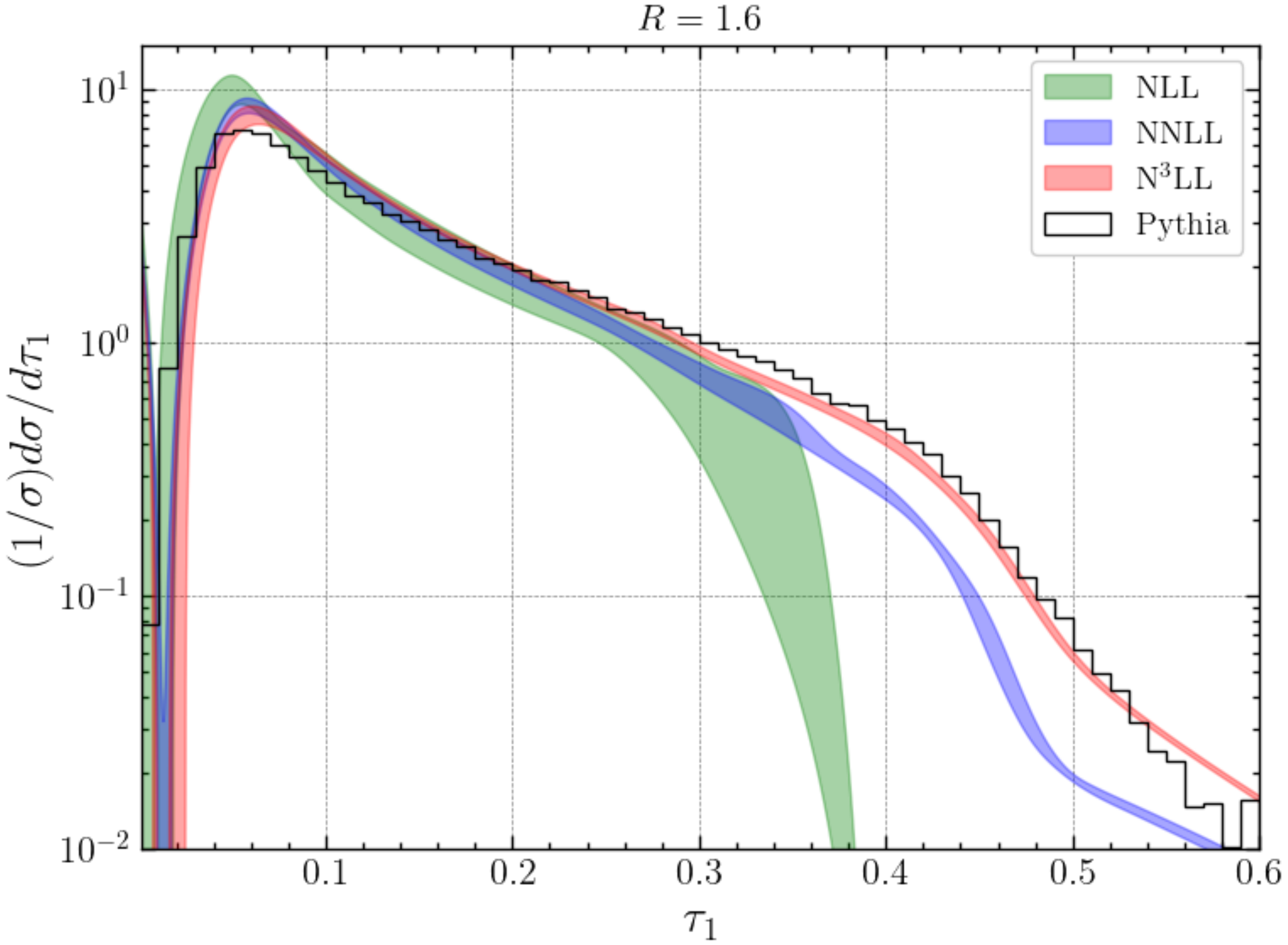
$R = 1.7$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0



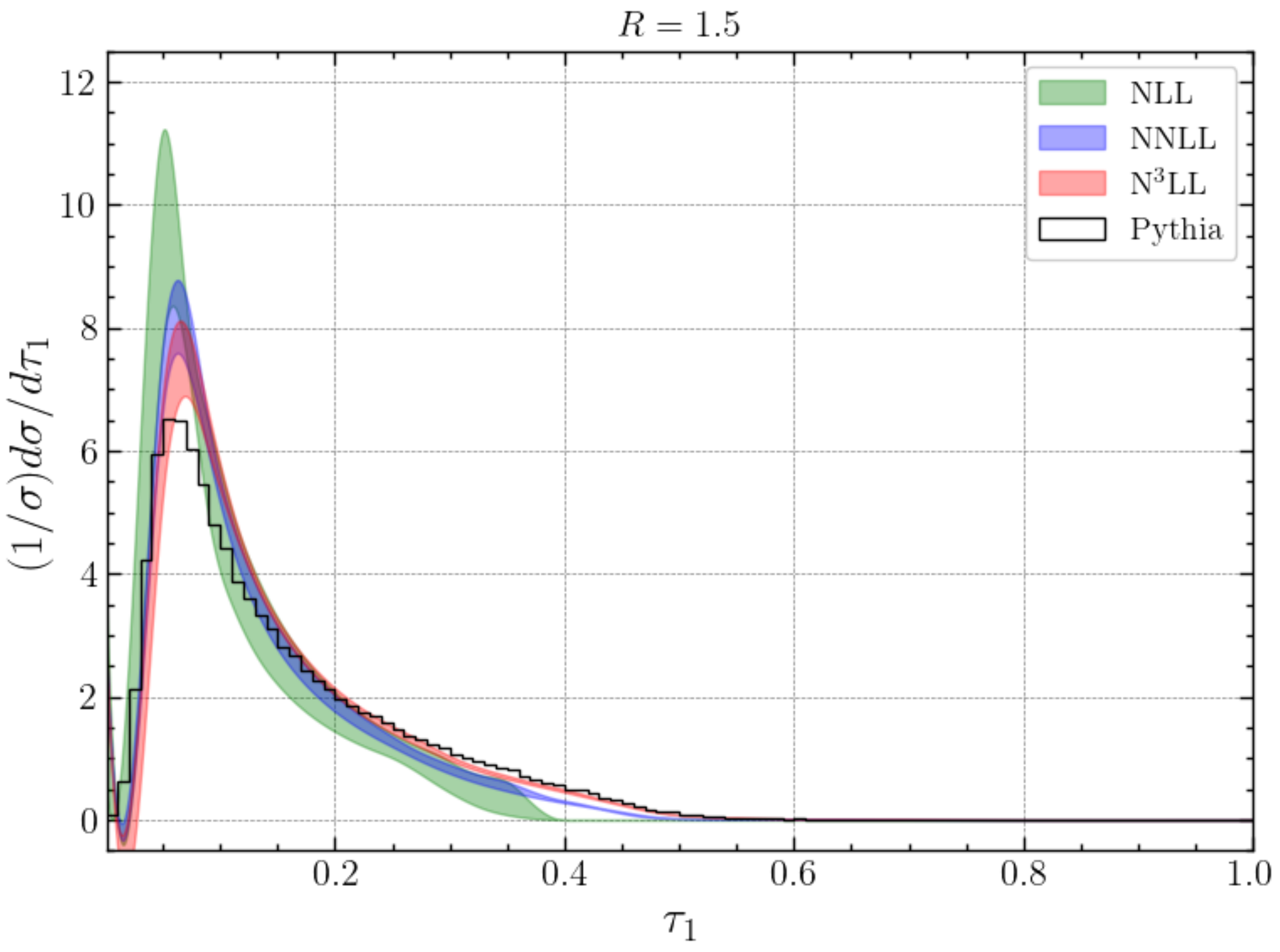
$R = 1.7$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
10^1
10^0
10^{-1}
10^{-2}
0.1
0.2
0.3
0.4
0.5
0.6

R=1.6

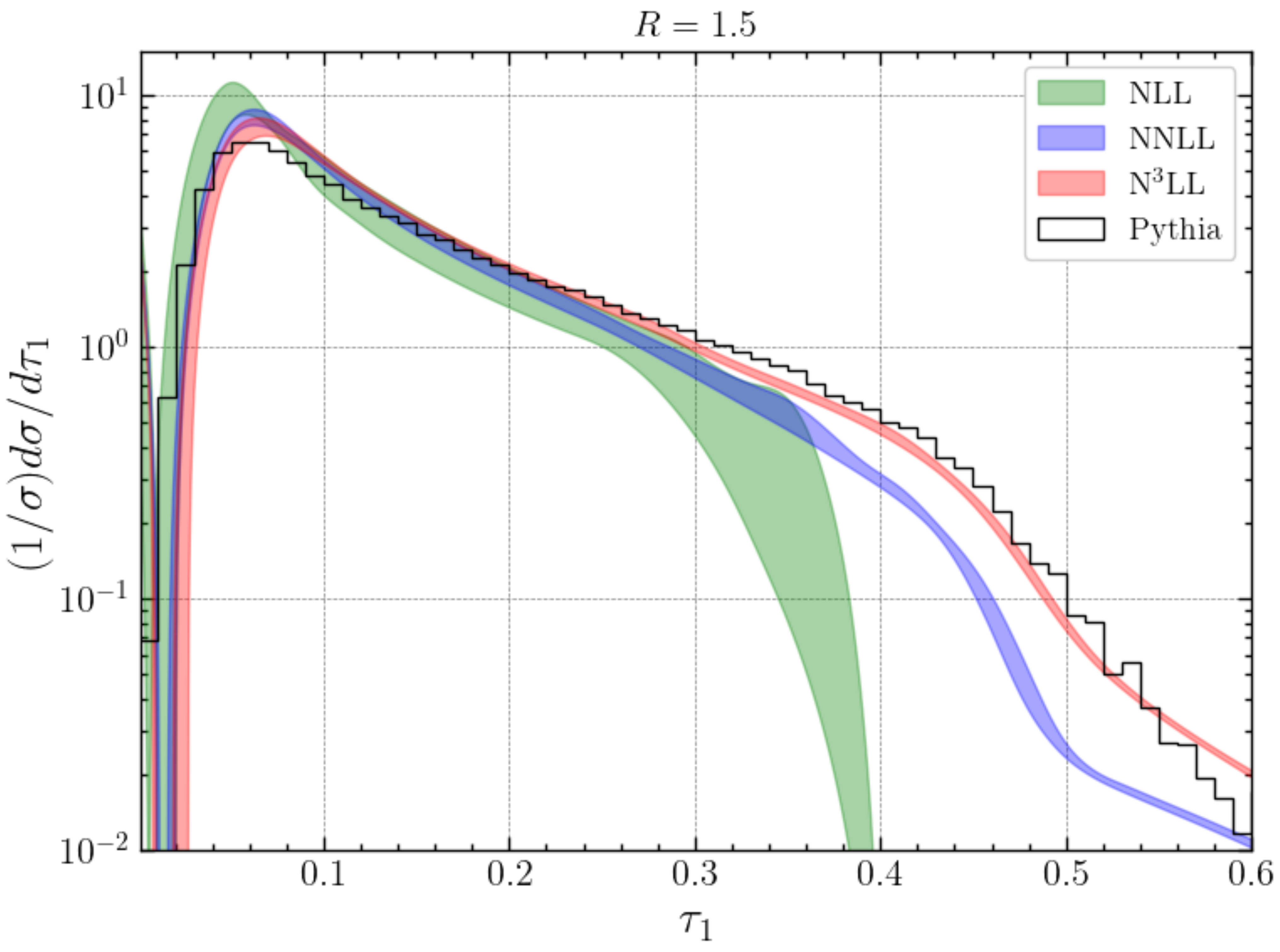

$R = 1.6$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0


$R = 1.6$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
10^{1}
10^{0}
10^{-1}
10^{-2}
0.1
0.2
0.3
0.4
0.5
0.6

R=1.5

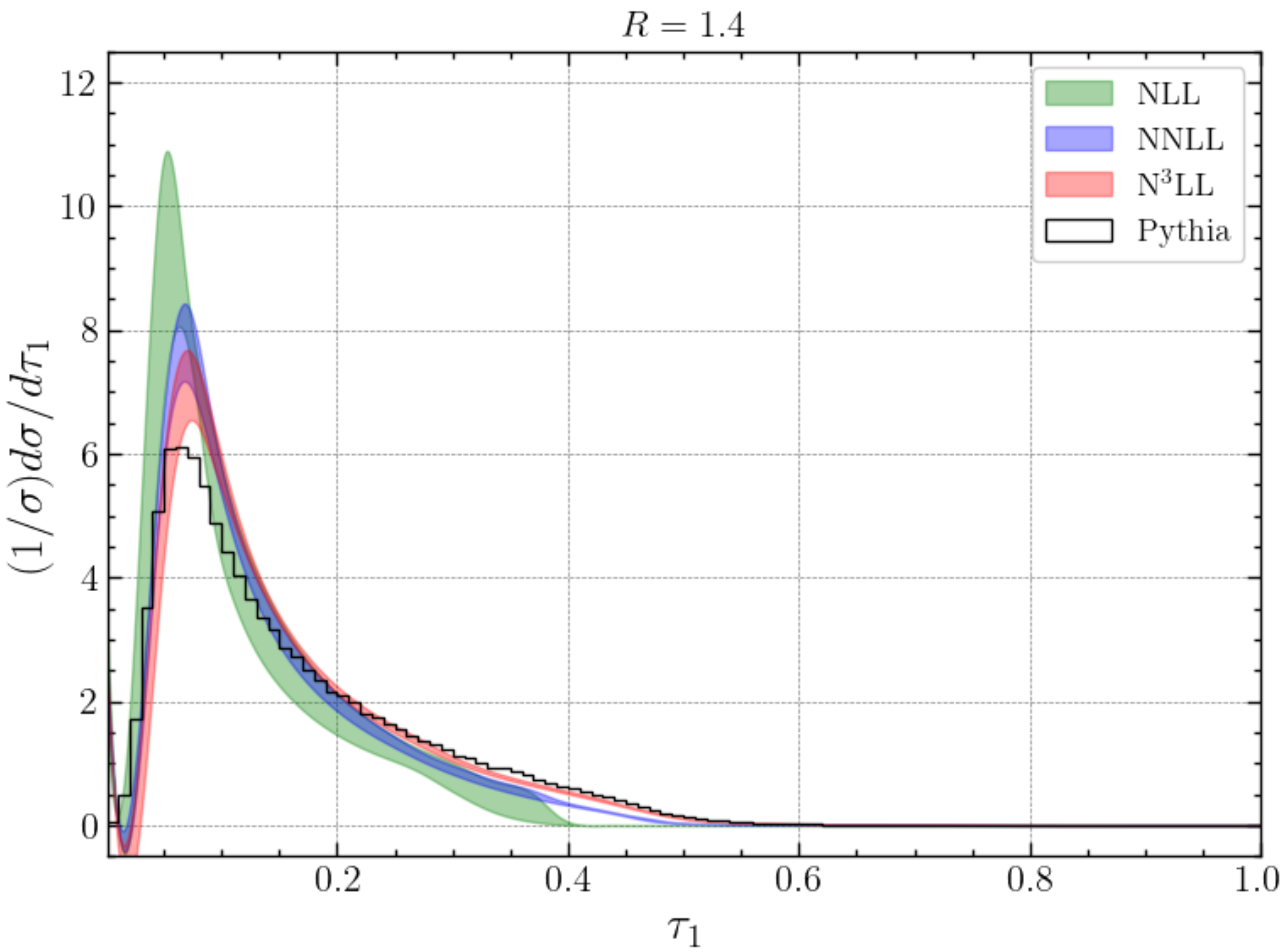


R = 1.5
NLL
NNLL
N³LL
Pythia
(1/σ)dσ/dτ₁
τ₁

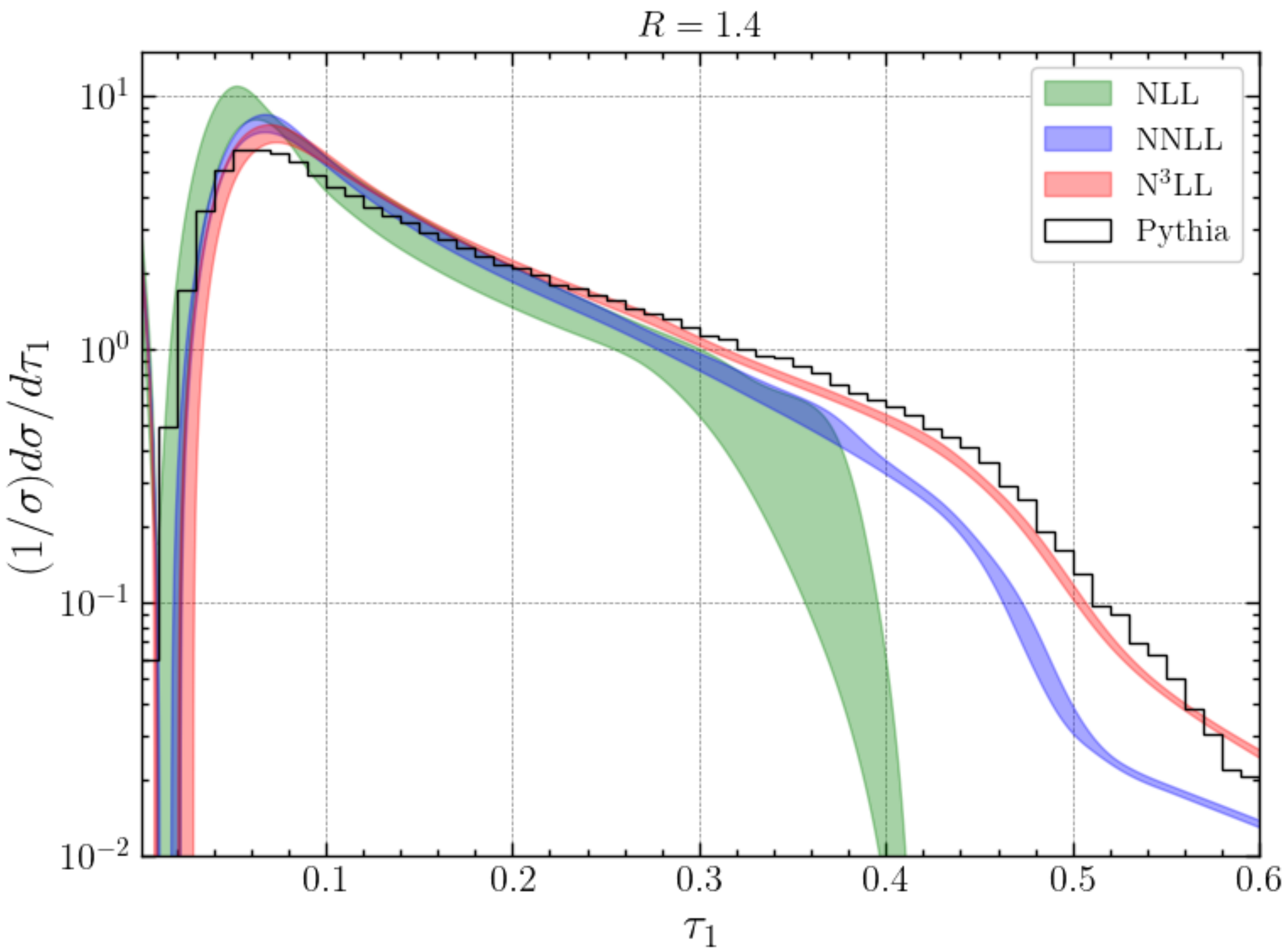


R = 1.5
NLL
NNLL
N³LL
Pythia
(1/σ)dσ/dτ₁
τ₁

R=1.4

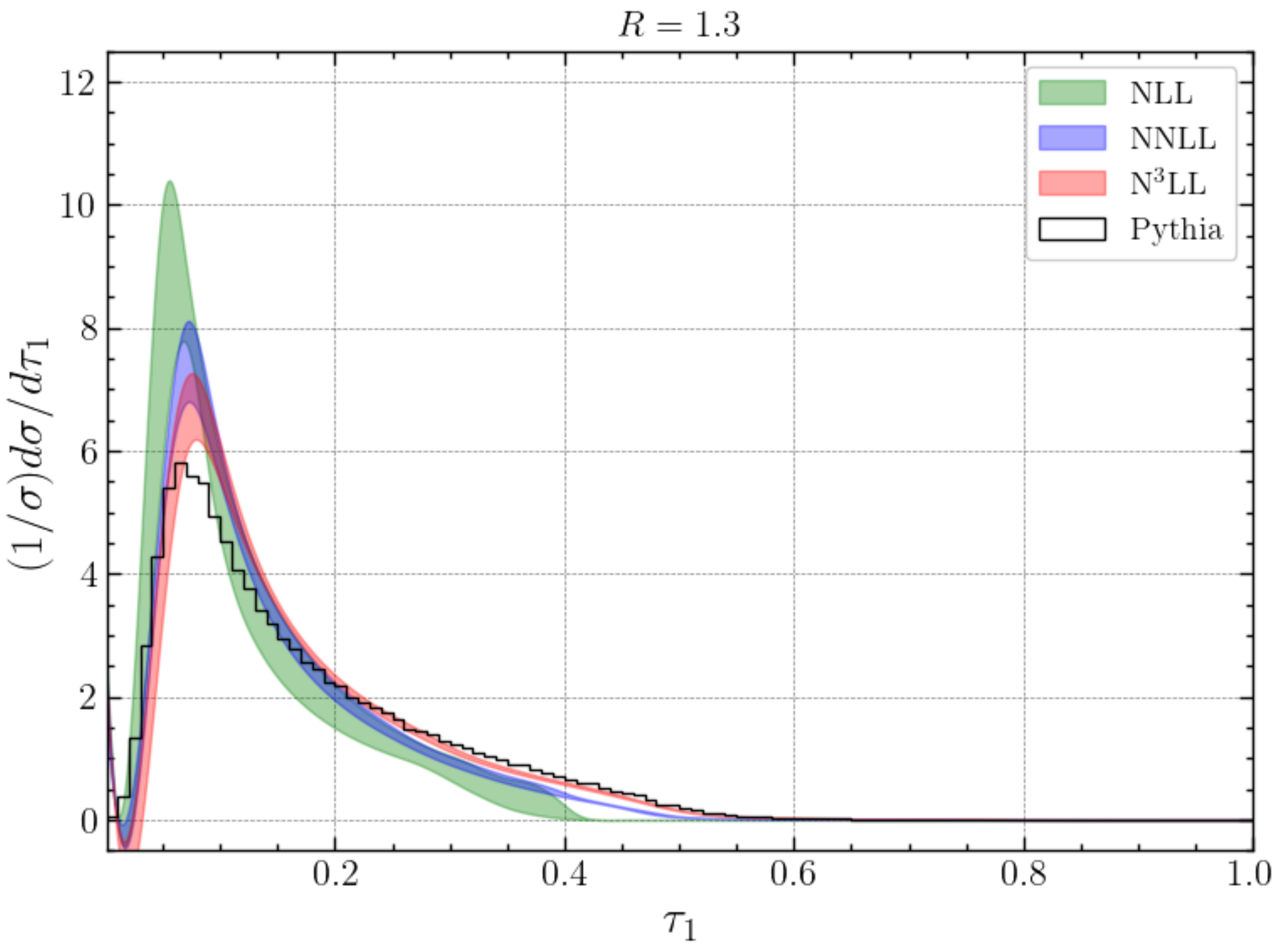


$R = 1.4$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0

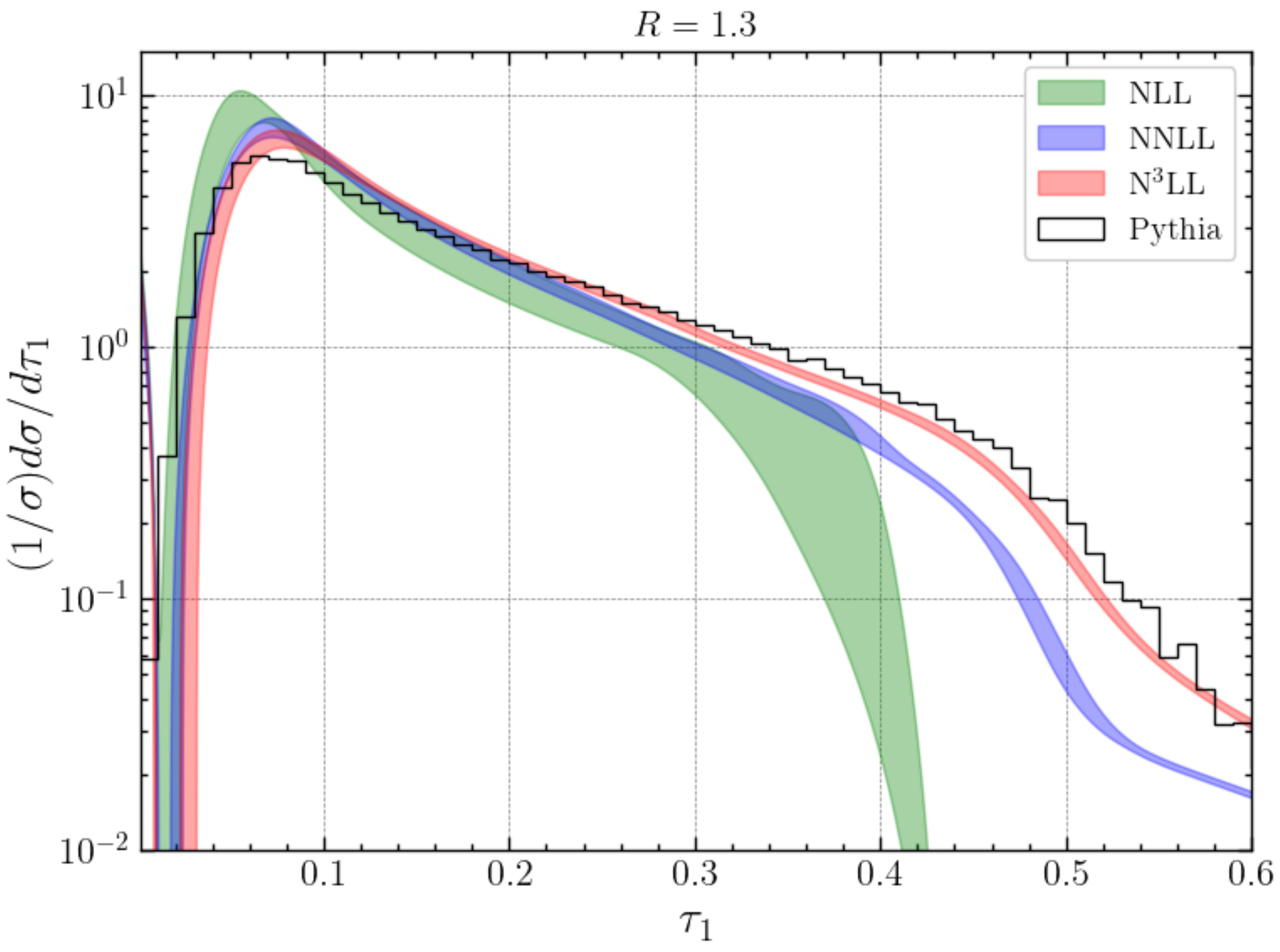


$R = 1.4$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
10^{1}
10^{0}
10^{-1}
10^{-2}
0.1
0.2
0.3
0.4
0.5
0.6

R=1.3

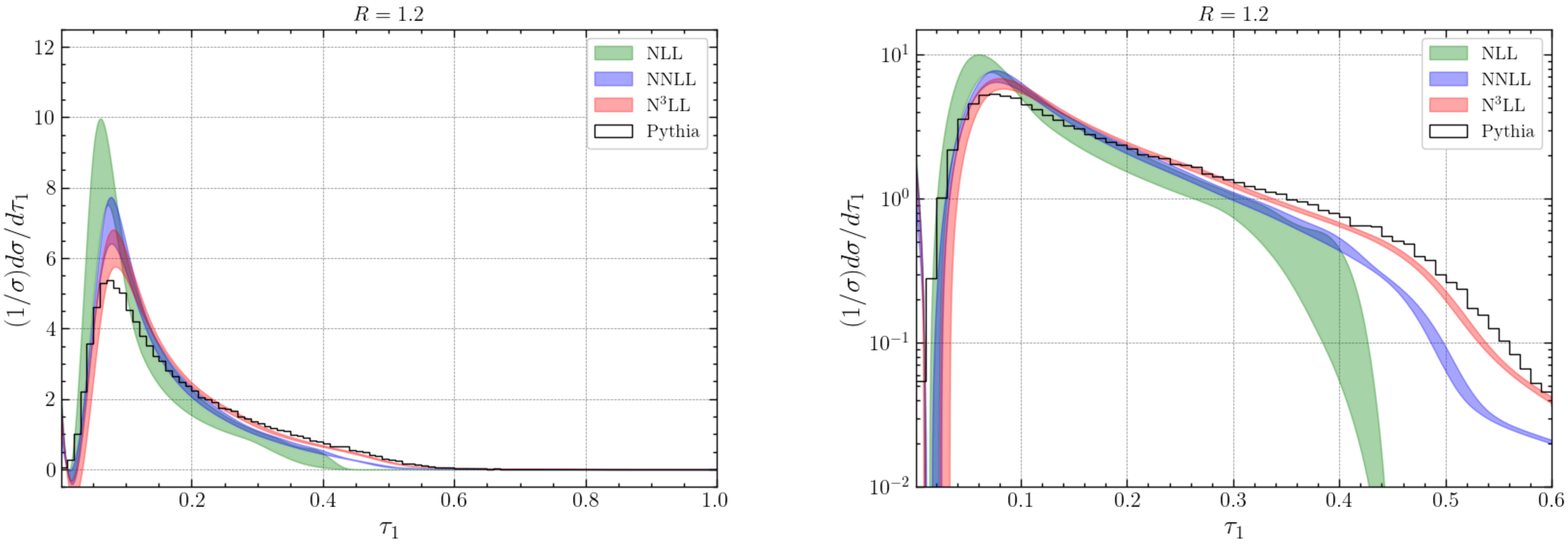


$R = 1.3$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0



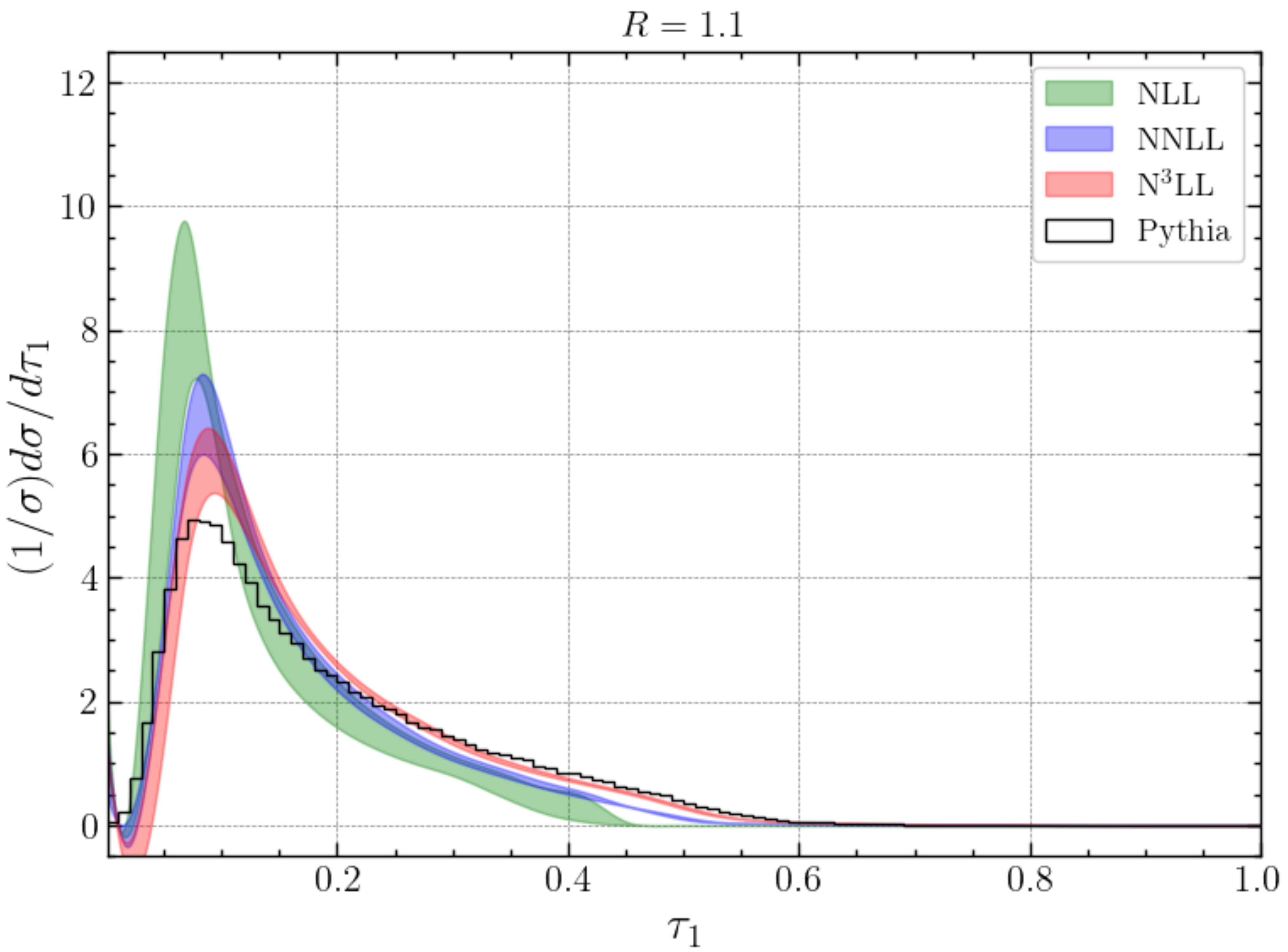
$R = 1.3$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
10^{1}
10^{0}
10^{-1}
10^{-2}
0.1
0.2
0.3
0.4
0.5
0.6

R=1.2

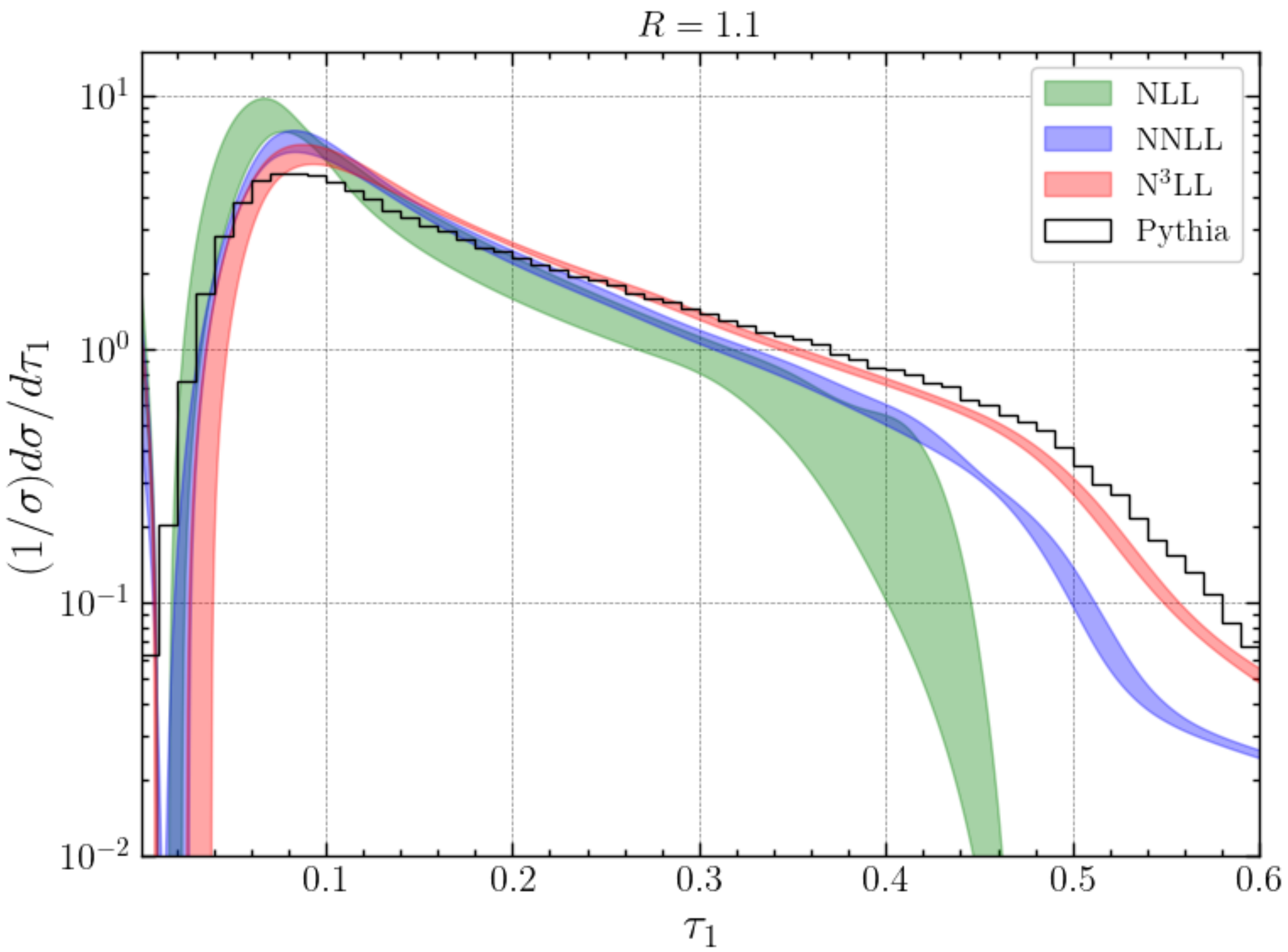


$R = 1.2$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0
$R = 1.2$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
10^{1}
10^{0}
10^{-1}
10^{-2}
0.1
0.2
0.3
0.4
0.5
0.6

R=1.1

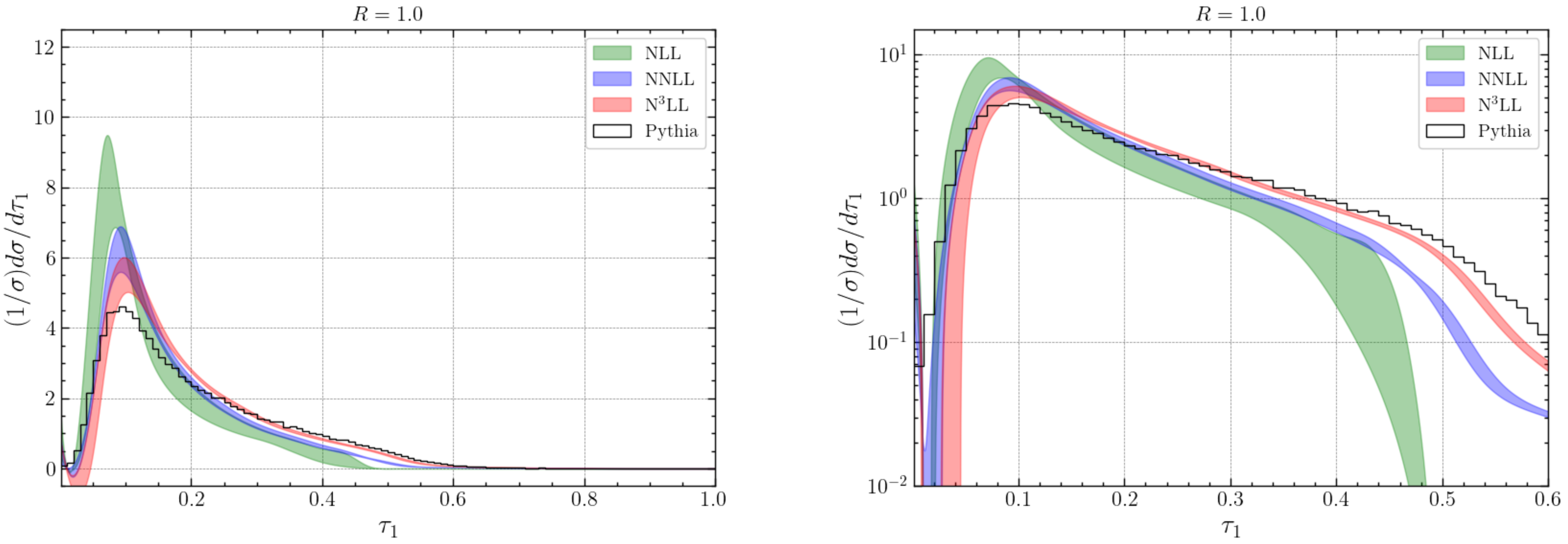


R = 1.1
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1



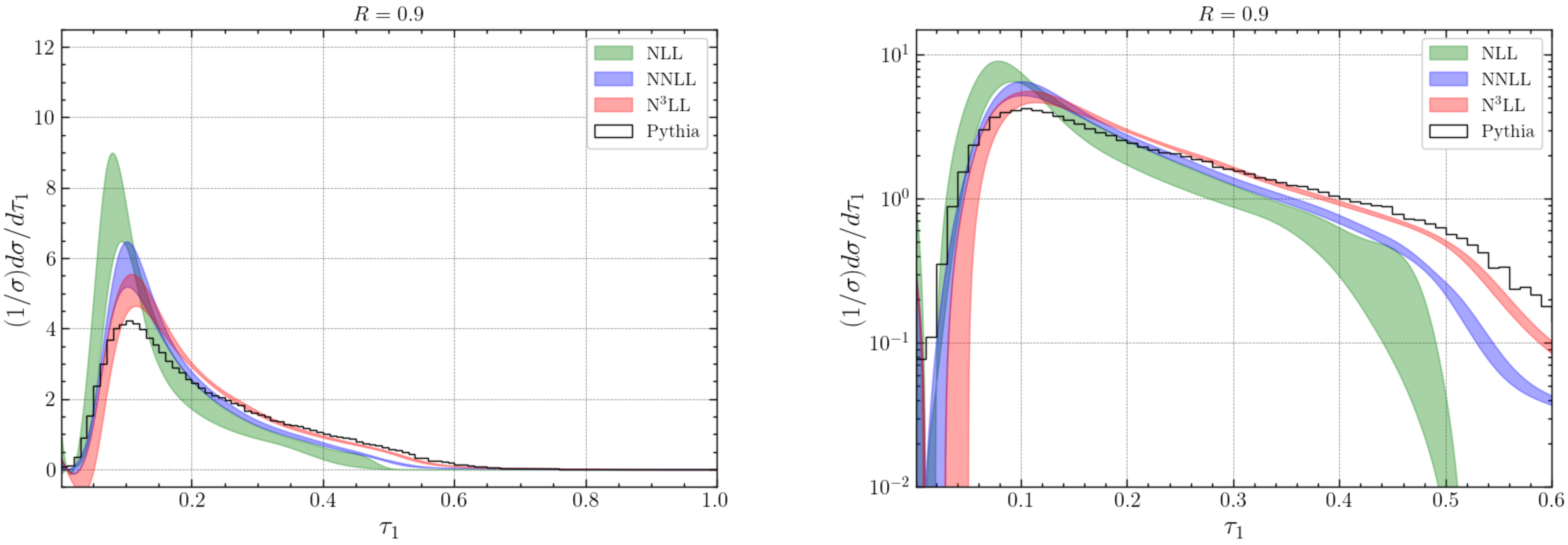
R = 1.1
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1

R=1.0



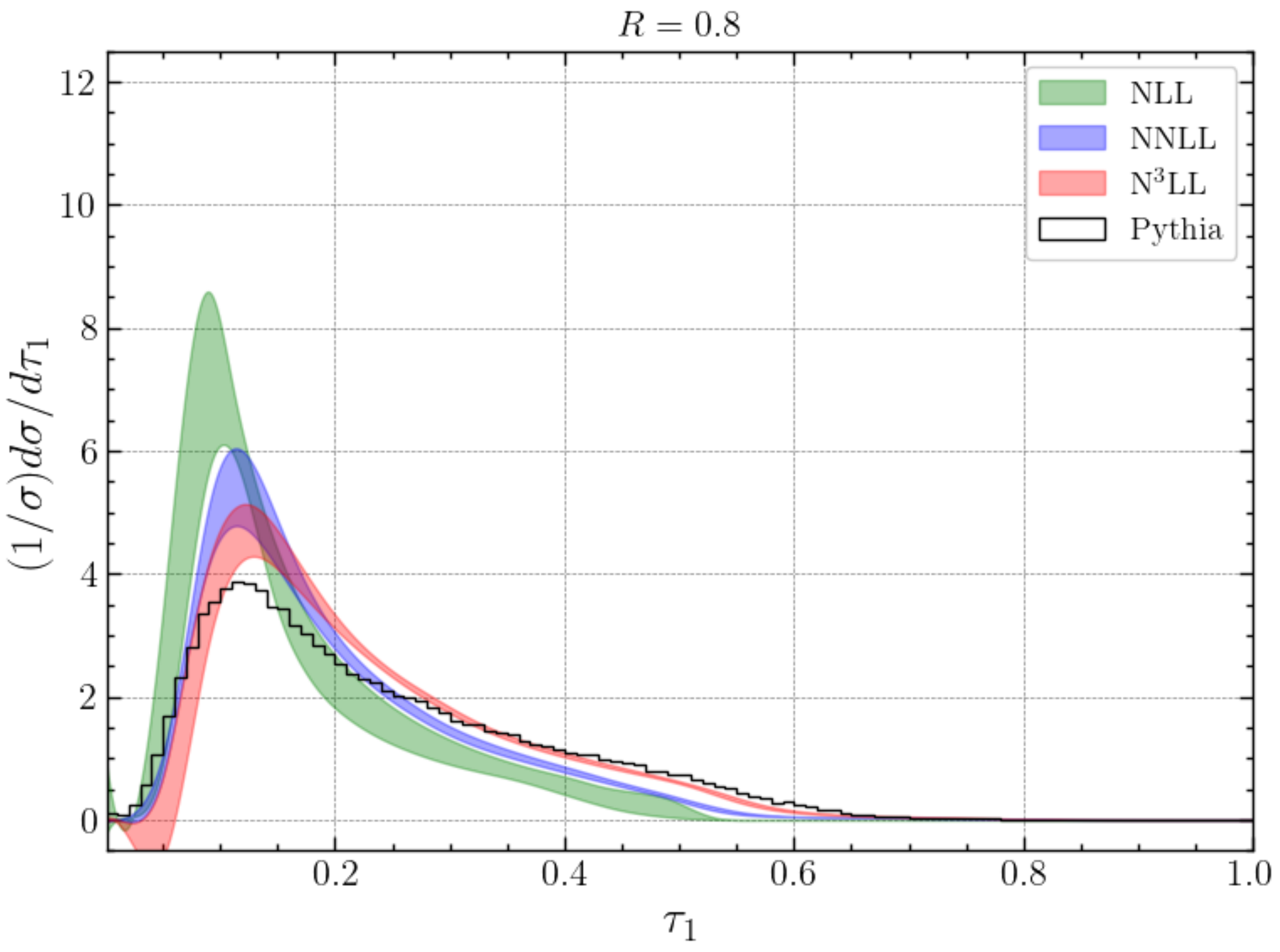
$R = 1.0$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0
$R = 1.0$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
10^{-2}
10^{-1}
10^{0}
10^{1}
0.1
0.2
0.3
0.4
0.5
0.6

R=0.9

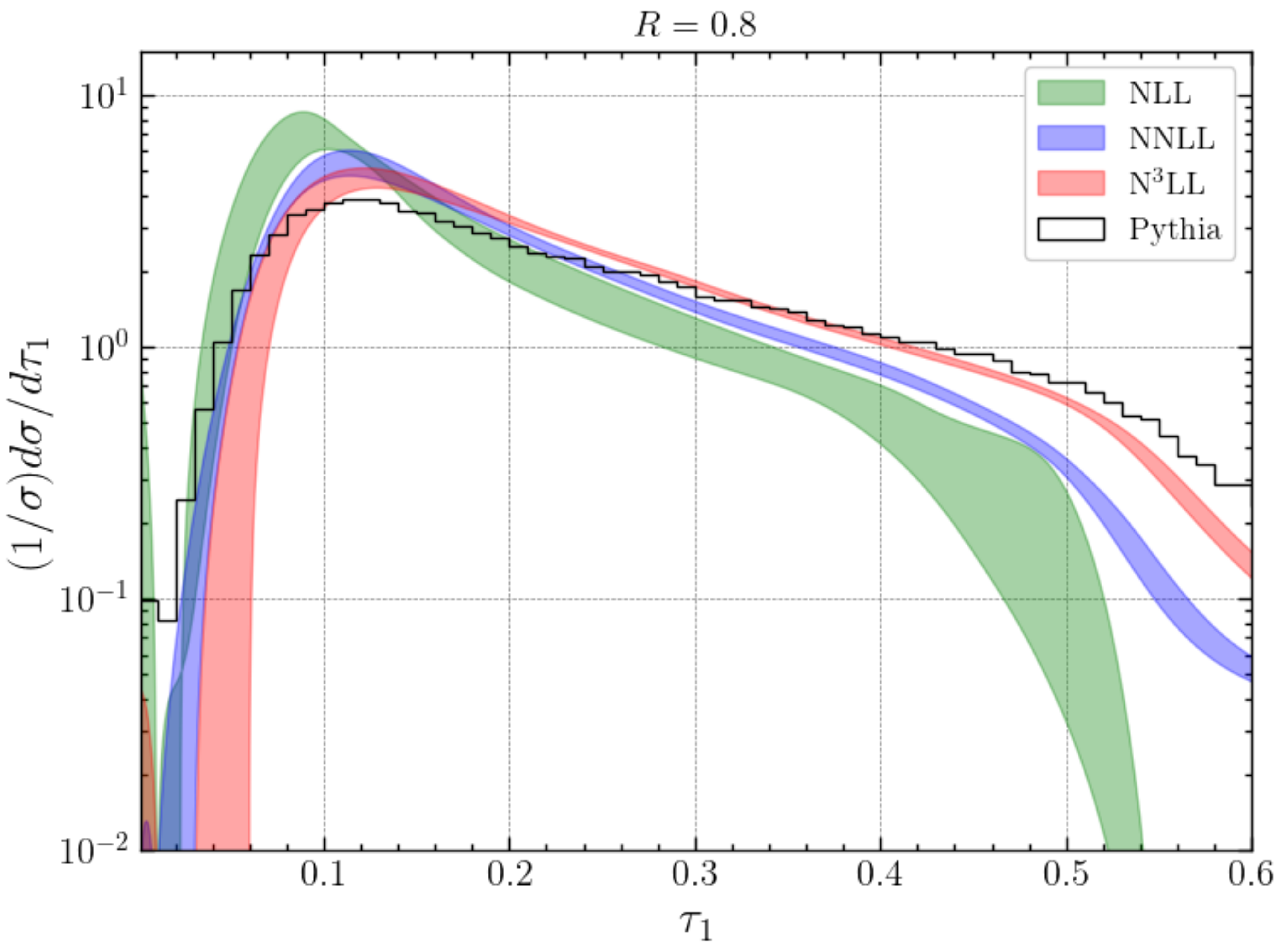


R = 0.9
NLL
NNLL
N³LL
Pythia
(1/σ)dσ/dτ₁
τ₁
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0
R = 0.9
NLL
NNLL
N³LL
Pythia
(1/σ)dσ/dτ₁
τ₁
10⁻²
10⁻¹
10⁰
10¹
0.1
0.2
0.3
0.4
0.5
0.6

R=0.8

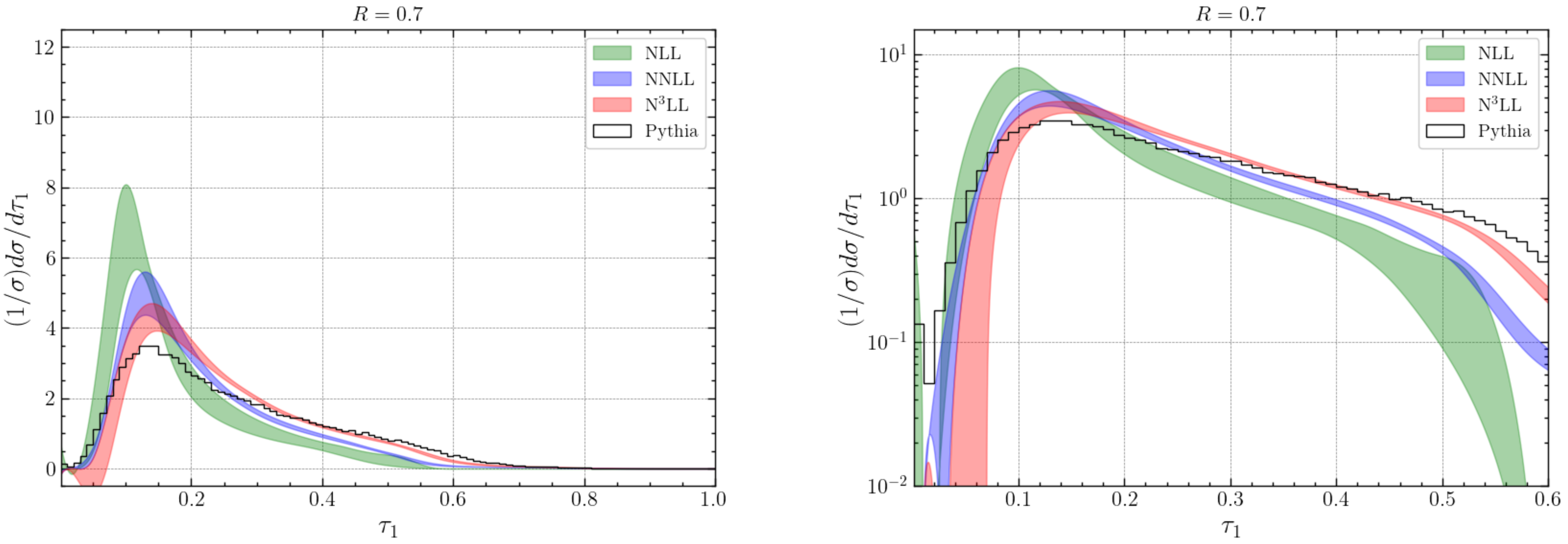


$R = 0.8$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0



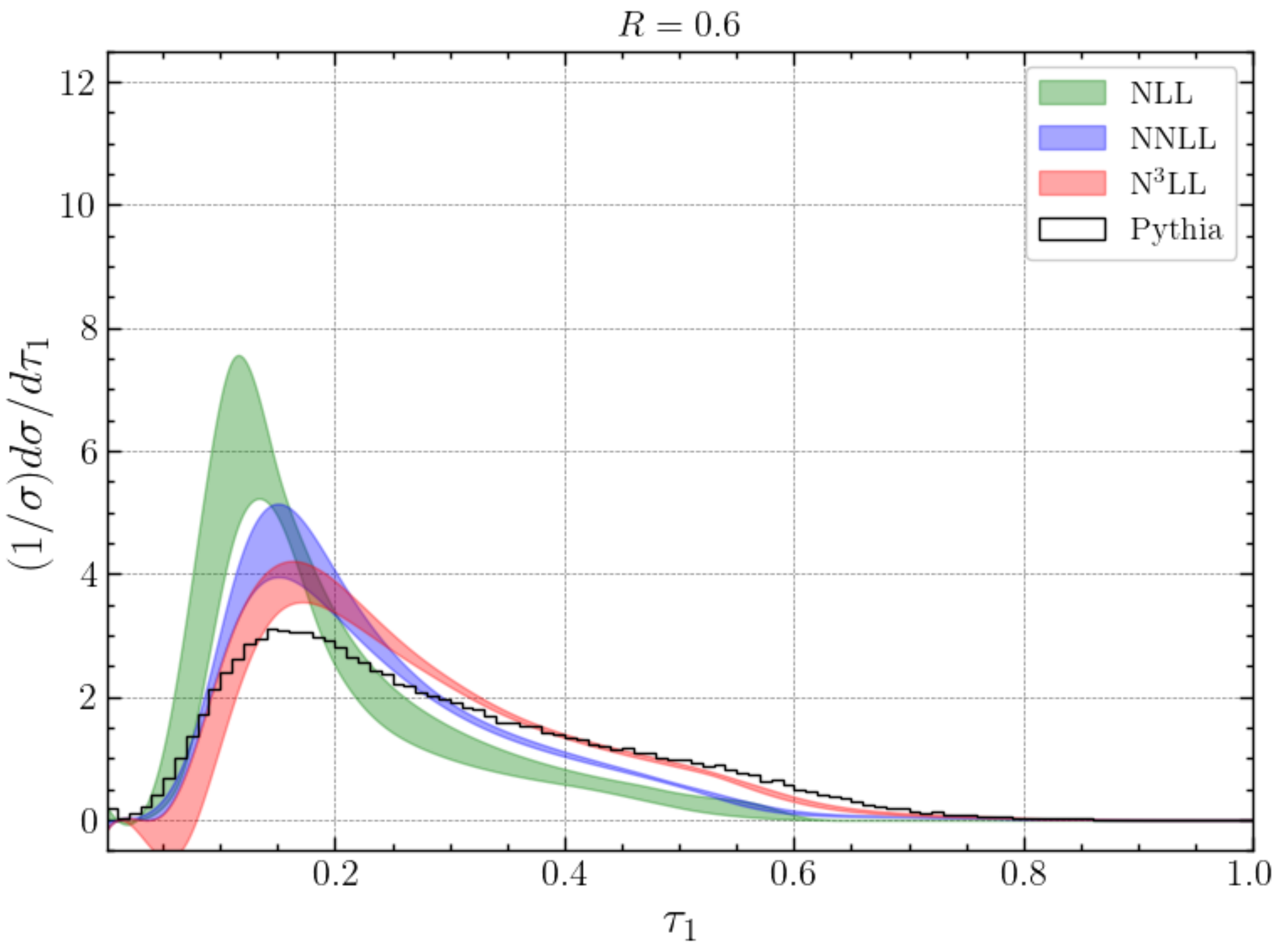
$R = 0.8$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
10^{-2}
10^{-1}
10^{0}
10^{1}
0.1
0.2
0.3
0.4
0.5
0.6

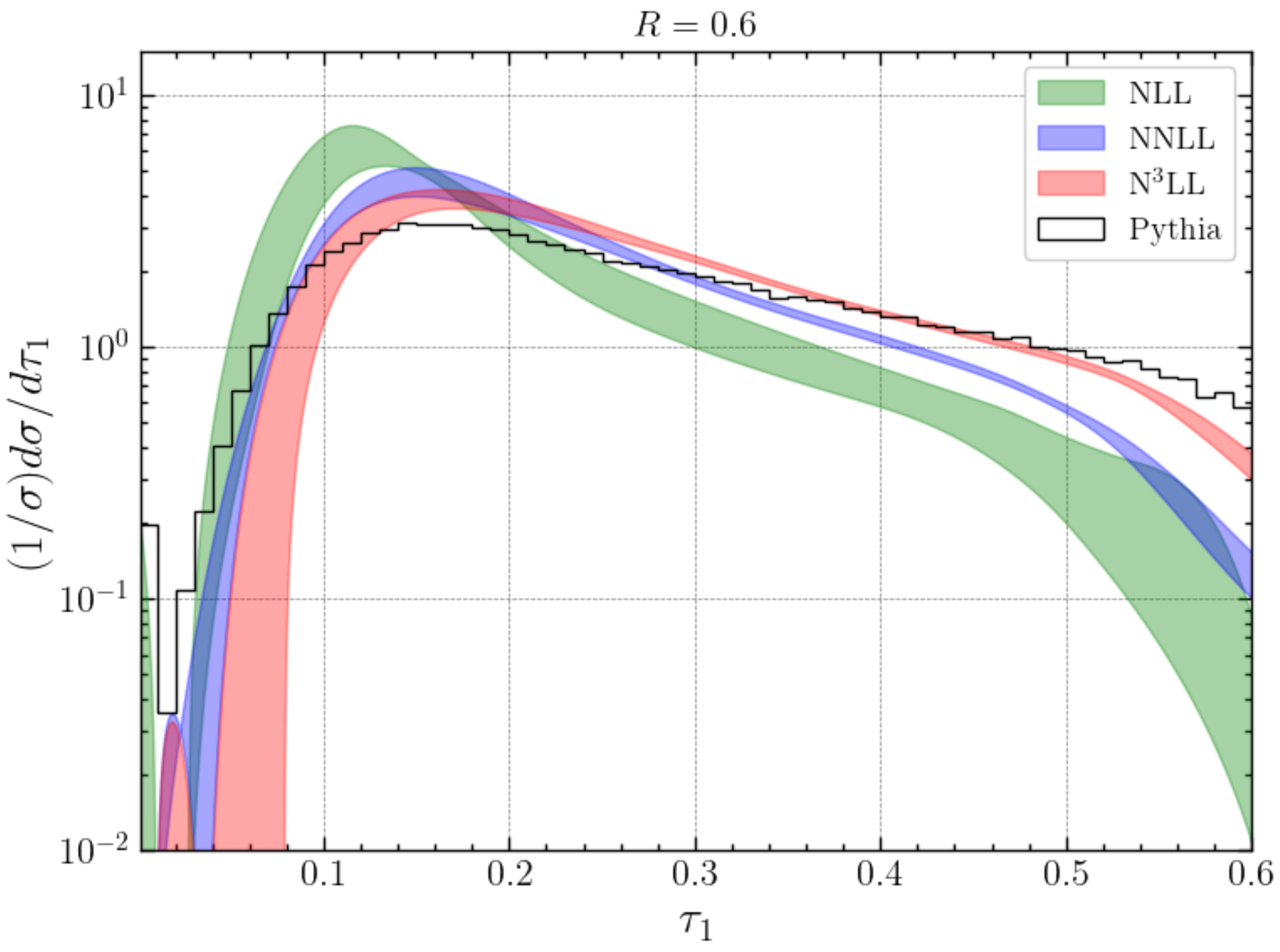
R=0.7



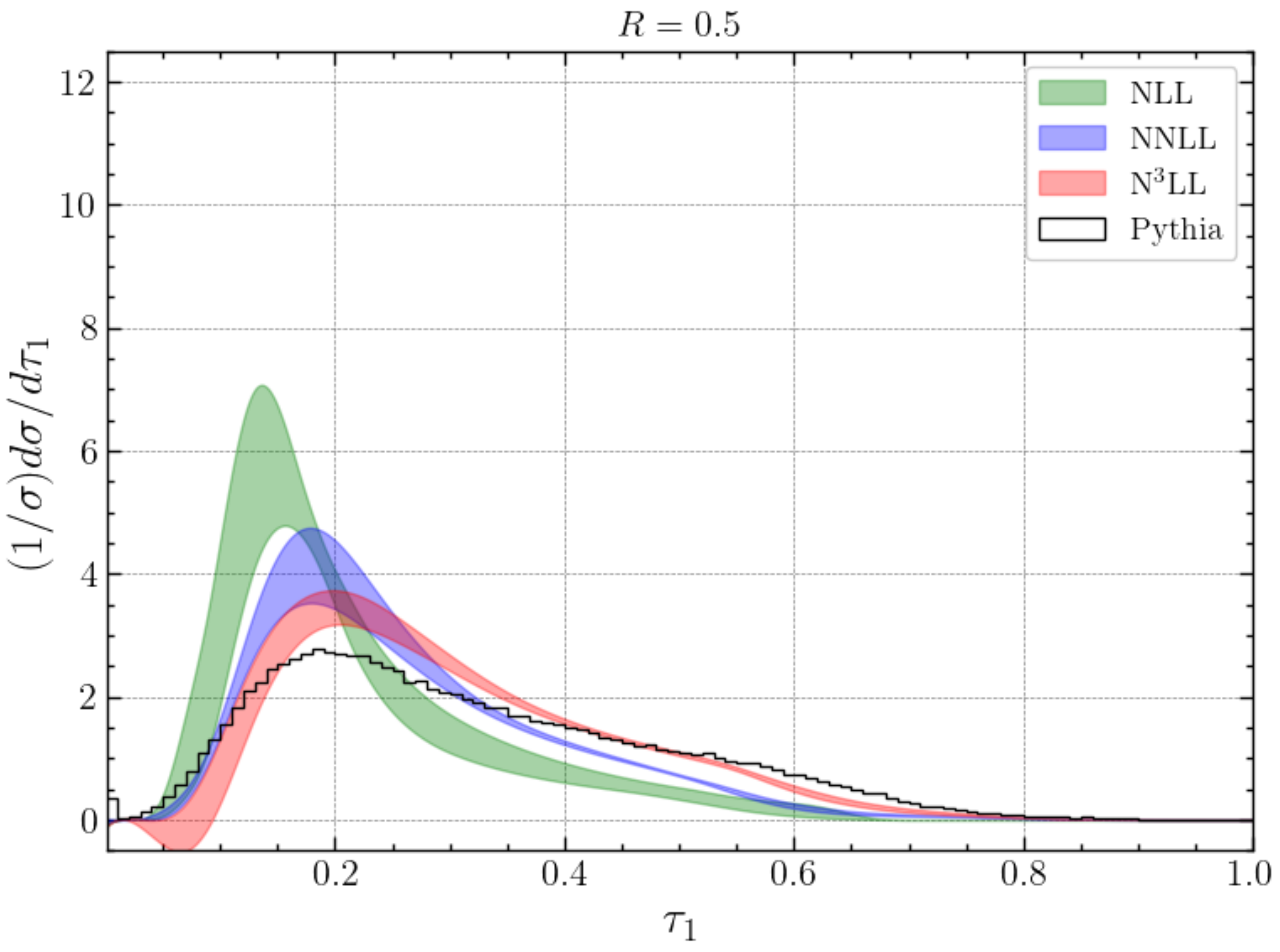
$R = 0.7$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
$R = 0.7$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1

R=0.6

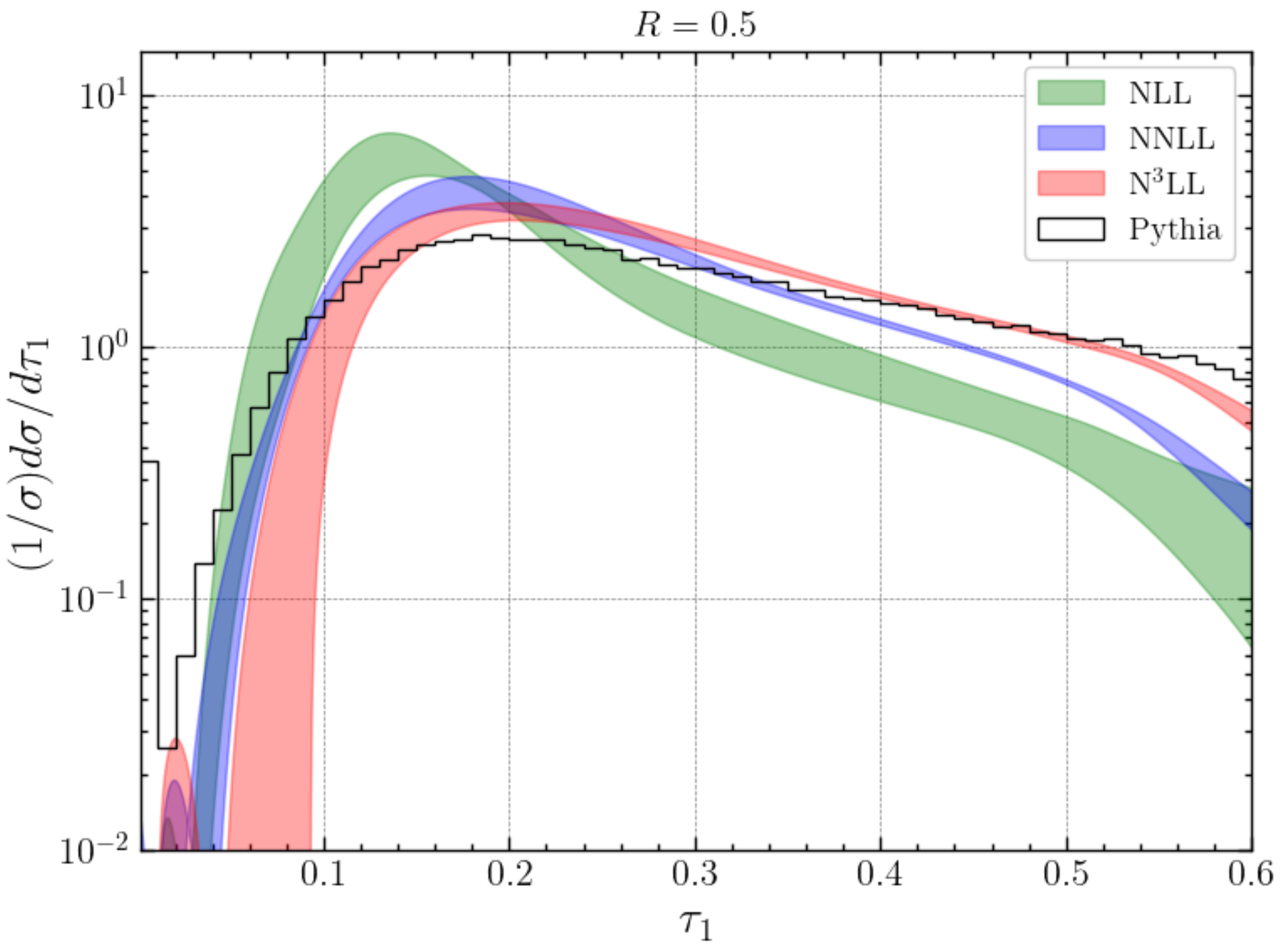

R = 0.6
NLL
NNLL
N³LL
Pythia
(1/σ)dσ/dτ₁
τ₁


R = 0.6
NLL
NNLL
N³LL
Pythia
(1/σ)dσ/dτ₁
τ₁

R=0.5



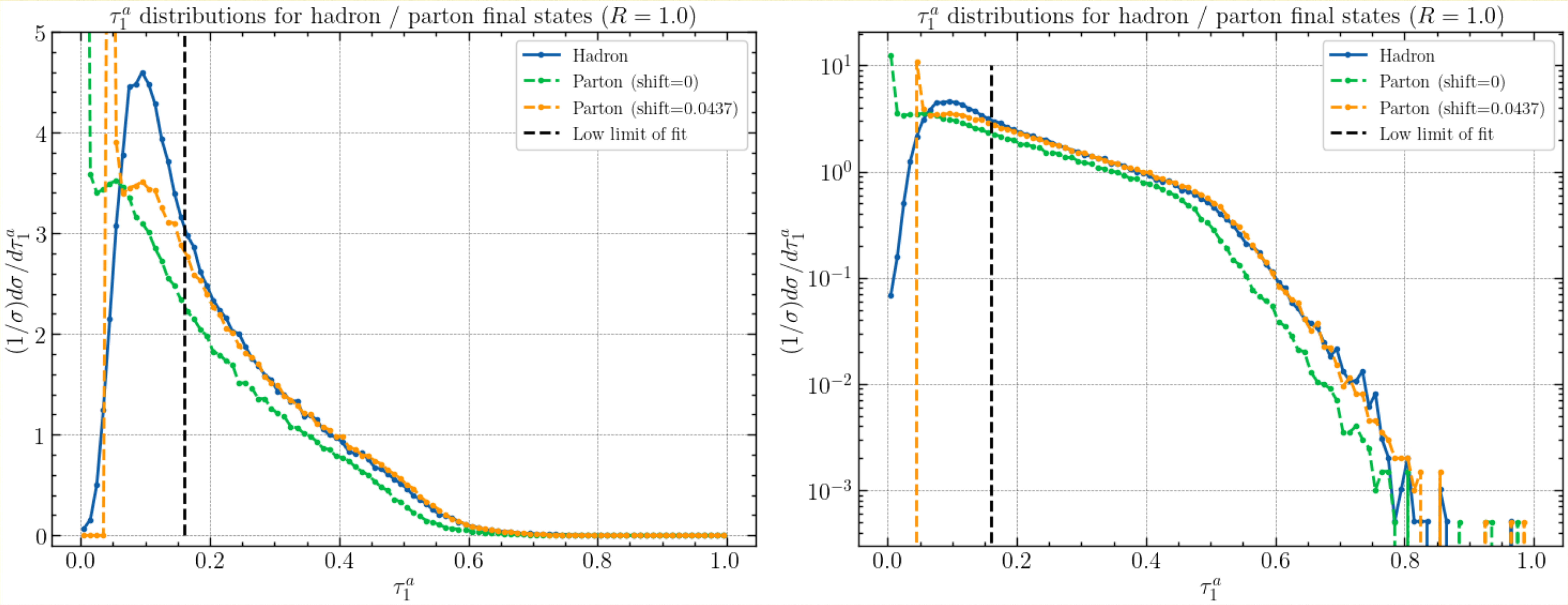
$R = 0.5$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0



$R = 0.5$
NLL
NNLL
N^{3}LL
Pythia
$(1/\sigma)d\sigma/d\tau_1$
τ_1
10^{1}
10^{0}
10^{-1}
10^{-2}
0.1
0.2
0.3
0.4
0.5
0.6

NONPERTURBATIVE SHIFT FROM PYTHIA

- Test effect of hadronization model in Pythia:

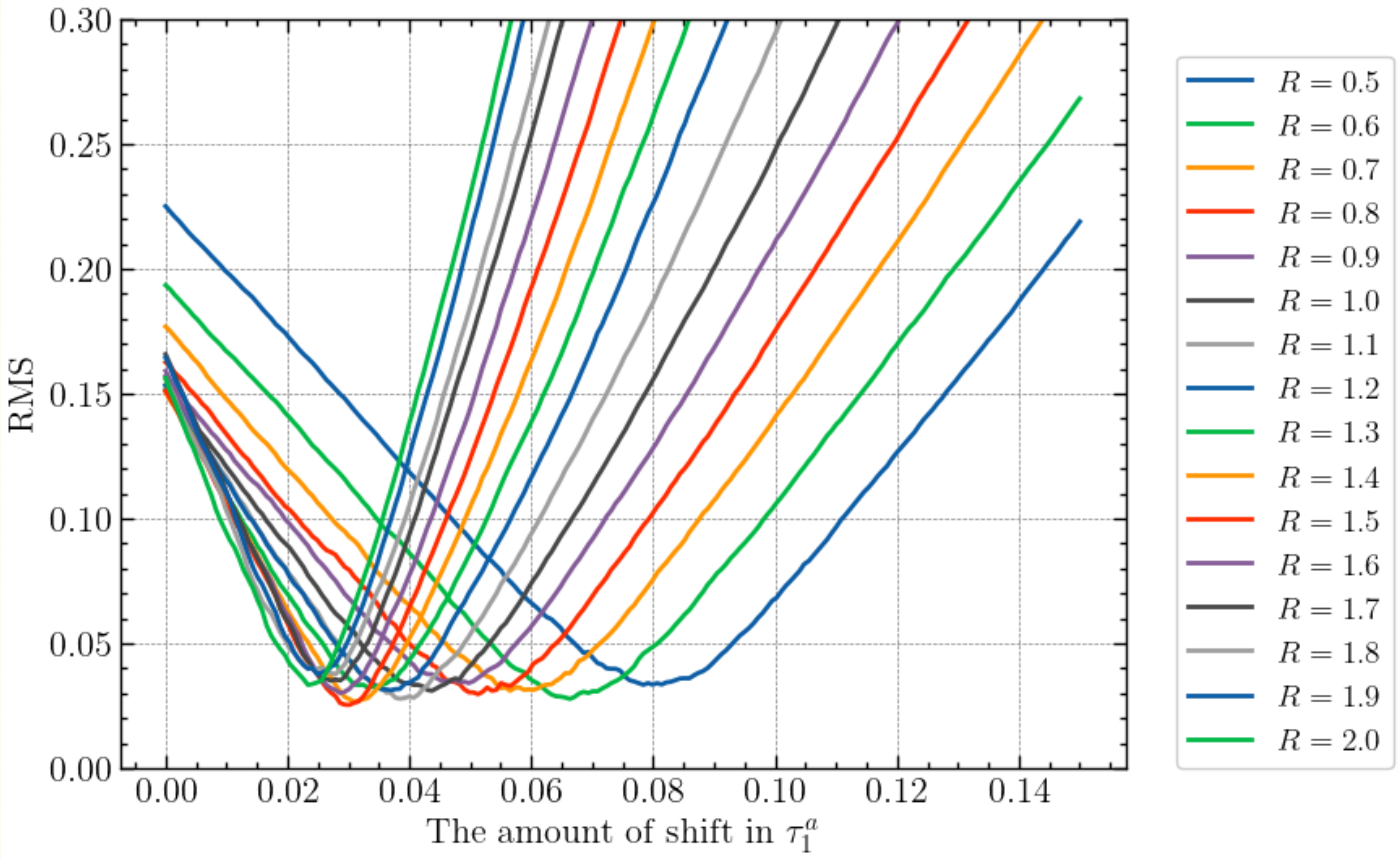


R DEPENDENCE OF SHIFT

- Perform fit for shift by RMS minimization:

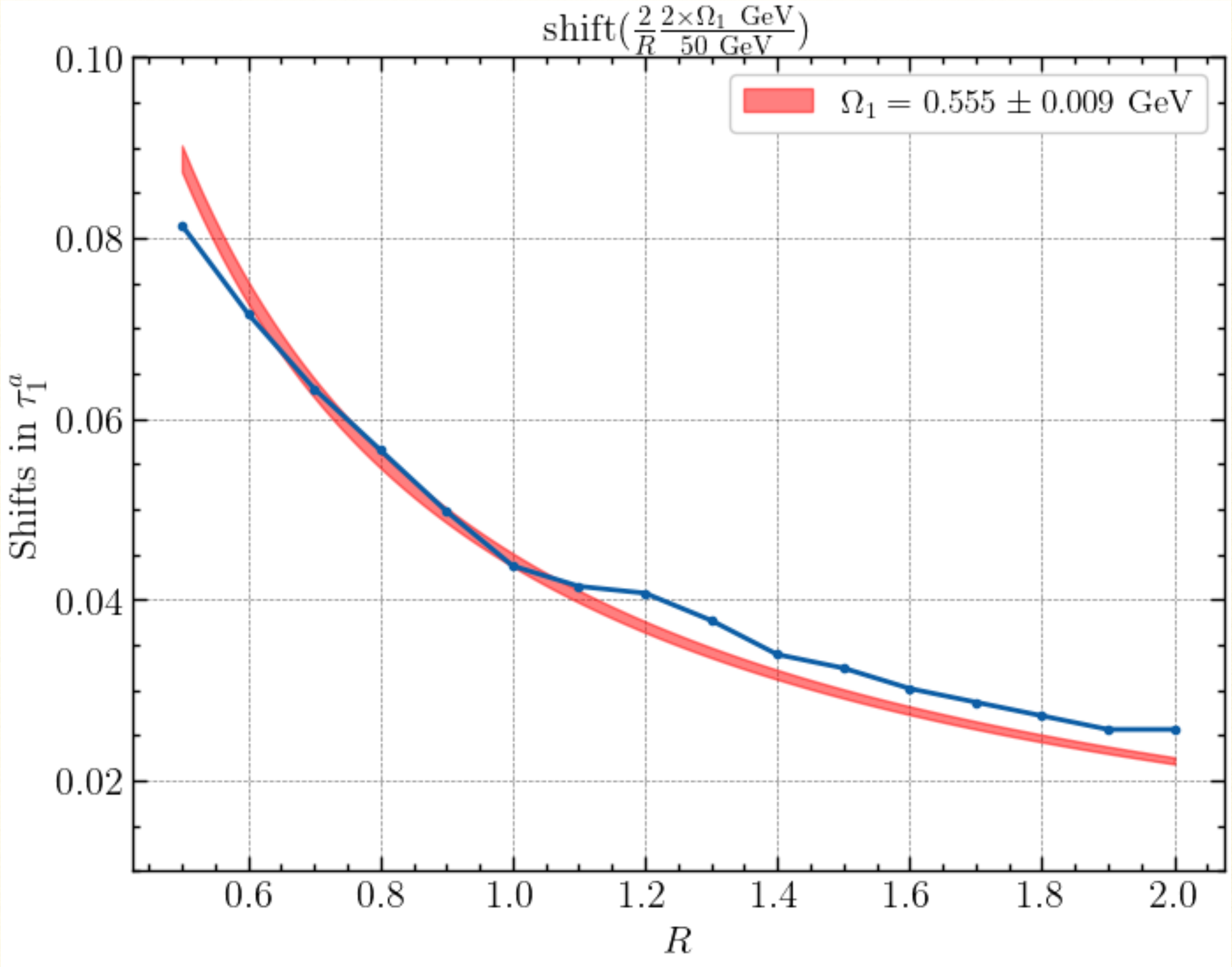
$$\frac{d\sigma}{d\tau_1^C}(\tau_1^C) \to \frac{d\sigma}{d\tau_1^C}(\tau_1^C - \Delta\tau)$$

$$\Delta\tau = \frac{4}{R}\frac{\Omega_1}{Q}$$



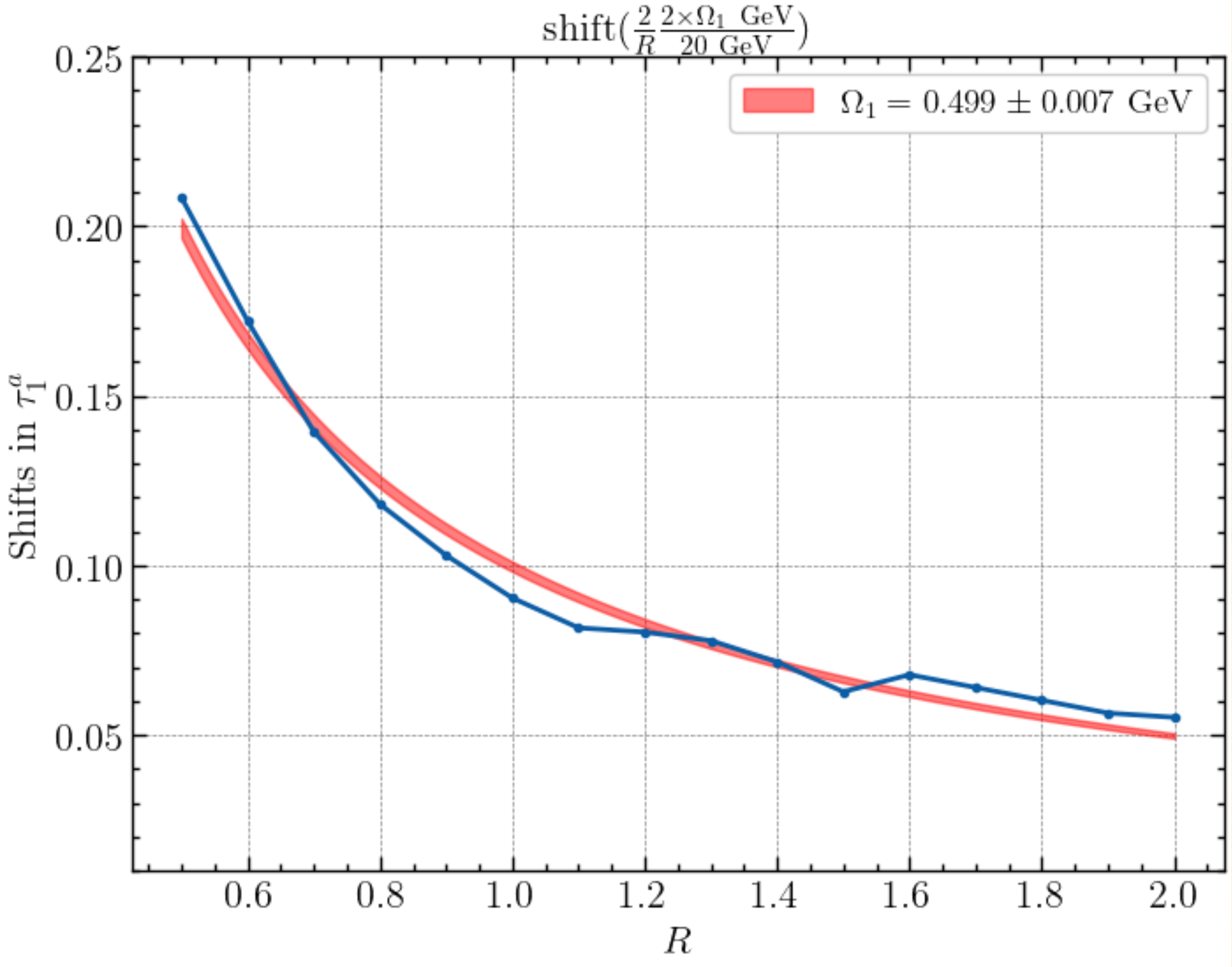
R DEPENDENCE OF SHIFT

- Q = 50 GeV, x = 0.05



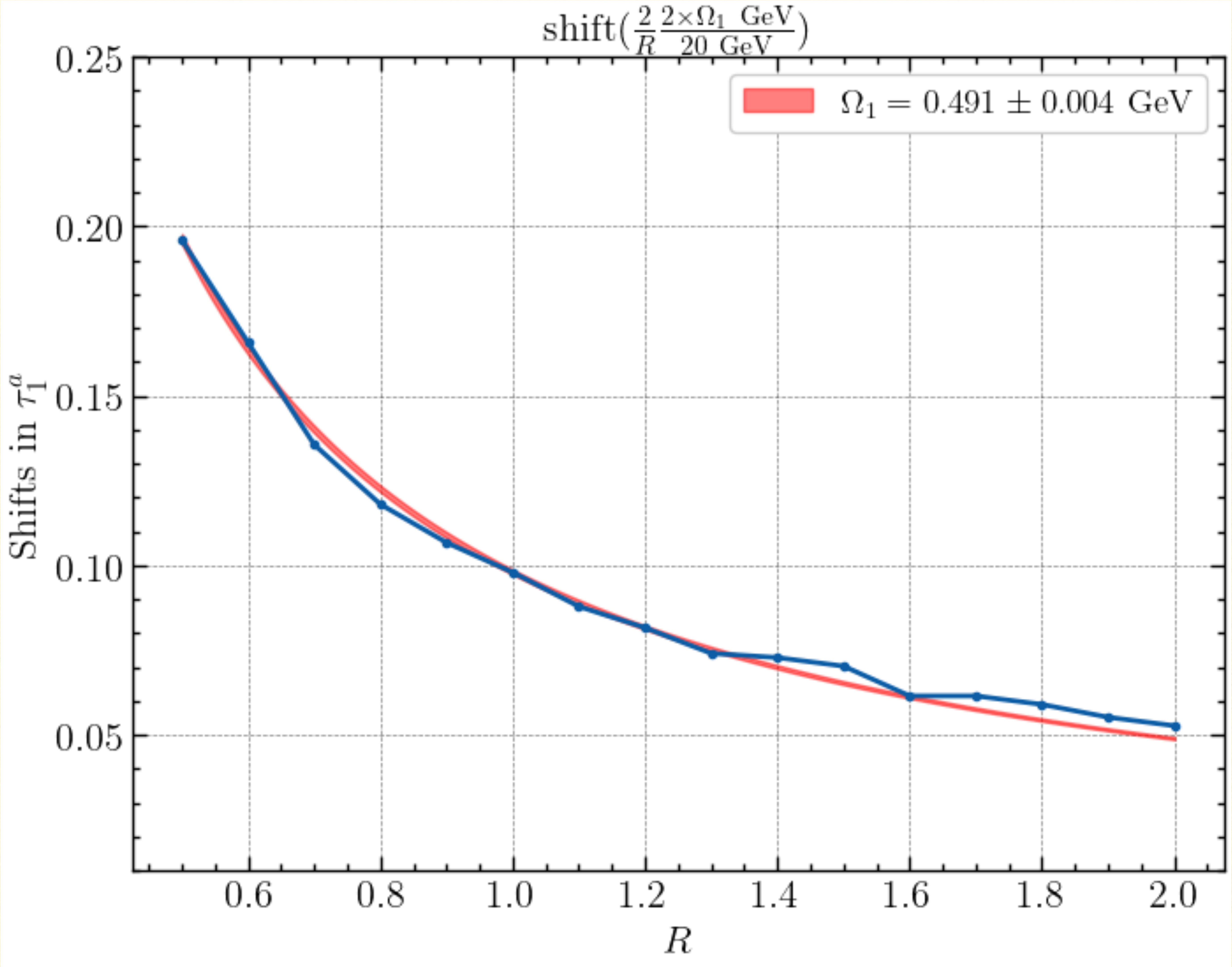
R DEPENDENCE OF SHIFT

- Q = 20 GeV, x = 0.05



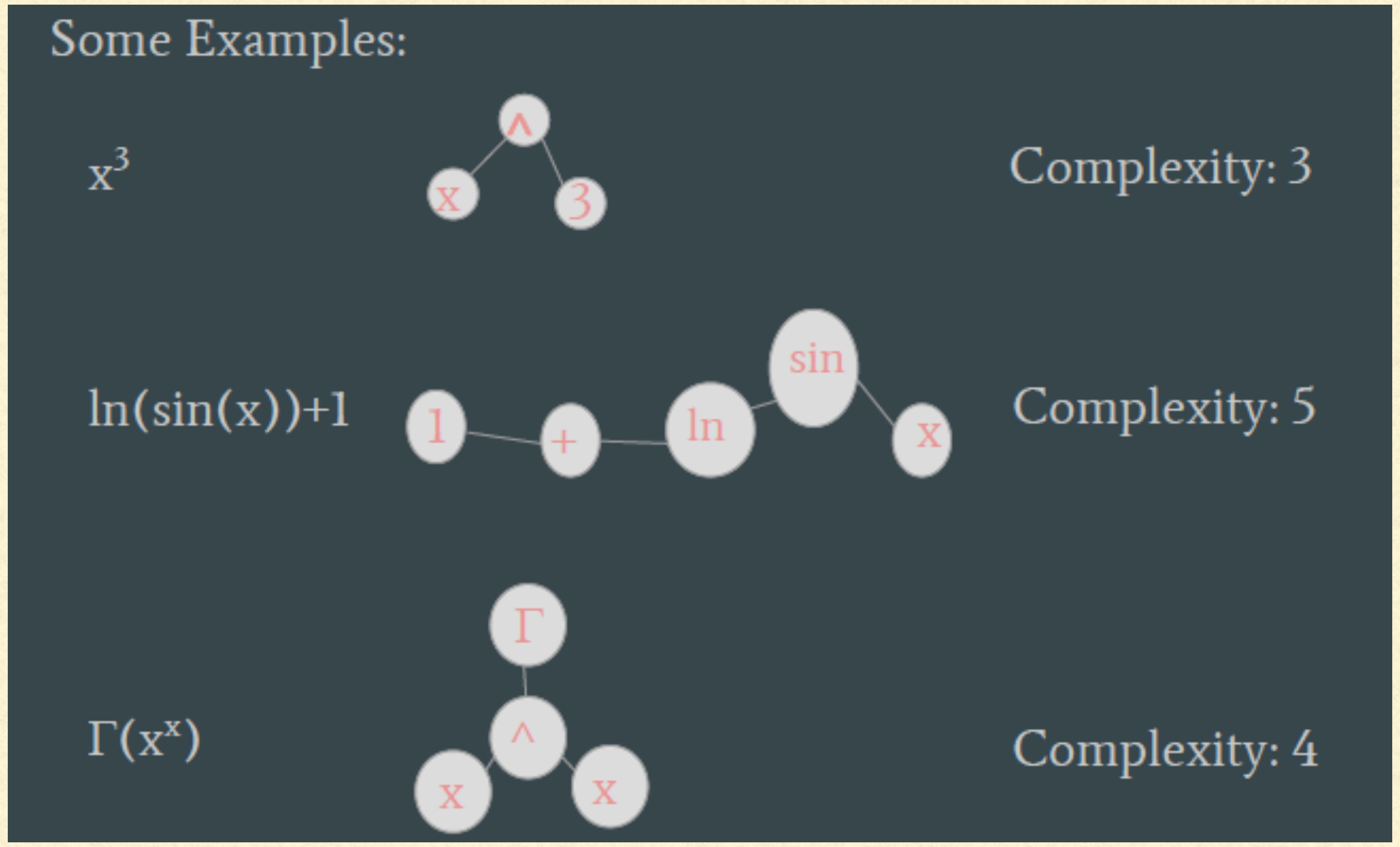
R DEPENDENCE OF SHIFT

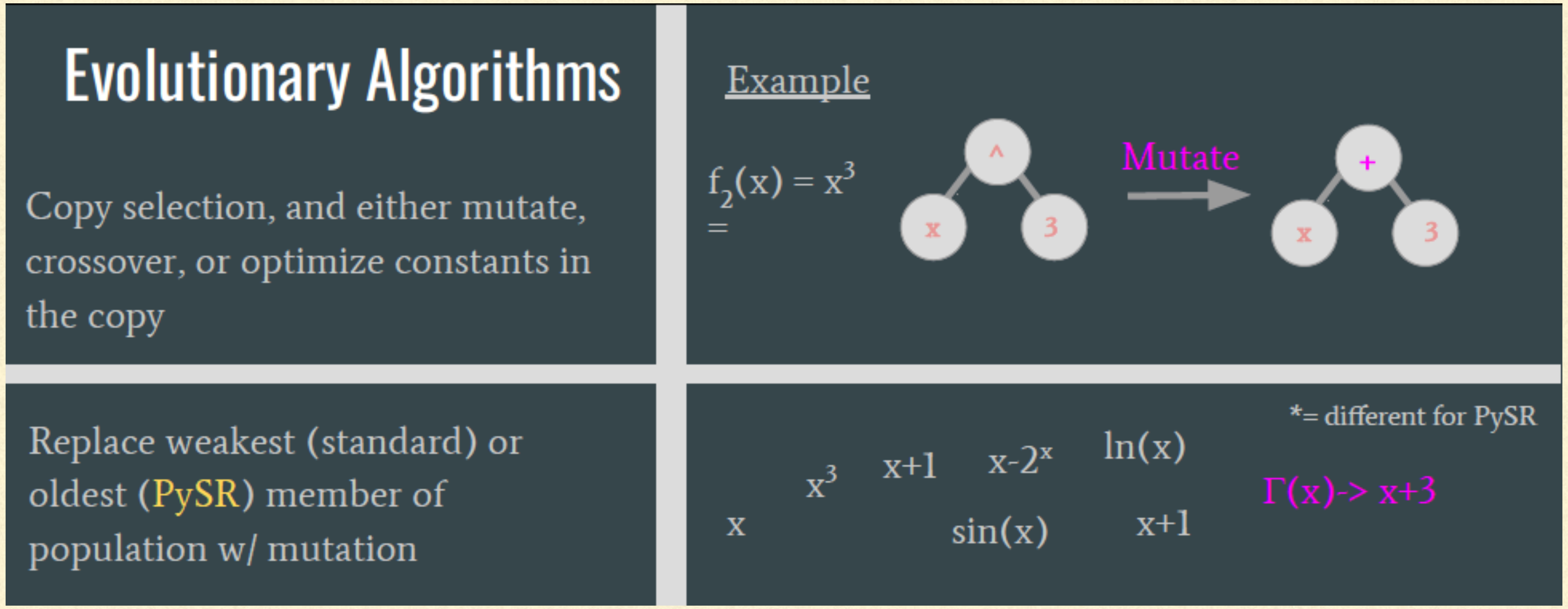
- Q = 50 GeV, x = 0.25



SYMBOLIC REGRESSION FOR FUNCTIONAL FORM

- Let Symbolic Regression learn the functional form of $\Delta\tau(R)$: PYSR library
- Minimize the Loss Function $$\mathcal{L}_{\text{best-fit}} = \frac{1}{N} ||\sigma_{\text{PT}}(\tau + \Delta(\tau, R), R) - \sigma(\tau, R)||^2$$
  - while limiting complexity for $\Delta(\tau, R)$
  - Learn and evolve:

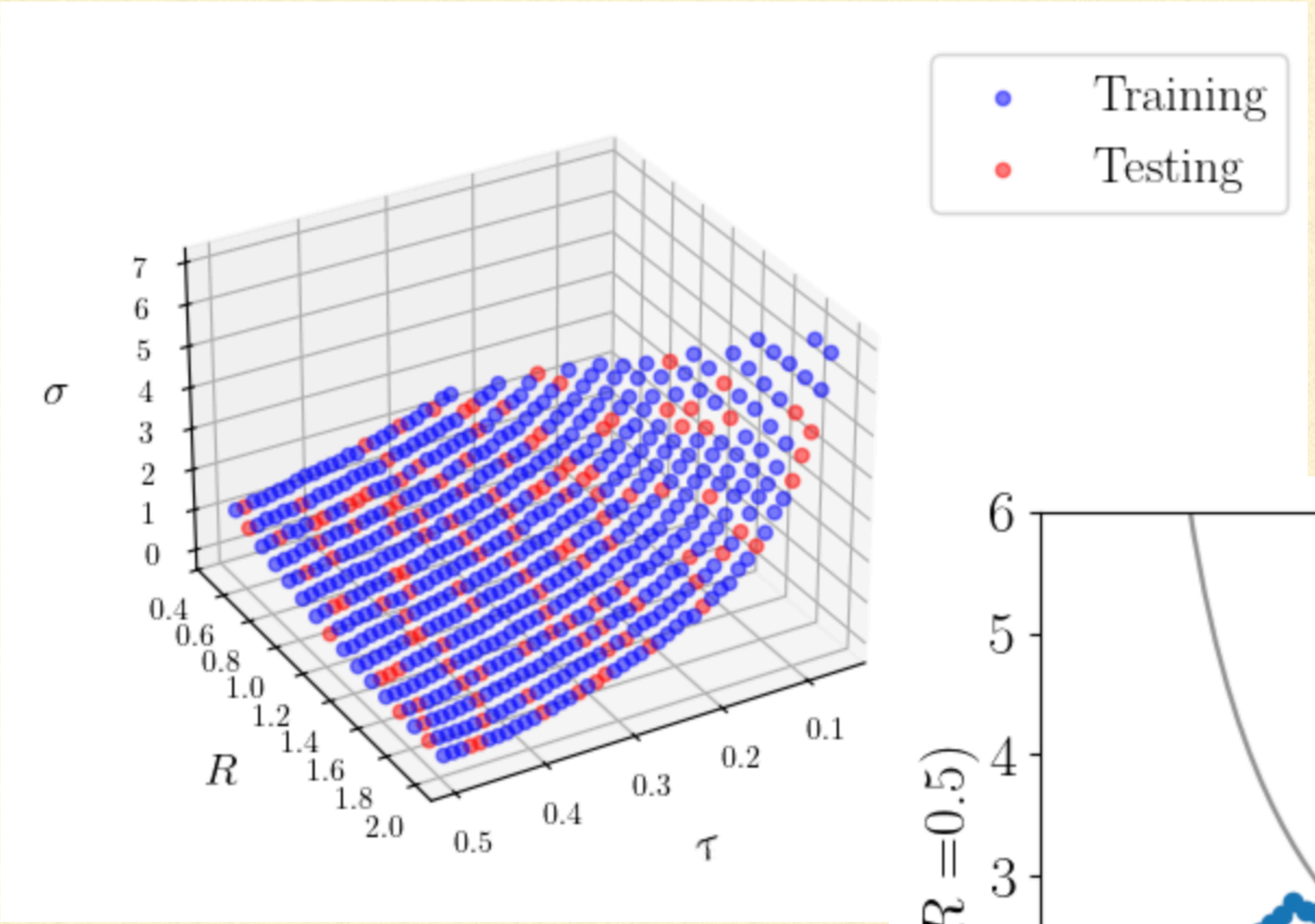


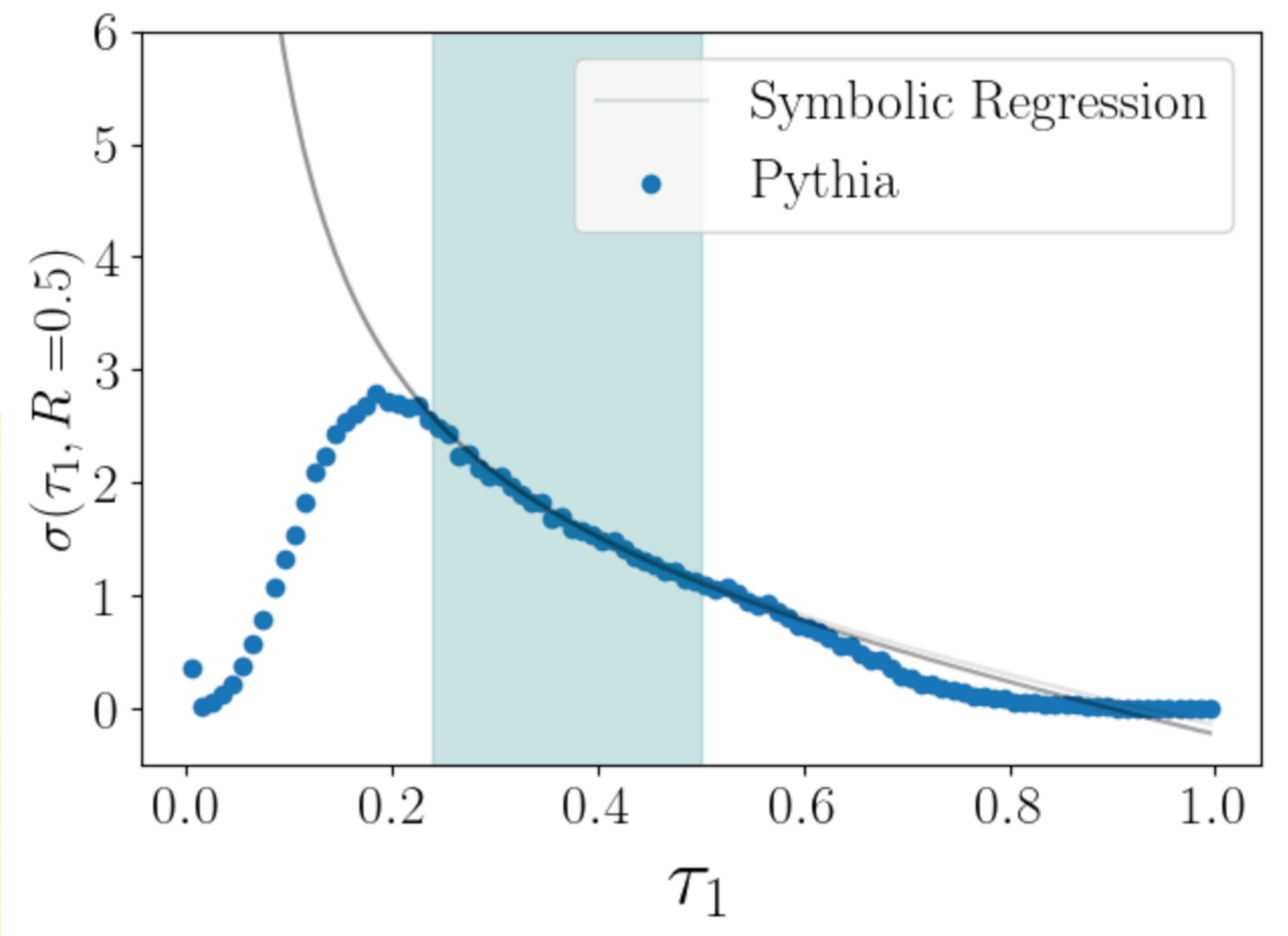


[from A. Dotson]

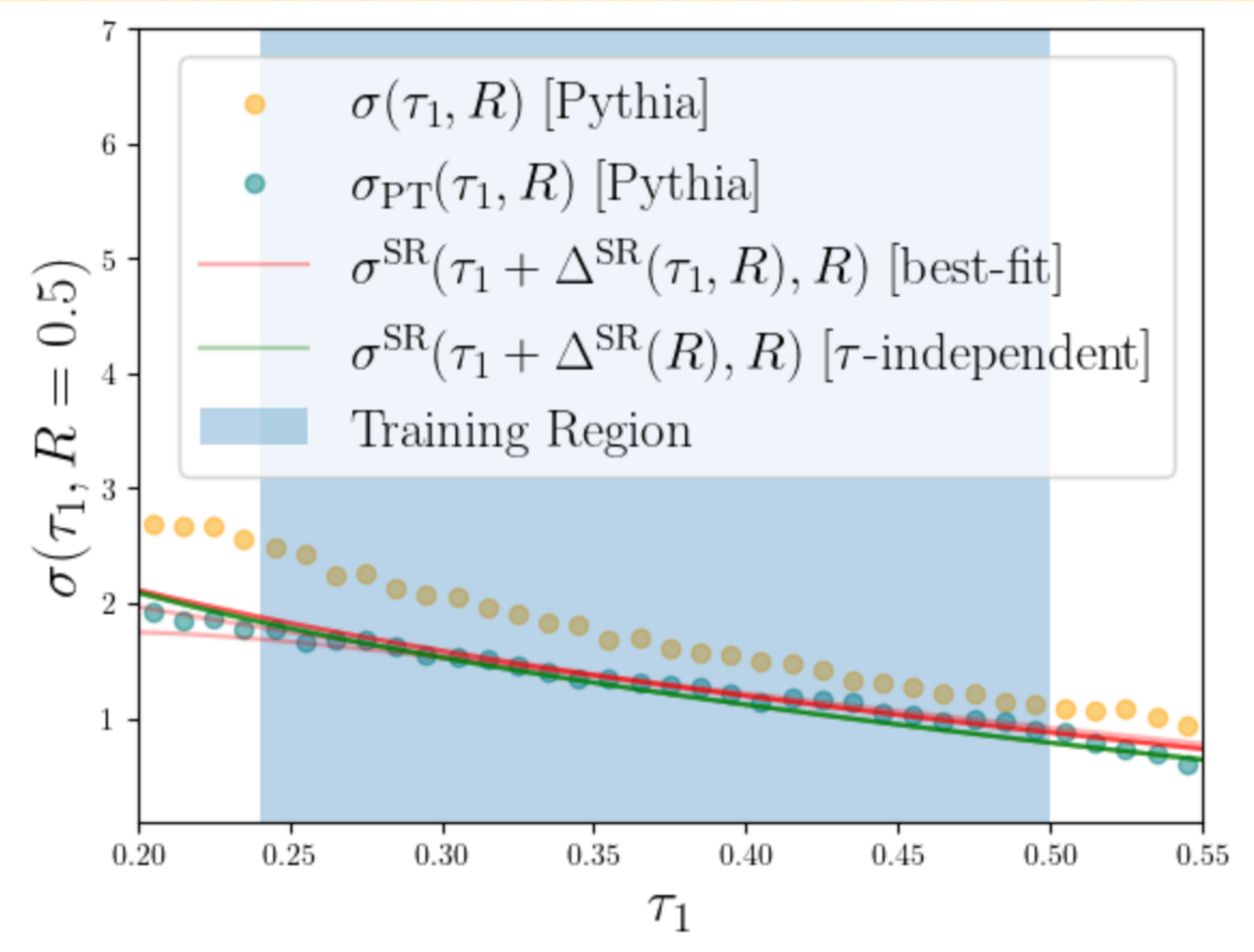
SYMBOLIC REGRESSION FOR FUNCTIONAL FORM

- Learn cross section:





- Learn shift:



SYMBOLIC REGRESSION FOR FUNCTIONAL FORM

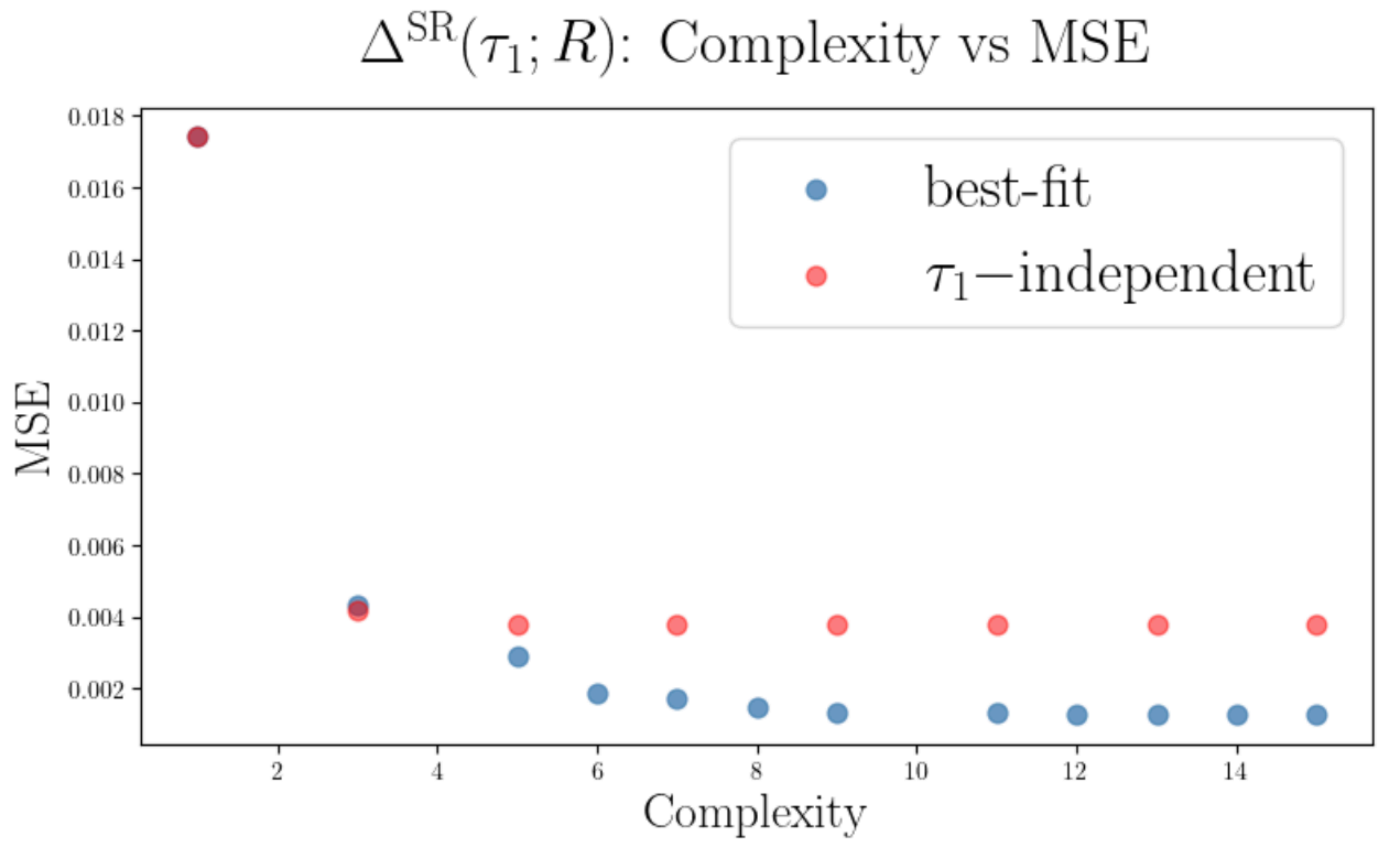
- Impose $\Delta = \Delta(R)$

```
— Complexity 1 | Loss 1.7420e-02 —
0.0346434860000000

— Complexity 3 | Loss 4.1875e-03 —
0.049257003
───────────
     R

— Complexity 5 | Loss 3.7773e-03 —
                0.041511405
0.005427174 + ───────────
                     R

— Complexity 7 | Loss 3.7632e-03 —
                 0.0416458475
0.0055539934 + ────────────
                      R

— Complexity 9 | Loss 3.7623e-03 —
                  0.043101504
0.004727854 + ───────────────
                    0.009582851
                R + ───────────
                         R

— Complexity 11 | Loss 3.7619e-03 —
                    0.043101504
0.004727854 + ────────────────────
                      0.009582851
                R + ──────────────
                    R - 0.008673818
```

- Allow $\Delta = \Delta(\tau, R)$

```
— Complexity 1 | Loss 1.7420e-02 —
0.0346445930000000

— Complexity 3 | Loss 4.3150e-03 —
0.049034517
───────────
     R

— Complexity 5 | Loss 2.9196e-03 —
0.056628764
───────────
  R + t1

— Complexity 6 | Loss 1.8540e-03 —
           -t1
0.0582787·e
────────────
     R

— Complexity 7 | Loss 1.7234e-03 —
  0.035091266
──────────────────
R·(t1 + 0.5521093)

— Complexity 8 | Loss 1.4738e-03 —
      0.02724789
─────────────────────────
t1 + log(R + 0.47777915)

— Complexity 9 | Loss 1.3034e-03 —
          0.024682902
──────────────────────────────────
R·(t1 + 0.42737383) - 0.10246847

— Complexity 11 | Loss 1.2979e-03 —
0.0005340574·R·(t1 + 0.39350095) + 0.0225029025064279
──────────────────────────────────────────────────────
        R·(t1 + 0.39350095) - 0.10505143
```

$$\Delta\tau \sim \frac{0.05}{R} = \frac{4\Omega_1}{QR} \Rightarrow \Omega_1 \sim 600\,\mathrm{MeV}$$



CONCLUSIONS & FUTURE DIRECTIONS

- Possibility to use R in a Centauric event shape to disentangle perturbative and nonperturbative contributions in QCD
- Have theoretical ingredients for high accuracy
- Experimental measurements from HERA and in future from EIC would be nice
- Resummation of logs of R will help extend applicability to smaller R
- Fun laboratory for ML techniques

BACKUPS

OTHER ALGORITHMS

- Spherically invariant (SI) anti-k_T
  - replace: $$f_{ij}^{SI}(R) = \frac{1-\cos\theta_{ij}}{1-\cos R} \Rightarrow \frac{1-(\cos\Delta\phi_{ij})/(\cosh y_i \cosh y_j) - \tanh y_i \tanh y_j}{1-\cos R}$$

$$\Theta_{\mathrm{J-SI}}(r,y,\phi) = \Theta\left(\ln\tan\frac{R}{2} - y\right) \Rightarrow \Omega^{J,B}_{\mathrm{J-SI}} = C^{J,B}_{\mathrm{J-SI}}(R)\Omega_1^{\mathrm{J-scheme}}$$

$$C^J_{\mathrm{J-SI}}(R) = \tan\frac{R}{2}\,,\, C^B_{\mathrm{J-SI}}(R) = \cot\frac{R}{2}$$

- Longitudinally invariant (LI) anti-k_T
  - Have to take small *R* limit and neglect hadron masses:

$$C^J_{\mathrm{LI}}(R) = R\sqrt{1-y}\,,\, C^B_{\mathrm{LI}}(R) = \frac{1}{R\sqrt{1-y}} + \mathcal{O}(R^3) + \mathrm{masses}$$

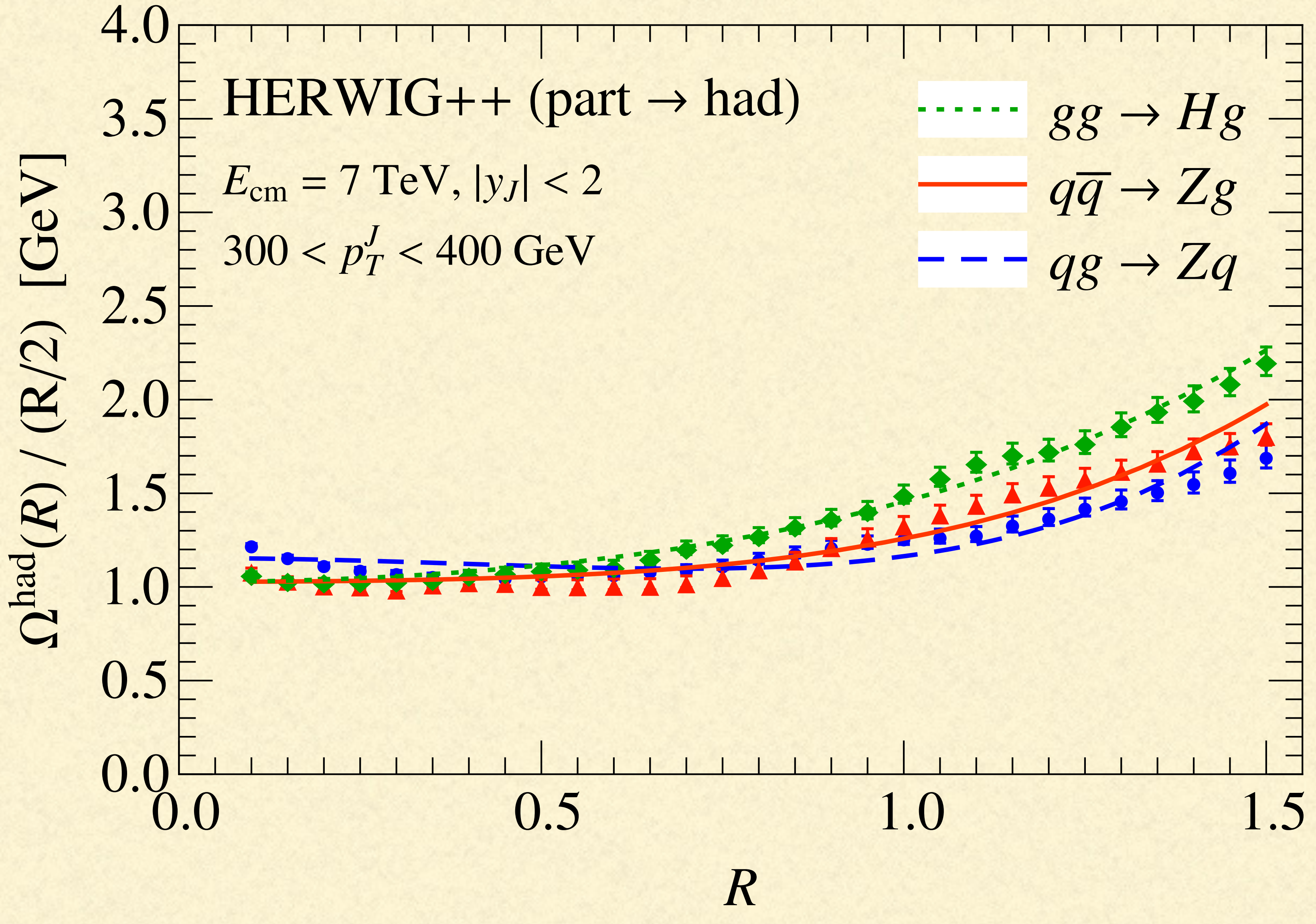
PP JET MASS

[1405.6722]

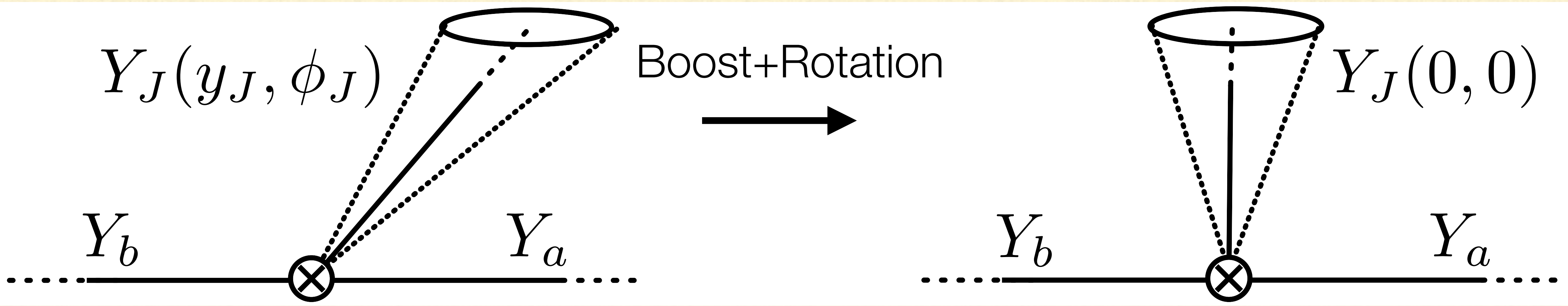
- Compare to NP corrections to m_J^2 in $pp \to J + X$

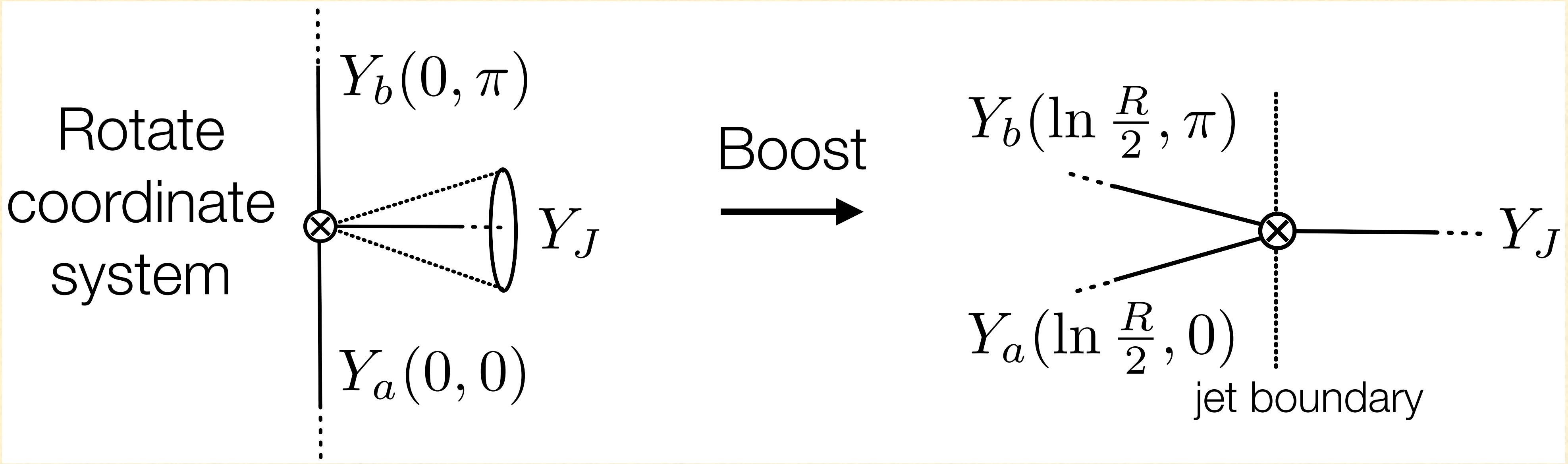
$$\Omega_\kappa(R) = \int_0^1 \mathrm{d}r \int_{-\infty}^{\infty} \mathrm{d}y \int_0^{2\pi} \mathrm{d}\phi \, f(r, y - y_J, \phi - \phi_J, R) \times \langle 0 | \bar{T}[Y_J^\dagger Y_b^\dagger Y_a^\dagger] \, \hat{\mathcal{E}}_T(r, y, \phi) T[Y_a Y_b Y_J] | 0 \rangle. \quad (8)$$

$$\Omega_\kappa(R) = \frac{R}{2}\,\Omega_\kappa^{(1)} + \frac{R^3}{8}\,\Omega_\kappa^{(3)} + \frac{R^5}{32}\,\Omega_\kappa^{(5)} + \mathcal{O}\Big[\Big(\frac{R}{2}\Big)^7\Big]$$



- Universality of shift parameter Ω_1 in small-*R* limit:

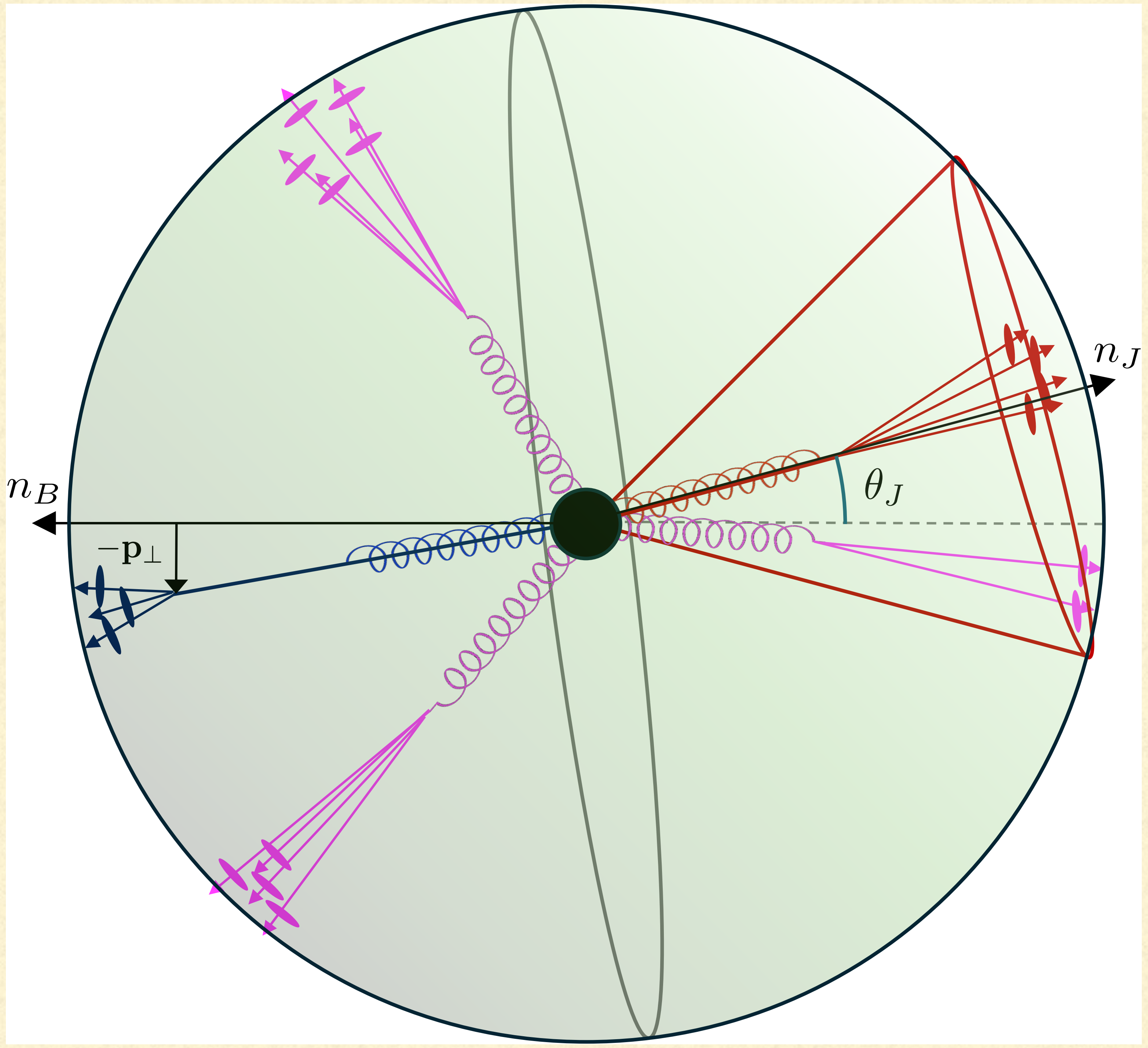


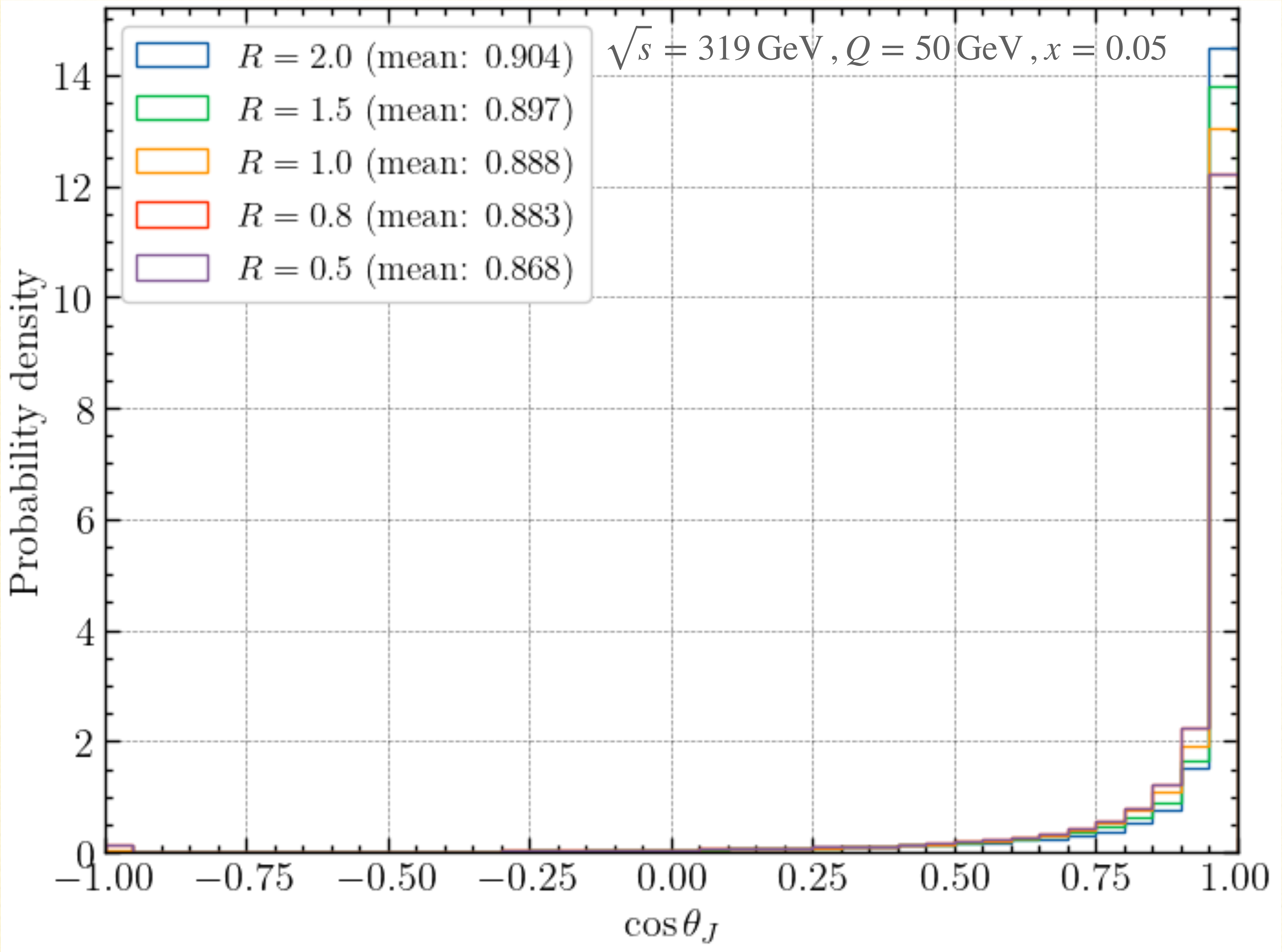


$$\Omega_q^{pp(1)} = \Omega_1^{ep}$$

θ DEPENDENCE OF FOUND JET AXIS

- Factorization required θ_J of found jet axis to be small





PERTURBATIVE SCALES

- Natural scales to minimize logs:

$$\mu_H \sim Q\,, \mu_B \sim Q\sqrt{\tau_1^C}\,, \mu_J \sim \frac{QR}{2}\sqrt{\tau_1^C}\,, \mu_S \sim \frac{QR}{2}\tau_1^C$$

- These do not meet at a common scale as $\tau_1^C \to \tau_1^{C,\max} \sim 1$
  (signal of *R* resummation needed for small *R*)

- To facilitate fixed-order matching, we revert to $\mu_H \sim Q\,, \mu_{B,J} \sim Q\sqrt{\tau_1^C}\,, \mu_S \sim Q\tau_1^C$

- And incorporate R dependence into transition parameters so regions track *R* dependence of $\tau_1^{C,\max}$:

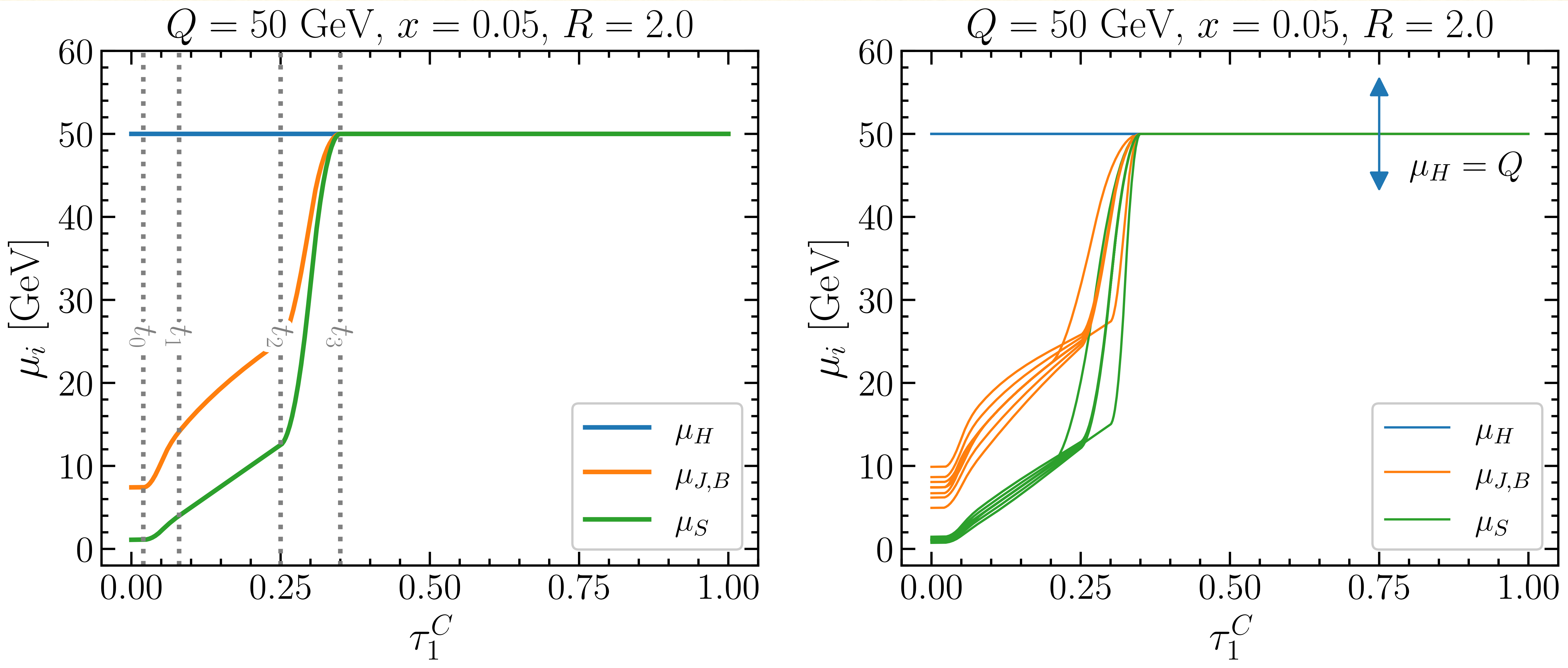
$$\mu = Q, \quad \mu_0 = 1.1 \text{ GeV}, \quad r = 1,$$

$$t_3 = 0.35\sqrt{2/R}, \quad t_2 = 0.25\sqrt{2/R},$$

$$t_1 = \min\left\{\frac{2}{R}\frac{4\text{ GeV}}{Q}, 0.6t_2\right\}, \quad t_0 = \min\left\{\frac{2}{R}\frac{1\text{ GeV}}{Q}, 0.6t_1\right\}.$$

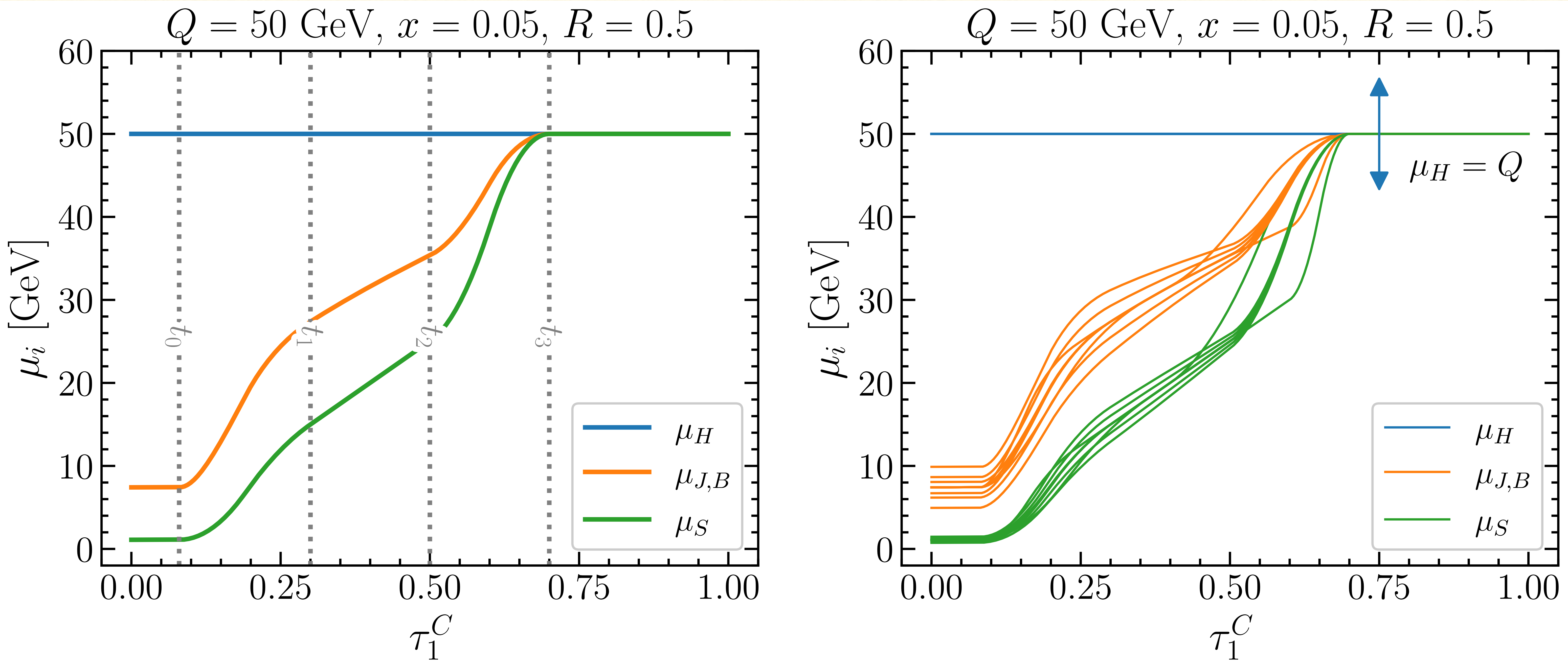
PROFILE FUNCTIONS

- Adjust to make scales merge for large τ to match to fixed-order result



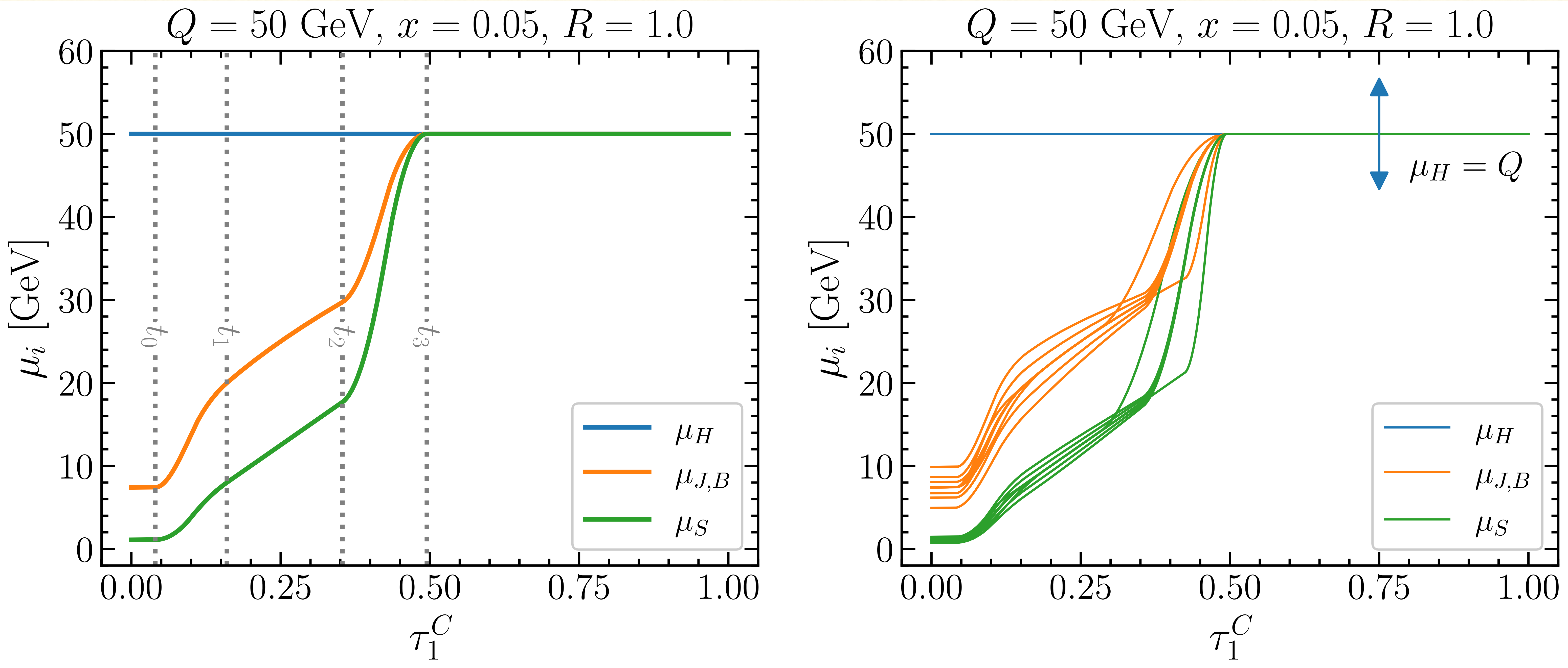
PROFILE FUNCTIONS

- Adjust to make scales merge for large τ to match to fixed-order result



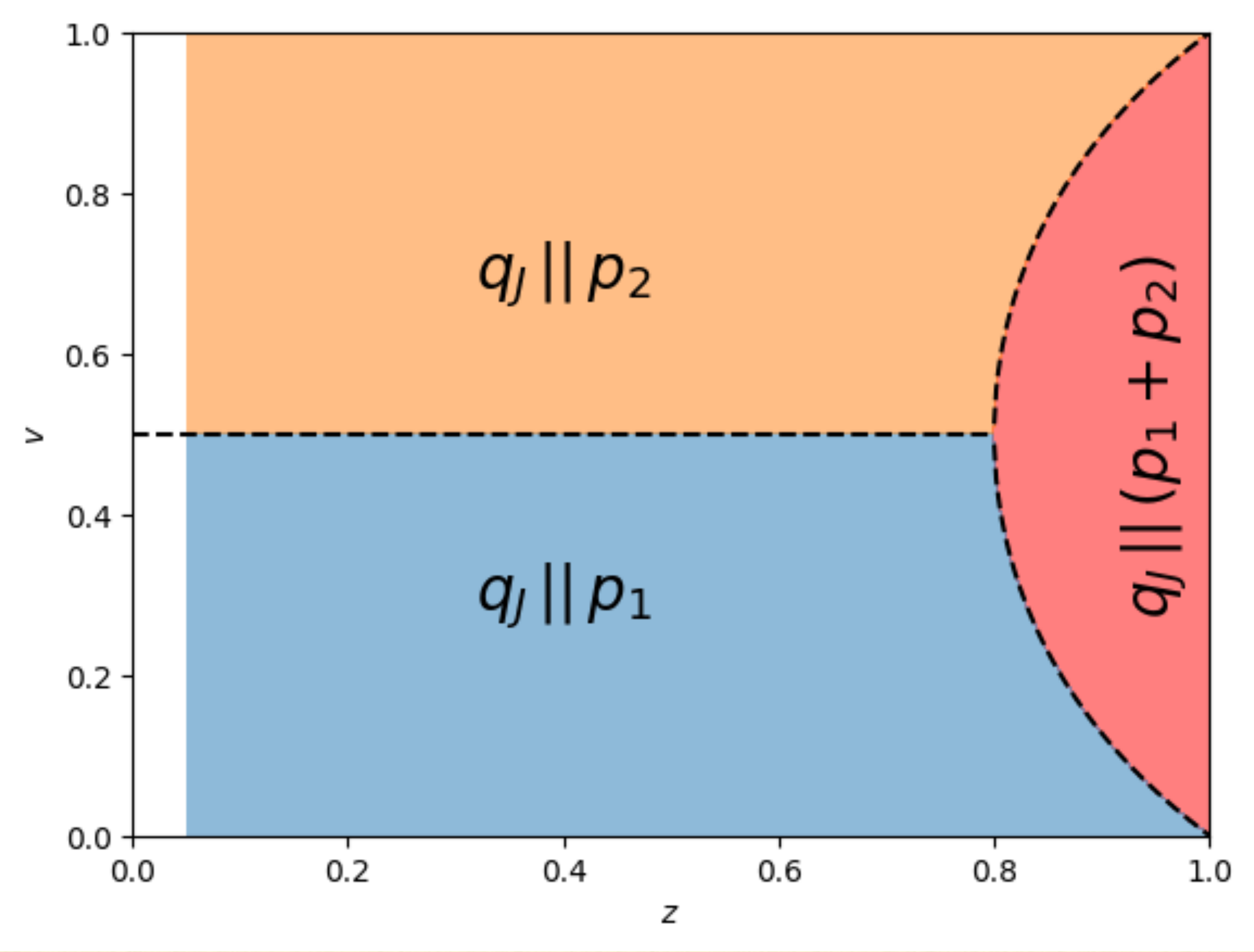
PROFILE FUNCTIONS

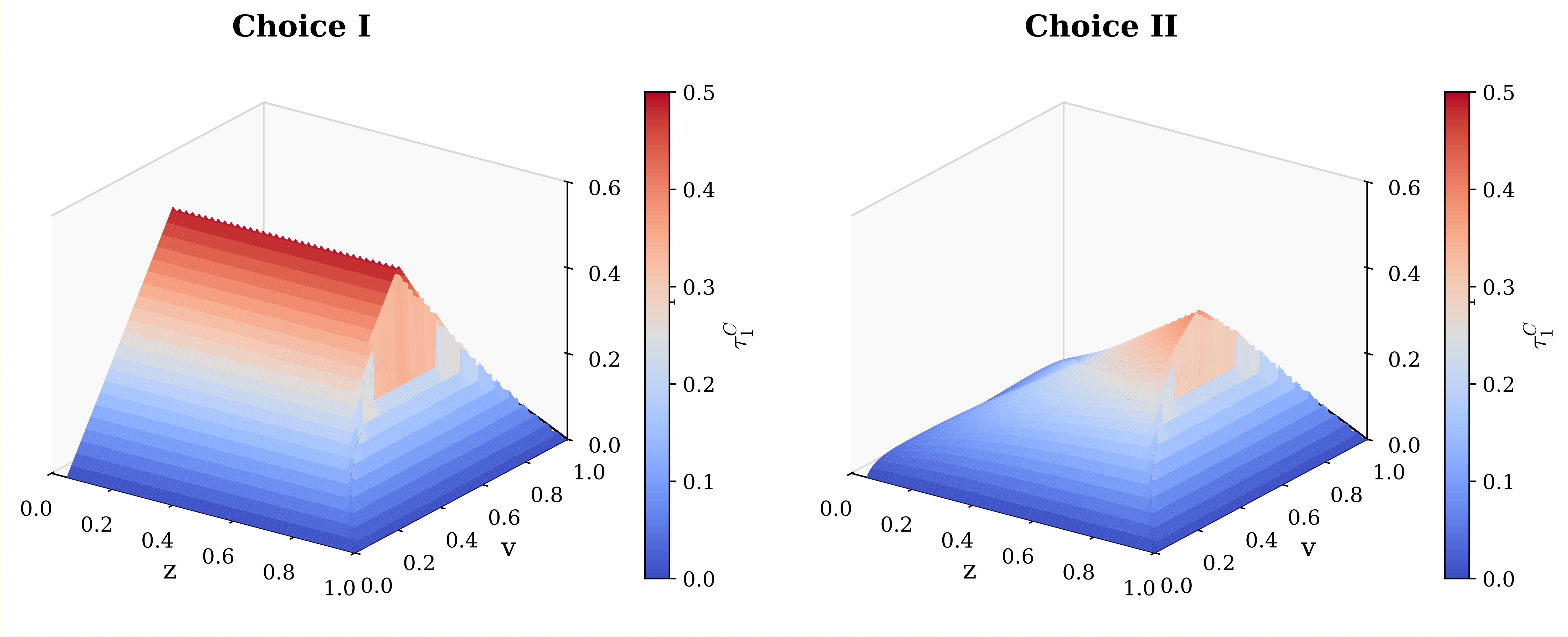
- Adjust to make scales merge for large τ to match to fixed-order result



PILEUP OF EVENTS AT $\tau_1^{C,\max}$

- τ_1^C with choice I weights pile events up at $\tau_1^{C,\max} = 0.5$ at LO, leads to discontinuity in differential cross section. Choice II smooths this out.





$$p_1^\mu = Q(1-v)\frac{n^\mu}{2} + Q\frac{1-z}{z}v\frac{\bar{n}^\mu}{2} - p_\perp^\mu$$

$$p_2^\mu = Qv\frac{n^\mu}{2} + Q\frac{1-z}{z}(1-v)\frac{\bar{n}^\mu}{2} + p_\perp^\mu$$