# An effective hadronic field theory for B-meson decays at high recoil

[in preparation]

Thorsten Feldmann, Jack Jenkins, Björn O. Lange, JdPL

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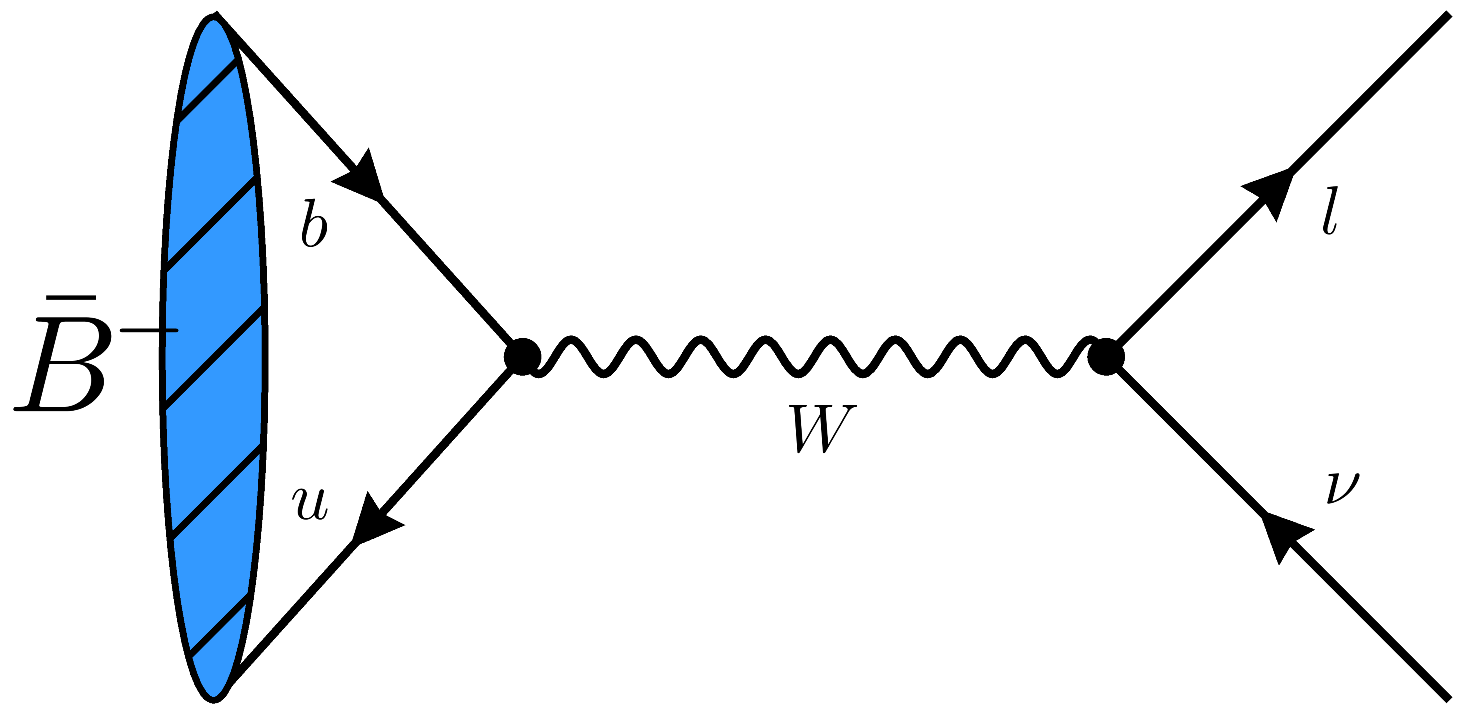
Jaime del Palacio Lirola

Universität Siegen

TP1 Theoretical Particle Physics

# Non-perturbative QCD: Exclusive decays

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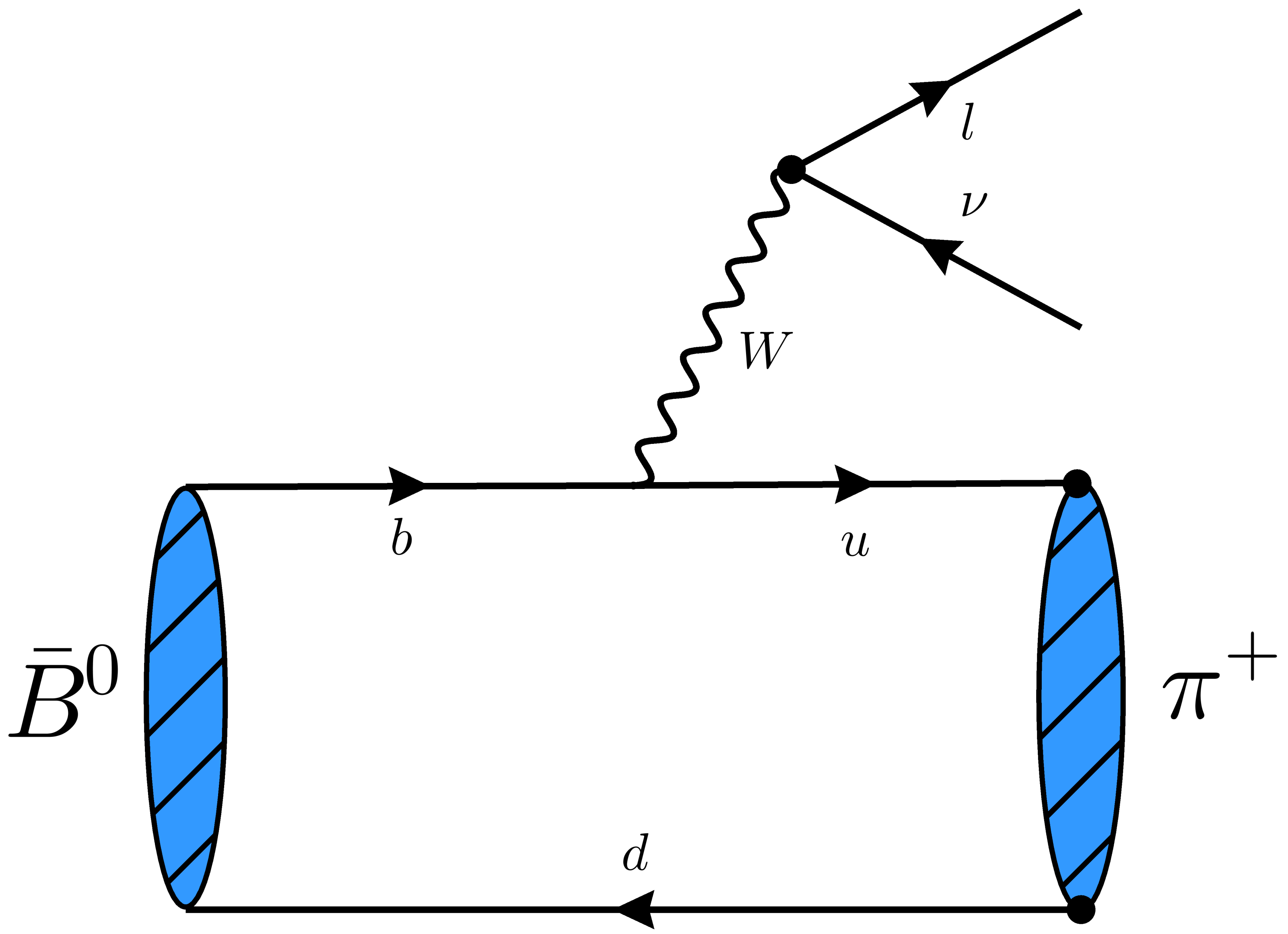


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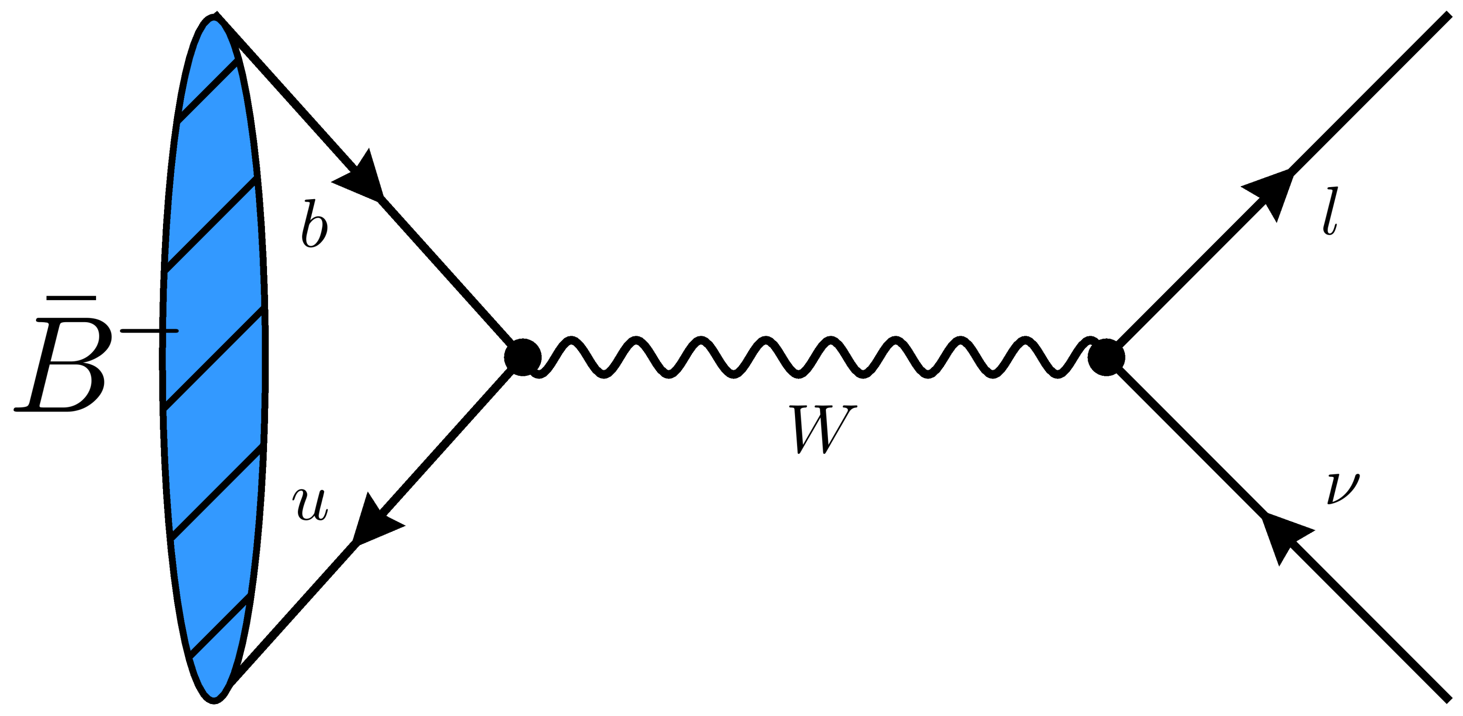


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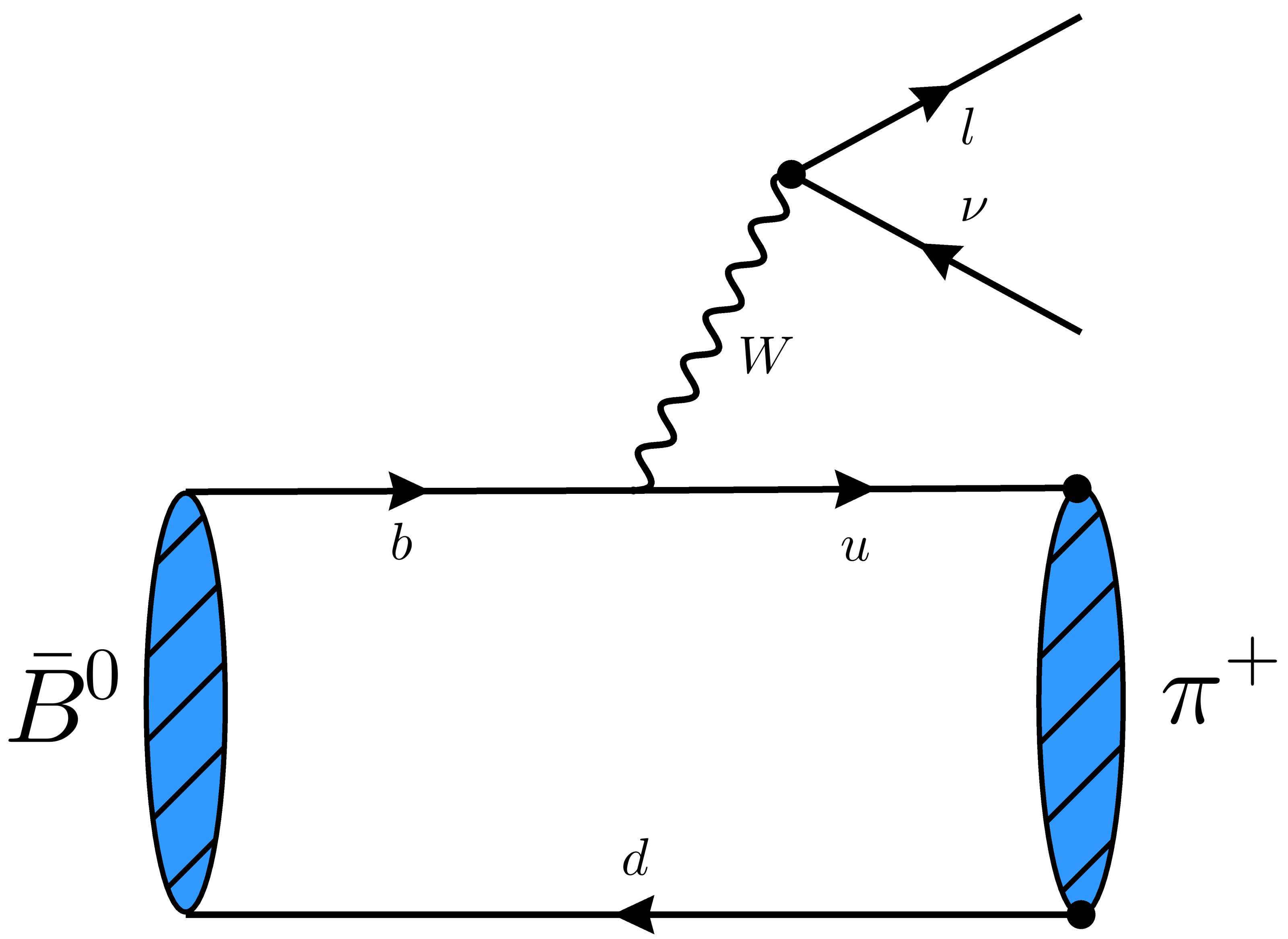


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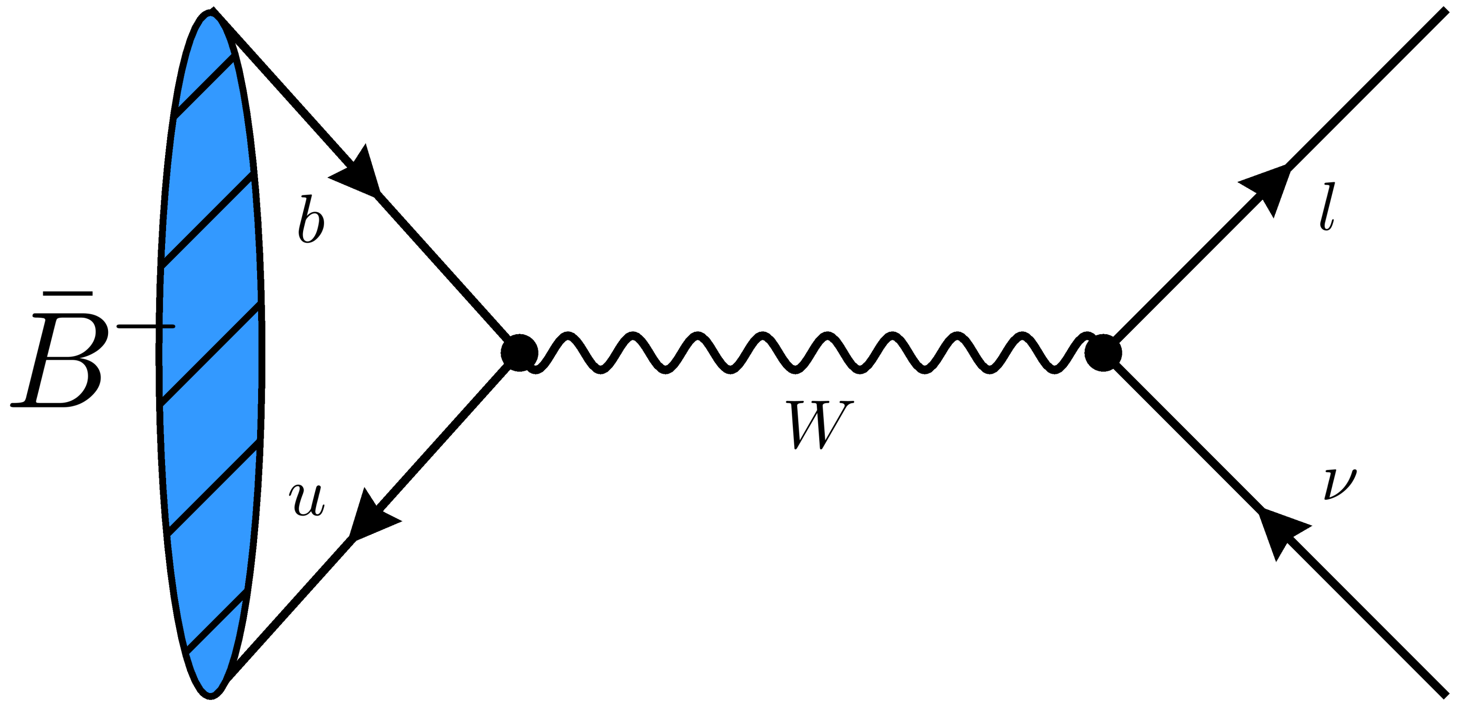


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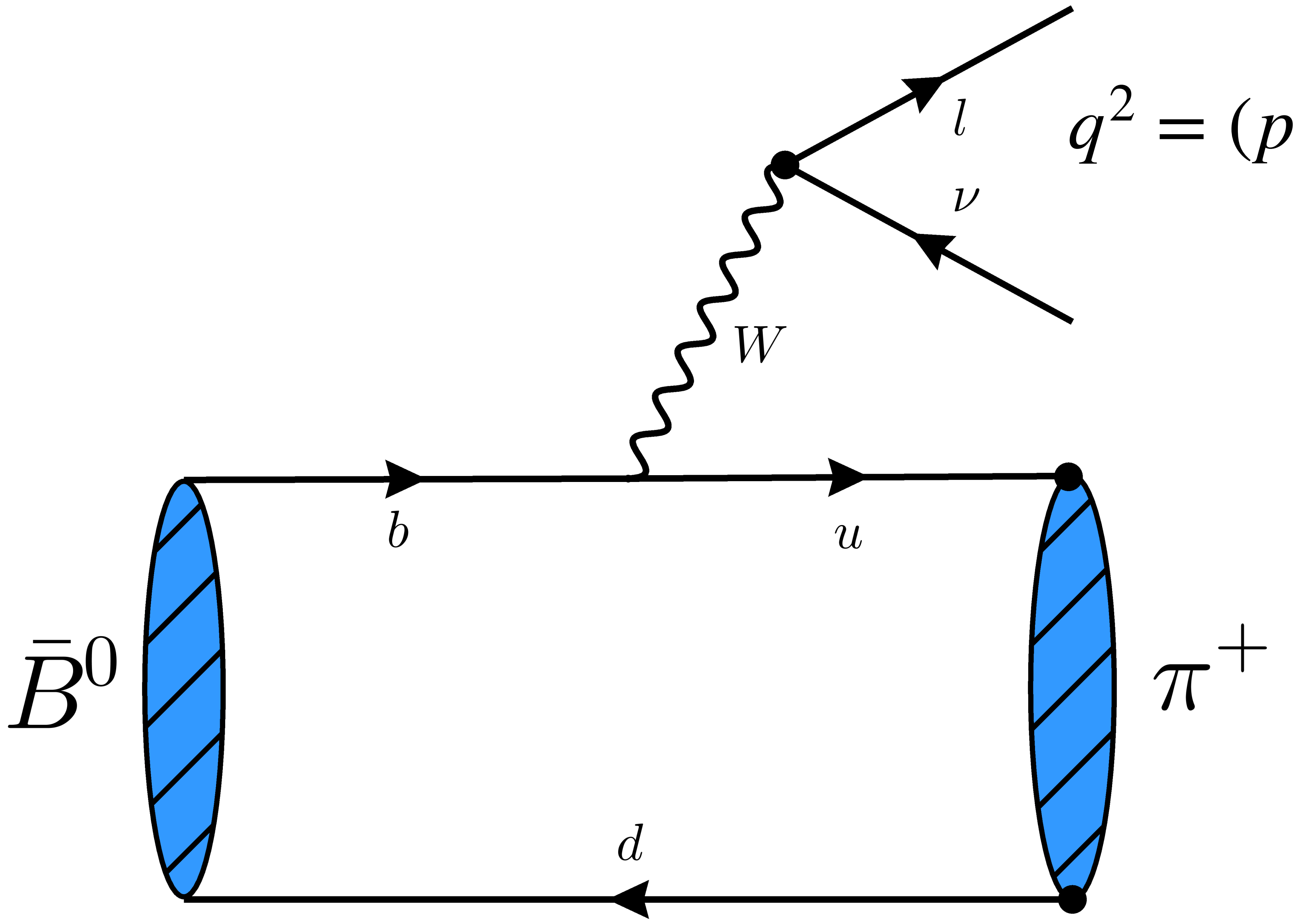


$$q^2 = (p-k)^2 = M_B^2 + m_\pi^2 - 2M_B E_\pi$$

$$\langle \pi^+(k) | \bar{u}\gamma_\mu(1-\gamma_5)b | \bar{B}^0(p)\rangle$$

$$= f_+^{B\to\pi}(q^2)(p_\mu + k_\mu) + f_-^{B\to\pi}(q^2)(p_\mu - k_\mu)$$

# QCD through EFTs

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$m_Q \gg \Lambda_{QCD}$

→*symmetries!*

*Heavy Quark Effective Theory*

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  [Falk and Grinstein, 9306310]
  [Fleischer, TUM-T31-34-92]

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- Chiral extrapolation at high $q^2$ (for lattice)
  [Becirevic, Prelovsek, Zupan, 0305001]

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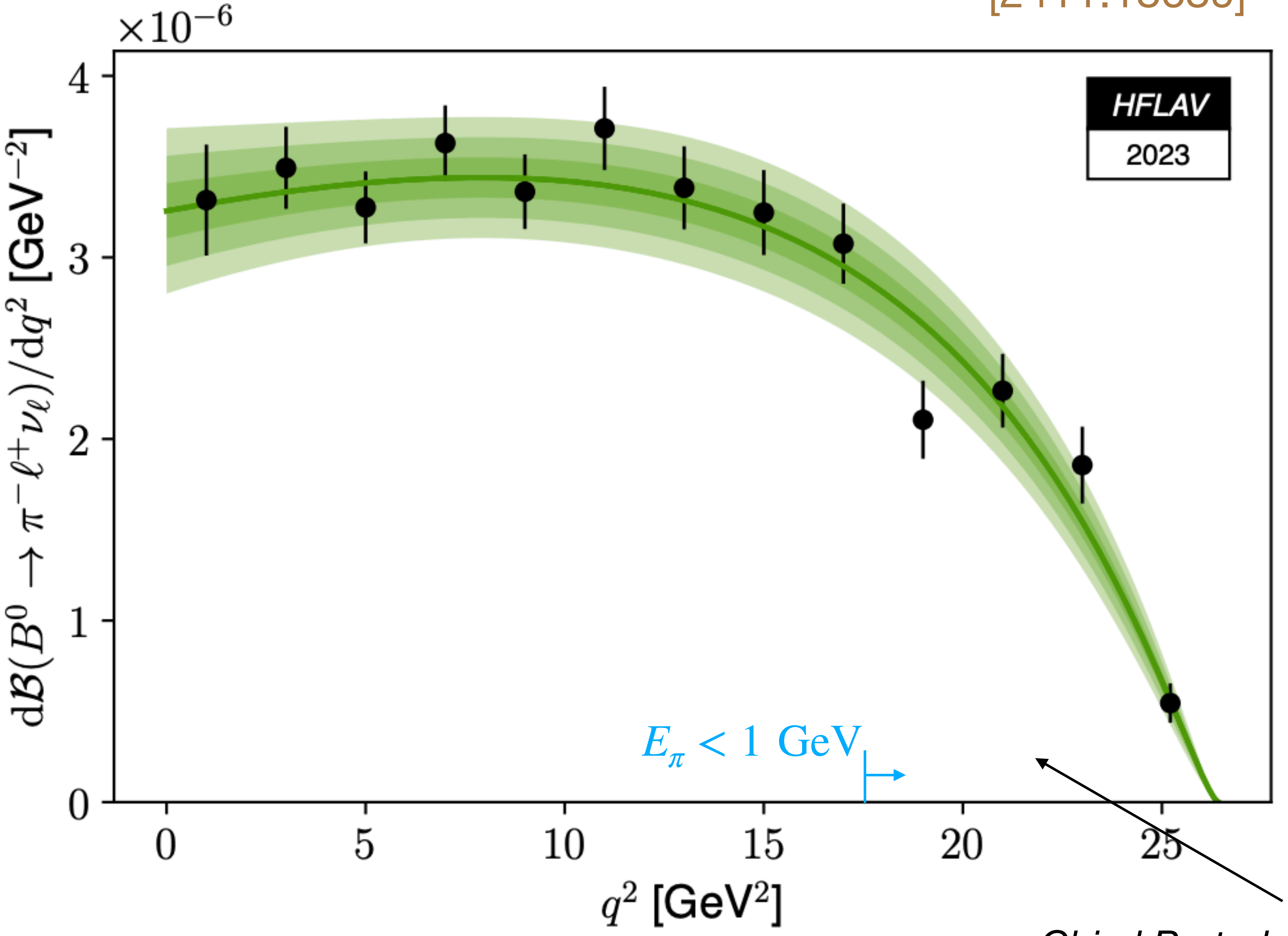
# High / low recoil regions

[2411.18639]




*Chiral Perturbation Theory only works here!*

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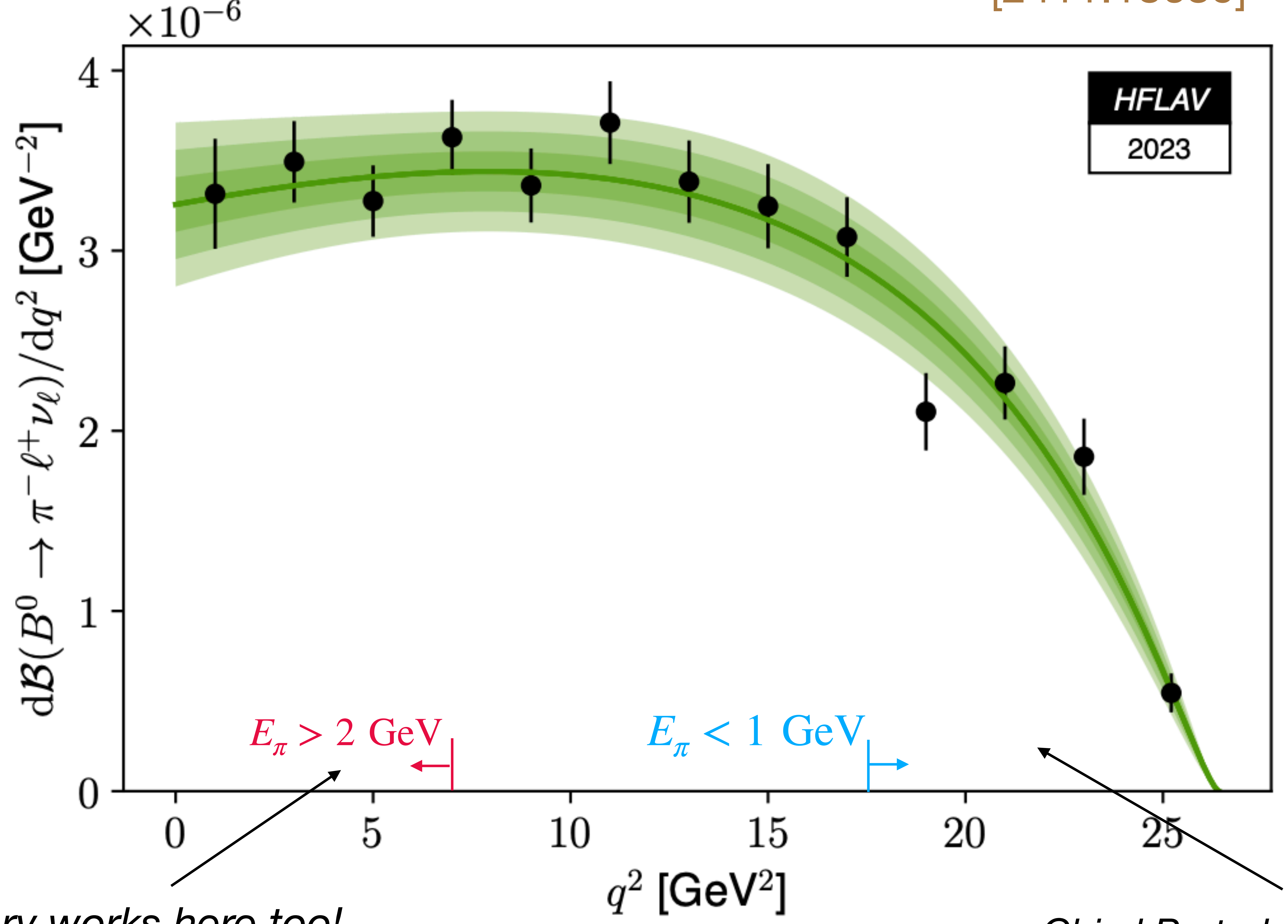


*New Theory works here too!*

*Chiral Perturbation Theory only works here!*

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*SCET (II) for ChPT*

$$\mathscr{L}^{(0)} = \mathscr{L}^{(0)}_{HH\pi_s} + \mathscr{L}^{(0)}_{\pi_s} + \mathscr{L}^{(0)}_{\pi_c} +$$

*External Currents: Non-local operators, energy dependent couplings*

$$\mathscr{L}^{(0)}_{\pi_s} = \mathscr{L}^{(0)}_{\pi_c} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu \Sigma^\dagger_{s/c}\partial^\mu \Sigma_{s/c}\right]$$

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### Covariant formulation of ChPT

$$\xi(x) \equiv \exp\left[\frac{i\pi^a(x)t^a}{f_\pi}\right]$$

$$\Sigma(x) = \xi^2(x) \to L\xi^2(x)R^\dagger$$

$$\xi(x) \to L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger$$

$$V^\mu = \frac{1}{2}\left[\xi^\dagger iD^\mu_L\xi + \xi iD^\mu_R\xi^\dagger\right] \quad V^\mu \to UV^\mu U^\dagger + U[iD^\mu, U^\dagger]$$

$$A^\mu = \frac{1}{2}\left[\xi^\dagger iD^\mu_L\xi - \xi iD^\mu_R\xi^\dagger\right] \quad A^\mu \to UA^\mu U^\dagger$$

### SCET (II) for ChPT

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| $A_s^\mu \sim (\epsilon,\ \epsilon,\ \epsilon)$ | $A_c^\mu \sim (\lambda^{-1}, \epsilon^2\lambda,\ \epsilon)$ |
| $V_s^\mu \sim (\epsilon,\ \epsilon,\ \epsilon)$ | $V_c^\mu \sim (\lambda^{-1}, \epsilon^2\lambda,\ \epsilon)$ |
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All chiral loops in the effective theory count as $\sim \epsilon^2$.

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$$A_c^2 = \frac{1}{2}\{\bar{n}\cdot A, n\cdot A\} + A_\perp^2 \sim \epsilon^2$$

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HHChPT

$$\bar{q}_L \Gamma h_v \overset{!}{=} C(\mu) \mathrm{Tr}\left[H_v P_R \Gamma\right] \xi^\dagger$$

$$C(\mu = m_b) = \sqrt{M_B} f_B + \mathcal{O}(1/m_b, \alpha_s, 1/f_\pi)$$

# External currents: Matching SCET → HHChPT

SCET I

$$\langle \pi_{ij}(k)|\, \bar{\xi}_{L,i} h_v \,|B_j(p)\rangle \equiv 2E\, \zeta_\pi(E,\mu)$$

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[Lange and Neubert, 0311345]

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$$O_{ijk}^{(1)} = \left[\bar{\mathcal{X}}_{L,i}(0)\,\frac{\not{\bar{n}}}{2}\,\mathcal{X}_{L,k}(s\bar{n})\right]\left[\bar{\mathcal{Q}}_{s,L,j}(tn)\,\frac{\not{\bar{n}}\not{n}}{4}\,\mathcal{H}_v(0)\right],$$

$$O_{ijk}^{(2)} = \left[\bar{\mathcal{X}}_{L,i}(0)\,\frac{\not{\bar{n}}}{2}\,i\not{\partial}_\perp \mathcal{X}_{R,k}(s\bar{n})\right]\left[\bar{\mathcal{Q}}_{s,R,j}(tn)\,\frac{\not{n}}{4}\mathcal{H}_v(0)\right],$$

$$O_{ijk}^{(3)} = \left[\bar{\mathcal{X}}_{L,i}(0)\,\frac{\not{\bar{n}}}{2}\,\not{\mathcal{A}}_{c\perp}(r\bar{n})\mathcal{X}_{R,k}(s\bar{n})\right]\left[\bar{\mathcal{Q}}_{s,R,j}(tn)\,\frac{\not{n}}{4}\mathcal{H}_v(0)\right],$$

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} Eventually match to chirally subleading hadronic operators / contributions*

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$$W_s(x) = \mathrm{P}\exp\left[i\int_{-\infty}^{0} ds\, n\cdot V_s(ns+x)\right]$$

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$$J_c^{(2)}(s,s',s'') = [\xi_c W_c^\dagger](s\bar{n}) \times S_c(s'\bar{n}) \times S_c(s''\bar{n}) \times [W_c\xi_c^\dagger](0)$$

$$\vdots$$

# External currents: Matching SCET → HHChPT

SCET II

$$O^{(1)}_{ijk} = [\bar{\mathcal{X}}_{L,i}(0)\,\frac{\bar{n}\!\!\!/}{2}\,\mathcal{X}_{L,k}(s\bar{n})]\,[\bar{\mathcal{Q}}_{s,L,j}(tn)\,\frac{\bar{n}\!\!\!/ n\!\!\!/}{4}\,\mathcal{H}_v(0)],$$

$$W_s(x) = \mathrm{P}\exp\left[i\int_{-\infty}^{0} ds\, n\cdot V_s(ns+x)\right]$$

$$W_c(x) = \mathrm{P}\exp\left[i\int_{-\infty}^{0} ds\, \bar{n}\cdot V_c(\bar{n}s+x)\right]$$

$$J_s \mapsto J_s L_s^\dagger(tn)\,, \quad J_c \mapsto L_c(s\bar{n}) J_c L_c^\dagger(0)$$

$$J_s^{(0)}(t) = \mathrm{Tr}[H_v(0)\frac{\bar{n}\!\!\!/ n\!\!\!/}{4}(1-\gamma_5)]W_s^\dagger(0)W_s(tn)\xi_s^\dagger(tn)$$

$$J_c^{(1)}(s,s') = [\xi_c W_c^\dagger](s\bar{n}) \times [W_c\,\bar{n}\cdot A_c\,W_c^\dagger](s'\bar{n}) \times [W_c\xi_c^\dagger](0)$$

$S_c = [W_c\,\bar{n}\cdot A_c\,W_c^\dagger]$ is chirally unsuppressed and completely invariant!

$$J_c^{(2)}(s,s',s'') = [\xi_c W_c^\dagger](s\bar{n}) \times S_c(s'\bar{n}) \times S_c(s''\bar{n}) \times [W_c\xi_c^\dagger](0)$$

$$\vdots$$

Infinite tower of operators with the same power counting but different coefficients…

# $B \to \pi_c$: A bit simpler

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Only one operator contributes at tree level, and even at 1-loop level

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$$2E\,\zeta_\pi(E,\mu) = \int_0^\infty dsdt\ \tilde{D}_1^{(M)}(s,t,E,\mu)\ \langle\pi|\ O(s,t)\ |B\rangle$$

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# What can you actually calculate?

<u>*Heavy Hadron Chiral Perturbation Theory*</u>

- Relate form factors and decay constant high $q^2$

<u>*+ SCET*</u>

- Relate form factors and decay constant low $q^2$: <u>Cannot do</u>

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- $SU(3)$ relations at high $q^2$
  [Falk and Grinstein, 9306310]
  [Fleischer, TUM-T31-34-92]

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# Conclusions

- We built SCET for a hadronic theory: SCHHChPT (placeholder name)
- Matching based on SCET II symmetries
- Chiral corrections from high $q^2$ are different that those at low $q^2$ ( $\approx 20\,\%$ difference in the correction), this is at odds with current literature
- Our results can be used for chiral extrapolation and $SU(3)$ relations at low $q^2$

# Thank you!

Jaime del Palacio Lirola

# Backup slides

The four-quark operators in an 'un-Fierzed' basis have the form $\sim [\bar{\mathcal{X}}_{L,i}(0) M \mathcal{H}_v(0)] \times [\bar{\mathcal{Q}}_{s,j}(tn) N \mathcal{X}_k(s\bar{n})]$. In the massless limit, the chirality of the soft anti-quark ($\mathcal{Q}_{s,j}$) must be the same as the chirality of the collinear anti-quark ($\mathcal{X}_k$), which is effectively equivalent to the condition $\{N, \gamma_5\} = 0$ on the Dirac structure $N$. Finally, in the Fierzed basis of Ref. [23], the color-singlet[4] operators generated in the matching of the time ordered product of the scalar current and subleading Lagrangian to four-quark operators in SCET-II are

$$O^{(1)}_{ijk} = \left[\bar{\mathcal{X}}_{L,i}(0)\, \frac{\not{\bar{n}}}{2}\, \mathcal{X}_{L,k}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_{s,L,j}(tn)\, \frac{\not{\bar{n}}\not{n}}{4}\, \mathcal{H}_v(0)\right], \tag{3.4}$$

$$O^{(2)}_{ijk} = \left[\bar{\mathcal{X}}_{L,i}(0)\, \frac{\not{\bar{n}}}{2}\, i\not{\partial}_\perp \mathcal{X}_{R,k}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_{s,R,j}(tn)\, \frac{\not{n}}{4} \mathcal{H}_v(0)\right], \tag{3.5}$$

$$O^{(3)}_{ijk} = \left[\bar{\mathcal{X}}_{L,i}(0)\, \frac{\not{\bar{n}}}{2}\, \not{\mathcal{A}}_{c\perp}(r\bar{n}) \mathcal{X}_{R,k}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_{s,R,j}(tn)\, \frac{\not{n}}{4} \mathcal{H}_v(0)\right], \tag{3.6}$$

$$O^{(4)}_{ijk} = \left[\bar{\mathcal{X}}_{L,i}(0)\, \frac{\not{\bar{n}}}{2} \mathcal{X}_{L,k}(s\bar{n})\right] \left[\bar{\mathcal{Q}}_{s,L,j}(tn) \not{\mathcal{A}}_{s\perp}(un) \frac{\not{n}}{4} \mathcal{H}_v(0)\right]. \tag{3.7}$$

# SCET (II) ChiPT: Power counting

$$p_s \sim (p, p, p), \quad p_c \sim (m_b, p^2/m_b, p), \tag{2.28}$$

where $p_c$ has been boosted in the $n$ direction with a boost factor of $m_b/p$ with respect to $p_s$. We can now define two separate power counting parameters $\epsilon = p/(4\pi f_\pi)$ and $\lambda = 4\pi f_\pi/m_b$, consistent with the hierarchy of those scales, and write

$$p_s \sim 4\pi f_\pi(\epsilon, \epsilon, \epsilon), \quad p_c \sim 4\pi f_\pi(\lambda^{-1}, \epsilon^2\lambda, \epsilon). \tag{2.29}$$

Here $\lambda$ resembles the traditional power counting parameter in SCET. The common scale factored out to make the power counting the most intuitive is $\Lambda_\chi$, reflecting the expected size of higher order operators in the hadronic theory.

| Soft | Collinear |
|---|---|
| $A_s^\mu \sim (\epsilon,\ \epsilon,\ \epsilon)$ | $A_c^\mu \sim (\lambda^{-1}, \epsilon^2\lambda,\ \epsilon)$ |
| $V_s^\mu \sim (\epsilon,\ \epsilon,\ \epsilon)$ | $V_c^\mu \sim (\lambda^{-1}, \epsilon^2\lambda,\ \epsilon)$ |
| $H_v \sim 1$ | |
| $D^\mu H_v \sim \epsilon$ | |
| $B_0 m_q \sim \epsilon^2$ | |

| Soft | Collinear |
|---|---|
| $\mathcal{L}_{\chi s}^{(0)} \sim \epsilon^2$ | $\mathcal{L}_{\chi c}^{(0)} \sim \epsilon^2$ |
| $\mathcal{L}_{H}^{(0)} \sim \epsilon$ | $J_c^{(0)} \sim 1$ |
| $J_s^{(0)} \sim 1$ | $J_c^{(1)} \sim 1$ |
| | $J_c^{(2)} \sim 1$ |
| | $\vdots$ |

**Table 1**. Power counting in the hadronic theory, with $\epsilon = p/(4\pi f_\pi)$ and $\lambda = 4\pi f_\pi/m_b$. Left: fundamental building blocks. Right: selected derived operators. All chiral loops in the effective theory count as $\sim \epsilon^2$.

$$\mathcal{L}_q = \bar{q}(i\not{D} - s + ip\gamma_5 + \not{v} + \not{a}\gamma_5)q$$
$$= \bar{q}_L(i\not{D} + \not{\ell})q_L + \bar{q}_R(i\not{D} + \not{r})q_R - \bar{q}_L\chi q_R - \bar{q}_R\chi^\dagger q_L\,, \tag{2.1}$$

where $iD_\mu = i\partial_\mu + gA_\mu^a T^a$ is the QCD covariant derivative, the projected quark fields are $q_L = P_L q$ and $q_R = P_R q$ with $P_{L(R)} = (1 \mp \gamma_5)/2$, and the sources appear in the linear combinations $\chi = s + ip$, $\ell = v - a$ and $r = v + a$. The Lagrangian Eq. (2.1) is invariant under local transformations in the chiral group $G \simeq SU(N)_L \times SU(N)_R$ specified by

$$q_L \mapsto Lq_L\,, \qquad q_R \mapsto Rq_R \tag{2.2}$$

The leading SCET II strong-interaction Lagrangian in the presence of external sources can be written as

$$\mathcal{L} = \mathcal{L}_{\rm YM} + \mathcal{L}_c + \mathcal{L}_s\,. \tag{2.19}$$

The Yang-Mills Lagrangian involves only the (soft and collinear) gluon fields, together with the gauge-fixing terms, which are not critical to the following analysis since they do not transform under chiral symmetry. The soft and collinear quark terms have the generic form as in Eq. (2.1):

$$\mathcal{L}_s = \bar{q}_{s,L}(i\not{D} + \not{\ell}_s)q_{s,L} + \bar{q}_{s,R}(i\not{D} + \not{r}_s)q_{s,R} - \bar{q}_{s,L}\chi_s q_{s,R} - \bar{q}_{s,R}\chi_s^\dagger q_{s,L} \tag{2.20}$$
$$\mathcal{L}_c = \bar{q}_{c,L}(i\not{D} + \not{\ell}_c)q_{c,L} + \bar{q}_{c,R}(i\not{D} + \not{r}_c)q_{c,R} - \bar{q}_{c,L}\chi_c q_{c,R} - \bar{q}_{c,R}\chi_c^\dagger q_{c,L}\,, \tag{2.21}$$

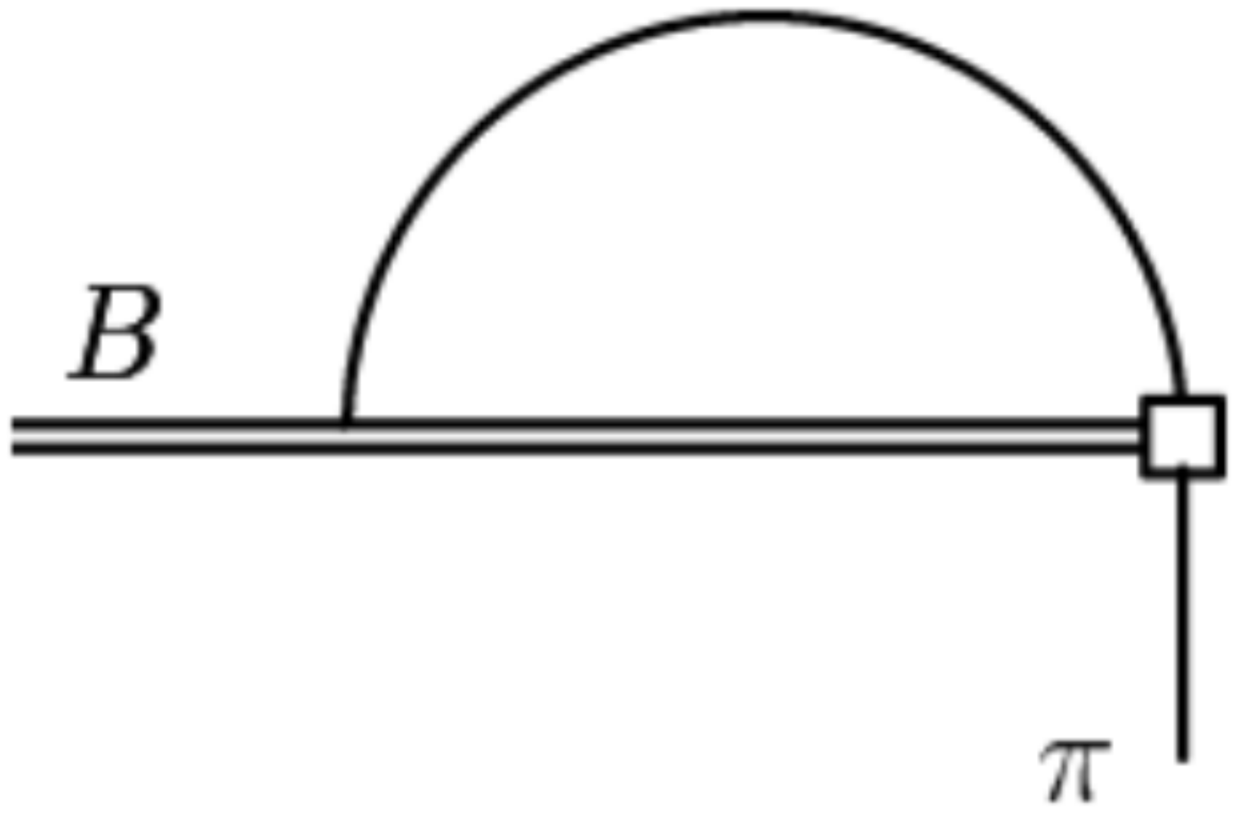
in terms of the QCD covariant derivatives. The full symmetry group of Eq. (2.19) above is therefore enlarged to $(SU(N)_L \times SU(N)_R)_s \times (SU(N)_L \times SU(N)_R)_c$, where

# Explicit results

$$M_{J_1}^{(1)} = M_{J_1}^{tree}\frac{1}{f_\pi^2}\left(\frac{1}{2}+\frac{9}{8}g_\pi^2\right)I_1 \approx M_{J_1}^{tree}\frac{7.8}{10f_\pi^2}I_1$$

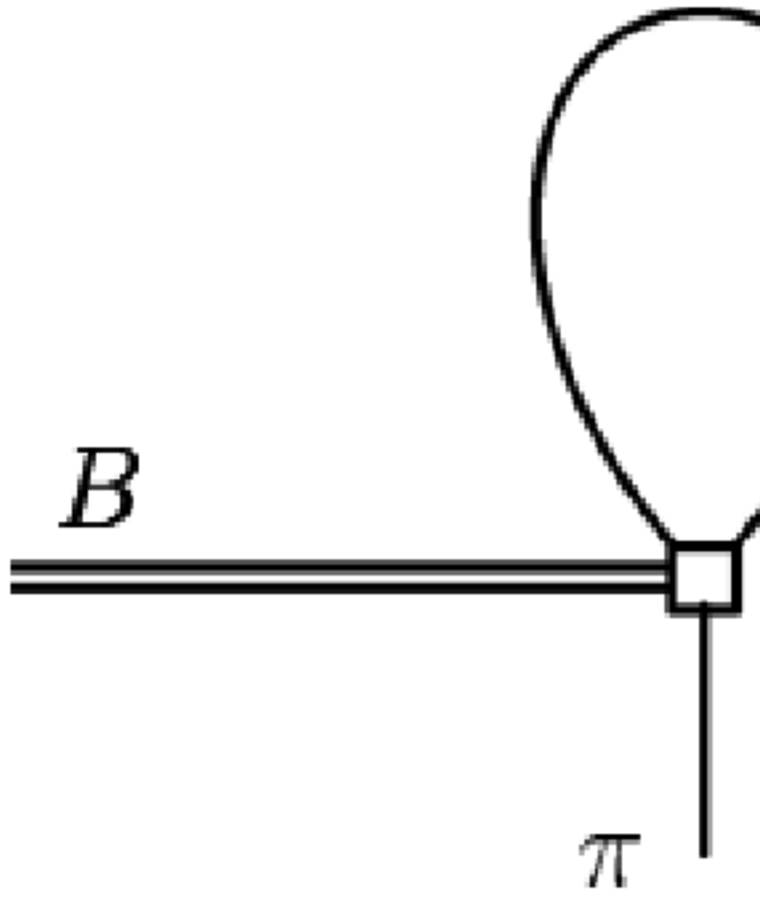
$\approx 20\,\%$ enhancement

$$M_{J_{soft}}^{(1)} = M_{J_{soft}}^{tree}\frac{1}{f_\pi^2}\left(\frac{3}{8}+\frac{9}{8}g_\pi^2\right)I_1 \approx M_{J_{soft}}^{tree}\frac{6.56}{10f_\pi^2}I_1$$




$$M_{J_1}^{(1)} = M_{J_1}^{tree}\left(\frac{1}{2}+\frac{9}{8}g_\pi^2\right)\frac{m_\pi^2}{f_\pi^2(4\pi)^2}\ln\frac{m_\pi^2}{\mu^2}$$

$$M_{J_1} \approx M_{J_1}^{tree}\left(1-\frac{8m_\pi^2}{10f_\pi^2(4\pi)^2}\ln\frac{m_\pi^2}{\mu^2}\right) \approx M_{J_1}^{tree}\,(1-0.028)$$



# SCET for ChPT

## Covariant formulation of ChPT

$$\xi(x) \equiv \exp\left[\frac{i\pi^a(x)t^a}{f_\pi}\right]$$

$$\Sigma(x) = \xi^2(x) \to L\xi^2(x)R^\dagger$$

$$\xi(x) \to L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger$$

$$V^\mu = \frac{1}{2}\left[\xi^\dagger iD_L^\mu \xi + \xi iD_R^\mu \xi^\dagger\right] \qquad V^\mu \to UV^\mu U^\dagger + U[iD^\mu, U^\dagger]$$

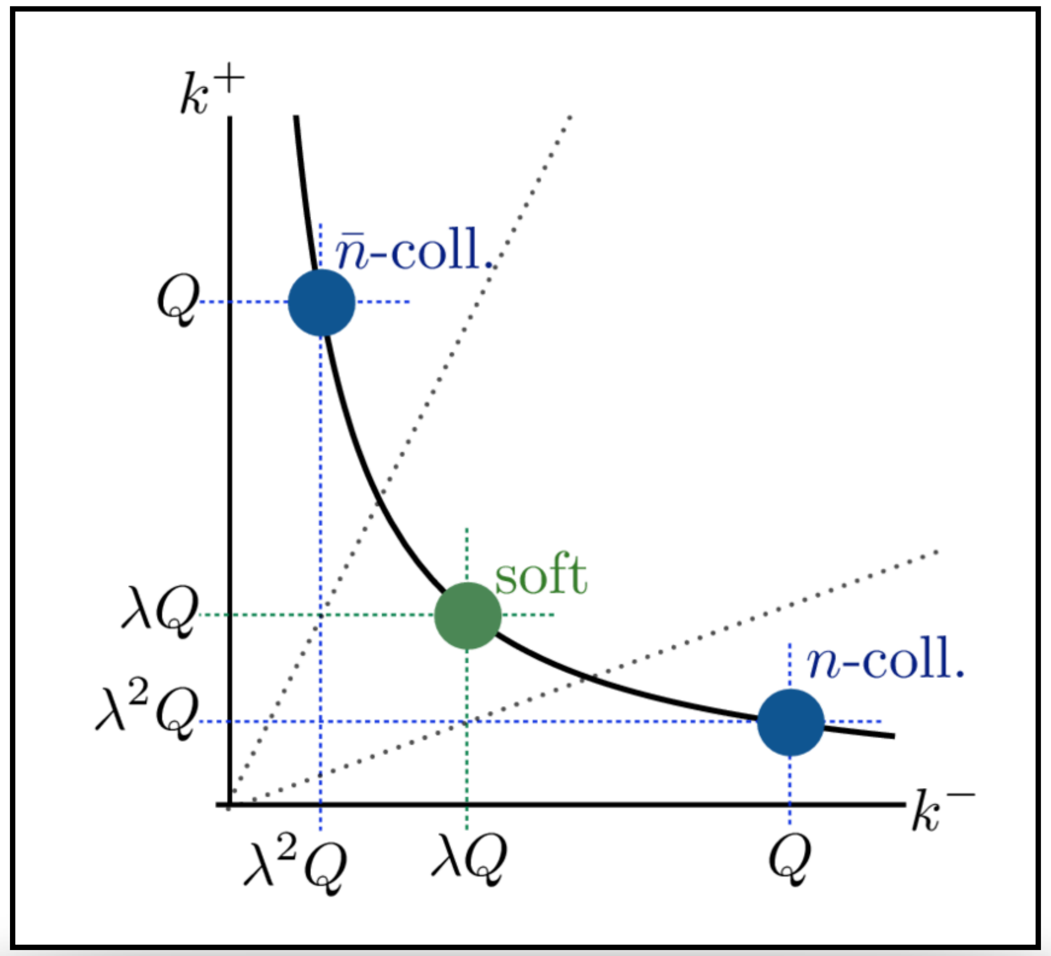
$$A^\mu = \frac{1}{2}\left[\xi^\dagger iD_L^\mu \xi - \xi iD_R^\mu \xi^\dagger\right] \qquad A^\mu \to UA^\mu U^\dagger$$

$$D^\mu_{L(R)} = \partial^\mu - i(V_{\text{ext}} \mp A_{\text{ext}})^\mu$$

$$D^\mu = \partial^\mu - iV^\mu_{\text{ext}}$$

## Introducing the soft and collinear sectors

$$V^\mu \to V^\mu_s, V^\mu_c$$



# QCD EFTs

$$m_q \ll 4\pi f_\pi$$

*→symmetries!*

*Chiral Perturbation Theory ($\pi, K, \eta$)*

$$\pi \to \ell\nu \qquad \pi\pi \to \pi\pi$$

$$q'_L = \xi^\dagger q_L \mapsto U q'_L, \quad q'_R = \xi q_R \mapsto U q'_R$$

$$\xi \mapsto L\xi U^\dagger = U\xi R^\dagger$$

*Non-linear representation:* $\xi_{ij}(x) = \exp\left[i\pi^a(x) t^a_{ij}/F_\pi\right]$

$$\mathcal{L}'_q = \bar{q}'(i\not{D} - S + iP\gamma^5 + \not{V} + \not{A}\gamma^5)q'$$

$$S = \xi^\dagger \chi \xi^\dagger + \text{h.c.}\,, \quad V_\mu = \frac{1}{2}(\xi^\dagger i D^\mu_L \xi + \xi i D^\mu_R \xi^\dagger)\,, \quad A_\mu = \frac{1}{2}(\xi^\dagger i D^\mu_L \xi - \xi i D^\mu_R \xi^\dagger)$$

$$Z[J] = \frac{1}{\mathcal{N}} \int [\mathcal{D}\xi \mathcal{D}\xi^\dagger][\mathcal{D}A]\ \det(i\not{D} - S + iP\gamma^5 + \not{V} + \not{A}\gamma^5) e^{iS_{\mathrm{YM}}}$$

# Hard Pion Chiral Perturbation Theory for $B \to \pi$ and $D \to \pi$ Formfactors

**Johan Bijnens and Ilaria Jemos**

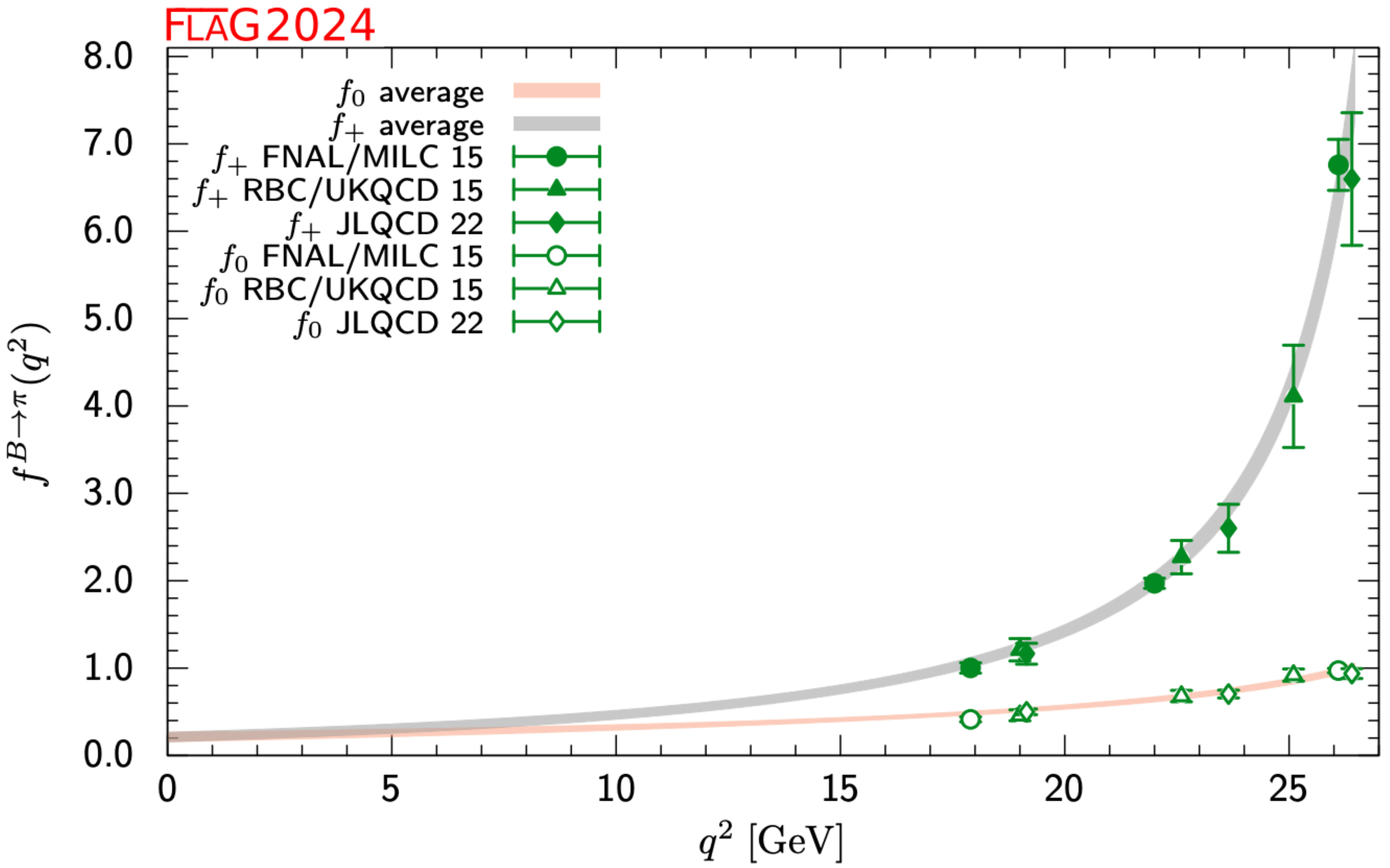
Department of Astronomy and Theoretical Physics, Lund University,
Sölvegatan 14A, SE 223-62 Lund, Sweden

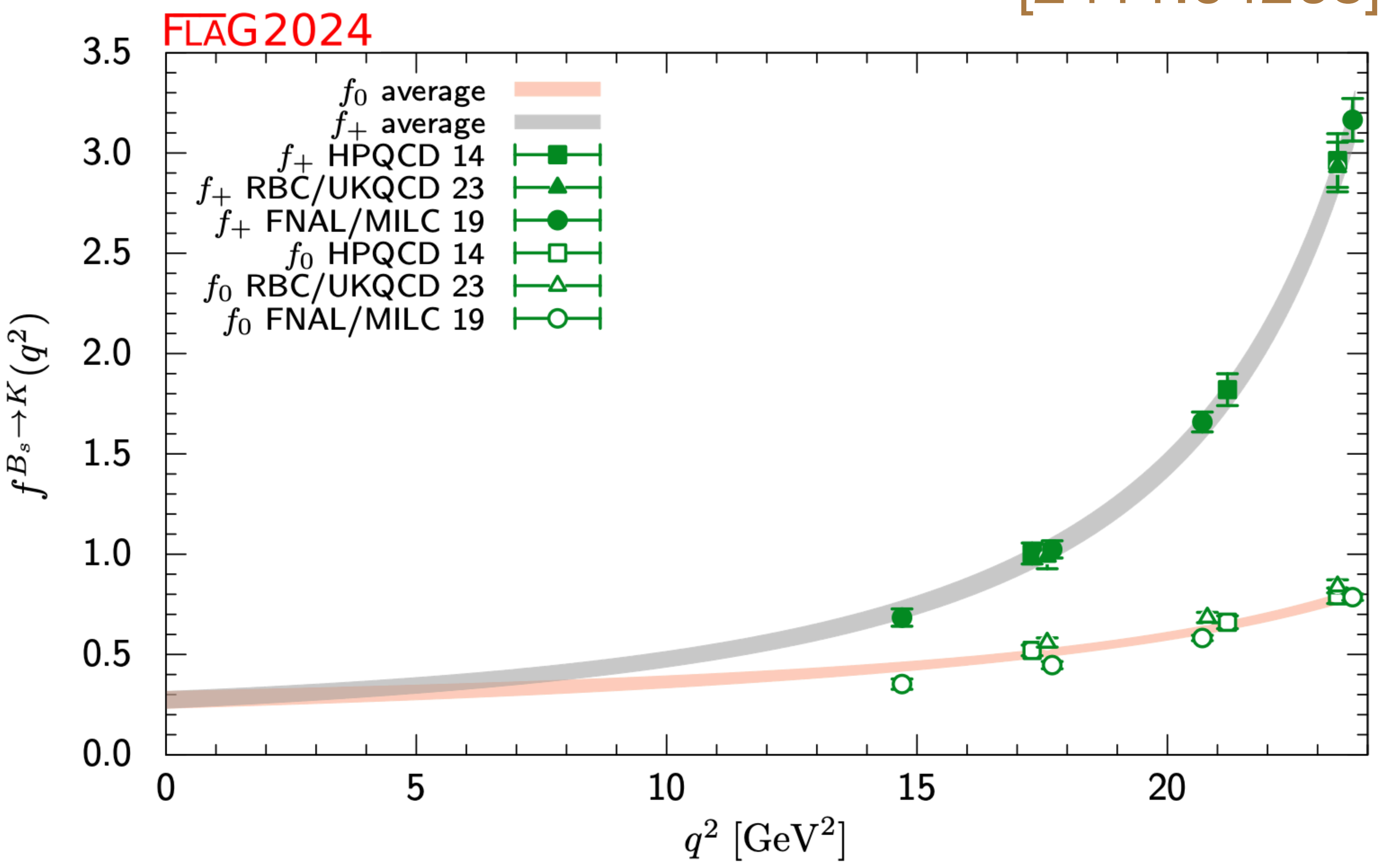
We should thus be able to describe the hard part of any diagram by an effective Lagrangian. This effective Lagrangian should include the most general terms allowed consistent with all the symmetries and have coefficients that depend on the hard kinematical quantities and can even be complex. A two-loop example will be given in [15]. We expect that a proof along the lines of SCET [20] should be possible. Once it is accepted that one can do this, a second step is to prove that the effective Lagrangian one uses is sufficient to describe the neighbourhood of the hard process and calculate chiral logarithms.

# What can you actually calculate?

$B \to \pi$ and $B_s \to K$ form factors are equivalent in the $SU(3)$ limit

[2411.04268]



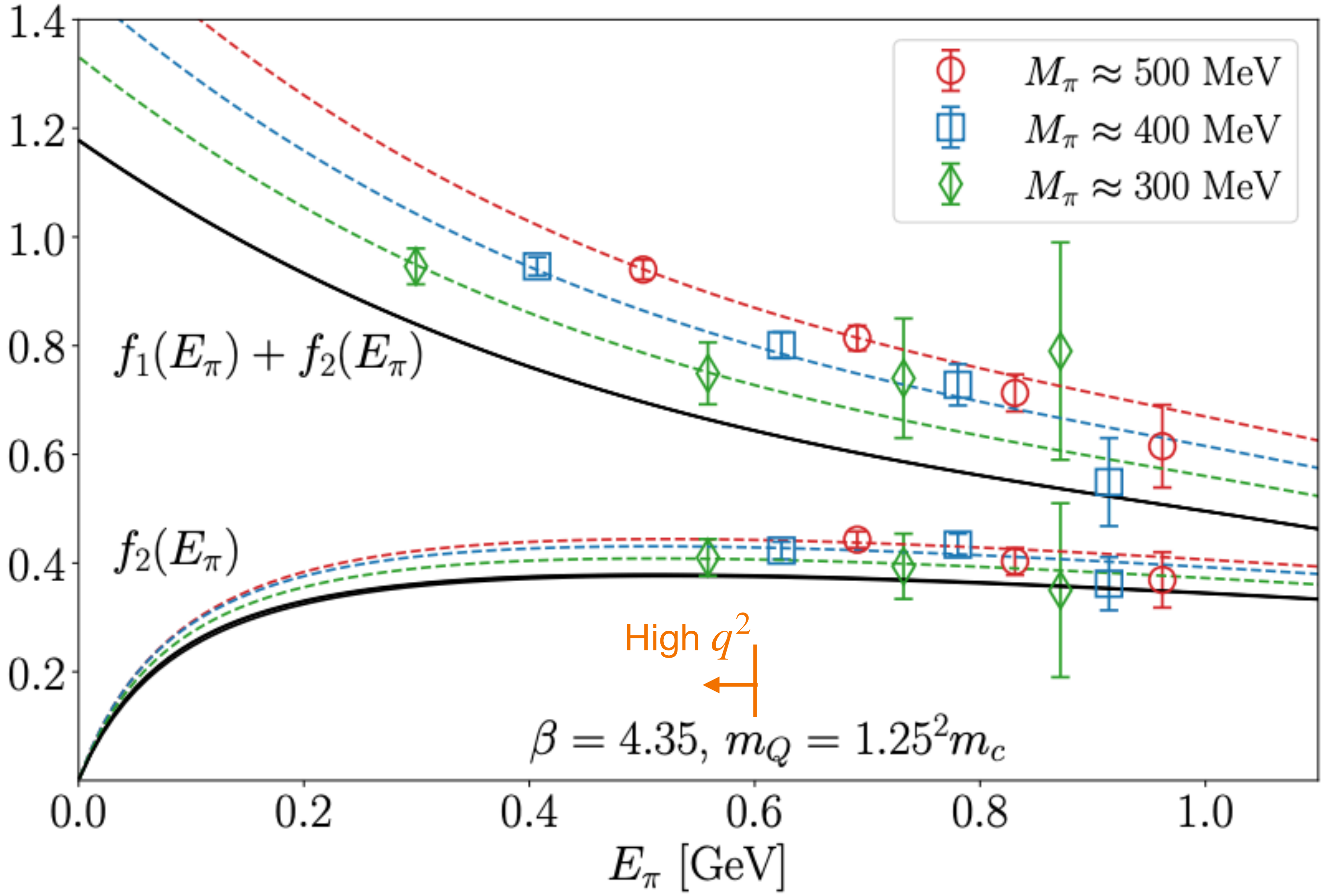




$$q^2 = M_B^2 + m_\pi^2 - 2M_B E_\pi$$

# What can you actually calculate?

Lattice at high $q^2$ with chiral extrapolation (HHChPT)

JLQCD [2203.04938]

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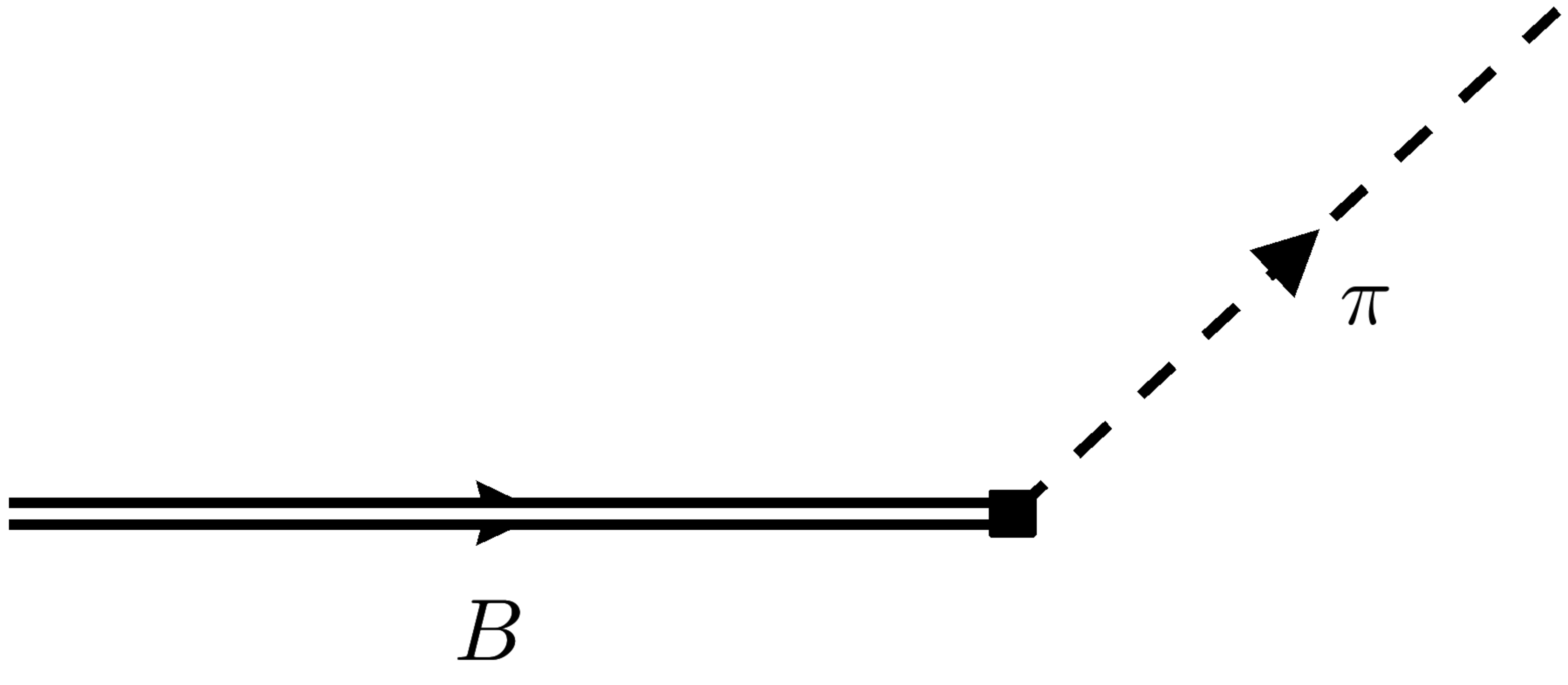
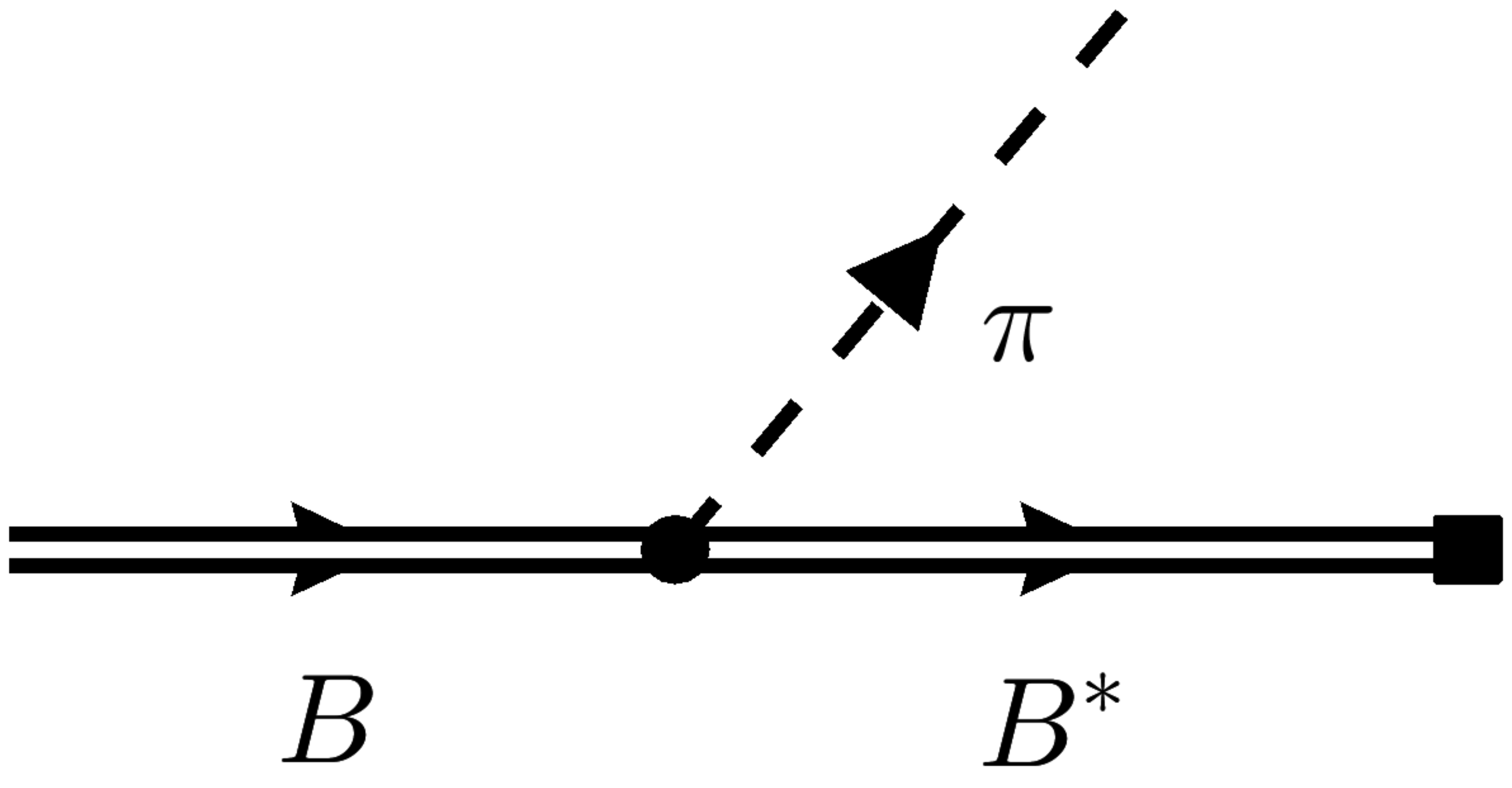
# HHChPT: Form factors




$$f_+^{B\to\pi} + f_-^{B\to\pi} = \left[\frac{f_B}{f_\pi}\right]\left[1 - \frac{g_\pi v\cdot p_\pi}{v\cdot p_\pi + \Delta^B}\right]$$

$$f_+^{B\to\pi} - f_-^{B\to\pi} = \frac{g_\pi f_B m_B}{f_\pi\left[v\cdot p_\pi + \Delta^B\right]}$$

$g_\pi = 0.492(29)$ [ALPHA, 1404.6951]

$\Delta^B = M_{B*} - M_B \approx 45\ \mathrm{MeV}$

# Heavy to light form factors

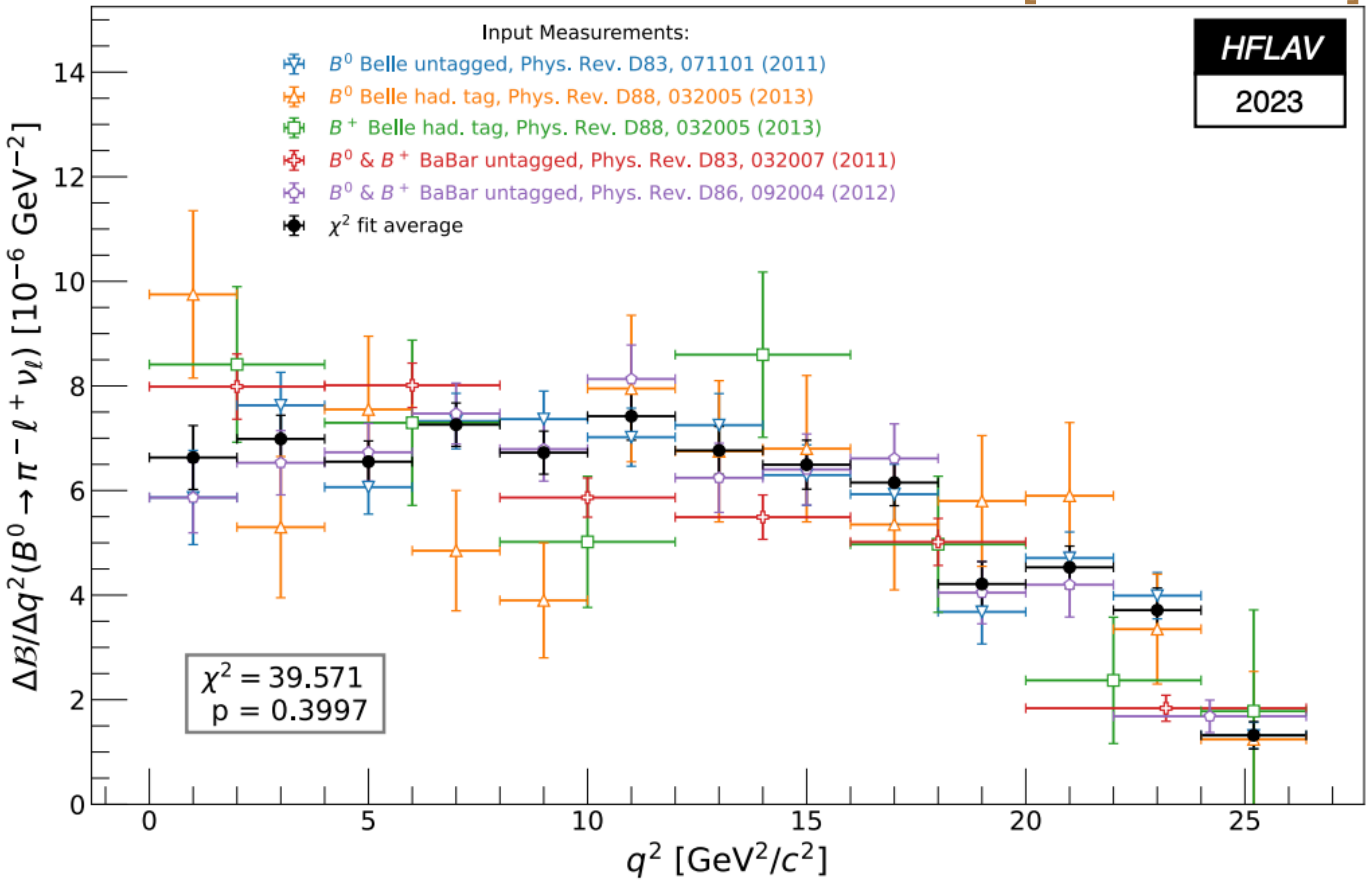
$|V_{ub}|$ Determination from $B_{(s)} \to \pi(K)\ell\nu$, need the form factors over full kinematic range

$$|V_{ub}| = (3.75 \pm 0.06_{exp} \pm 0.19_{theo}) \times 10^{-3}$$

HFLAV[2411.18639]

# Various $B \to \pi \ell \nu$ measurements

[2411.18639]




# HQ extrapolation

JLQCD [2203.04938]