q_T/Q correction for unpolarized Drell-Yan process in TMD factorization

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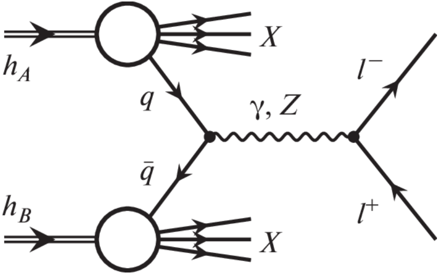
Outline

Introduction to TMD factorization theorem

Why TMD distributions?

- Low-q_T measurements. Transverse dependence of partonic distributions cannot be neglected.
- TMDPDFs encode non-perturbative information of the hadron than ordinary PDFs.
- In this work, we consider unpolarized Drell-Yan reaction.



Composite background field method

- Hadronic tensor for Drell-Yan:

$$W^{\mu\nu} = \sum_X \int \frac{d^4y}{(2\pi)^4} e^{-iqy} \langle p_1, p_2 | J^\mu_G(y) | X \rangle \langle X | J^\nu_{G'}(0) | p_1, p_2 \rangle$$

$$J^\mu_G = \bar{q}\gamma^\mu_G q, \qquad \gamma^\mu_G = g^G_R \gamma^\mu (1+\gamma^5) + g^G_L \gamma^\mu (1-\gamma^5)$$

- Write the hadronic tensor in the path integral formalism. Split original QCD fields into two background and one dynamical component

$$q(x) = q_{\bar{n}}(x) + q_n(x) + \psi(x),$$
$$A^\mu(x) = A^\mu_{\bar{n}}(x) + A^\mu_n(x) + B^\mu(x)$$

⋆ Background fields obey equations of motion.
⋆ Dynamical fields should be integrated out → leads to the computation of the electroweak (EW) current.

Power counting rules

- Let Λ be low-energy scale of QCD, such that $\lambda \sim \Lambda/Q$. TMD factorization is derived in the limit

$$Q^2 \gg \Lambda^2, \qquad Q^2 \gg \boldsymbol{q}_T^2,$$

where $Q^2 = \tau^2 - q_T^2 = 2q^+q^- + \boldsymbol{q}_T^2$.

$$q^\mu \sim \{1, 1, \lambda\}Q, \qquad y^\mu \sim \{1, 1, \lambda^{-1}\}Q^{-1}, \qquad q \cdot y \sim 1$$

- $\bar{n}$-collinear and n-collinear modes:

$$k_{\bar{n}}^\mu \lesssim \{1, \lambda^2, \lambda\}Q, \qquad k_n^\mu \lesssim \{\lambda^2, 1, \lambda\}Q$$

- "Good" and "bad" spinorial components ($\gamma^+ = n \cdot \gamma$, $\gamma^- = \bar{n} \cdot \gamma$):

$$\xi_{\bar{n}} = P_- q_{\bar{n}}, \;\; \eta_{\bar{n}} = P_+ q_{\bar{n}}, \qquad \xi_n = P_- q_n, \;\; \eta_n = P_+ q_n, \qquad P_\pm = \frac{\gamma^\pm \gamma^\mp}{2}$$

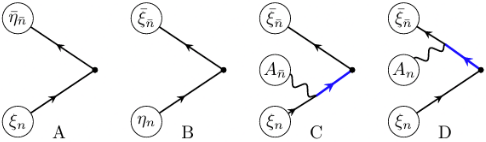
$$\xi_{\bar{n}/n} \sim \lambda, \qquad \eta_{\bar{n}/n} \sim \lambda^2$$

Electromagnetic current at NLP

- LP current is just two "good" components.

$$J^{\mu}_{\text{LP}} = \bar{\xi}_{\bar{n}} \gamma^{\mu}_{T} \xi_{n} + \bar{\xi}_{n} \gamma^{\mu}_{T} \xi_{\bar{n}}$$

- Diagrams contributing to NLP current are



$$J^{\mu}_{\text{kinematic NLP}} = -n^{\mu} \bar{\xi}_{\bar{n}} \frac{\overleftarrow{\not{\partial}}_{T}}{\overleftarrow{\partial}_{+}} \xi_{n} - n^{\mu} \bar{\xi}_{n} \frac{\overrightarrow{\not{\partial}}_{T}}{\overrightarrow{\partial}_{+}} \xi_{\bar{n}},$$

$$J^{\mu}_{\text{genuine NLP}} = \mathrm{i}g \bar{\xi}_{\bar{n}} \not{A}_{\bar{n},T} \left(\frac{\bar{n}^{\mu}}{\overrightarrow{\partial}_{-}} - \frac{n^{\mu}}{\overleftarrow{\partial}_{+}} \right) \xi_{n} - \mathrm{i}g \bar{\xi}_{n} \left(\frac{\bar{n}^{\mu}}{\overleftarrow{\partial}_{-}} - \frac{n^{\mu}}{\overrightarrow{\partial}_{+}} \right) \not{A}_{\bar{n},T} \xi_{\bar{n}}$$

TMD correlators (I)

- Twist-2. $\Gamma \in \{\gamma^+, \gamma^+\gamma^5, \mathrm{i}\sigma^{\alpha+}\gamma^5\}$

$$\Phi_{11}^{[\Gamma]}(z,b) = \langle p,s|\bar{\xi}W^\dagger(zn+b)\frac{\Gamma}{2}W\xi(0)|p,s\rangle$$

- Twist-3.

$$\Phi_{21,\mu}^{[\Gamma]}(\{z\},b) = g\langle p,s|\bar{\xi}[z_1n+b,z_2n+b]F_{\mu+}W^\dagger(z_2n+b)\frac{\Gamma}{2}W^\dagger\xi(z_3n)|p,s\rangle,$$

$$\Phi_{12,\mu}^{[\Gamma]}(\{z\},b) = g\langle p,s|\bar{\xi}W^\dagger(z_1n+b)\frac{\Gamma}{2}W(z_2n)F_{\mu+}[z_2n,z_3n]\xi|p,s\rangle$$

- Wilson line in the light-cone direction:

$$W(x) = P\exp\left(\mathrm{i}g\int_0^{-\infty} d\sigma\, A_+(x+\sigma n)\right)$$

Renormalized TMD correlators

- TMD correlators are parametrized in terms of TMDPDFs (in momentum-fraction space).

$$\Phi_{11}^{[\gamma^+]}(x,b) = f_1(x,b) + \mathrm{i}\epsilon_T^{\mu\nu} b_\mu s_{T\nu} M f_{1T}^{\perp}(x,b)$$

- Renormalized TMD correlators depend on two renormalization scales.
  1. Ultraviolet (UV) scale, μ.
  2. Rapidity scale: $\zeta/\bar{\zeta}$ for $\bar{n}/n$-collinear hadron.

- Evolution equations.

$$\mu^2 \frac{d}{d\mu^2}\Phi_{NM}^{[\Gamma]}(\{z\},b;\mu,\zeta) = (\tilde{\gamma}_N(\{z\}_N;\mu,\zeta) + \tilde{\gamma}_M(\{z\}_M;\mu,\zeta))\Phi_{NM}^{[\Gamma]}(\{z\},b;\mu,\zeta),$$

$$\zeta \frac{d}{d\zeta}\Phi_{NM}^{[\Gamma]}(\{z\},b;\mu,\zeta) = -\mathcal{D}(b,\mu)\Phi_{NM}^{[\Gamma]}(\{z\},b;\mu,\zeta)$$

Correlators with different twist configurations do not mix in the evolution.

Hadronic tensor up to NLP (I)

- Factorization: $W^{\mu\nu} \sim$ (Hard coefficient) $\otimes$ ($\bar{n}$-coll. hadron) $\otimes$ (n-coll. hadron).

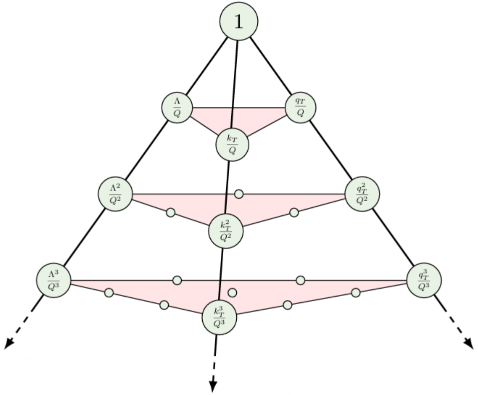
$$\widetilde{W}^{\mu\nu}_{\rm LP} = \frac{1}{4}\mathbb{C}_{\rm LP}\left(\frac{\tau^2}{\mu^2}\right)\sum_{n,m}\Big\{ \mathrm{Tr}\left(\gamma^{\mu}_{G}\overline{\Gamma}^{+}_{m}\gamma^{\nu}_{G'}\overline{\Gamma}^{-}_{n}\right)\Phi_{11}^{[\Gamma_n^+]}(x_1,b;\mu,\zeta)\overline{\Phi}_{11}^{[\Gamma_m^-]}(x_2,b;\mu,\bar{\zeta})+\dots\Big\},$$

$$\widetilde{W}^{\mu\nu}_{\rm kNLP} = \frac{\mathrm{i}}{4}\mathbb{C}_{\rm LP}\left(\frac{\tau^2}{\mu^2}\right)\sum_{n,m}\Big\{\frac{\bar{n}^{\mu}\mathrm{Tr}(\gamma^{\rho}_{G}\overline{\Gamma}^{+}_{m}\gamma^{\nu}_{G'}\overline{\Gamma}^{-}_{n}) + \bar{n}^{\nu}\mathrm{Tr}(\gamma^{\mu}_{G}\overline{\Gamma}^{+}_{m}\gamma^{\rho}_{G'}\overline{\Gamma}^{-}_{n})}{q^-} \times \Phi_{11}^{[\Gamma_n^+]}(x_1,b;\mu,\zeta)\left(\partial_\rho + \frac{\partial_\rho\mathcal{D}}{2}\ln\left(\frac{\bar{\zeta}}{\zeta}\right)\right)\overline{\Phi}_{11}^{[\Gamma_m^-]}(x_2,b;\mu,\bar{\zeta})+\dots\Big\},$$

$$\widetilde{W}^{\mu\nu}_{\rm gNLP} = \frac{-\mathrm{i}}{4}\int_{-1}^{1} du_1 du_2 du_3 \delta(u_1+u_2+u_3)\Big\{ \left[\mathbb{C}_R T^{\mu\nu\rho}_{-}(\bar{n},n) - \mathrm{i}\pi\mathbb{C}_I T^{\mu\nu\rho}_{+}(\bar{n},n)\right]\Big(\delta(x_2-u_3)\Phi_{11}^{[\Gamma_n^+]}\overline{\mathbf{\Phi}}_{\rho,\oplus}^{[\Gamma_m^-]}+\dots\Big\}$$

Power corrections

- Three types of power corrections: Λ/Q, k_T/Q, q_T/Q. [A. Vladimirov, 2307.13054]
  - ⋆ $\Lambda/Q \to$ higher-twist contributions.
  - ⋆ $k_T/Q \to$ parton transverse momentum.



Kinematic Power Corrections

- For the unpolarized TMDPDF f_1:

$$W^{\mu\nu}_{\text{KPC},f_1 f_1} \sim \mathbb{C}_{\text{LP}}\left(\frac{Q^2}{\mu^2}\right) \int d\xi_1 d\xi_2 \int d^4k_1 d^4k_2 \, \delta^{(4)}(q - k_1 - k_2)$$
$$\delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-)\delta(k_1^2)\delta(k_2^2)$$
$$\boxed{((k_1 k_2)g^{\mu\nu} - k_1^\mu k_2^\nu - k_2^\mu k_1^\nu)} f_1(\xi_1, k_T; \mu, Q^2) f_1(\xi_2, k_T; \mu, Q^2)$$

where $\xi_{1,2} = \xi_{1,2}(\boldsymbol{k}_{1T}^2, \boldsymbol{k}_{2T}^2)$.

- Comparison with LP.

$$W^{\mu\nu}_{\text{LP},f_1 f_1} \sim \mathbb{C}_{\text{LP}}\left(\frac{\tau^2}{\mu^2}\right) \int d^2\boldsymbol{k}_{1T} d^2\boldsymbol{k}_{2T} \, \delta^{(2)}(\boldsymbol{q}_T - \boldsymbol{k}_{1T} - \boldsymbol{k}_{2T})$$
$$\boxed{g_T^{\mu\nu}} f_1(x_1, k_T; \mu, \zeta) f_1(x_2, k_T; \mu, \bar{\zeta})$$

- Restoration of gauge invariance and frame invariance.

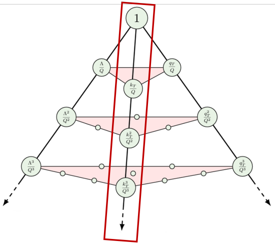
$$q_\mu W^{\mu\nu}_{\text{KPC},f_1 f_1} \sim (k_{1\mu} + k_{2\mu})((k_1 k_2)g^{\mu\nu} - k_1^\mu k_2^\nu - k_2^\mu k_1^\nu) = 0$$

Why q_T/Q correction

- Compute the angular coefficients.

$$d\sigma \sim \sum \Sigma_n S_n(\theta,\phi), \qquad \Sigma_n = \frac{4\pi\alpha_{\text{em}}^2}{3s}(-Q^2)\sum_{G,G'} \mathfrak{L}_n^{\mu\nu} W_{\mu\nu} z_{n\ell}^{GG'} \Delta_G^* \Delta_{G'}$$

- Twist-3 TMDPDFs are almost impossible to extract from data.
- So far, the cross-section is described in terms of the KPCs.



- We desire to improve the theoretical prediction for higher values of q_T.

Leading TMD approximation

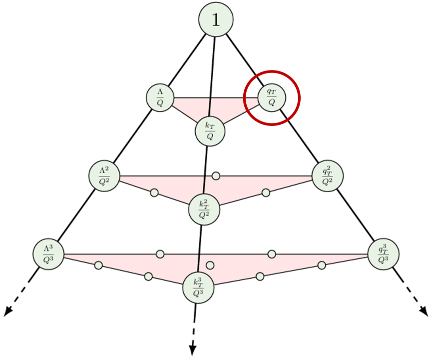
Introduction to leading TMD approximation

- Twist-three TMD correlators exhibit singular behaviour at $b \to 0$.

$$\Phi_{\text{twist-3}} = \widehat{\Phi}_{\text{twist-3}} + \frac{1}{b}\Phi_{\text{twist-2}}, \qquad \lim_{b\to 0} \widehat{\Phi}_{\text{twist-3}} = \text{finite}$$

- Second term produces q_T/Q correction.

$$d\sigma^{\text{gNLP}} = \widehat{d\sigma}^{\text{gNLP}} + \frac{q_T}{Q} d\sigma^{\text{L-TMD}}, \qquad \widehat{d\sigma}^{\text{gNLP}} \sim \frac{\Lambda}{Q}$$



- New twist-3 TMD correlators:

$$\widehat{\Phi}_{\bullet,\mu}(x_{1,2,3},b;\mu) = \Phi_{\bullet,\mu}(x_{1,2,3},b;\mu) - \frac{b^\nu}{b^2}[S_\bullet \otimes \Phi_{11}]_{\mu\nu}(x_{1,2,3},b;\mu)$$

such that $\lim_{b\to 0} \widehat{\Phi}^{[\Gamma]}_{\bullet,\mu}(x_{1,2,3},\mu) = \text{finite}$.

- Operator Product Expansion (OPE) for the twist-2 TMD correlator:

$$\Phi_{11}(x,b) = C_{2/f}(\mathbf{L}_b;\mu_{\text{OPE}}) \otimes f_{\text{coll}}(x;\mu_{\text{OPE}}) + \mathcal{O}(b^2), \quad \mathbf{L}_b = \ln(\mu_{\text{OPE}}^2 b^2)$$

This relation can be inverted perturbatively (set $\mu_{\text{OPE}} = \mu$):

$$f_{\text{coll}}(x;\mu) = C^{-1}_{2/f}(\mathbf{L}_b;\mu) \otimes \Phi_{11}(x,b) + \mathcal{O}(b^2)$$

- Twist-3 OPE:

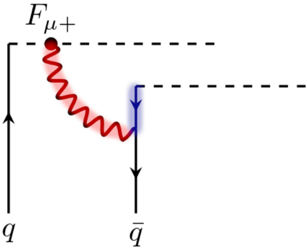
$$\Phi^\mu_\bullet(x_{1,2,3},b;\mu) = \frac{b_\nu}{b^2} C^{\mu\nu}_{3/f}(\mathbf{L}_b;\mu) \otimes f_{\text{coll}}(x;\mu) + \mathcal{O}(b^0)$$

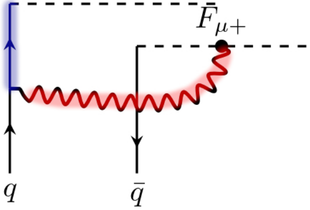
- Altogether:

$$[S_\bullet \otimes \Phi_{11}]^{\mu\nu}(x_{1,2,3},b;\mu) = C^{\mu\nu}_{3/f}(\mathbf{L}_b;\mu) \otimes C^{-1}_{2/f}(\mathbf{L}_b;\mu) \otimes \Phi_{11}(x,b)$$

LO calculation (I)

- Quark contribution.





Left:

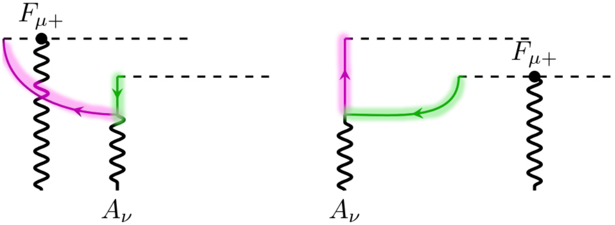
$$ig^2 C_F \int d^d y\, \bar{\xi}(z_1 n + b)\frac{\Gamma}{2}P_- \$(z_3 n - y)(\delta^\alpha_\mu \gamma^+ - n^\alpha \gamma_{T\mu})q(y)\partial_\alpha D(y - z_2 n - b)$$

Right:

$$ig^2 \int d^d y\, \bar{q}(y)(\delta^\alpha_\mu \gamma^+ - n^\alpha \gamma_{T\mu})\$(z_1 n + b - y)P_+\frac{\Gamma}{2}\xi(z_3 n)\partial_\alpha D(y - z_2 n)$$

LO calculation (II)

- Gluon contribution.



Left:

$$-ig^2 T_F \int d^d y \ \mathrm{Tr}\left\{P_- \$(z_3 n - y)\gamma^\nu \$(y - z_1 n - b) P_+ \frac{\Gamma}{2}\right\} F_{\mu+}(z_2 n + b) A_\nu(y)$$

Right:

$$-ig^2 T_F \int d^d y \ \mathrm{Tr}\left\{P_- \$(z_3 n - y)\gamma^\nu \$(y - z_1 n - b) P_+ \frac{\Gamma}{2}\right\} F_{\mu+}(z_2 n) A_\nu(y)$$

- We got some expression from previous diagrams. For example:

$$\Phi^{[\Gamma]}_{21,\mu}(x_{1,2,3}, b; \mu)\Big|_{\text{quark}} = a_s(\mu)\frac{C_F}{2}\frac{b^\nu}{b^2}(\theta(x_2, x_3) - \theta(-x_2, -x_3))$$
$$\times \sum_k \text{Tr}\left[\overline{\Gamma}_k^- \left(\gamma_\mu\gamma_\nu - \frac{x_3}{x_1}\gamma_\nu\gamma_\mu\right)\Gamma\right]\Phi_{11}^{[\Gamma_k^+]}(-x_1, b) + \mathcal{O}(b^0, a_s^2)$$

- First, substitute parametrization for the twist-2 TMD correlator. Then, compare with the parametrization for the twist-3 TMD correlator.

  We get twist-3 TMDPDFs in terms of quark and gluon twist-2 TMDPDFs. Notation: $\theta_{ij} = \theta(x_i, x_j) - \theta(-x_i, -x_j)$.

$$\mathbf{f}_\ominus^\perp\Big|_{\text{L-TMD}} = \frac{1}{b^2M^2}\left[2a_sC_F\theta_{23}\left(\frac{x_1 - x_3}{x_1}f_1(-x_1, b) - 2f_1(x_3, b)\right)\right.$$
$$\left. + \frac{a_s}{2}\theta_{13}f_g(-x_2, b)\right] + \mathcal{O}(a_s^2)$$

Final result

- gNLP coefficients in the L-TMD approximation.

$$\Sigma_1^{\text{L-TMD}} = \frac{8\pi\alpha_{\text{em}}^2}{3N_c s Q^2} \sum_{f,G,G'} Q^4 \Delta_G^* \Delta_{G'} \boxed{\frac{-2|\boldsymbol{q}_T|}{Q}} z_{+\ell}^{GG'} z_{+f}^{GG'} \mathcal{J}_+^{\text{L-TMD}},$$

$$\Sigma_3^{\text{L-TMD}} = \frac{8\pi\alpha_{\text{em}}^2}{3N_c s Q^2} \sum_{f,G,G'} Q^4 \Delta_G^* \Delta_{G'} \boxed{\frac{-4|\boldsymbol{q}_T|}{\tau}} z_{-\ell}^{GG'} z_{-f}^{GG'} \mathcal{J}_-^{\text{L-TMD}}$$

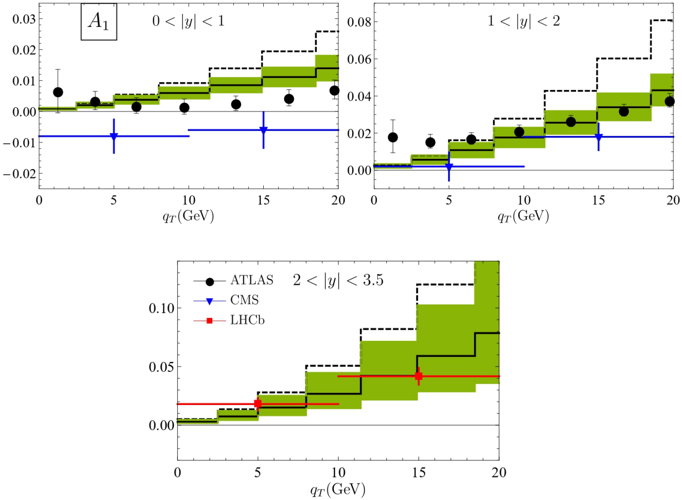
- Convolutions finite at $q_T \to 0$. First TMDPDF evaluated at (x_1, b). Second one evaluated at (x_2, b).

$$\mathcal{J}_\pm^{\text{L-TMD}} = \frac{1}{2}\int_0^\infty \frac{b\,db}{2\pi}(J_0(b|\boldsymbol{q}_T|) + J_2(b|\boldsymbol{q}_T|))\left(\frac{\tau^2}{\zeta_\mu}\right)^{-2\mathcal{D}(b,\mu)} \Big\{ s_{qq}\otimes f_1\bar{f}_1 \pm s_{qq}\otimes \bar{f}_1 f_1 - f_1 s_{qq}\otimes \bar{f}_1 \mp \bar{f}_1 s_{qq}\otimes f_1 + s_{qg}\otimes f_g(f_1\pm\bar{f}_1) - (f_1\pm\bar{f}_1)s_{qg}\otimes f_g \Big\}$$

- Quark and gluon coefficient functions (at a_s order):

$$s_{qq}(x) = \frac{4a_s C_F}{(1-x)_+}, \quad s_{qg}(x) = a_s(1-x), \quad s\otimes f(x) = \int_x^1 \frac{dy}{y} s(y) f\left(\frac{x}{y}\right)$$

- Coefficient $A_1 = \Sigma_1/\Sigma_U$. [A. Arroyo-Castro, I. Scimemi, A. Vladimirov, 2503.24336]. Measurement at $Q \in [80, 100]$ GeV made by ATLAS, CMS and LHCb during the $\sqrt{s} = 8$ TeV run.



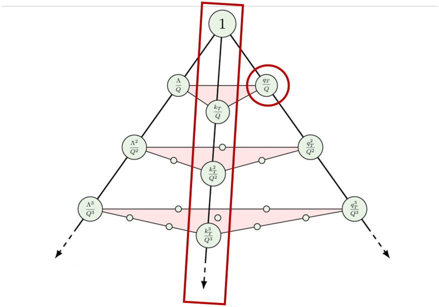
Final remarks

Final remarks (I)

- **Conclusions**.

q_T/Q correction is "hidden" in twist-three terms

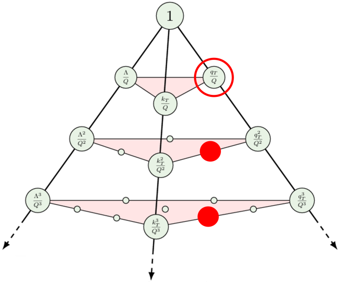
Leading TMD approximation is in complete agreement with data



Final remarks (II)

- **What are we working on?**

Leading TMD approximation is complete agreement with data



- **Long-term goals?** Compute q_T^2/Q, q_T^3/Q^3, etc.

More of TMD physics at...

- https://indico.fis.ucm.es/event/37/.
- Hadron tomography and EIC and JLaB physics.

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Thank you for your attention!

Back up

Composite background field method

- Path integral. (Anti-)causal sectors.
- We split fields into background (which obey EOMs) and dynamical components:

$$q^{(\pm)} = q_{\bar{n}}^{(\pm)} + q_n^{(\pm)} + \psi^{(\pm)},$$
$$A_\mu^{(\pm)} = A_{\bar{n},\mu}^{(\pm)} + A_{n,\mu}^{(\pm)} + B_\mu^{(\pm)}.$$

- Background field gauge:

$$D_\mu[A_{\bar{n}}^{(\pm)} + A_n^{(\pm)}]B^{(\pm)\mu} = 0.$$

- Hadronic tensor is matrix element of effective operator:

$$\mathcal{J}_{GG'}^{\mu\nu}(y) \sim J_G^{\mu(-)}[q](y) J_{G'}^{\nu(+)}[q](0) e^{iS_{\text{int}}^{(+)} - iS_{\text{int}}^{(-)}}\Big|_{q=\psi+q_{\bar{n}}+q_n}.$$

- Light-cone gauge:

$$A_{\bar{n}}^{+(\pm)} = 0, \quad A_n^{-(\pm)} = 0.$$

gNLP angular coefficients

$$\frac{Q}{2M}\Pi_1^{\mathrm{gNLP}} = z_{+q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_A] - 2r_{+q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_A]$$
$$+z_{+q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_A] - 2r_{+q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_A],$$

$$\frac{\mathcal{T}}{4M}\Pi_3^{\mathrm{gNLP}} = z_{-q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_S] + 2ir_{-q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_S]$$
$$+z_{-q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_S] - 2ir_{-q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_S],$$

$$\frac{\mathcal{T}}{2M}\Pi_6^{\mathrm{gNLP}} = z_{-q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_S] - 2ir_{-q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_S]$$
$$-z_{-q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_S] - 2ir_{-q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_S],$$

$$\frac{Q}{4M}\Pi_7^{\mathrm{gNLP}} = z_{+q}^{GG'}\mathcal{J}_1^R[\{f_1\mathbf{g}_2^\perp + \mathbf{g}_2^\perp f_1\}_A] - 2r_{+q}^{GG'}\mathcal{J}_1^R[\{h_1^\perp\mathbf{h}_\oplus + \mathbf{h}_\oplus h_1^\perp\}_A]$$
$$-z_{+q}^{GG'}\mathcal{J}_1^I[\{f_1\mathbf{f}_2^\perp + \mathbf{f}_2^\perp f_1\}_A] + 2r_{+q}^{GG'}\mathcal{J}_1^I[\{h_1^\perp\mathbf{h}_\ominus + \mathbf{h}_\ominus h_1^\perp\}_A]$$

Matching of twist-three TMD distributions

$$f_2^{\perp} = \frac{1}{b^2 M^2}\left[4a_s C_F \theta_{23}\left(f_1(-x_1, b) - f_1(x_3, b)\right) + a_s \theta_{13}\frac{x_1}{x_2} f_g(-x_2, b)\right],$$

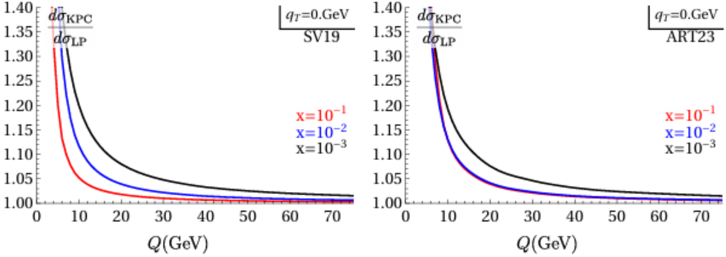
$$\mathbf{g}_2^{\perp} = 0,$$

$$\bar{\mathbf{f}}_2^{\perp} = \frac{-1}{b^2 M^2}\left[4a_s C_F \theta_{23}\left(\bar{f}_1(-x_1, b) - \bar{f}_1(x_3, b)\right) + a_s \theta_{13}\frac{x_1}{x_2} f_g(-x_2, b)\right],$$

$$\bar{\mathbf{g}}_2^{\perp} = 0$$

Kinematic Power Corrections

- Ratio between KPCs and LP Drell-Yan cross section at $q_T = 0$ [A. Vladimirov, 2307.13054].



- Fraction momentum.

$$\xi_1 = \frac{x_1}{2}\left(1 + \frac{\boldsymbol{k}_{1T}^2}{\tau^2} - \frac{\boldsymbol{k}_{2T}^2}{\tau^2} + \frac{\sqrt{\lambda(\boldsymbol{k}_{1T}^2, \boldsymbol{k}_{2T}^2, \tau^2)}}{\tau^2}\right)$$