Renormalon analysis of SCET-II observables

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Cluster of Excellence PRISMA++ | Precision Physics, Fundamental Interactions and Structure of Matter

Motivation

- Typical factorisation theorem for e^+e^--observables in SCET

$$d\sigma \simeq H(\mu_F) \cdot J(\mu_F) \otimes \bar{J}(\mu_F) \otimes S(\mu_F)$$

- Study impact of non-perturbative effects with renormalon techniques

- Previous studies focused on observables where the leading non-perturbative effects are generated by the the soft function

  ➡ Thrust, C-Parameter, … [Mateu,Stewart,Thaler;12]

  ➡ Very few studies of renormalons in collinear functions [Scimemi,Vladimirov;17] [Gracia,Mateu;21]

- SCET-II observables not as well studied [Becher,Bell;13]

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- Our goal is to analyse the renormalon structure for generic SCET-II observables

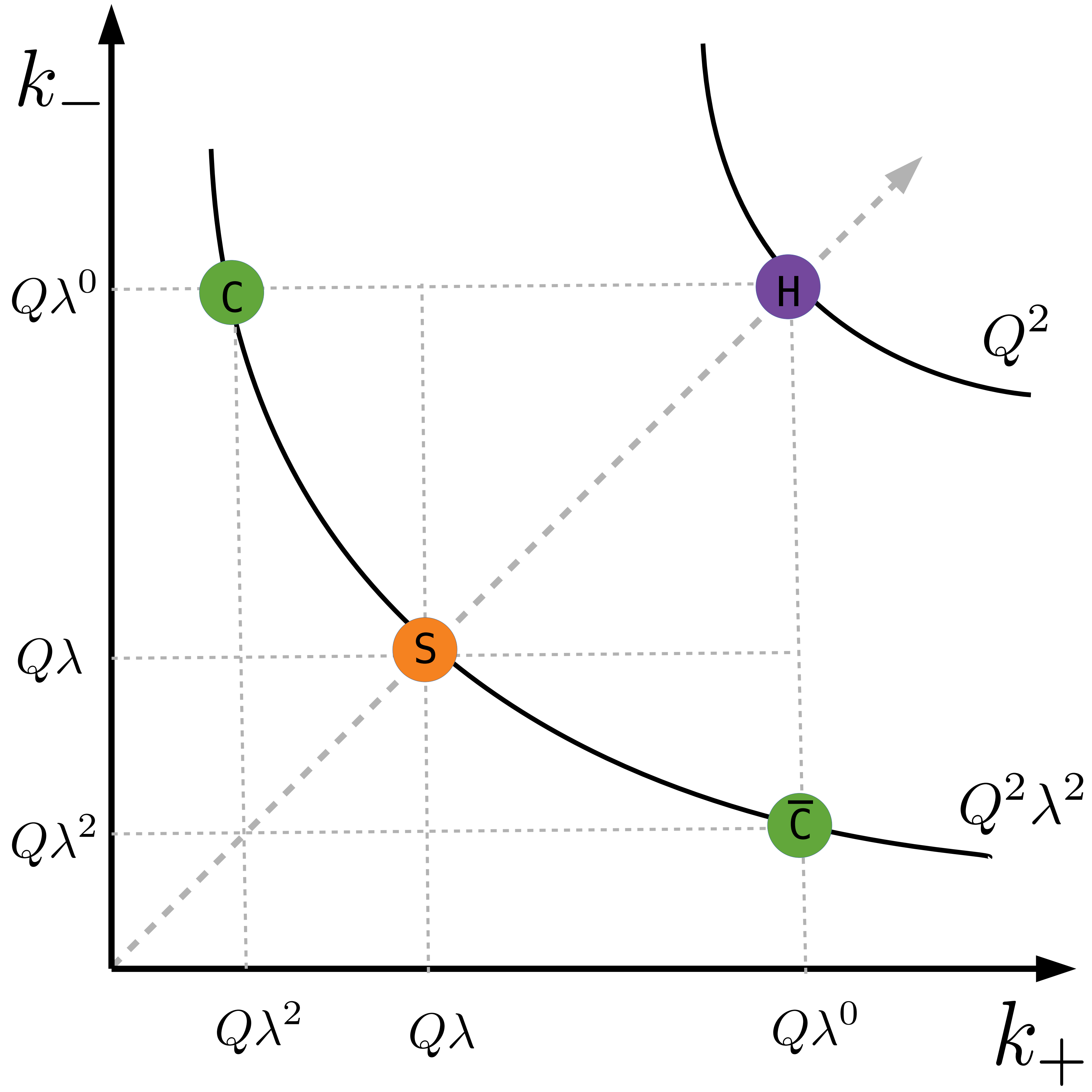
SCET-II

- Typical scaling
  - Hard region: $k_H^\mu \sim (1, 1, 1)Q$
  - Collinear region: $k_C^\mu \sim (1, \lambda^2, \lambda)Q$
  - Soft region: $k_S^\mu \sim (\lambda, \lambda, \lambda)Q$
- Soft and collinear modes have same virtuality

  ➡Additional rapidity divergences

- Introduce additional regulator [Becher,Bell;12]

$$\prod_i \int \frac{d^d k_i}{(2\pi)^d} \left(\frac{\nu}{k_i^- + k_i^+} \right)^\alpha \delta(k_i^2)\, \theta(k_i^0)$$

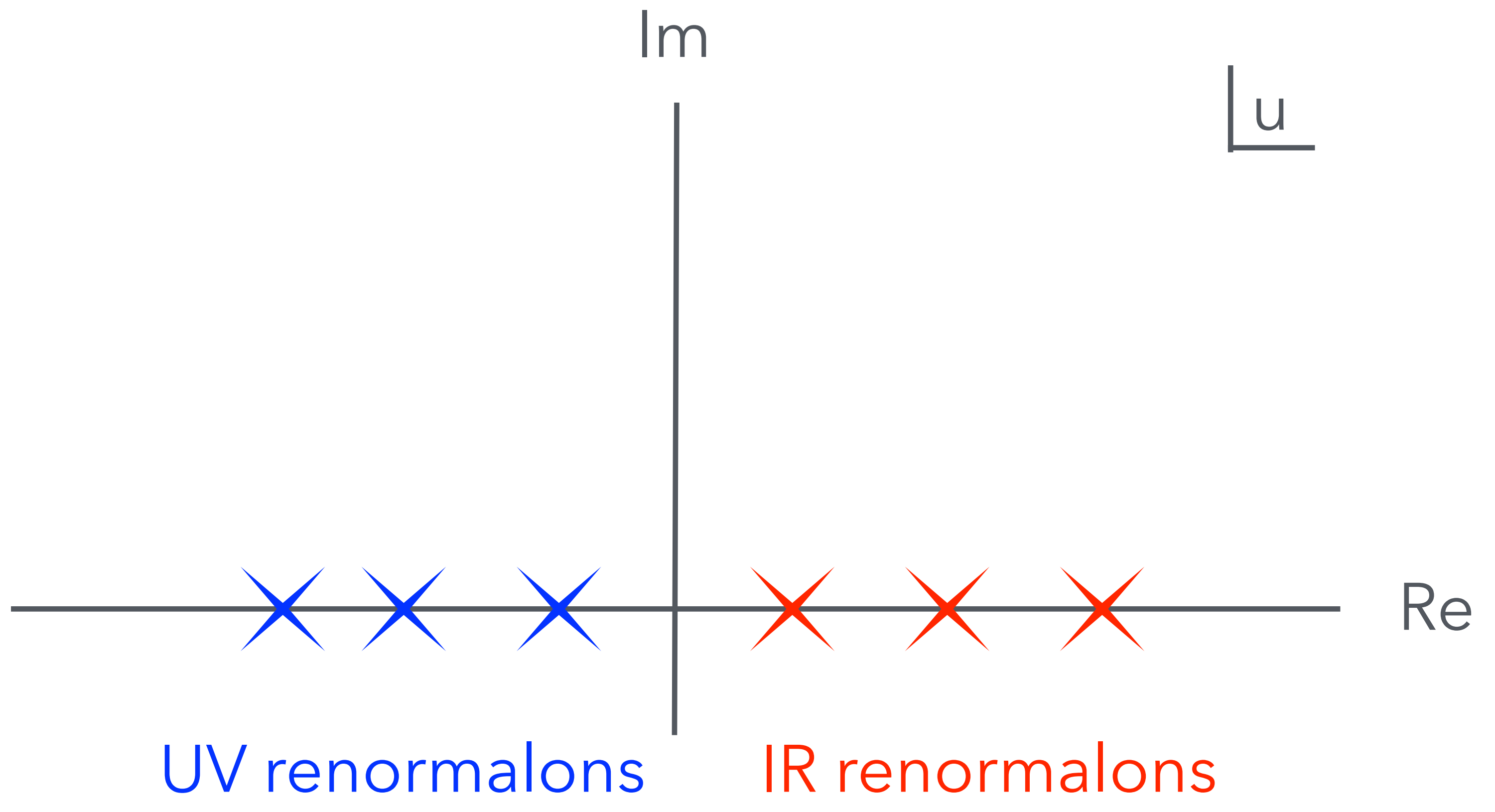


Renormalons

- Perturbative series in QCD are asymptotic

  ➡Behaviour is caused by sensitivity to soft momenta

- Borel transform enlarges convergence radius



$$f(a_s) = \frac{1}{\beta_0} \sum_{n=0}^{\infty} f_{n+1}\, a_s^{n+1} \quad \longrightarrow \quad B[f](u) = \frac{1}{\beta_0} \sum_{n=0}^{\infty} \frac{f_{n+1}}{n!} u^n$$

- Renormalons are poles on the real axis in the complex Borel plane

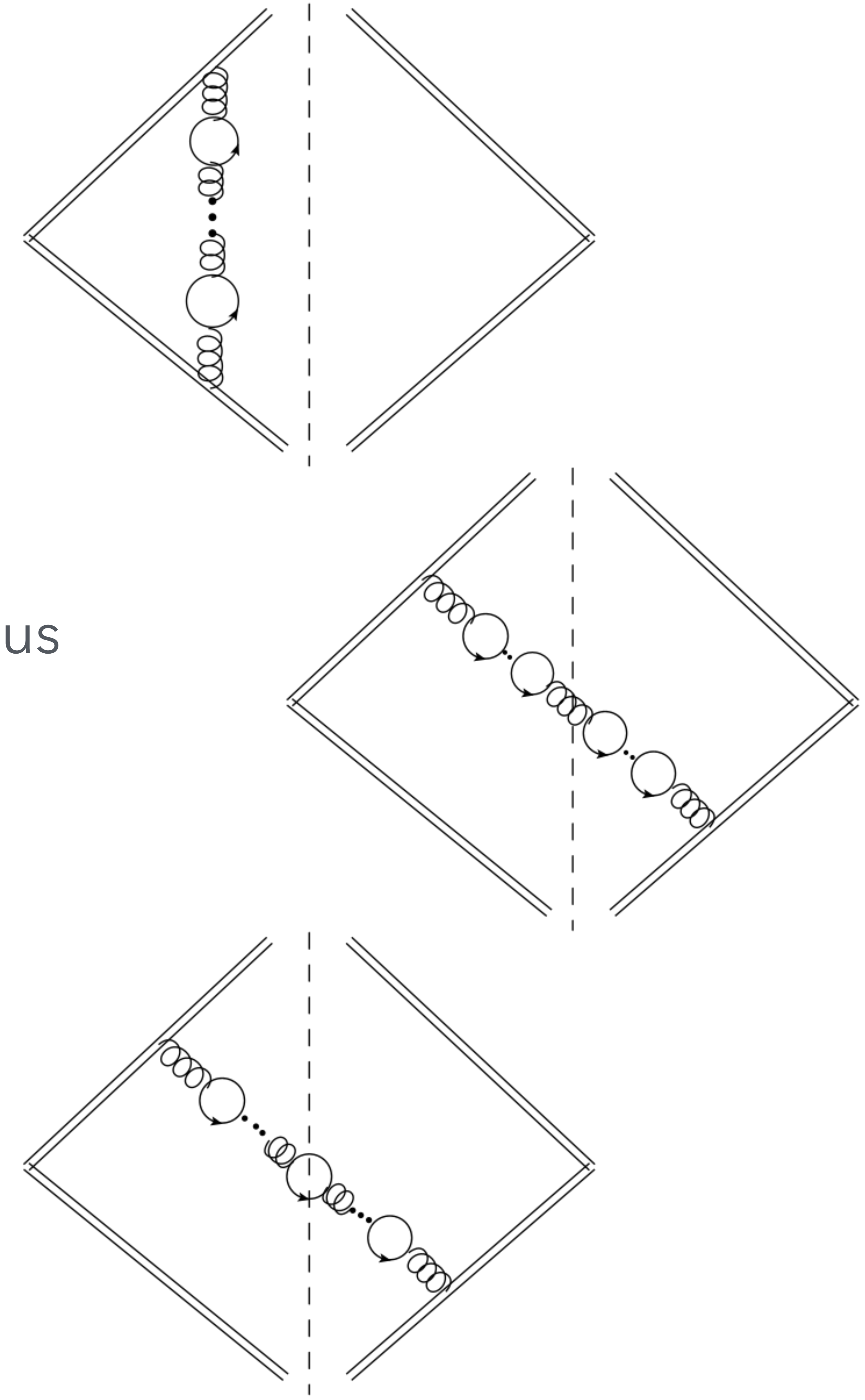
$$[f(a_s)]_B = \int_0^\infty \mathrm{d}u\; e^{-u/a_s} B[f](u) = \int_0^\infty \mathrm{d}u \left(\frac{\Lambda_{\mathrm{QCD}}}{\mu}\right)^{2u} B[f](u)$$

- Inverse transformation relates pole position to Λ_{QCD}

Large-β_0 approximation

- Approximation is defined by the condition $\alpha_s \beta_0 \sim \mathcal{O}(1)$
  - ➡ Tightly connected to bubble sum approximation
- Two differences compared to standard QCD renormalon calculus
  - Only real emissions contribute
  - Presence of rapidity regulator
- Additional approximation on measurement function

$$\mathcal{M}(p_q, p_{\bar{q}}) = \mathcal{M}(p_q + p_{\bar{q}})$$



Soft function

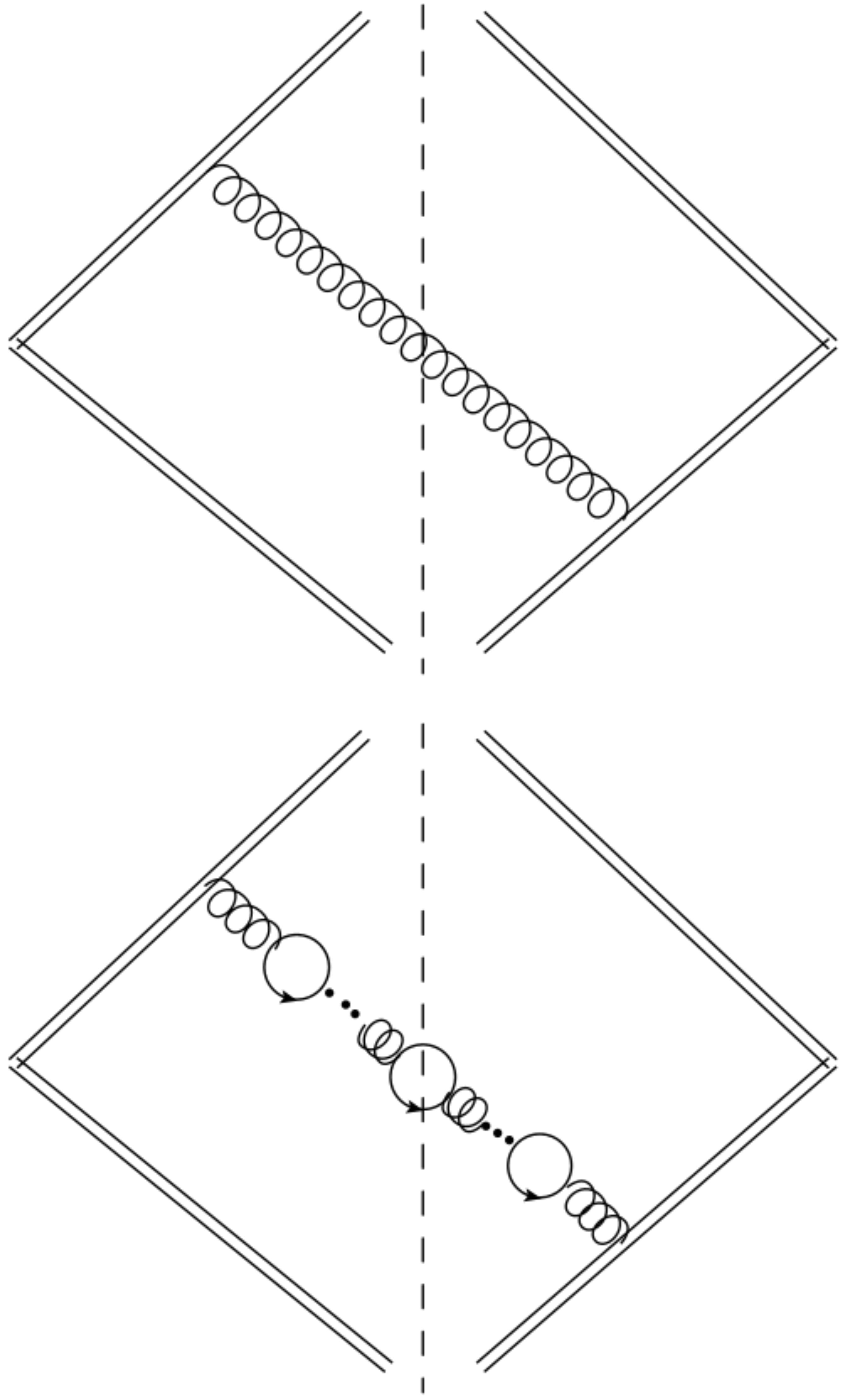
- Laplace-transformed soft function

$$S(\tau,\nu) = 1 + \frac{\alpha_s}{4\pi}\frac{e^{\epsilon\gamma_E}\mu^{2\epsilon}\nu^{\alpha}}{2\pi^{1-\epsilon}}\int \mathrm{d}^d p\, \delta(p^2)\,\theta(p^0)\,\frac{\mathscr{A}_S(p)}{(n\cdot p + \bar{n}\cdot p)^{\alpha}}\,\mathscr{M}_S(\tau;p) + \mathcal{O}(\alpha_s^2)$$

- Borel transformation results in

$$\alpha_s\delta(p^2) \to -\frac{4\pi(\mu^2 e^C)^u\theta(p^2)}{\beta_0\Gamma(1-u)\Gamma(u)}\frac{1}{(p^2)^{1+u}} \qquad \text{and} \qquad d \to 4$$

$$\Rightarrow B[S](u) = -\frac{8C_F}{\pi\beta_0}\frac{(\mu^2e^C)^u\nu^{\alpha}}{\Gamma(1-u)\Gamma(u)}\int \mathrm{d}^4 p\,\frac{\theta(p^2)\,\theta(p^0)\,\mathscr{M}_S(\tau;p)}{(n\cdot p)(\bar{n}\cdot p)\,(n\cdot p+\bar{n}\cdot p)^{\alpha}\,(p^2)^{1+u}}$$



Soft function

- Borel-transformed soft function

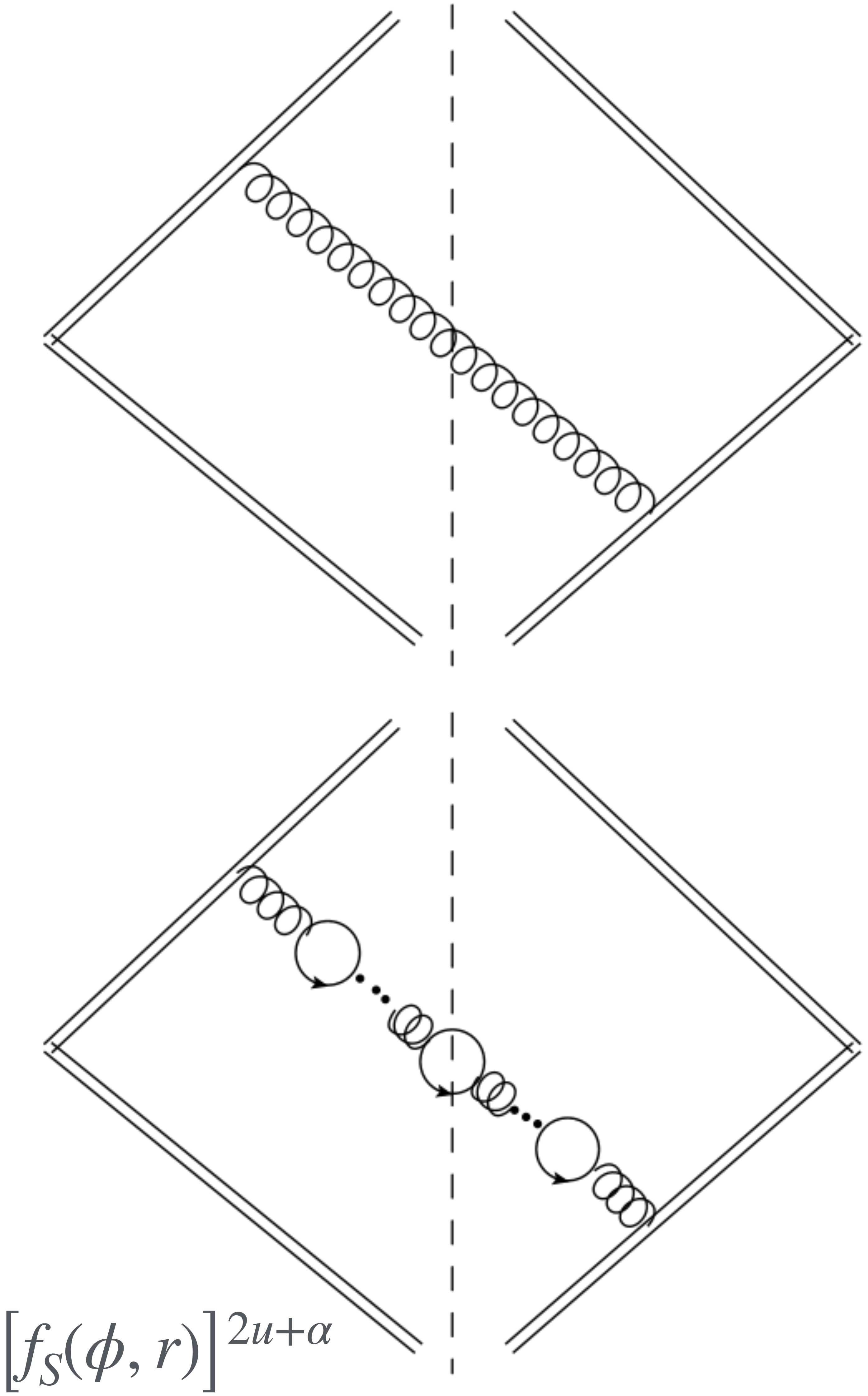
$$B[S](u) = -\frac{8C_F}{\pi\beta_0}\frac{(\mu^2 e^C)^u \nu^\alpha}{\Gamma(1-u)\Gamma(u)}\int \mathrm{d}^4p\, \frac{\theta(p^2)\,\theta(p^0)\,\mathscr{M}_S(\tau;p)}{(n\cdot p)(\bar{n}\cdot p)\,(n\cdot p+\bar{n}\cdot p)^\alpha\,(p^2)^{1+u}}$$

- Momentum mapping: $p_\pm = p_T\, e^{\pm y}/r$

- Measurement function: $\mathscr{M}_S(\tau;p) = \exp\left[-\tau\, p_T f_S(\phi,r)\right]$

- Master formula for Borel-transformed soft function

$$B[S](u) = -\frac{4C_F}{\pi\beta_0}\frac{(\mu^2 e^C)^u \nu^\alpha \tau^{2u+\alpha}}{\Gamma(1-u)\Gamma(u)}\frac{\Gamma^2(\alpha/2)\,\Gamma(-2u-\alpha)}{\Gamma(\alpha)}\int_0^1 \frac{\mathrm{d}r\, r^{1+2u+\alpha}}{(1-r^2)^{1+u}}\int_{-\pi}^{\pi}\mathrm{d}\phi\, \left[f_S(\phi,r)\right]^{2u+\alpha}$$



Jet function

- Similar strategy as for soft function with off-shell matrix element gives

$$B[J](u) = -\frac{4C_F}{\pi\beta_0}\frac{(\mu^2 e^C)^u\,\tau^{2u}}{\Gamma(1-u)\Gamma(u)}\left(\frac{\nu}{Q}\right)^{\alpha}\Gamma(-2u)\int_0^1 \mathrm{d}z\int_0^1 \mathrm{d}r\,\frac{z^{-1-\alpha}\,r^{1+2u}}{(1-r^2)^{1+u}}\,\frac{2-z(2-z)(2-r^2)}{[1-z(1-r^2)]^2}\int_{-\pi}^{\pi}\mathrm{d}\phi\,\left[f_J(\phi,r,z)\right]^{2u}$$

  - Measurement function $\qquad \mathscr{M}_J(\tau;p) = \exp\left[-\tau\,p_T f_J(\phi,r,z)\right]$
  - Momentum mapping $\qquad p_- = zQ \qquad p_+ = p_T^2/(zQr^2)$

Collinear anomaly framework

- Product of soft and jet functions can be written as

$$J_n(\tau,\mu,\nu)\,J_{\bar{n}}(\tau,\mu,\nu)\,S(\tau,\mu,\nu) = (Q\bar{\tau})^{-2F(\tau,\mu)}\;W(\tau,\mu)$$

- $F(\tau,\mu)$ collinear anomaly exponent
- $W(\tau,\mu)$ remainder function

- Turns into a sum in Borel space since there is at most a single bubble chain

$$B[J_n](u) + B[J_{\bar{n}}](u) + B[S](u) = B[W](u) - 2\ln(Q\bar{\tau})\,B[F](u)$$

- Via the previous definitions we can define master formulas for these functions
  - ➡ Rapidity divergences cancel if the observable satisfies the condition $f_J(\phi, r, 0) = f_S(\phi, r)$

Collinear anomaly framework

- Master formulas for generic SCET-2 observables

- Collinear anomaly exponent

$$B[F](u) = \frac{8C_F}{\pi\beta_0}\frac{(\mu^2\tau^2 e^C)^u\,\Gamma(-2u)}{\Gamma(1-u)\Gamma(u)}\int_0^1 \frac{\mathrm{d}r\, r^{1+2u}}{(1-r^2)^{1+u}}\int_{-\pi}^{\pi}\mathrm{d}\phi\,\left[f_S(\phi,r)\right]^{2u}$$

- Remainder function

$$B[W](u) = -\frac{8C_F}{\pi\beta_0}\frac{(\mu^2\tau^2 e^C)^u\,\Gamma(-2u)}{\Gamma(1-u)\Gamma(u)}\int_0^1 \frac{\mathrm{d}r\, r^{1+2u}}{(1-r^2)^{1+u}}\int_{-\pi}^{\pi}\mathrm{d}\phi$$
$$\times\left\{2\Big(\ln\big(rf_S(\phi,r)\big)-H_{-1-2u}\Big)\left[f_S(\phi,r)\right]^{2u}+\int_0^1\mathrm{d}z\left[\frac{1}{z}\right]_+\frac{2-z(2-z)(2-r^2)}{[1-z(1-r^2)]^2}\left[f_J(\phi,r,z)\right]^{2u}\right\}$$

Collinear anomaly framework

- IR renormalon located at $u = 1/2$
- Collinear anomaly exponent

$$B[F](u) = -\frac{4C_F}{\pi^2\beta_0}\frac{\mu\tau e^{C/2}}{(u-1/2)}\int_{-\pi}^{\pi}\mathrm{d}\phi\left\{\lim_{r\to 0}\left[rf_S(\phi,r)\right] + \int_0^1 \frac{\mathrm{d}r}{\sqrt{1-r^2}}\frac{\mathrm{d}}{\mathrm{d}r}\left[rf_S(\phi,r)\right]\right\} + \mathcal{O}((u-1/2)^0)$$

- Remainder function

$$B[W](u) = -\frac{4C_F}{\pi^2\beta_0}\frac{\mu\tau e^{C/2}}{(u-1/2)}\int_{-\pi}^{\pi}\mathrm{d}\phi\left\{\frac{1}{(u-1/2)}\left[\lim_{r\to 0}\left[rf_S(\phi,r)\right] + \int_0^1 \frac{\mathrm{d}r}{\sqrt{1-r^2}}\frac{\mathrm{d}}{\mathrm{d}r}\left[rf_S(\phi,r)\right]\right]\right.$$
$$+\lim_{r\to 0}\left[rf_S(\phi,r)\right]\Big(2\ln(\mu\bar{\tau}) + C - 2\Big) - 2\int_0^1 \mathrm{d}z\left[\frac{1}{z}\right]_+ \lim_{r\to 0}\left[rf_J(\phi,r,z)\right]\int_0^1 \frac{\mathrm{d}r}{\sqrt{1-r^2}}\frac{\mathrm{d}}{\mathrm{d}r}\left[rf_S(\phi,r)\right]\left(2\ln\left(\frac{\mu\bar{\tau}}{\sqrt{1-r^2}}\right) + C - 2\right)$$
$$\left.+\int_0^1 \frac{\mathrm{d}r}{\sqrt{1-r^2}}\int_0^1 \mathrm{d}z\left[\frac{1}{z}\right]_+ \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{2-z(2-z)(2-r^2)}{[1-z(1-r^2)]^2}\, rf_J(\phi,r,z)\right)\right\} + \mathcal{O}((u-1/2)^0)$$

WTA-axis jet broadening

- Scalar sum of transverse momenta with respect to the winner-take-all (WTA)-axis [Bertolini,Chan,Thaler;13]

$$f_S(\phi, r) = \frac{1}{2} \qquad\qquad f_J(\phi, r, z) = \frac{1}{2}\min\left[\frac{1}{1-z}, \frac{1}{z}\right]$$

- WTA-axis within our approximation
  - ➡ $q\bar{q}$ -pair always pre-clustered
  - ➡ Points in either primary quark or off-shell gluon direction
  - ➡ No recoil effects
- Leading IR renormalon

$$B[F](u) = -\frac{2C_F}{\beta_0}\frac{e^{C/2}\mu\tau}{u-1/2} + \mathcal{O}((u-1/2)^0)$$

$$B[W](u) = -\frac{2C_F}{\beta_0}\frac{e^{C/2}\mu\tau}{u-1/2}\left(\frac{1}{u-1/2} - 5\sqrt{2} + 10\ln(1+\sqrt{2}) + C + 2\ln\left(\frac{\mu\bar{\tau}}{2}\right)\right) + \mathcal{O}((u-1/2)^0)$$

Thrust-axis jet broadening

- Factorisation theorem more complicated because of recoil effects

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = H(Q^2)\int \mathrm{d}^{d-2}p_L^\perp \int \mathrm{d}^{d-2}p_R^\perp \; J_n(\tau, p_L^\perp)\, J_{\bar{n}}(\tau, p_R^\perp)\, S(\tau, -p_L^\perp, -p_R^\perp)$$

- Recoil effects are trivial in the large-β_0 approximation since there is only a single bubble chain

$$\int \mathrm{d}^{d-2}p_L^\perp \int \mathrm{d}^{d-2}p_R^\perp \; J_n(\tau, p_L^\perp)\, J_{\bar{n}}(\tau, p_R^\perp)\, S(\tau, -p_L^\perp, -p_R^\perp)$$

$$= 1 + \delta J_n(\tau, p_L^\perp = 0) + \delta J_{\bar{n}}(\tau, p_R^\perp = 0) \quad + \int \mathrm{d}^{d-2}p_L^\perp \int \mathrm{d}^{d-2}p_R^\perp \; \exp\left[-\tfrac{\tau}{2}\,|p_L^\perp|\right]\, \exp\left[-\tfrac{\tau}{2}\,|p_R^\perp|\right] \delta S(\tau, -p_L^\perp, -p_R^\perp)$$

Thrust-axis jet broadening

$$\int \mathrm{d}^{d-2}p_L^\perp \int \mathrm{d}^{d-2}p_R^\perp \; J_n(\tau, p_L^\perp)\, J_{\bar{n}}(\tau, p_R^\perp)\, S(\tau, -p_L^\perp, -p_R^\perp)$$

$$= 1 + \delta J_n(\tau, p_L^\perp = 0) + \delta J_{\bar{n}}(\tau, p_R^\perp = 0) + \int \mathrm{d}^{d-2}p_L^\perp \int \mathrm{d}^{d-2}p_R^\perp \; \exp\left[-\tfrac{\tau}{2}\,|p_L^\perp|\right]\; \exp\left[-\tfrac{\tau}{2}\,|p_R^\perp|\right] \delta S(\tau, -p_L^\perp, -p_R^\perp)$$

- Jet functions evaluated at 0-recoil $\Rightarrow f_J(\phi, r, z) = 1$
- Soft function measurement $\Rightarrow \mathscr{M}_S(\tau; p) = \exp\left[-\tfrac{\tau}{2}\left(\,|p^\perp| + |p_L^\perp| + |p_R^\perp|\,\right)\right]$
  - Assume gluon emission into left hemisphere $\Rightarrow p_L^\perp = p^\perp \quad p_R^\perp = 0$

    $\Rightarrow \mathscr{M}_S(\tau; p) = \exp\left[-\tau\,|p^\perp|\right] \quad \Rightarrow f_S(\phi, r) = 1$
  - Similar arguments for right hemisphere

Thrust-axis jet broadening

- Surprisingly the large-β_0 calculation simplifies in the recoil-sensitive case

- Calculation now follows exactly the same as for the WTA-broadening

- Leading IR renormalon

$$B[F](u) = -\frac{4C_F}{\beta_0}\frac{e^{C/2}\mu\tau}{u-1/2} + \mathcal{O}((u-1/2)^0)$$

$$B[W](u) = -\frac{4C_F}{\beta_0}\frac{e^{C/2}\mu\tau}{u-1/2}\left(\frac{1}{u-1/2} + \frac{44}{15} + C + 2\ln\left(\frac{\mu\bar{\tau}}{2}\right)\right) + \mathcal{O}((u-1/2)^0)$$

- Similar leading renormalon structure as for WTA-broadening

Thrust-axis jet broadening

- Leading IR renormalon

$$B[F](u) = -\frac{4C_F}{\beta_0}\frac{e^{C/2}\mu\tau}{u-1/2} + \mathcal{O}((u-1/2)^0) \qquad B[W](u) = -\frac{4C_F}{\beta_0}\frac{e^{C/2}\mu\tau}{u-1/2}\left(\frac{1}{u-1/2} + \frac{44}{15} + C + 2\ln\left(\frac{\mu\bar{\tau}}{2}\right)\right) + \mathcal{O}((u-1/2)^0)$$

- „Mass-scheme" dependence $\frac{1}{2}\sum_{i\in X}|p_i^\perp| \Rightarrow \frac{1}{2}\sum_{i\in X}\sqrt{p_i^+p_i^-}$
  - Logic from different scheme still valid $\hat{f}_S(\phi, r) = \hat{f}_J(\phi, r, z) = \frac{1+r}{2r}$

- Leading IR renormalon

$$B[\hat{F}](u) = -\frac{2(2+\pi)C_F}{\pi\beta_0}\frac{e^{C/2}\mu\tau}{u-1/2} + \mathcal{O}((u-1/2)^0)$$

$$B[\hat{W}](u) = -\frac{2(2+\pi)C_F}{\pi\beta_0}\frac{e^{C/2}\mu\tau}{u-1/2}\left(\frac{1}{u-1/2} - \frac{4(1-11\pi-37\ln(2))}{15(2+\pi)} + C + 2\ln\left(\frac{\mu\bar{\tau}}{2}\right)\right) + \mathcal{O}((u-1/2)^0)$$

Dijet transverse-momentum decorrelation

- Vector sum of transverse momenta with respect to the WTA-axis [Gutierrez-Reyes,Scimemi,Waalewijn,Zoppi;18]

$$f_S(\phi, r) = -2i\cos\phi \qquad f_J(\phi, r, z) = -2i\cos\phi\left[\frac{\theta(1/2-z)}{1-z} - \frac{\theta(z-1/2)}{z}\right]$$

➡Non-trivial azimuthal integration

•Everything else is the same as for WTA broadening

- Leading IR renormalon

$$B[F](u) = -\frac{4C_F}{\beta_0}\frac{e^C\mu^2\tau^2}{u-1} + \mathcal{O}((u-1)^0)$$

$$B[W](u) = -\frac{4C_F}{\beta_0}\frac{e^C\mu^2\tau^2}{u-1}\left(\frac{1}{u-1} + 2 + C + 2\ln\left(\frac{\mu\bar{\tau}}{4}\right)\right) + \mathcal{O}((u-1)^0)$$

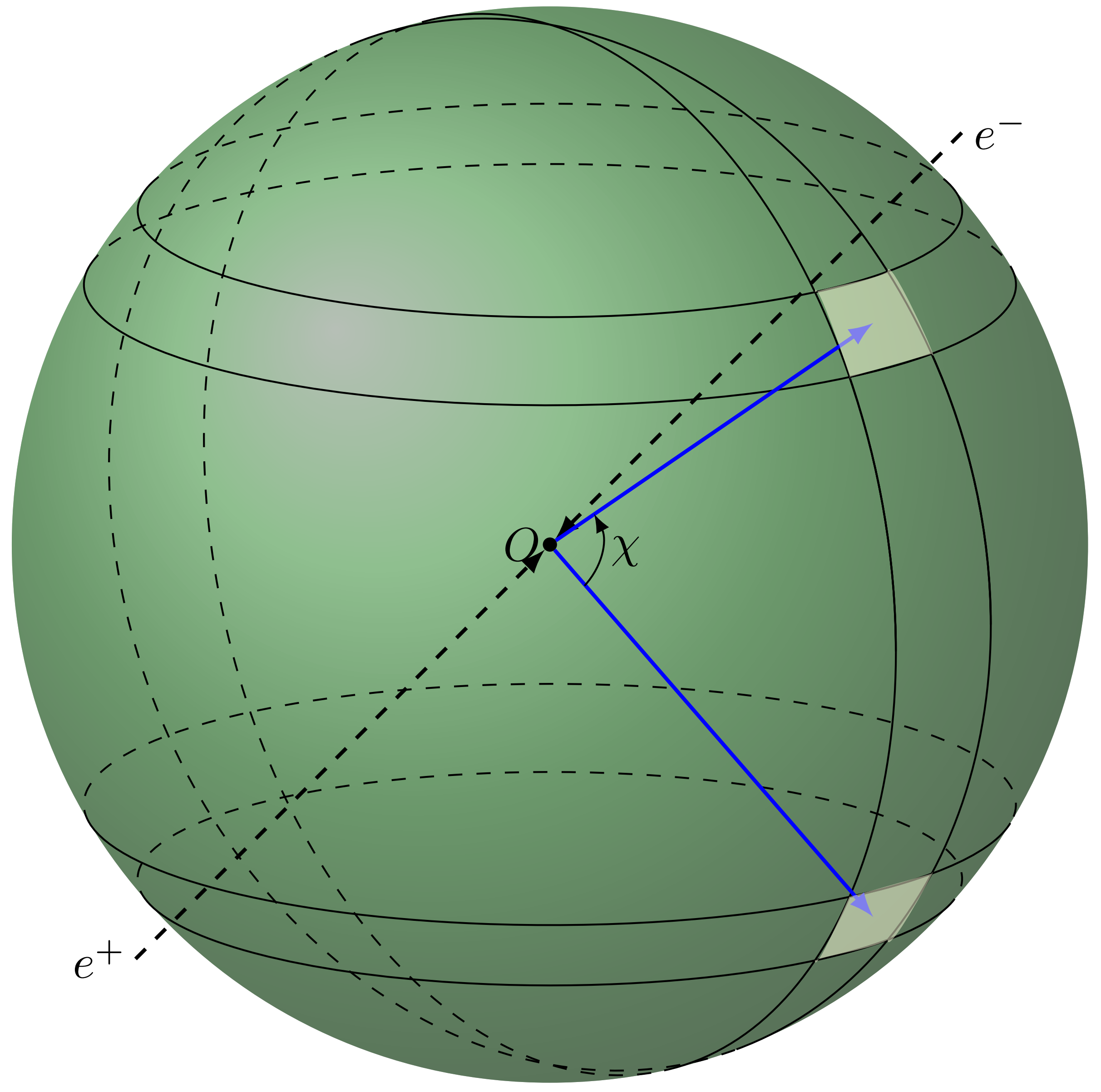
- Renormalon singularities at half-integer values of u have disappeared under the azimuthal Integration

Energy-energy correlator

- Factorisation of EEC in back-to-back Limit Fragmenting jet function [Moult,Zhu;18]

$$\frac{d\Sigma}{dz} = \frac{1}{2}\int d^2\vec{k}_\perp \int \frac{d^2\vec{b}_\perp}{(2\pi)^2}\, e^{-i\vec{b}_\perp\cdot\vec{k}_\perp} H(Q,\mu) J_n(\vec{b}_\perp,\mu,\nu) J_{\bar{n}}(\vec{b}_\perp,\mu,\nu) S(\vec{b}_\perp,\mu,\nu)\,\delta\left(1 - z - \frac{\vec{k}_\perp^2}{Q^2}\right),$$

$$D_{h/q}(x,\tau,\mu,\nu) = \sum_k \int_x^1 \frac{\mathrm{d}x'}{x'} K_{q\to k}(x',\tau,\mu,\nu)\; d_{h/k}\left(\frac{x}{x'},\mu\right)$$

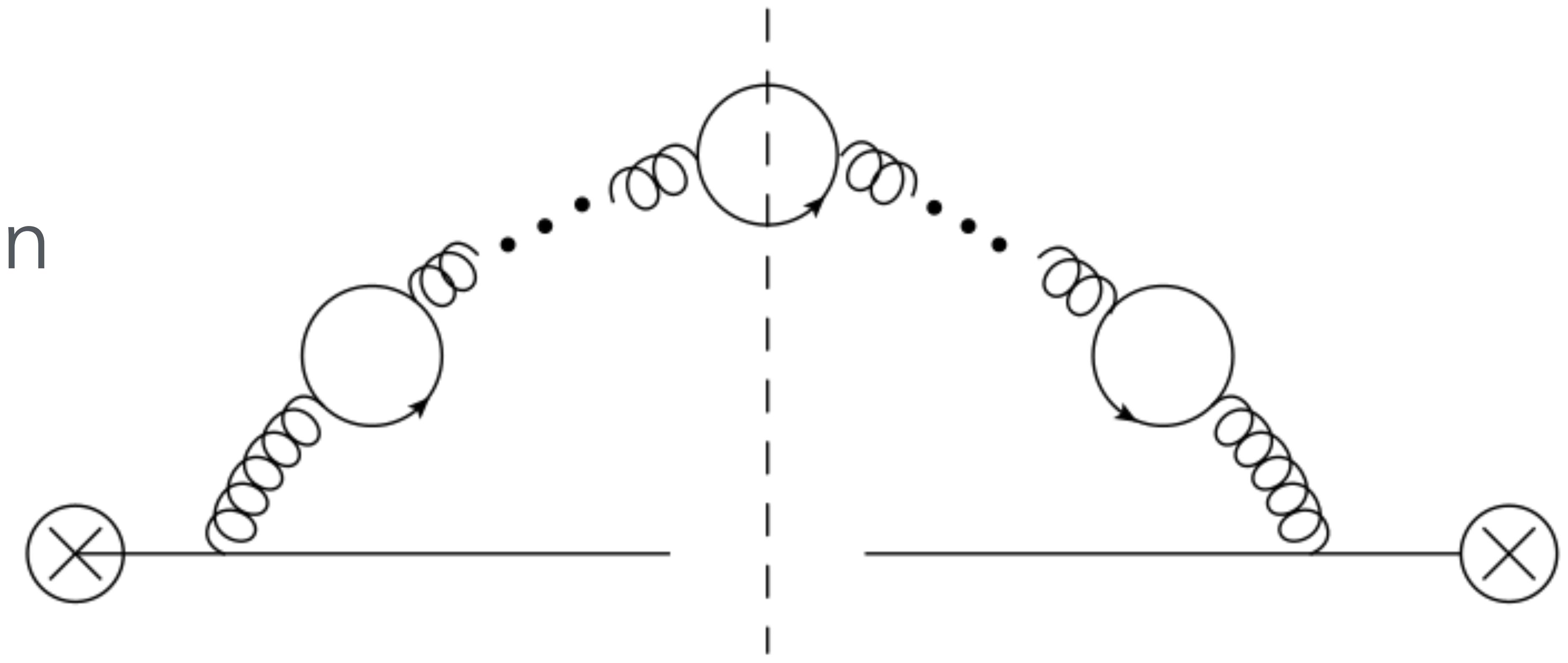


- EEC jet function is given by first moment

$$J_n(\vec{b}_\perp,\mu,\nu) = \sum_k \int_0^1 \mathrm{d}x\, x\, K_{q\to k}(x,\vec{b}_\perp,\mu,\nu)\,,$$

- In our case $k = \begin{cases} q & \text{diagonal channel} \\ g & \text{off-diagonal channel} \end{cases}$

EEC : Diagonal channel



- Diagonal channel similar to standard jet function calculation

    ➡Slightly different matrix element

    ➡ Suffers from rapidity divergences

- Same soft function as for transverse-momentum resummation

$$B[F](u) = -\frac{4C_F}{\beta_0}\frac{e^C\mu^2\tau^2}{u-1} + \mathcal{O}((u-1)^0)$$

$$B[W^{(\text{diag})}](u) = -\frac{4C_F}{\beta_0}\frac{e^C\mu^2\tau^2}{u-1}\left(\frac{1}{u-1} - 1 + C + 2\ln(\mu\bar{\tau})\right) + \mathcal{O}((u-1)^0)$$

- Diagonal channel $\Rightarrow$ Similar renormalon structure as for transverse-momentum resummation

[Jaarsma,Li,Moult,Waalewijn,Zhu;25]

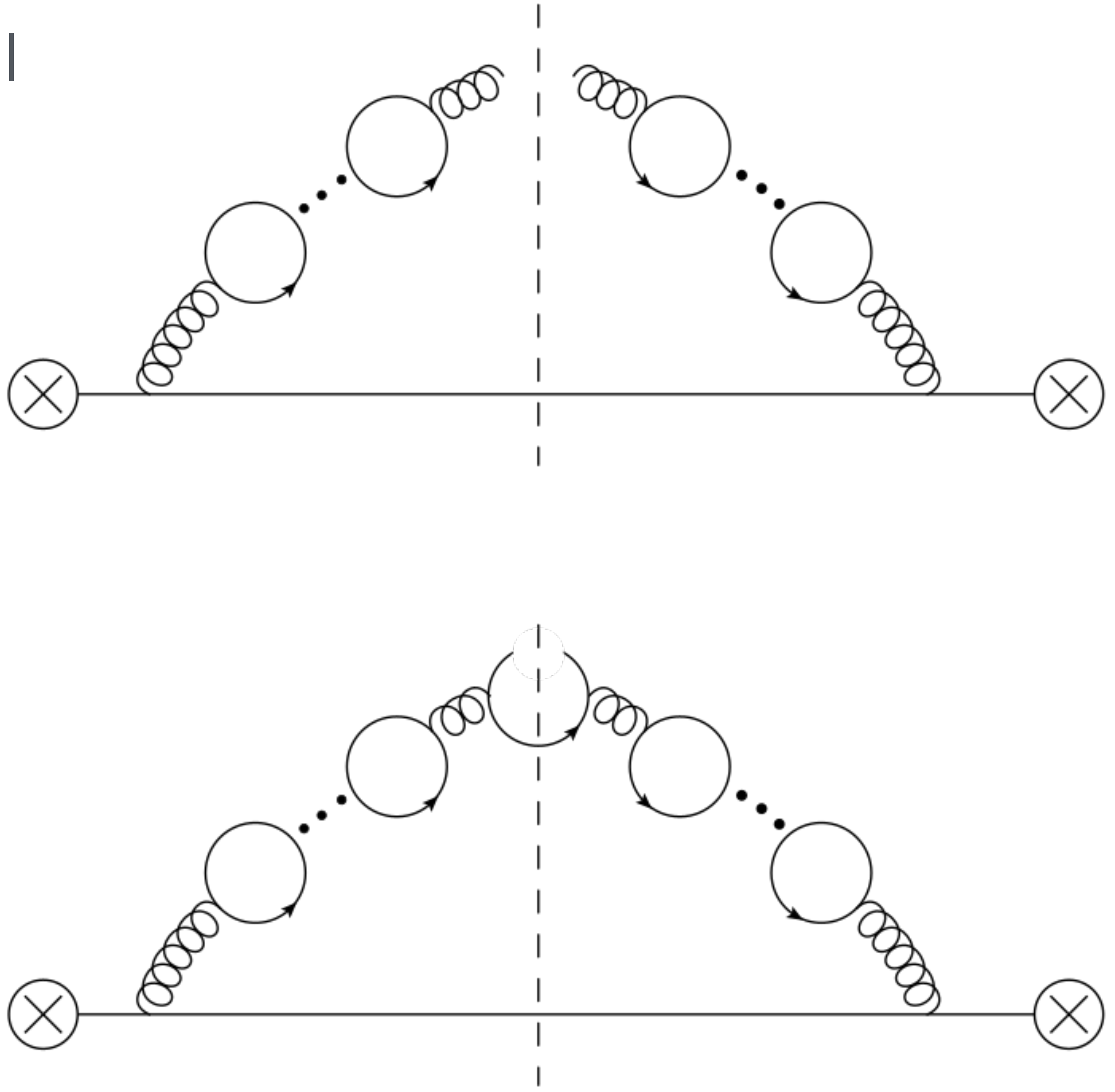
EEC : Off-diagonal channels

- Off-diagonal channels very different from diagonal channel
  - ➡ No rapidity divergence
  - ➡ No soft function required
- Proper $q \to g$ channel with bubbles is scaless

$$K_{q\to g}(x,\tau,\nu) = \frac{\alpha_s}{4\pi}\frac{x}{2\pi}\int \mathrm{d}^4k\ \delta(k^2)\,\theta(k_-)\,\delta\!\left(\frac{1-x}{x}p_- - k_-\right)\mathscr{M}_D(\tau;k)$$

$$\times \int \mathrm{d}p^2\ \mathscr{A}^{q\to g}(k;p^2)\frac{1}{\pi}\,\mathrm{Im}\left[\frac{1}{-p^2-i\eta}\right]$$

- Only $\mathcal{O}(\alpha_s)$ contribution from $q \to g$ channel
  - ➡ $q \to \bar{q}$ channel
- Instead of exact computation $\Rightarrow$ undo the gluon splitting



EEC : Off-diagonal channel

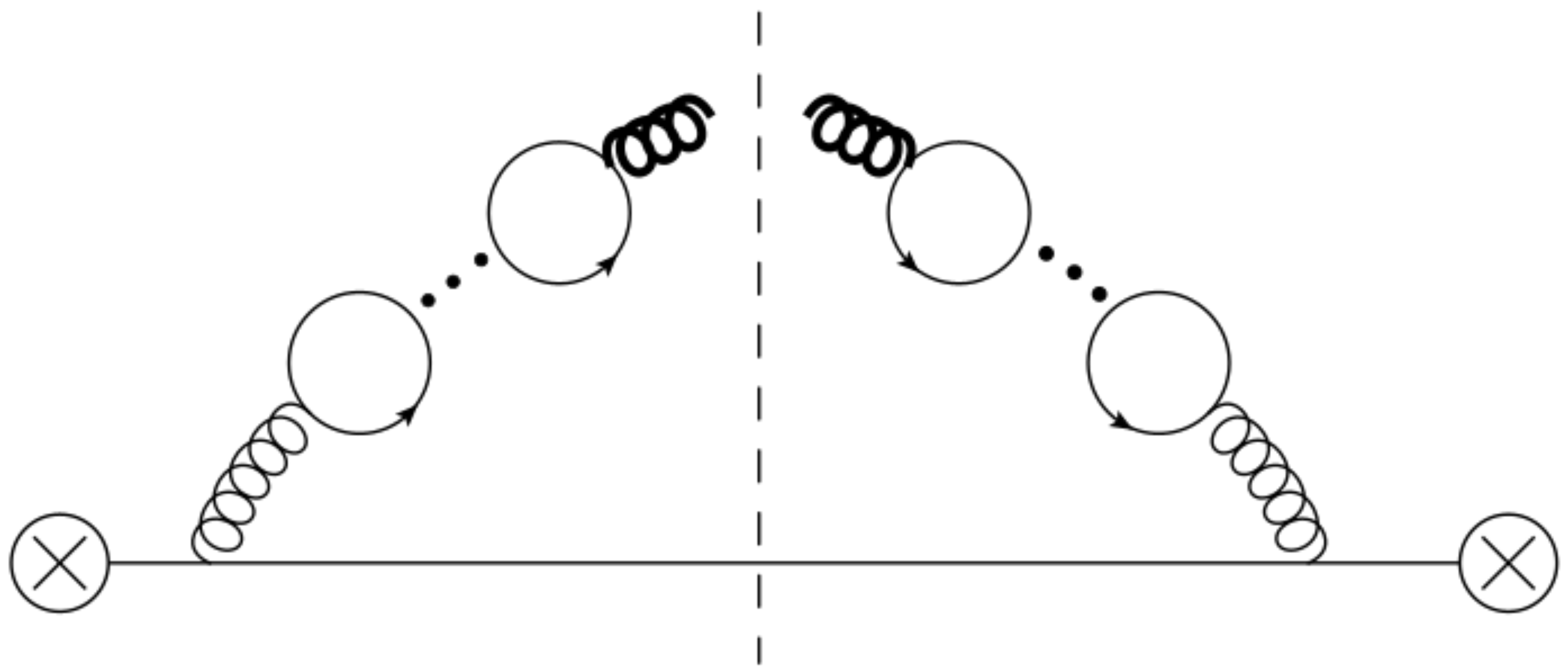
$$B[K^{app}_{q\to g}](x,u) = -\frac{(\mu^2 e^C)^u}{\Gamma(1-u)\Gamma(u)\beta_0}\frac{x}{2\pi}\int d^4k\,\delta(k^2)\,\theta(k_-)\,\delta\!\left(\frac{1-x}{x}p_- - k_-\right)\mathcal{M}(\tau;k)\int dp^2\,\frac{\mathcal{A}^{q\to g}(k;p^2)}{(p^2)^{1+u}}\theta(p^2)$$

- Matrix element scaling

$$\mathcal{A}^{q\to g}(k;p^2) \sim \frac{1}{x} \qquad \mathcal{A}^{q\to q}(k;p^2) \sim \frac{1}{1-x}$$

- Leading IR renormalon

$$B[W^{(\mathrm{off-diag})}](u) = \frac{8C_F}{\beta_0}\frac{e^C\mu\tau}{u-1/2} + \mathcal{O}((u-1/2)^0)$$



- Off-diagonal channel $\Rightarrow$ Leading IR renormalon for EEC

[Schindler,Stewart,Sun;24][Jaarsma,Li,Moult,Waalewijn,Zhu;25]

Renormalon ambiguity

- Ambiguity generated by poles in the Borel function

$$\mathrm{Res}_{u=k/2}\left[\frac{e^{-u/a_s(\mu)}}{(u-\frac{k}{2})^n}\right] = \frac{(-1)^{n-1}}{(n-1)!\,a_s^{n-1}(\mu)}\left(\frac{\Lambda_{\mathrm{QCD}}}{\mu}\right)^k$$

- Pole positions:

$$u = 1/2 \Rightarrow \Lambda_{\mathrm{QCD}}$$

$$u = 1 \Rightarrow \Lambda_{\mathrm{QCD}}^2$$

- General structure for SCET-II observables

$$\delta O = -\,2\delta F \ln(Q\bar{\tau}) + \delta W$$

δF → $$c_F\,(\tau\Lambda_{\mathrm{QCD}})^{k/2}$$

δW → $$\left(c_W - \frac{c_F}{a_s(1/\tau)}\right)(\tau\Lambda_{\mathrm{QCD}})^{k/2}$$

Summary

- In the past renormalon studies were mostly performed for soft functions
- Analysed the renormalon structure of generic SCET-II observables
  - ➡Requires information on soft and jet functions in the presence of rapidity regulator
- Rapidity logarithms handled via collinear anomaly
  - ➡ Collinear anomaly exponent $\Rightarrow$ single pole in Borel plane
  - ➡ Remainder function $\Rightarrow$ double pole in Borel plane
- Validated this pattern for different observables
  - ➡ EEC slightly different but coincides in appropriate channel

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