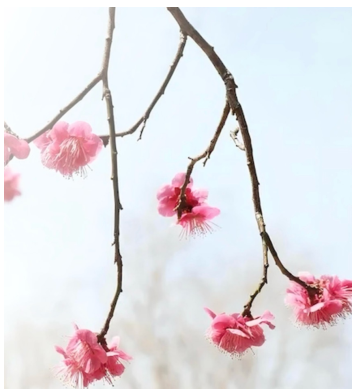
Factorization for Semileptonic Decays of Boosted and Off-shell Top Quarks with a Large-angle Bottom Jet

Based on work with Christoph Regner
arXiv:2507.17872



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$\int dk\,\Pi$ Doktoratskolleg Particles and Interactions

universität wien

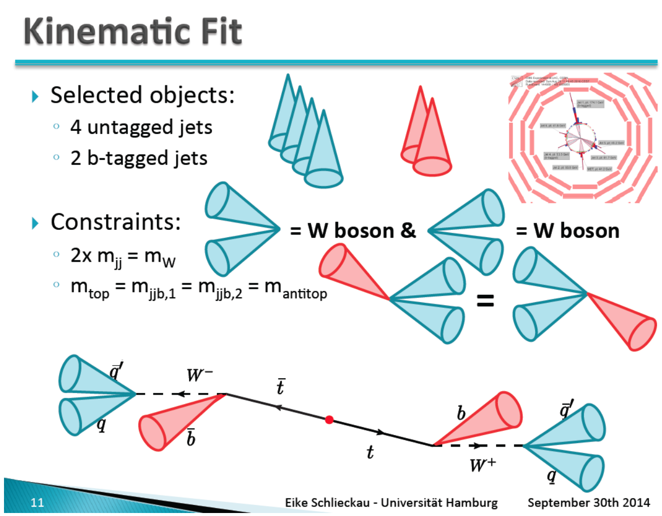
FWF Der Wissenschaftsfonds.

Content

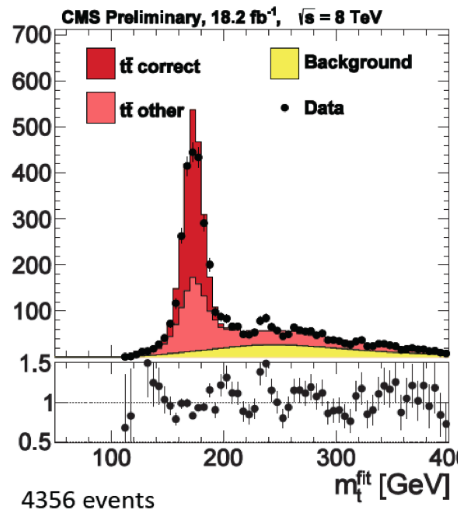
- Motivation: Understanding the meaning of m_t^{MC} for decay sensitive sensitive observables
- Factorization for semi-leptonic decays of boosted and off-shell top quarks in e^+e^- annihilation
- NLO numerical results
- Conclusions & outlook

Most Precise Top Mass Measurements Method

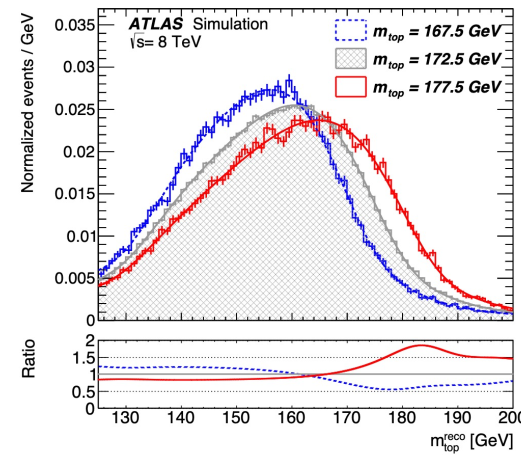
LHC+Tevatron: Direct top mass measurements



CMS-PAS-TOP-14-002



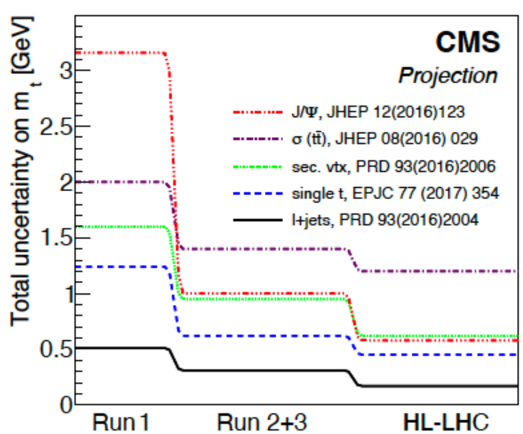
4356 events



kinematic mass determination

based on the picture of a top quark particle

Determination of the best-fit value of the Monte-Carlo top quark mass parameter



$$m_t^{\mathrm{MC}} = 171.77 \pm 0.37 \text{ GeV}$$

CMS collaboration. arXiv: 2302.01967

⊕ High top mass sensitivity

⊖ Precision of MC ?

⊖ Meaning of m_t^{MC} ?

← Δm_t ~ 200 MeV (projection)

→ Alternative option: use EECs as new observable that can be computed

Holguin, Moult, Pathak, Procura, Schöfbeck, 2407.12900

What is m_t^{MC} ?

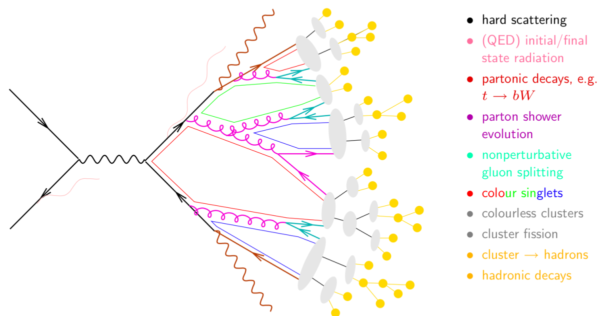
What does the question mean in the first place?

→ It means that we can provide the relation $$m_t^{\rm MC} = m_t^{\rm scheme}(\mu) + \frac{\alpha_s(\mu)}{\pi}\delta m^{\rm scheme} + \ldots$$

where δm^{scheme} can be **computed in pQCD to (at least) NLO**

The issue is complicated as we must understand and control the interplay of the different components of MC event generators

- Parton shower
- Matching
- Hadronization model



Direct measurements are based on the picture of a top quark particle

- Direct measurements are based on reconstructed top quark decay products
- Employed MCs (Pythia, Herwig) are based on the narrow width limit
- MCs model QCD, hadronization and unstable particle effects & m_t^{MC} = mass in propagator

⇨ m_t^{MC} is close to the top quark pole mass m_t^{pole}

Conservative but insufficient conclusion: $$m_t^{\rm MC} = m_t^{\rm pole} + \mathcal{O}(\Gamma_t, \Lambda_{\rm QCD})$$

What is m_t^{MC} ?

There are 3 essential ingredients to achieve a more rigorous and definite answer:

1) At least NLL precise parton shower
2) Observable with factorization of non-perturbative and perturbative contributions and summation of large logarithms
3) Factorization compatible hadronization model

Plätzer, Samitz, AHH, 2404.09856

→ also relevant when estimating NP effects from MCs

The crucial MC property that fixes the meaning of m_t^{MC} is the cut-off prescription of the parton shower

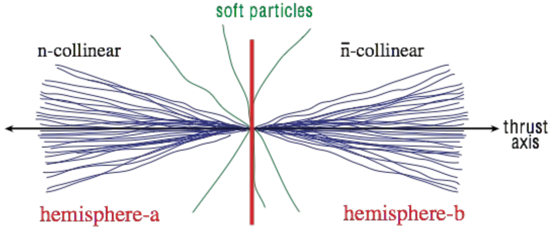
Plätzer, Samitz, AHH, 1807.06617

→ cutoff Q_0 is a factorization scale at the interface to non-perturbative effects

→ fixes the renormalization scheme of m_t^{MC} as it affects soft radiation in the top quark rest frame (treatment of virtual versus real soft radiation)

Currently there is only 1 observable class where all these aspects are understood

Jet-mass based event-shape observables in e^+e^- collisions for boosted top pair production in the dijet region:



→ 2-jettiness, thrust, fat jet mass … (decay insensitive, global)

→ Limitations of the narrow width limit do not play an important role

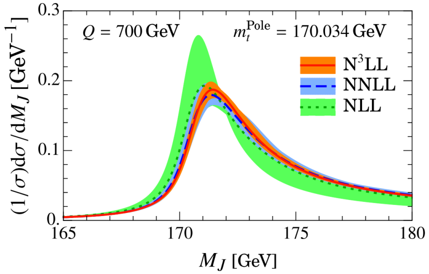
Boosted Top Eventshapes

Factorized cross section (uses effective theories SCET, bHQET):

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar t}^2} = \sigma_0\, H_Q(Q)\, H_m\Big(m_t, \frac{Q}{m_t}\Big) \int d\ell^+ \, d\ell^- \times J_{B_t}^{\Gamma_t}\Big(s_t^* - v_n^- \ell^+, \Gamma_t\Big)\, J_{B_{\bar t}}^{\Gamma_t}\Big(s_{\bar t}^* - v_{\bar n}^+ \ell^-, \Gamma_t\Big)\, S_{\rm hemi}(\ell^+, \ell^-)\,.$$

pert. ultra-collinear soft

pert. large-angle soft



known at N^{3}LL order

Fleming, Mantry, Stewart, AHH (2007)
Bachu, Mateu, Pathak, Stewart, AHH (2022)

Hadron level: $$S_{\rm hemi}(\ell^+, \ell^-) = \int dk^+ \, dk^- \, \hat S_{\rm hemi}(\ell^+ - k^+, \ell^- - k^-) F(k^+, k^-)$$

partonic soft function

shape function

$$S_{\rm hemi}(\ell^+, \ell^-) = \frac{1}{N_c} {\rm tr}_c \big[\langle 0 | [(\overline{Y}_{\bar n,-})^{\intercal}\, Y_{n,-}](0)\, \delta(\ell^+ - n\cdot \hat P_a)\, \delta(\ell^- - \bar n \cdot \hat P_b) [Y_{n,+}^\dagger\, (\overline{Y}_{\bar n,+}^\dagger)^{\intercal}](0) | 0\rangle\big]$$

→ only large-angle soft radiation is sensitive to linear NP power corrections

Monte-Carlo Top Quark Mass

Dependence of the parton-level peak position from CB on the shower cut Q_0

Plätzer, Samitz, AHH, 1807.06617
2404.09856

$$\frac{d\sigma^{cb}}{d\tau}(\tau, Q, m, Q_0) = \frac{d\sigma^{cb}}{d\tau}\left(\tau + \left[16\frac{Q_0}{Q} - 8\pi\frac{Q_0 m}{Q^2}\right]\frac{C_F\alpha_s(Q_0)}{4\pi}, Q, m, Q_0 = 0\right)$$

large-angle soft

should be compensated by hadronization corrections

ultra-collinear

(self-energy absorbed into generator mass)

Comparison to factorization formula with the same cutoff implemented shows:

- Coherent branching compatible with analytic factorization with Q_0 cutoff
- Ultra-collinear term implies that the generator mass is

$$m_t^{\text{Herwig}} = m_t^{\text{CB}}(Q_0) = m_t^{\text{pole}} - \frac{2}{3}\alpha_s(Q_0)Q_0 + \mathcal{O}(\alpha_s^2\, Q_0)$$

Ultra-collinear linear Q_0-dependence cancels in the observable.

→ Next logical steps:

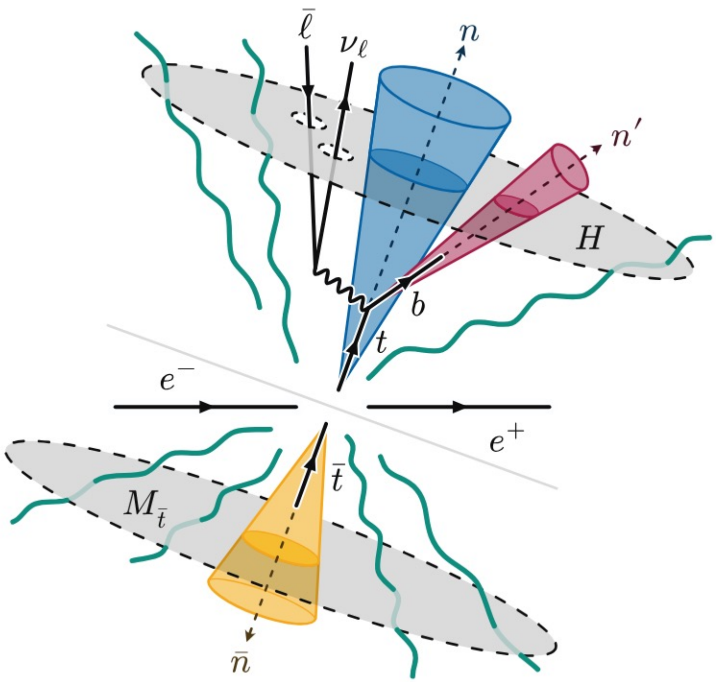
(Result universal?)

- top decay sensitive observables ←
- non-global observables
- other parton showers

Including Semileptonic Top Quark Decay

Regner, AHH, 2507.17672

$$e^-e^+ \to t^\star\ \bar{t}^\star, \quad t^\star \longrightarrow b\ell^+\nu_\ell + X'$$



- Boosted top-antitop production
- semileptonic top quark decay, antitop inclusive
- b-jet: all hadrons in top-hemisphere
- Measurement of hemisphere masses M_t and $M_{\bar{t}}$
  Double resonant kinematics i.e. $M_{t/\bar{t}} \approx m_t$

$$\frac{d\sigma}{dM_t^2 dM_{\bar{t}}^2} \longrightarrow \frac{d\sigma}{dM_t^2 dM_{\bar{t}}^2 dX} \quad \text{@ hadron level}$$

- X constructed from lepton and b-jet momenta
- Hierarchy of scales

$$E_{\rm cm} = Q \gg m_t \gg M_{b\text{-jet}} \gg M_{t/\bar{t}} - m_t \sim \Gamma_t$$

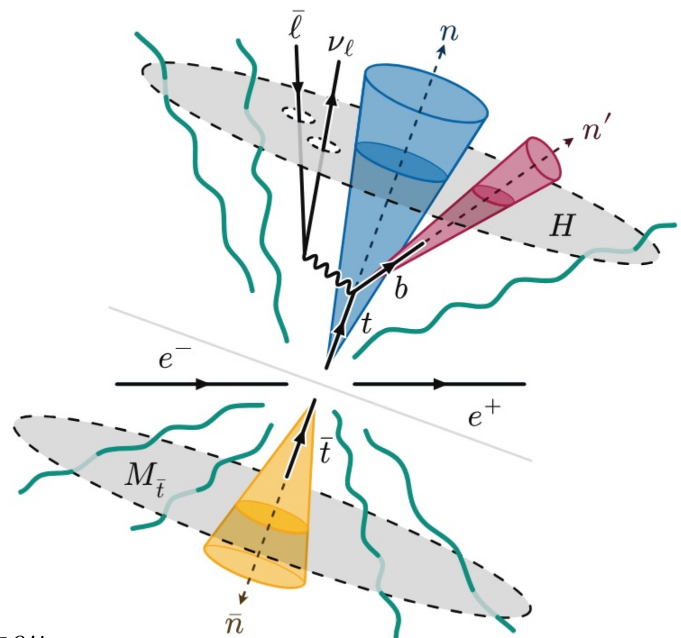
- Endpoint region: $M_{b\text{-jet}} \sim m_t\Gamma_t \ll m_t$

Four distinct QCD radiation modes (no interference)

- ultracollinear radiation (top)
- large-angle soft radiation
- hard-collinear radiation (b-jet)
- ultracollinear radiation (antitop)

(first three:) contribute to b-jet

Global context of the new factorization formula



Inclusive dijet factorization (top resonance)

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} \sim H_Q \times H_m \times J_{B_{\bar{t}}}^{\Gamma_t} \otimes S_{\text{hemi}} \otimes J_{B_t}^{\Gamma_t}$$

Inclusive semileptonic B-decay (endpoint region)

$$d\Gamma(\bar{B} \to X_u \ell \bar{\nu}_\ell) = d\text{PS} \times G_F^2 \times L_{\mu\nu} \times W^{\mu\nu}$$

$$W^{\mu\nu} \sim H_d \times S_{\text{shape}} \otimes J_{n'}$$

Semileptonic decay of boosted resonant top (endpoint)

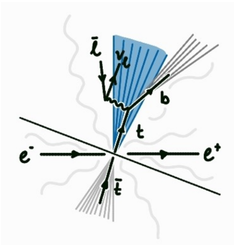
$$\frac{d^3\sigma}{dM_t^2 dM_{\bar{t}}^2 dX} \sim d\text{PS} \times L^{\sigma\nu} \times \frac{\tilde{g}_{\rho\sigma}\tilde{g}_{\mu\nu}}{|q^2 - M_W^2 + iM_W\Gamma_W|^2} \times W^{\rho\mu}$$

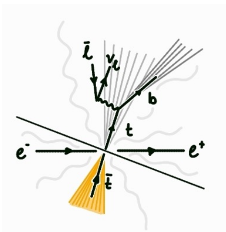
$$W^{\rho\mu} \sim H_Q \times H_m \times H_d^{\rho\mu} \times J_{B_{\bar{t}}}^{\Gamma_t} \otimes S_{\text{hemi}} \tilde{\otimes} S_{ucs} \otimes J_{n'}$$

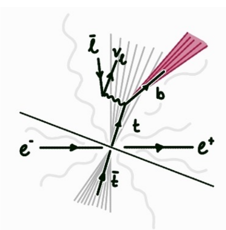
- Merges dijet and inclusive semileptonic meson decay factorization
- “Fermi-motion” of decaying top fixed by hemisphere mass measurement
- Leading power hadronization effects & summation of large logs (Q, m_t, Γ_t)

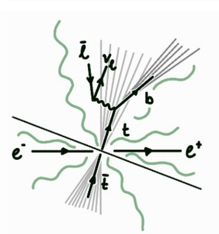
Factorization Modes

Regner, AHH, 2507.17672









$\frac{m_t}{Q} \ll 1\,, \quad \frac{\Gamma}{m_t} \ll 1$ $s^*_{t,\bar t} \sim \Gamma\,, \quad \Delta \sim \frac{m_t}{Q}\Gamma$	e^+e^- frame	top rest frame	virtuality
n-ultra-colllinear (top) $(h_{v_n}, A_n^{\mathrm{uc},\mu})$	$k_n^\mu \sim \Gamma\left(\frac{m_t}{Q}, \frac{Q}{m_t}, 1\right)$	$k_n^\mu \sim \Gamma(1,1,1)$ $\sim \Gamma(1,1,1)'$	$\mu_{B_t} \sim \Gamma$
$\bar n$-ultra-collinear (antitop) $(h_{v_{\bar n}}, A_{\bar n}^{\mathrm{uc},\mu})$	$k_{\bar n}^\mu \sim \Gamma\left(\frac{Q}{m_t}, \frac{m_t}{Q}, 1\right)$	$k_{\bar n}^\mu \sim \Gamma\left(\frac{Q^2}{m_t^2}, \frac{m_t^2}{Q^2}, 1\right)$	$\mu_{B_{\bar t}} \sim \Gamma$
n'-collinear (b-jet) $(\xi_{n'}, A^\mu_{n'})$		$p^\mu_{n'} \sim m_t\left(\frac{\Gamma}{m_t}, 1, \sqrt{\frac{\Gamma}{m_t}}\right)'$	$\mu_J \sim \sqrt{m_t\Gamma}$
large-ang. hemisphere soft (q_s, A_s^μ)	$k_s^\mu \sim \Gamma\frac{m_t}{Q}(1,1,1)$	$k_s^\mu \sim \Gamma\left(1, \frac{m_t^2}{Q^2}, \frac{m_t}{Q}\right)$ $\sim m_t\left(\frac{\Gamma}{m_t}, \frac{\Gamma m_t}{Q^2}, \frac{\Gamma}{Q}\right)'$	$\mu_S \sim \frac{m_t}{Q}\Gamma$

→ Separation of modes in QFT

- Soft-Collinear Effective Theory – SCET (hard-colinear and soft)
- Boosted Heavy Quark Effective Theory – bHQET (ultra-collinear)

Factorization Derivation

→ Step 1: Factorization as for $\dfrac{d\sigma}{dM_t^2 dM_{\bar t}^2}$

$$\mathrm{d}\sigma = \sum_X^{\text{res.}} (2\pi)^4 \delta^{(4)}(q_{ee} - P_X) \sum_{i=v,a} L^i_{\mu\nu} \langle 0| \mathcal{J}^{\mu\dagger}_{t\bar t,i}(0) |X\rangle \langle X| \mathcal{J}^{\nu}_{t\bar t,i}(0) |0\rangle$$

QCD → SCET: $\mathcal{J}^{\mu}_{t\bar t,i}(0) = C_Q(Q)\, \mathcal{J}^{\mu}_{i,\mathrm{SCET}}(0)\,, \qquad \mathcal{J}^{\mu}_{i,\mathrm{SCET}}(0) = [\bar\chi_n\, S^\dagger_{n,+}\, \Gamma^\mu_i\, S_{\bar n,-}\, \chi_{\bar n}](0)$

SCET → bHQET: $\mathcal{J}^{\mu}_{i,\mathrm{SCET}}(0) = C_m\Big(m_t, \dfrac{Q}{m_t}\Big)\, \mathcal{J}^{\mu}_{i,\mathrm{bHQET}}(0)\,, \quad \mathcal{J}^{\mu}_{i,\mathrm{bHQET}}(0) = [\bar h_{v_n} W^{\mathrm{uc}}_{n,-}\, Y^\dagger_{n,+}\, \Gamma^\mu_i\, Y_{\bar n,-}\, W^{\mathrm{uc}\,\dagger}_{\bar n,+} h_{v_{\bar n}}](0)$

Fierz +
measurement function

$$M_t^2 = (m_t v_n + k_n + k_{s,a})^2$$

$$M_{\bar t}^2 = (m_t v_{\bar n} + k_{\bar n} + k_{s,b})^2$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_t^2 \mathrm{d}M_{\bar t}^2} = \sigma_0\, H_Q(Q)\, H_m\Big(m_t, \frac{Q}{m_t}\Big) \int \mathrm{d}\ell^+\, \mathrm{d}\ell^- \times J^{\Gamma_t}_{B_t}\Big(s_t^* - \frac{Q}{m_t}\ell^+\Big)\, J^{\Gamma_t}_{B_{\bar t}}\Big(s_{\bar t}^* - \frac{Q}{m_t}\ell^-\Big)\, S_{\mathrm{hemi}}(\ell^+, \ell^-)$$

Only top ultra-collinear jet function needs to be considered further

Large-angle soft radiation insensitive to top decay.

Including Top Quark Decay

→ Step 2: Generalization of inclusive jet function adding the top decay (top rest frame)

$$J_{B_t}^{\Gamma_t}(k^+,k^-) = \frac{1}{8\pi\, N_c\, m_t} \sum_X (2\pi)^4\, \delta^{(4)}(m_t\, v + k - P_X)\, \mathrm{Tr}\Big[\langle 0|\, \overline{T}\, (W_{n,+}^{\mathrm{uc},\dagger}\, h_v)(0)\,|X\rangle\, \langle X|\, T(\bar{h}_v\, W_{n,-}^{\mathrm{uc}})(0)\,|0\rangle\Big]$$

Implement top decay final states and SM decay operator via T-product

$$|X\rangle \to |\ell^+\nu_\ell\rangle\, |X_b\rangle$$

$$T[\bar{h}_v W_{n,-}^{\mathrm{uc}}](0) \to \int \mathrm{d}^4 z_1\, T\, \tilde{\mathcal{O}}_\Gamma(z_1)\, [\bar{h}_v W_{n,-}^{\mathrm{uc}}](0)$$

$$\tilde{\mathcal{O}}_\Gamma(z) = -i\, \frac{4G_F}{\sqrt{2}} V_{tb}\, \frac{M_W^2\, \tilde{g}_{\mu\sigma}}{M_W^2 - q^2 - iM_W\Gamma_W}\, \big[(\bar{b}\,\gamma^\mu P_L\, t)\, (\bar{\nu}_\ell\, \gamma^\sigma P_L\, \ell)\big](z)$$

Match SM decay operator on (b)HQET

$$(\bar{b}\gamma^\mu P_L t)(0) = \sum_{j=1}^{3} C_j(m_t, \hat{v}^+\hat{H}^-)\, \big[\bar{\chi}_{n'}\, Y_{n',+}^{\mathrm{uc},\dagger}\, \Gamma_j^\mu\, h_v\big](z)$$

$$\mathrm{d}J_{B_t}^{\Gamma_t}(k^+,k^-) = \left(\frac{e}{\sqrt{2}\, s_w}\right)^4 |V_{tb}|^2\, \mathrm{d}H^2\, \mathrm{d}\Pi_3(M_t; p_\ell, p_{\nu_\ell}, H)\, \tilde{L}^{\sigma\nu}(p_\ell, p_{\nu_\ell})\, \frac{\tilde{g}_{\rho\sigma}\, \tilde{g}_{\mu\nu}}{|q^2 - M_W^2 + iM_W\Gamma_W|^2}\, W^{\rho\mu}(n,v,k,q)$$

$$W^{\rho\mu}(n,v,k,q) = H_d^{\rho\mu}(q,v,m_t) \int \mathrm{d}\hat{k}_{n'}^+\, \mathrm{d}\hat{k}_{uc}^+\, \delta(m_t + \hat{k}^+ - \hat{q}^+ - \hat{k}_{n'}^+ - \hat{k}_{uc}^+)\, J_{n'}(\hat{H}^-\hat{k}_{n'}^+)\, \tilde{S}_{ucs}(\gamma^2, k^+ + k^-, \hat{k}_{uc}^+)$$

↑ novel factorization function (soft radiation in top rest frame)

The Ultra-Collinear-Soft-Function

Regner, AHH, 2507.17672

Describes **coherent soft radiation in boosted top's rest frame**

- Gauge invariant
- Non-local matrix element
- Can be computed perturbatively

- Sensitive to top production, propagation and decay
  → contains non-factorizable contributions
- Depends on $\bar{n}^\mu, n'^\mu, v^\mu$
  → dependence on frame-dependent b-jet emission angle

$k^+ = k.n$

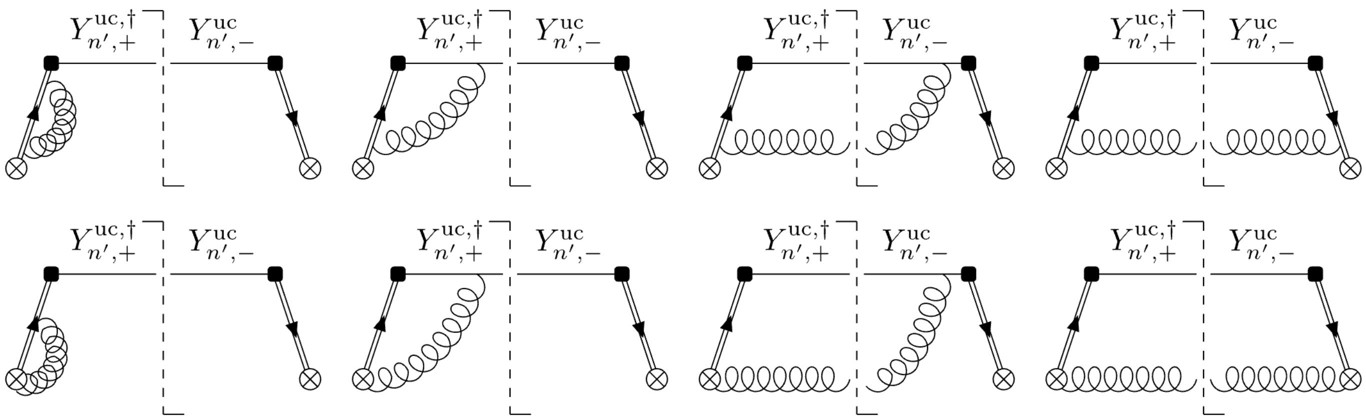
$\hat{a}^+ = a.n'$ (b-jet direction)

$$\gamma^2 = \frac{2}{\bar{n}\cdot n'}$$

$$\tilde{S}_{\rm ucs}(2v\cdot k, \hat{a}^+) = \frac{1}{8\pi N_C m_t}\sum_{X_{\rm uc}} \delta(\hat{a}^+ - \hat{K}^+_{X_{\rm uc}}) \int {\rm d}^4 z_1\, {\rm d}^4 z_1\, e^{ik\cdot(z_2 - z_1)} \times$$

$$\times\ \langle 0|\overline{\mathbf{T}}\Big\{[(W^{{\rm uc},\dagger}_{n,+})^{bc}(h_{v_n})^c_\beta](z_2)[(\bar{h}_{v_n})^d_\alpha(Y^{\rm uc}_{n',-})^{de}](0)\Big\}\,|X_{\rm uc}\rangle\langle X_{\rm uc}|\,\mathbf{T}\Big\{\underbrace{[(Y^{{\rm uc},\dagger}_{n',+})^{ef}(\overbrace{h_{v_n})^f_\alpha](0)[(\bar{h}_{v_n}}^{\text{top propagation}}}_{\text{top decay}}\underbrace{)^a_\beta(W^{\rm uc}_{n,-})^{ab}](z_1)}_{\text{top production}}\Big\}|0\rangle$$

gauge cancellations



shape-function type diagrams (but off-shell)

+ c.c.

→ cannot be written as a forward scattering matrix element

The Ultra-Collinear-Soft-Function

Regner, AHH, 2507.17672

Describes **coherent soft radiation in boosted top's rest frame**

- Gauge invariant
- Non-local
- Perturbative: computed at NLO

- Sensitive to top production, propagation and decay
  → contains non-factorizable contributions
- Depends on $\bar n^\mu, n'^\mu, v^\mu$
  → dependence on frame-dependent b-jet emission angle

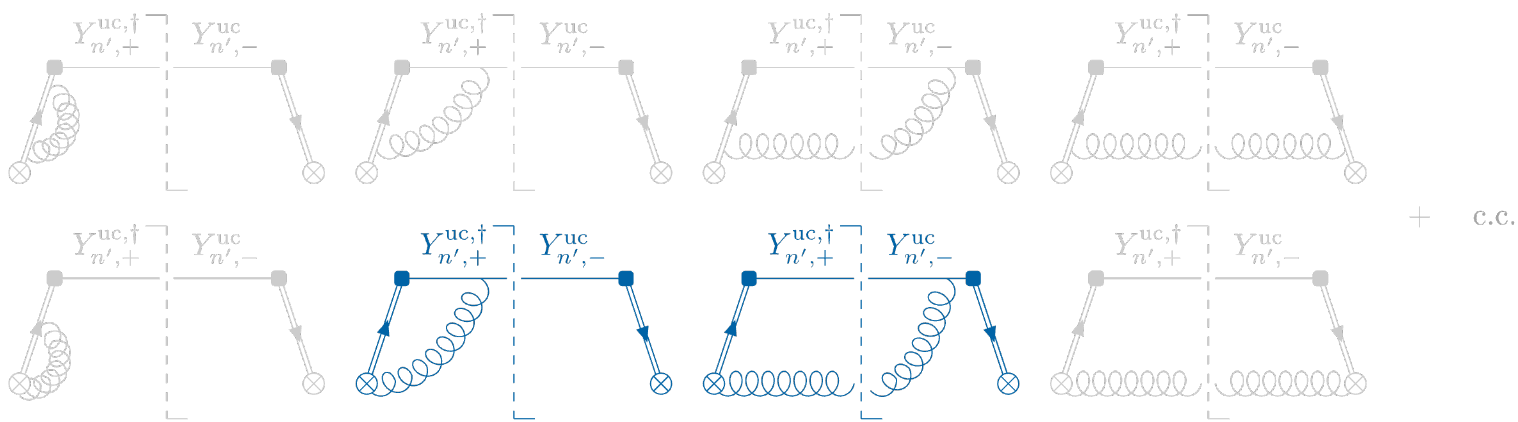
$k^+ = k.n$

$\hat a^+ = a.n'$ (b-jet direction)

$\gamma^2 = \dfrac{2}{\bar n \cdot n'}$

$$\tilde S_{\rm ucs}(2v\cdot k,\hat a^+) = \frac{1}{8\pi N_C m_t}\sum_{X_{\rm uc}} \delta(\hat a^+ - \hat K^+_{X_{\rm uc}}) \int \mathrm{d}^4 z_1\, \mathrm{d}^4 z_1\, e^{ik\cdot(z_2-z_1)} \times$$

$$\times\ \langle 0|\overline{\mathbf T}\Big\{[(W^{\rm uc,\dagger}_{n,+})^{bc}(h_{v_n})^c_\beta](z_2)[(\bar h_{v_n})^d_\alpha (Y^{\rm uc}_{n',-})^{de}](0)\Big\}\,|X_{\rm uc}\rangle\langle X_{\rm uc}|\,\mathbf T\Big\{\underbrace{[(Y^{\rm uc,\dagger}_{n',+})^{ef}(h_{v_n})^f_\alpha](0)}_{\text{top decay}}\overset{\text{top propagation}}{}\underbrace{[(\bar h_{v_n})^a_\beta (W^{\rm uc}_{n,-})^{ab}](z_1)}_{\text{top production}}\Big\}|0\rangle$$



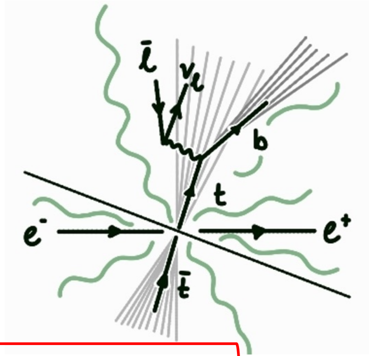
non-factorizable Feynman diagrams

Hadronization effects, renormalization, RG

The large-angle soft momenta contribute to

hemisphere mass & b-jet momentum

This implies an entangled and angle dependent form of the convolution involving the large-angle soft radiation function for the top hemisphere.



$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_t^2\mathrm{d}M_{\bar t}^2} = \sigma_0\, H_Q(Q)\, H_m\Big(m_t, \frac{Q}{m_t}\Big) \int \mathrm{d}\ell^+\, \mathrm{d}\ell^- \times J_{B_t}^{\Gamma_t}\big(s_t^* - v_n^-\ell^+\big)\, J_{B_{\bar t}}^{\Gamma_t}\big(s_{\bar t}^* - v_{\bar n}^+\ell^-\big)\, S_{\mathrm{hemi}}(\ell^+, \ell^-)$$

$$\sim H_d^{\rho\mu} \int \mathrm{d}\hat\ell^+\, J_{n'}\big(\hat H^-(m_t - \hat q^+ - \hat\ell^+)\big)\, S_{ucs}\Big(\gamma^2, s_t^* - \frac{Q}{m_t}\ell^+, \hat\ell^+ + \frac{s_t^+}{2} - \frac{Q}{\gamma^2 m_t}\ell^+\Big)$$

$$\gamma^2 = \frac{2}{\bar n\cdot n'}$$

- The hadronization corrections from the hemisphere soft function S_{hemi} modify the hemisphere mass M_t and the b-jet momentum
- ucs function known at NLO. All other FO ingredients are known to at least 2 loops, all anomalous dimensions to 3 loops.
- Consistency condition connects $Z_{S_{ucs}}$ to Z_{shapefct} and $Z_{J_{Bt}}$ but in a more involved entangled convolution.
- RG evolution factors known at N^3LL but enter in a non-standard way in cross section

First Numerical Results

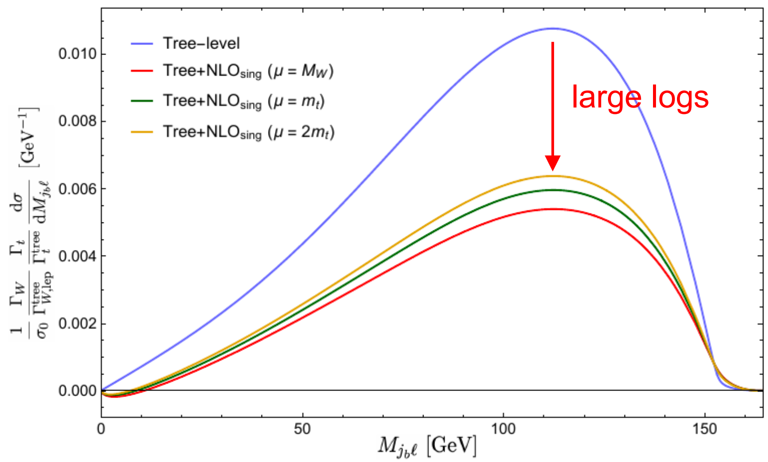
Regner, AHH, 2507.17672

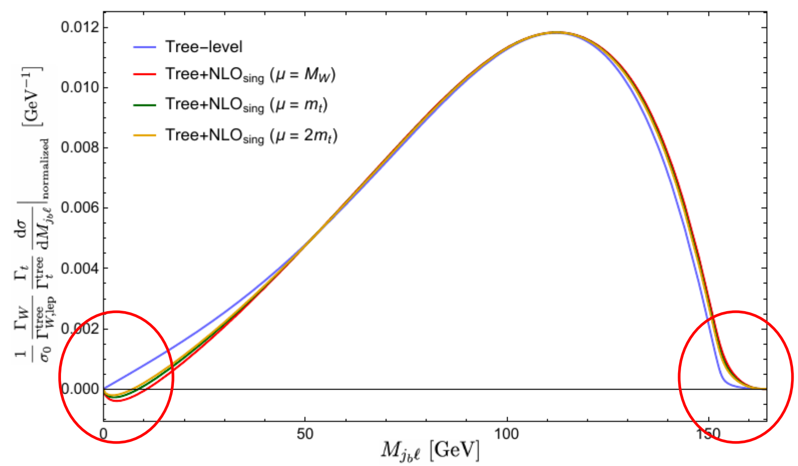
- Resummation of logarithms and implementation of hadronization effects are very involved due to complexity of the convolutions.

→ NLO fixed-order results for the b-jet lepton invariant mass

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{j_b\ell}}(\Delta M_t)\bigg|_{\mathrm{NLO,sing}} \equiv \int_{(m_t-\Delta M_t)^2}^{(m_t+\Delta M_t)^2} \mathrm{d}M_t^2 \int_{(m_t-\Delta M_t)^2}^{(m_t+\Delta M_t)^2} \mathrm{d}M_{\bar t}^2 \, \frac{\mathrm{d}^3\sigma}{\mathrm{d}M_t^2\mathrm{d}M_{\bar t}^2\mathrm{d}M_{j_b\ell}}\bigg|_{\mathrm{NLO,sing}}$$

$$Q = 700\ \mathrm{GeV},\ m_t^{\mathrm{pole}} = 173\ \mathrm{GeV},\ \Delta M_t = 10\ \mathrm{GeV}$$





- Resummation of large logarithms is essential
- NLO effects in the endpoint regions due to corrections of the hemisphere resonance mass the b-jet mass.

Summary and Outlook

- We have extended the factorization for inclusive fat top jets adding the inclusive semileptonic top decay which allows us to extend these studies to observables such as decay $M_{\text{b-jet lepton}}$

- Upcoming work:
  - analysis of non-factorizable QCD corrections versus the narrow width limit
  - summation of large logs
  - analysis of shower cut dependence (off-shell vs narrow width)
  - tests of Herwig shower and hadronization model

- Future directions:
  - towards observables suitable for LHC (narrower b-jet)
  - off-shell effects in top-spin observables
  - ultrasoft effects in semileptonic decays at the top threshold

Global context of the new factorization formula

Inclusive dijet factorization (top resonance)

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_t^2\mathrm{d}M_{\bar t}^2} = \sigma_0\, H_Q(Q)\, H_m\Big(m_t,\frac{Q}{m_t}\Big) \int \mathrm{d}\ell^+\,\mathrm{d}\ell^- \times J_{B_t}^{\Gamma_t}\Big(s_t^* - v_n^-\ell^+\Big)\, J_{B_{\bar t}}^{\Gamma_t}\Big(s_{\bar t}^* - v_{\bar n}^+\ell^-\Big)\, S_{\rm hemi}(\ell^+,\ell^-)$$

Inclusive semileptonic B-decay (endpoint region)

$$\mathrm{d}\Gamma(\bar B \to X_u \ell \bar\nu_\ell) = \mathrm{d}H^2\,\mathrm{d}\Pi_3(p_B; p_\ell, p_{\nu_\ell}, H)\,\Big(\frac{4G_F}{\sqrt 2}\Big)^2 |V_{ub}|^2\, \tilde L_{\mu\nu}(p_\ell,p_{\nu_\ell})\, W^{\mu\nu}(p_B,q)$$

$$W^{\mu\nu}(v,\,p_B,q) = H_d^{\mu\nu}(q,v,m_b)\int \mathrm{d}\hat\ell^+\, J_{n'}(\hat H^-(m_b\hat v^+ + \overline{\Lambda}\hat v^+ - \hat q^+ - \hat\ell^+))\, S_{\rm shape}(\hat v^+,\hat\ell^+)$$

Inclusive semileptonic B-decay (endpoint region)

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}M_t^2\mathrm{d}M_{\bar t}^2\mathrm{d}X} = \sigma_0 \int \mathrm{d}H^2\,\mathrm{d}\Pi_3(M_t = m_t v_n + k^*; p_\ell, p_{\nu_\ell}, H)\,\delta(X - \mathcal{X}(M_t,p_\ell,p_{\nu_\ell},H))$$

$$\times \Big(\frac{e}{\sqrt2\, s_w}\Big)^4 |V_{tb}|^2\, \tilde L^{\sigma\nu}(p_\ell,p_{\nu_\ell})\,\frac{\tilde g_{\rho\sigma}\,\tilde g_{\mu\nu}}{|q^2 - M_W^2 + iM_W\Gamma_W|^2}\, W^{\rho\mu}(n,v,k,q)$$

$$W^{\rho\mu}(n,v,k,q) = H_Q(Q)\, H_m\Big(m_t,\frac{Q}{m_t}\Big)\, H_d^{\rho\mu}(q = p_\ell + p_{\nu_\ell}, v, m_t)$$

$$\times \int \mathrm{d}\ell^+\,\mathrm{d}\ell^-\mathrm{d}\hat\ell^+\, J_{B_{\bar t}}^{\Gamma_t}\Big(s_{\bar t}^* - \frac{Q}{m_t}\ell^-\Big)\, S_{\rm hemi}(\ell^+,\ell^-)$$

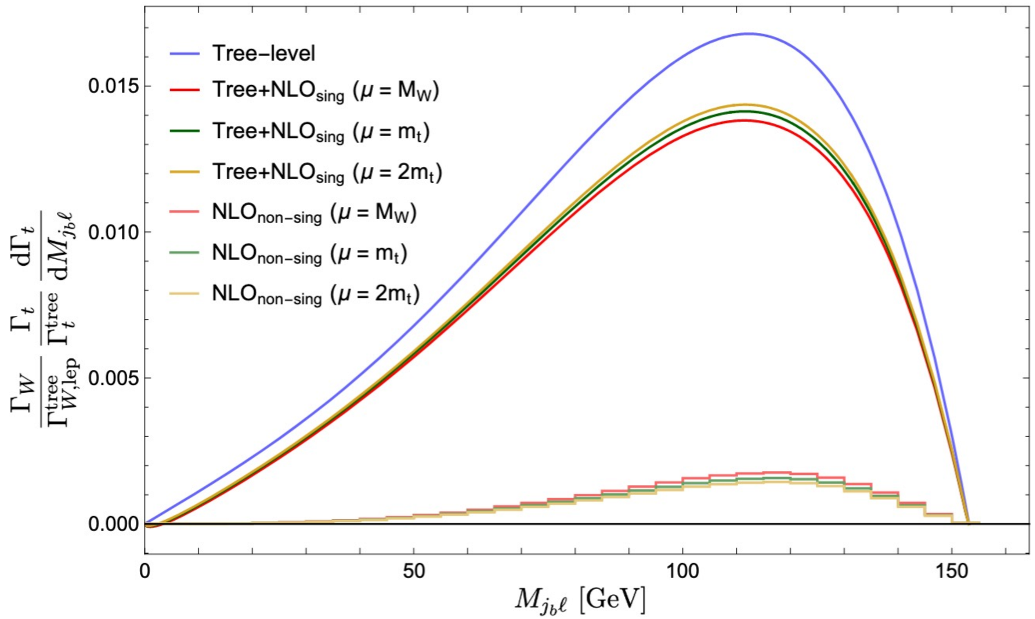
$$\times J_{n'}\Big(\hat H^-(m_t\hat v_n^+ - \hat q^+ - \hat\ell^+)\Big)\, S_{ucs}\Big(\hat v_n^+, v_n^-, \gamma^2, \frac{s_t^*}{2} - \frac{Q}{m_t}\ell^+, \frac{s_t^*}{2}, \hat\ell^+\Big)$$

First Numerical Results

Regner, AHH, 2507.17672

- The factorization formula is applicable in the lower and upper endpoint region
- Non-singular need to be extracted from NLO off-shell calculations

→ NLO non-singular corrections for the decay of an on-shell top quark



- Non-singular effects are expected not to be important to correctly quantify parton shower's cut-off effects.

Ultra-Collinear Soft Funktiion at NLO

Regner, AHH, 2507.17672

$$
\begin{aligned}
&\tilde{S}_{ucs}(\hat{v}^+, v^-, \gamma^2, s, \bar{a}^+) = \frac{1}{4\pi m_t \hat{v}^+}\,\frac{1}{|\Delta|^2}\Bigg\{\delta(\bar{a}^+) \\
&+\frac{\alpha_s C_F}{4\pi}\Bigg\{\delta(\bar{a}^+)\Bigg[-\frac{4\pi^2}{6}+8-4\log\left(\frac{-2\Delta}{\mu}\right)-4\log\left(\frac{-2\Delta^*}{\mu}\right)+4\,\mathrm{Li}_2\left(\frac{1}{\tilde{\gamma}^2}\right) \\
&\quad+4\log\left(\frac{-2\Delta}{\mu}\right)\log\left(\tilde{\gamma}^2\right)+4\log\left(\frac{-2\Delta^*}{\mu}\right)\log\left(\tilde{\gamma}^2\right)-2\log^2\left(\tilde{\gamma}^2-1\right)+2\log^2\left(1-\frac{1}{\tilde{\gamma}^2}\right)\Bigg] \\
&\quad-\frac{2}{\mu}\mathcal{L}_0\left(\frac{\bar{a}^+}{\mu}\right)\left[\log\left(\frac{\bar{a}^+-2\Delta}{\mu}\right)+\log\left(\frac{\bar{a}^+-2\Delta^*}{\mu}\right)\right] \\
&\quad+\frac{|\Delta|^2}{|\bar{a}^+\tilde{\gamma}^2-2\Delta|^2}\,\frac{8}{\mu}\mathcal{L}_0\left(\frac{\bar{a}^+}{\mu}\right)\Bigg[2\log\left(\frac{\bar{a}^+\tilde{\gamma}^2-2\Delta}{\mu}\right) \\
&\qquad+2\log\left(\frac{\bar{a}^+\tilde{\gamma}^2-2\Delta^*}{\mu}\right)-\log\left(\frac{\bar{a}^+-2\Delta}{\mu}\right)-\log\left(\frac{\bar{a}^+-2\Delta^*}{\mu}\right)\Bigg] \\
&\quad+\frac{1}{\mu}\mathcal{L}_1\left(\frac{\bar{a}^+}{\mu}\right)\left[\frac{16|\Delta|^2}{|\bar{a}^+\tilde{\gamma}^2-2\Delta|^2}-4\right]+\frac{2}{\Delta-\Delta^*}\left[\log\left(\frac{\bar{a}^+-2\Delta}{\mu}\right)-\log\left(\frac{\bar{a}^+-2\Delta^*}{\mu}\right)\right] \\
&\quad+\frac{2\tilde{\gamma}^2}{|\bar{a}^+\tilde{\gamma}^2-2\Delta|^2}\,\frac{\Delta+\Delta^*}{\Delta-\Delta^*}\times\Bigg[(\bar{a}^+\tilde{\gamma}^2-2\Delta^*)\left(2\log\left(\frac{\bar{a}^+\tilde{\gamma}^2-2\Delta}{\mu}\right)-\log\left(\frac{\bar{a}^+-2\Delta}{\mu}\right)\right) \\
&\qquad-(\bar{a}^+\tilde{\gamma}^2-2\Delta)\left(2\log\left(\frac{\bar{a}^+\tilde{\gamma}^2-2\Delta^*}{\mu}\right)-\log\left(\frac{\bar{a}^+-2\Delta^*}{\mu}\right)\right)\Bigg] \\
&\quad+\frac{4\tilde{\gamma}^2}{|\bar{a}^+\tilde{\gamma}^2-2\Delta|^2}\Bigg[\Delta\left(\log\left(\frac{\bar{a}^+-2\Delta}{\mu}\right)-2\log\left(\frac{\bar{a}^+\tilde{\gamma}^2-2\Delta}{\mu}\right)\right) \\
&\quad+\Delta^*\left(\log\left(\frac{\bar{a}^+-2\Delta^*}{\mu}\right)-2\log\left(\frac{\bar{a}^+\tilde{\gamma}^2-2\Delta^*}{\mu}\right)\right)\Bigg]\Bigg\}+\left[\delta(\bar{a}^+)\,\frac{\Delta+\Delta^*}{|\Delta|^2}+\delta'(\bar{a}^+)\right]\delta m_t(R)\Bigg\}.
\end{aligned}
$$