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SCET 2026 – KIAS



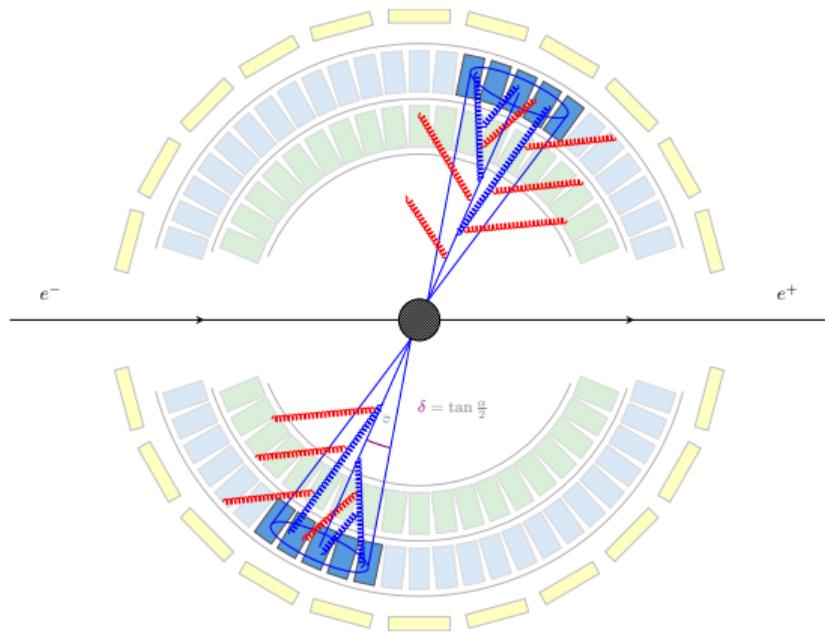
Bridging Multiple Roads to Non-Global Resummation

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Large logarithms in dijet processes



- We find a twofold pattern of logarithmic enhancements in dijet production

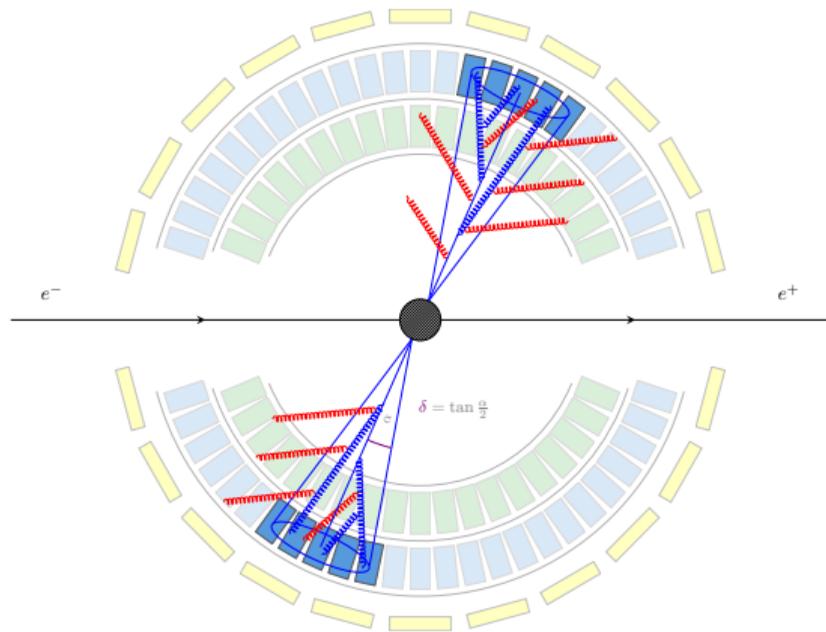
$$e^+ e^- \rightarrow \gamma^* \rightarrow 2 \text{ jets}$$

when (hard) radiation is restricted to be within the jets, since only (soft) radiation below Q_0 is allowed outside of the jets.

$$\sigma \sim 1 + \frac{\alpha_s}{\pi} C_F \left(3 \log \delta - 4 \log \delta \log \frac{Q}{Q_0} + \text{const.} \right)$$

[Sterman and Weinberg, 1977]

Large logarithms in dijet processes



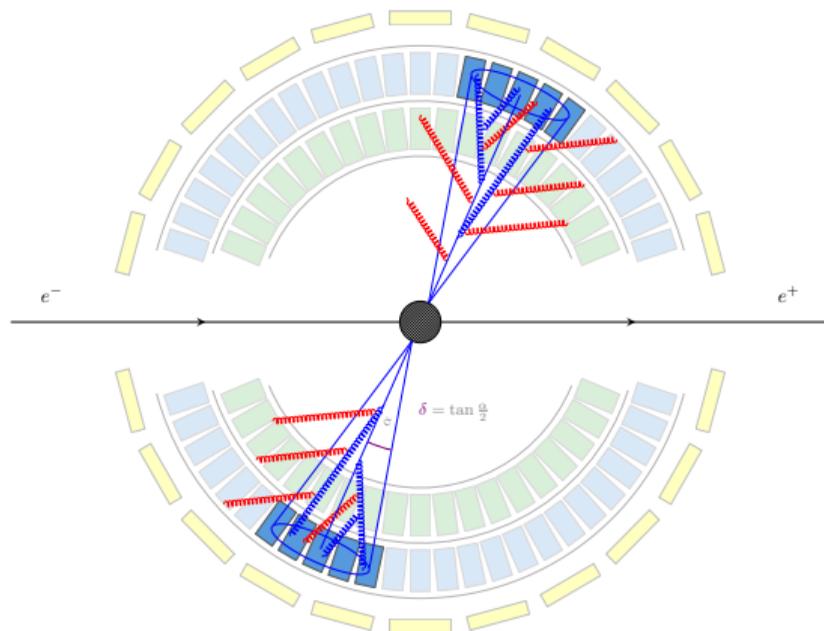
- We find both **soft** and **collinear** logarithmic enhancements

We assume for the rest of this talk

$$\log \delta \sim 1$$

$$\log \left(\frac{Q}{Q_0} \right) \gg 1$$

Large logarithms in dijet processes



o Example:

$$Q_0 \sim 5 \text{ GeV}$$

$$Q \sim 1 \text{ TeV}$$

then the product; $\alpha_s \sim 0.1$ and $L = \log \frac{Q}{Q_0}$

$$\alpha_s L \sim \mathcal{O}(1)$$

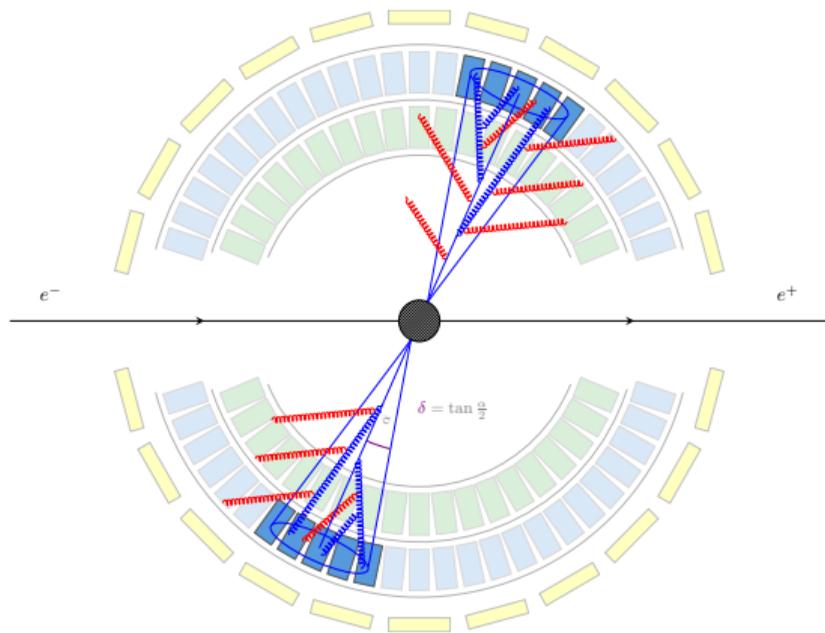
$$(\alpha_s L)^2 \sim \mathcal{O}(1)$$

$$(\alpha_s L)^n \sim \mathcal{O}(1)$$

spoils the perturbative expansion!!!

$$\sigma_{\text{LO}} + (\alpha_s L) \sigma_{\text{NLO}} + (\alpha_s L)^2 \sigma_{\text{NNLO}} + \dots$$

Large logarithms in dijet processes



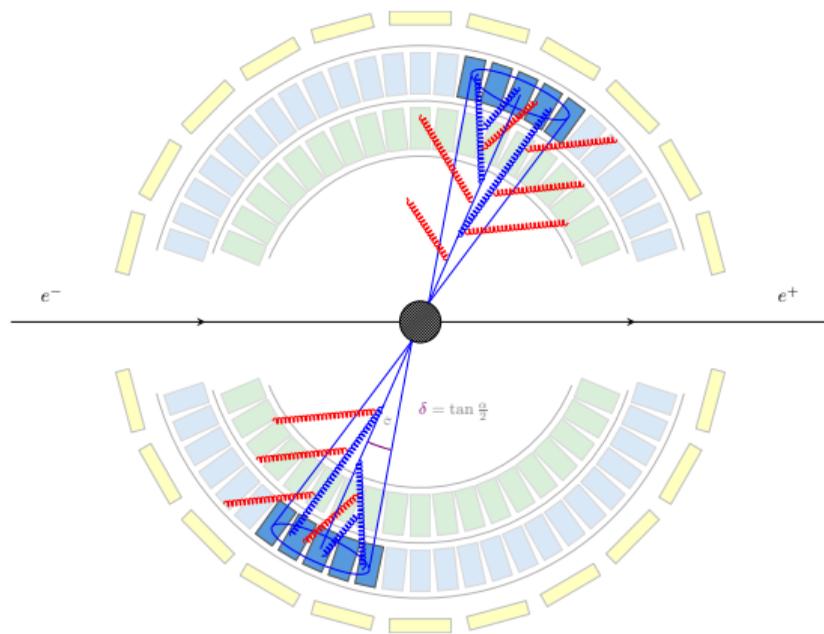
- Fixed order perturbation theory breaks down due to logarithmically enhanced corrections

$$\alpha_s^n L^m \quad \text{with} \quad L = \log\left(\frac{Q}{Q_0}\right)$$

Identify

- 1 $(\alpha_s L)^n$ Leading Logarithms (LL)
 - 2 $\alpha_s (\alpha_s L)^n$ Next-to-Leading Logarithms
- ⇒ (Re)assign LL, NLL, ... → LO, NLO, ...

Large logarithms in dijet processes



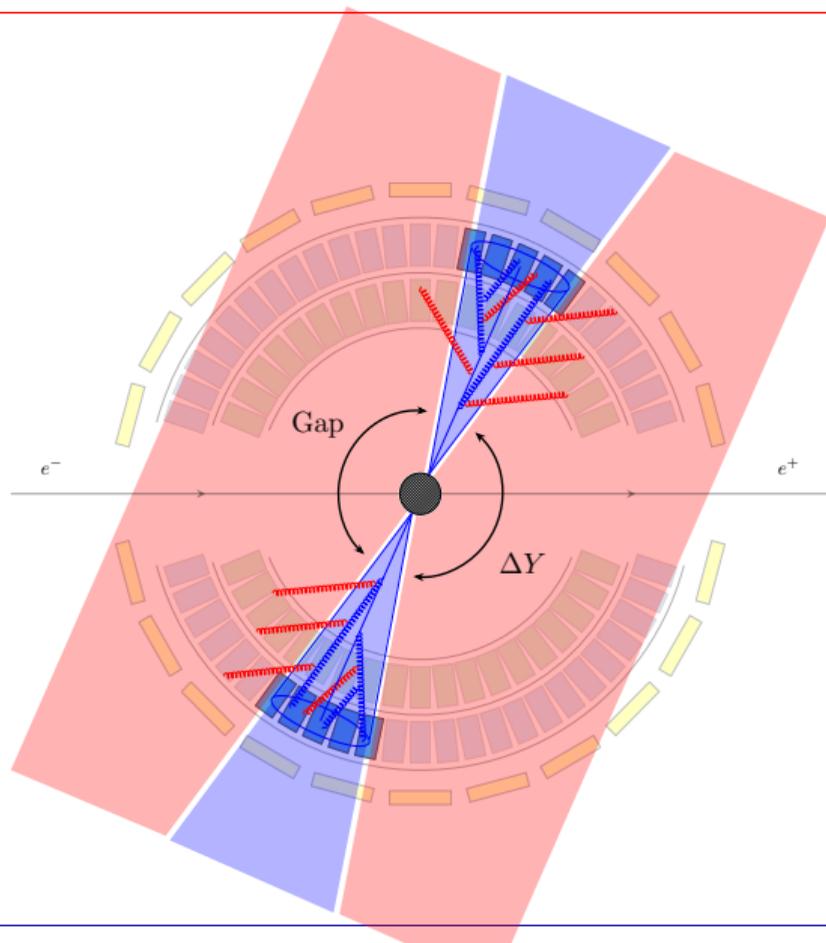
- Fixed order perturbation theory breaks down due to logarithmically enhanced corrections

$$\alpha_s^n L^m \quad \text{with} \quad L = \log \left(\frac{Q}{Q_0} \right)$$

To obtain reliable predictions across disparate scales, it is necessary to capture the entire tower of logarithms

⇒ *Resummation*

Higher-order effects: Non-Global Logarithms



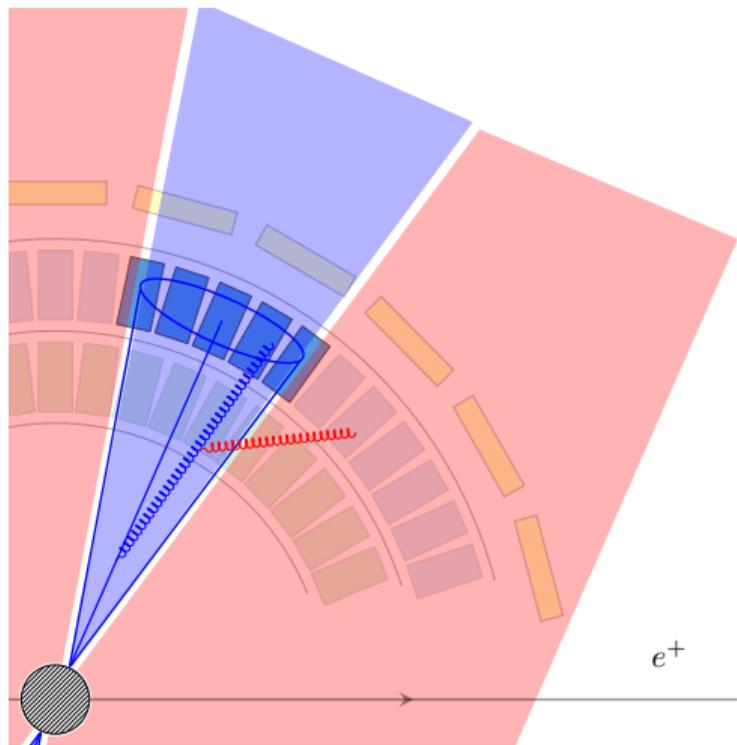
Jet cross sections involve angular cuts which constrain radiation within a corner of the phase space. As a consequence, logarithmically enhanced higher-order corrections known as

Non-Global Logarithms (NGLs)

arise.

[Dasgupta and Salam, hep-ph/0104277]

Higher-order effects: Non-Global Logarithms



The leading NGLs start at two loops

$$\begin{aligned}\sigma \sim & 1 + \frac{\alpha_s}{\pi} C_F \left(3 \log \delta - 4 \log \delta \log \frac{Q}{Q_0} + \text{const.} \right) \\ & + \left(\frac{\alpha_s}{\pi} \right)^2 \left[C_F^2 B_F(Q, Q_0, \delta) \right. \\ & \left. + C_F T_F n_F B_{n_F}(Q, Q_0, \delta) + C_F C_A B_A(Q, Q_0, \delta) \right] \\ & \left[-\zeta_2 + \text{Li}_2(e^{-2\Delta Y}) \right] \log^2 \frac{Q}{Q_0} + \dots\end{aligned}$$

[Dasgupta and Salam, hep-ph/0203009]

Outline: running across the most effective bridge

Parton Shower

EFT Methods

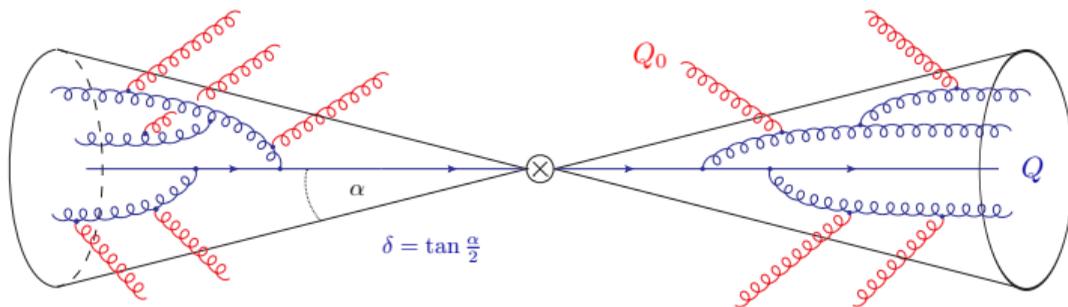


Generating Functionals

Factorisation & resummation by RG evolution

- Cross section for jet production in e^+e^- collisions with veto on radiation factorises into hard \mathcal{H}_m and soft \mathcal{S}_m functions [Becher et. al., 1508.06645]

$$\sigma(Q, Q_0) = \sum_{m=m_0}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\}, Q, \mu) \otimes \mathcal{S}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$



Factorisation & resummation by RG evolution

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$$\sigma(Q, Q_0) = \sum_{m=m_0}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\}, Q, \mu) \otimes \mathcal{S}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$
$$\frac{d}{d \log \mu} \mathcal{H}_m(Q, \mu) = - \sum_{l=2}^m \mathcal{H}_l(Q, \mu) \Gamma_{lm}(Q, \mu)$$

The RG evolution is governed by the anomalous dimension which has been extracted up to two-loops

$$\Gamma = \frac{\alpha_s}{4\pi} \Gamma^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \Gamma^{(2)} + \dots \quad \Rightarrow \text{See Juerg Haag's talk!}$$

❓ How to solve complicated RGEs ❓

⇒ Monte Carlo Methods

- In practice coupled RGEs for hard functions \mathcal{H}_m , however these simplify at LL due to the form of $\mathbf{\Gamma}^{(1)}$

$$\frac{d}{dt} \mathcal{H}_m(t) = \mathcal{H}_m(t) \mathbf{V}_m + \mathcal{H}_{m-1}(t) \mathbf{R}_{m-1}$$

$$\mathcal{H}_m(t) = \mathcal{H}_m(t_0) e^{(t-t_0) \mathbf{V}_m} + \int_{t_0}^t dt' \mathcal{H}_{m-1}(t') \mathbf{R}_{m-1} e^{(t-t') \mathbf{V}_m}$$

$$\mathbf{\Gamma}^{(1)} = \begin{pmatrix} \mathbf{V}_2 & \mathbf{R}_2 & 0 & 0 & \dots \\ 0 & \mathbf{V}_3 & \mathbf{R}_3 & 0 & \dots \\ 0 & 0 & \mathbf{V}_4 & \mathbf{R}_4 & \dots \\ 0 & 0 & 0 & \mathbf{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Iterative solution

$$\mathcal{H}_k(t) = \mathcal{H}_k(0) e^{t \mathbf{V}_k},$$

$$\mathcal{H}_{k+1}(t) = \int_0^t dt' \mathcal{H}_k(t') \mathbf{R}_k e^{(t-t') \mathbf{V}_{k+1}},$$

$$\mathcal{H}_{k+2}(t) = \int_0^t dt' \mathcal{H}_{k+1}(t') \mathbf{R}_{k+1} e^{(t-t') \mathbf{V}_{k+2}},$$

$$\mathcal{H}_{k+3}(t) = \dots$$

- o In practice coupled RGEs for hard functions \mathcal{H}_m , however these simplify at LL due to the form of $\mathbf{\Gamma}^{(1)}$



$$\frac{d}{dt} \mathcal{H}_m(t) = \mathcal{H}_m(t) \mathbf{V}_m + \mathcal{H}_{m-1}(t) \mathbf{R}_{m-1}$$

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Iterative solution

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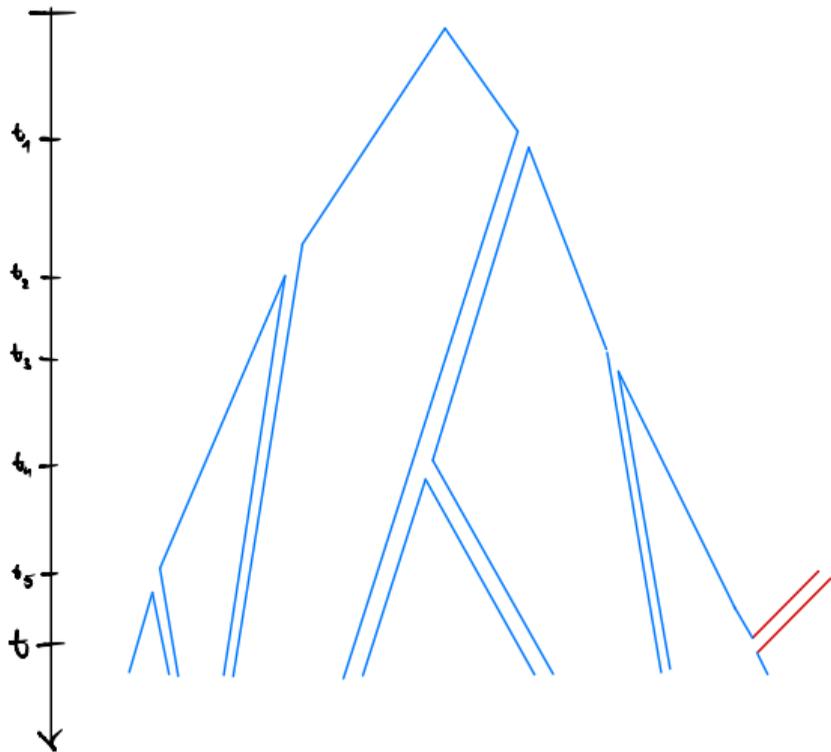
$$\mathcal{H}_{k+2}(t) = \int_0^t dt' \mathcal{H}_{k+1}(t') \mathbf{R}_{k+1} e^{(t-t') \mathbf{V}_{k+2}},$$

$$\mathcal{H}_{k+3}(t) = \dots$$

Algorithm

- 1 Generate shower time t “ordering variable” and pick an emitting dipole
- 2 Generate directions (θ, ϕ) and construct new emission n_k
- 3 Repeat.
Until an emission enters the gap.

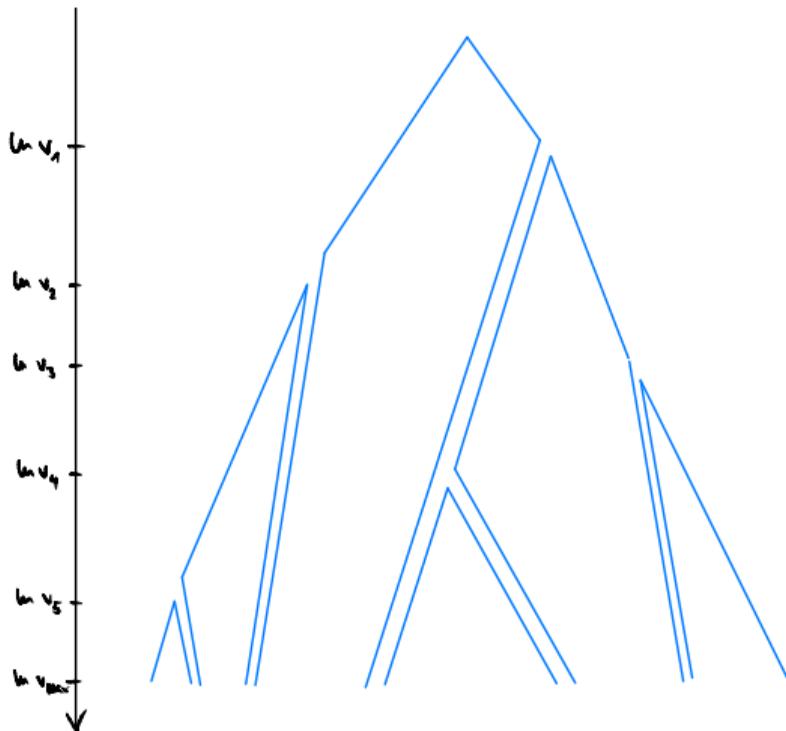
Soft Radiation



Parton showers are constructed using a Markovian algorithm, which probabilistically transforms an n -parton state into an $n + 1$ -parton state.

$$\frac{d\mathcal{P}_{n \rightarrow n+1}}{d \ln v} = \sum_{\text{dipoles } \{\tilde{i}, \tilde{j}\}} \int d\bar{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s(k_t)}{\pi} \times \left[g(\bar{\eta}) a_k P_{\tilde{i} \rightarrow ik}(a_k) + g(-\bar{\eta}) b_k P_{\tilde{j} \rightarrow jk}(b_k) \right]$$

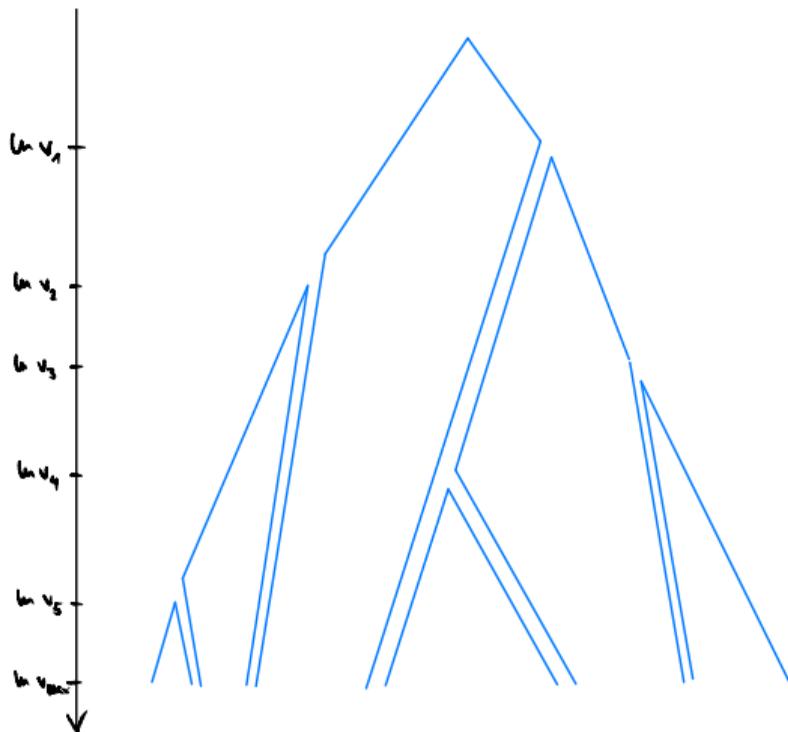
Soft + Collinear Radiation



Algorithm

- 1 Generate candidate $\ln v$ and pick emitting dipole
- 2 Generate directions (θ, ϕ) and construct new emission p_k
- 3 Redistribute recoil !
- 4 Accept/veto emission
- 5 Repeat.
Until we reach the shower cutoff.

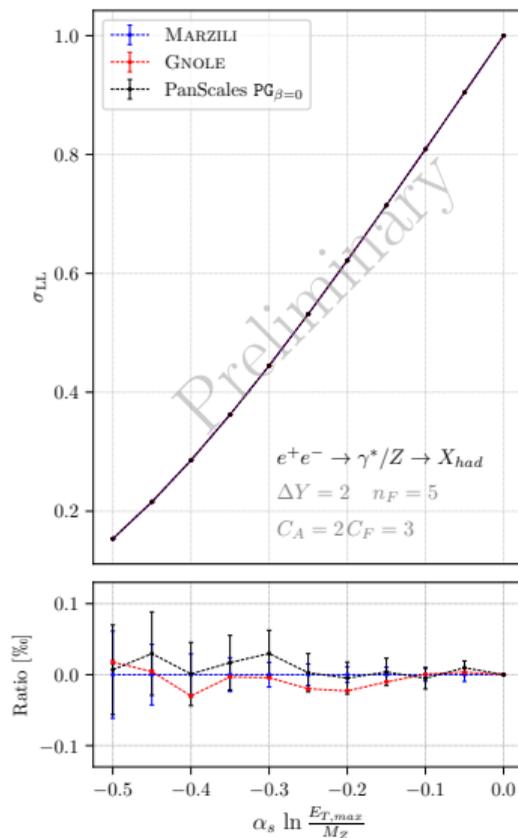
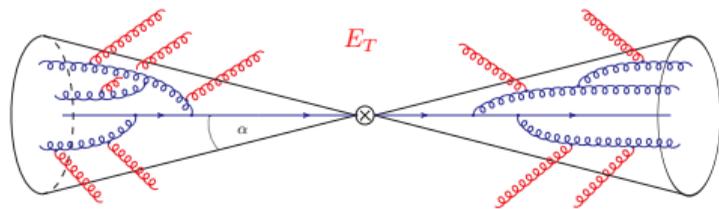
More versatile framework
⇒ Captures soft radiation



Validation: LL resummation interjet energy flow

Gap fraction: fraction of events with transverse energy E_T in gap below Q_0

$$R(Q_0) \equiv \frac{1}{\sigma_{\text{tot}}} \int_0^{E_{T,\text{max}}} dE_T \frac{d\sigma}{dE_T}$$



Validation: LL resummation interjet energy flow

Technical challenges in PanScales (and GNOLE)

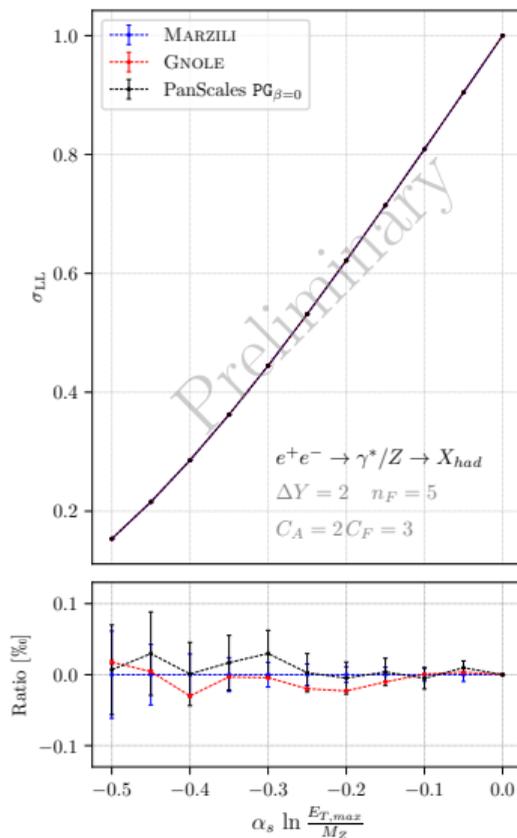
- Extract precise logarithmic correction via extrapolation of $\alpha_s \rightarrow 0$

$$\alpha_s \cdot \ln \frac{E_{T,max}}{M_Z} = \lambda \Leftrightarrow \frac{E_{T,max}}{M_Z} = e^{\lambda \cdot \alpha_s^{-1}}$$

```
1 import numpy as np
2
3 lam_val = -0.5
4 as_val = 10**(-5)
5
6 np.exp(lam_val * as_val**(-1))
```

[1] ✓ 0.0s Python

... np.float64(0.0)



Validation: LL resummation interjet energy flow

Technical challenges in PanScales (and GNOLE)

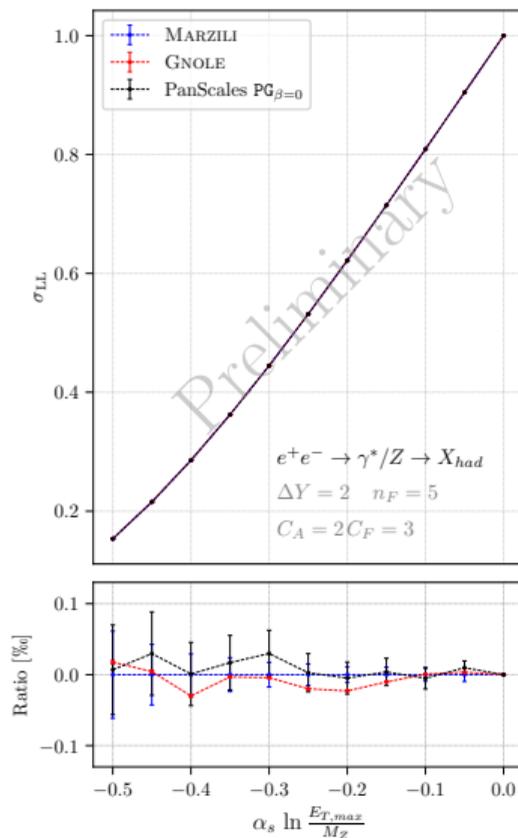
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```

[1] ✓ 0.0s Python

... np.float64(0.0) ← underflow rounding error



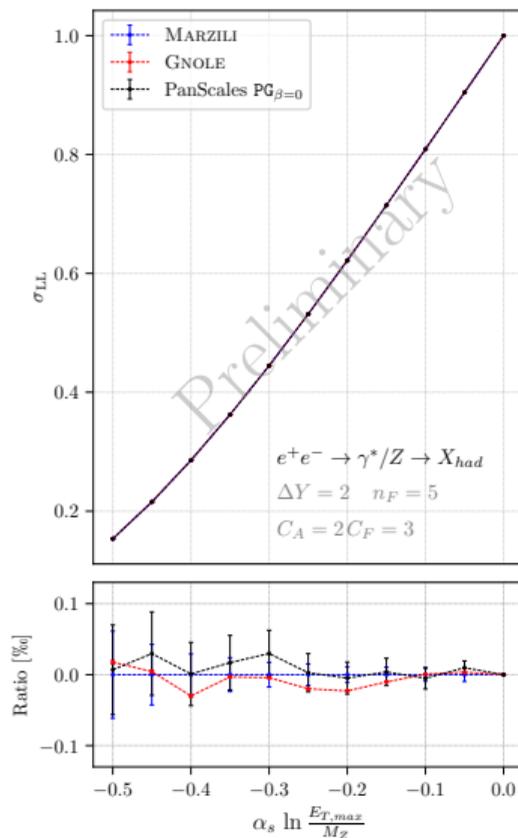
Validation: LL resummation interjet energy flow

Technical challenges in PanScales (and GNOLE)

- Extract precise logarithmic correction via extrapolation of $\alpha_s \rightarrow 0$

$$\alpha_s \cdot \ln \frac{E_{T,max}}{M_Z} = \lambda \Leftrightarrow \frac{E_{T,max}}{M_Z} = e^{\lambda \cdot \alpha_s^{-1}}$$

- Populate phase-space deep in the Lund plane



Non-global (resummation) frontier

Subleading Colour

[Forshaw et. al. ; CVOLVER]
[Nagy et. al. ; DEDUCTOR]
[PanScales ; NODS]
[Hatta et. al.]

Higher Log-accuracy

[Banfi et. al. ; Gnole]
[Becher et. al.]
[PanScales]
[Bell et. al.; Soft-Serve]
[S. Caron-Huot]

Super-Leading Logs

[Forshaw et. al.]
[Becher et. al.]
[Dasgupta et. al.]

Non-global (resummation) frontier

Rest of this talk

Subleading Colour

Higher Log-accuracy

Super-Leading Logs

[Forshaw et. al. ; CVOLVER]

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[Forshaw et. al.]

[Becher et. al.]

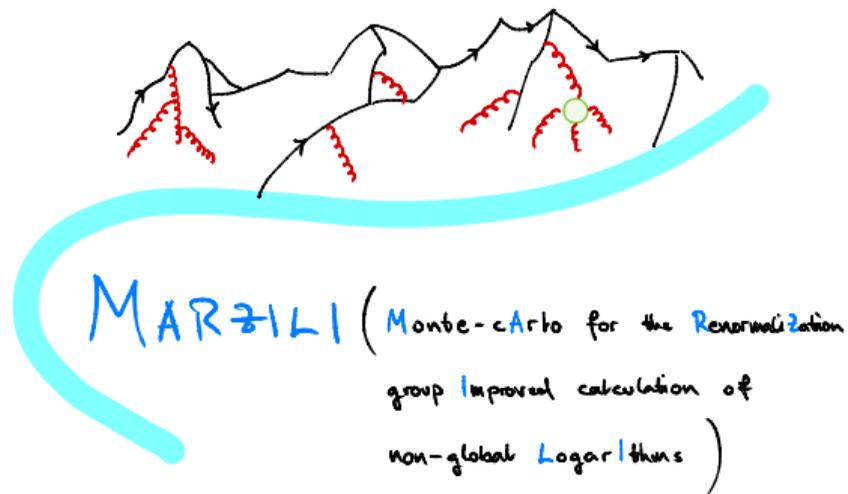
[Dasgupta et. al.]

Extend

- LL [1803.07045]
- LL' [1901.09038]

to NLL; using

- $\Gamma^{(2)}$ [2112.02108]



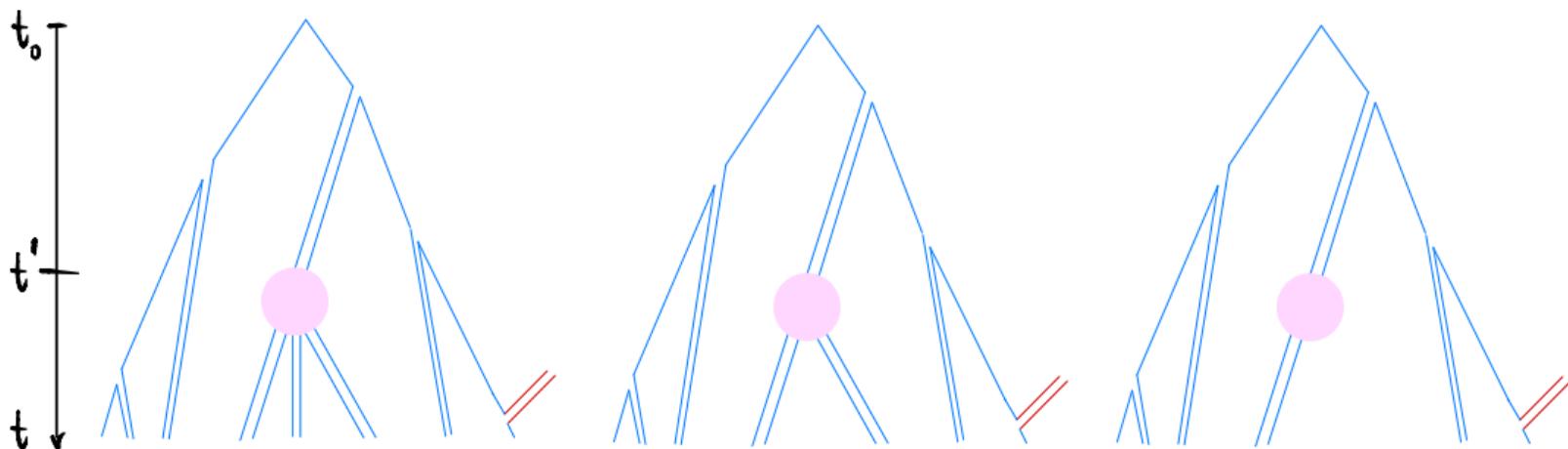
Include corrections due to $\Gamma^{(2)}$ in the evolution

$$\mathcal{H}_k(t_0) \int_{t_0}^t dt' \mathbf{U}_{kl}(t' - t_0) \cdot \frac{\alpha(t')}{4\pi} \left(\Gamma_{ll'}^{(2)} - \frac{\beta_1}{\beta_0} \Gamma_{ll'}^{(1)} \right) \cdot \mathbf{U}_{l'm}(t - t')$$

LL evolution

Insertion of $\Gamma^{(2)}$

LL evolution

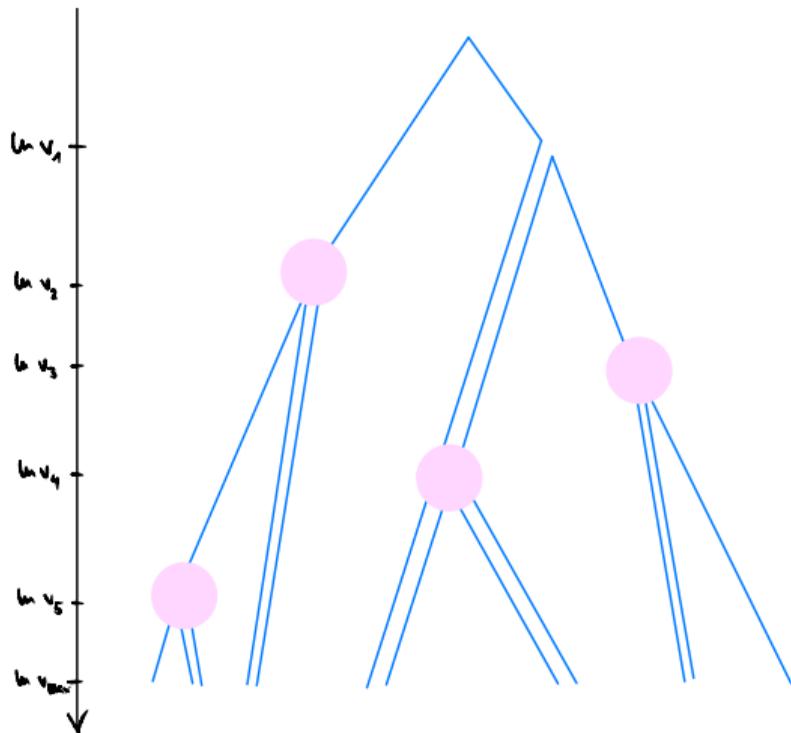


Splitting probability contains a virtual piece

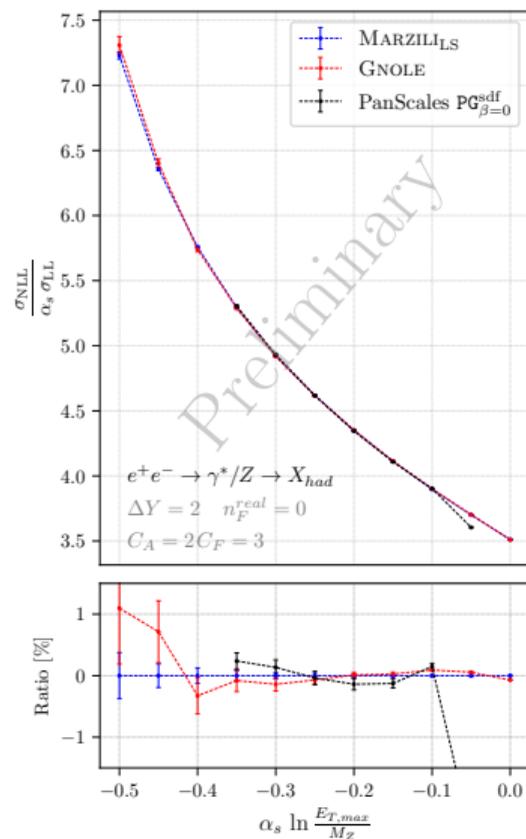
$$\frac{d\mathcal{P}_{n \rightarrow n+1}}{d \ln v} = \sum_{\{\tilde{i}, \tilde{j}\}} \int d\bar{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s(k_t)}{\pi} \left(1 + \frac{\alpha_s(k_t) K_{\text{CMW}}}{2\pi} \right) \times \left[g(\bar{\eta}) a_k P_{\tilde{i} \rightarrow ik}(a_k) + g(-\bar{\eta}) b_k P_{\tilde{j} \rightarrow jk}(b_k) \right]$$

Additional acceptance for soft partners

$$P_{\text{accept}} = \frac{|M_{\text{DS}}|^2}{\sum_h |M_{\text{shower},h}|^2}$$



Comparison at NLL



Agreement between frameworks

- MARZILI [2307.02283 + upcoming]
- GNOLE [2111.02413]
- PanScales [2307.11142]

Detailed comparison with Gnole at NLL_[upcoming]

Divide the NLL contribution in different pieces

$$\sigma_{\text{NLL}} \sim \sigma_{\text{hard}} + \sigma_{\text{soft}} + \Delta\sigma_{\text{run.}} + \sigma_{\Gamma(2)}$$

Detailed comparison with Gnole at NLL_[upcoming]

Divide the NLL contribution in different pieces

Scheme – dep.

$$\begin{array}{cccccccc} \sigma_{\text{NLL}} & \sim & \sigma_{\text{hard}} & + & \sigma_{\text{soft}} & + & \Delta\sigma_{\text{run.}} & + & \sigma_{\Gamma(2)} \\ \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow \\ \checkmark & & \checkmark & & \times & & \times & & \times \\ \sigma_{\text{NLL}}^{\text{GNOLE}} & \sim & \sigma_{\text{hard}} & + & \sigma_{\text{soft}} & + & \Delta\sigma_{\text{run.}} & + & \sigma_{\Gamma(2)} \end{array}$$

Detailed comparison with Gnole at NLL [upcoming]

Add a piece proportional to ϵ to the anomalous dimension

$$\tilde{\Gamma}^{(1)} = \Gamma^{(1)} + \epsilon \Delta \Gamma^{(1)}$$

See Juerg Haag's talk!

LS – Scheme [Caron-Huot, 1501.03754]

[Catani, Marchesini, Webber, 1991]

 Affects NLL contributions through renormalisation

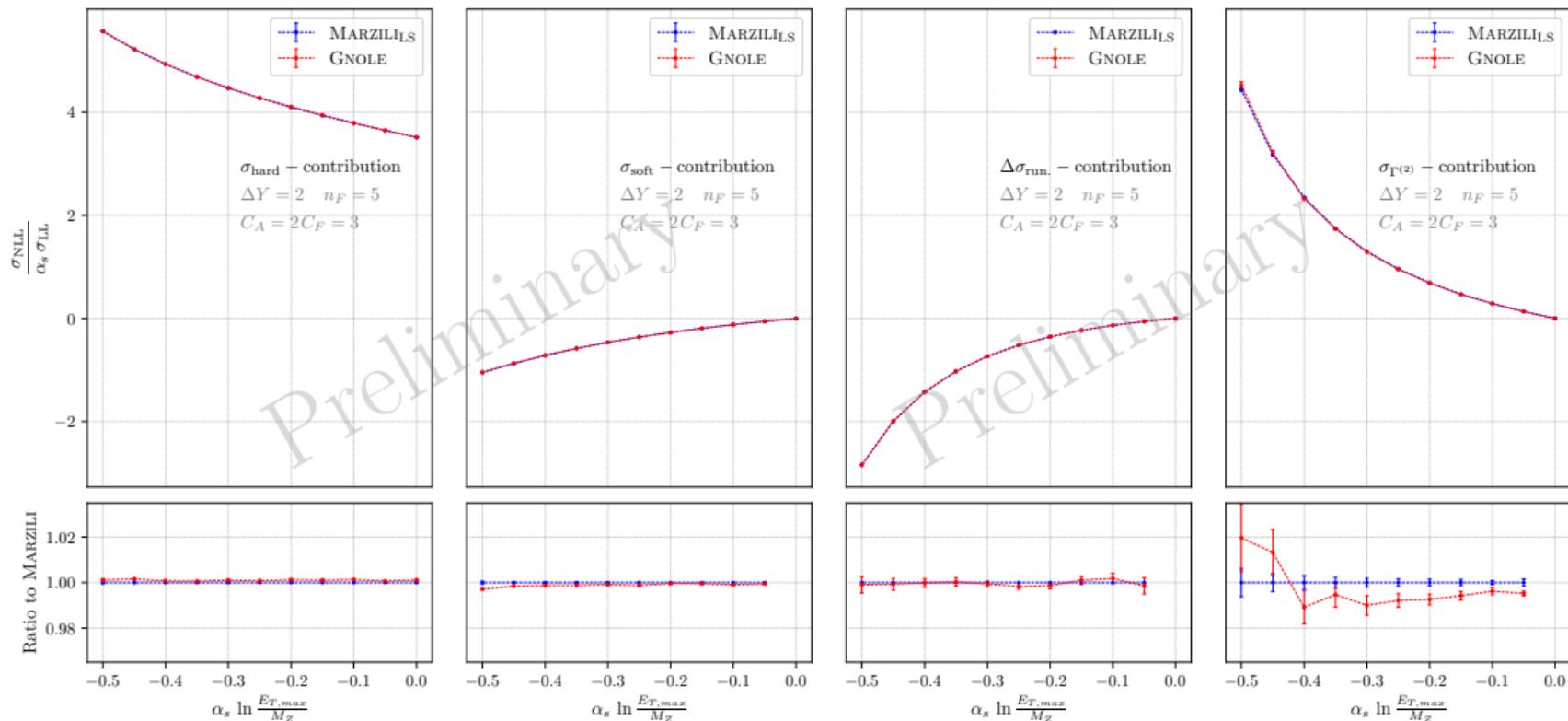
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LS – Scheme

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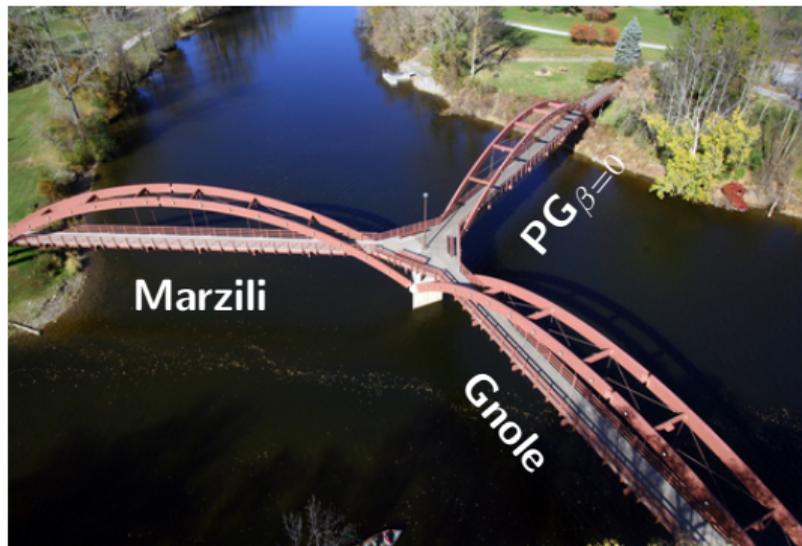
Detailed comparison with Gnole at NLL_[upcoming]



Conclusions: good agreement between frameworks

Parton Shower

EFT Methods



Generating Functionals