Nonperturbative Power Corrections to Event Shapes

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Goals

We want to study Ω_n, a **universal nonperturbative** matrix element, to enable

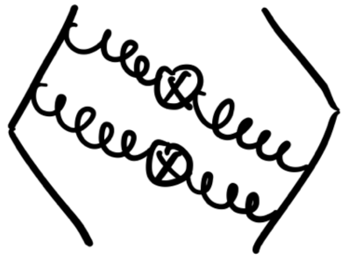
- Field theory predictions of nonperturbative hadronization effects
- Precision calculations of event shape observables

[Abbate, Becher, Bell, Fickinger, Hoang, Mateu, Korchemsky, C. Lee, K. Lee, Pathak, Schindler, Sterman, Stewart, Sun, etc.]

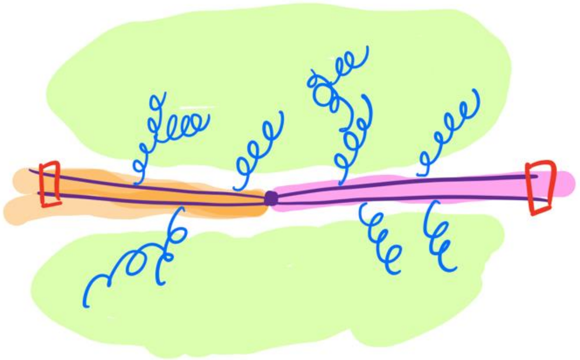
Our main result: The design of a new observable ($\mathbf{EEC}^{n}_{b2b}$) that is sensitive to, and thus enables the extraction of, Ω_n.

Outline

1. Nonperturbative corrections $\Omega_n(\zeta_1 \ldots \zeta_n)$



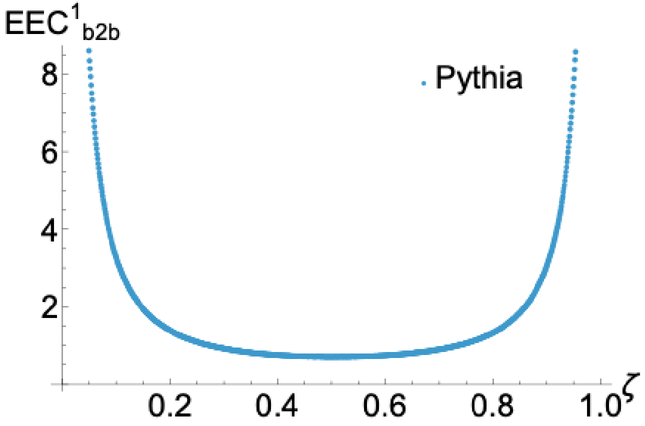
2. Factorization of the back-to-back energy correlator ($\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}$)



3. OPE of $\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}$

$$\boldsymbol{EEC^{n}_{b2b}(z,\{\zeta_i\}) = f(\{\zeta_i\} \times \sum_{q=0}^{n} \frac{\Omega_q(\{\zeta_i\})}{Q^q} h(z) + \ \ldots}$$

4. Comparison to Pythia



Nonperturbative Corrections

$$\Omega_n(\zeta_1 \cdots \zeta_n)$$

Event Shape Observables

Goal: Precision determination of **event shape observables** (semi-inclusive observables)

from $e^+e^- \to \text{hadrons}$

Useful for e.g. extracting α_s [CERN alphas-2025], top mass [Fleming, Hoang, Monty, and Stewart 2008], etc.

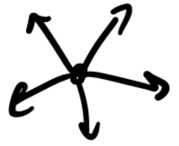
Nonperturbative corrections to event shapes are captured by Ω_n!

Example: Thrust

$$\tau = \left(\frac{1}{Q}\right) \min_t \sum_i (|\vec{p}_i| - |\vec{p}_i \cdot \hat{t}|)$$

Thin jet: $\tau \sim 0$

Spherical jet: $\tau \sim$ ½

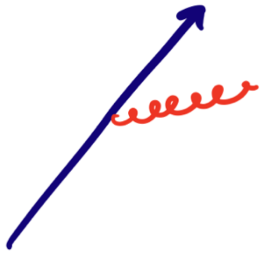


Nonperturbative Power Corrections

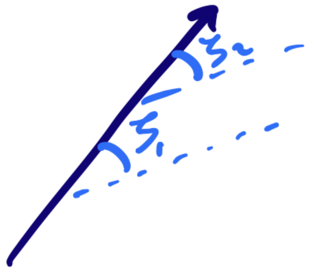
n nonperturbative (transverse) energy measurements of soft radiation at fixed angles, $\zeta_i = \frac{1-\cos\theta_i}{2}$, contribute corrections $\Omega_n(\{\zeta_i\})$ to event shapes

 Non-perturbative

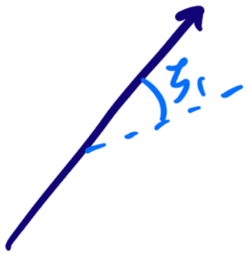
 Perturbative



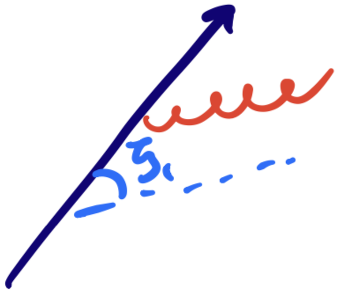
$\sim \alpha_s$



$\sim \frac{\Omega_2(\zeta_1, \zeta_2)}{Q^2} \sim \frac{\Lambda_{QCD}^2}{Q^2}$



$\sim \frac{\Omega_1(\zeta_1)}{Q} \sim \frac{\Lambda_{QCD}}{Q}$

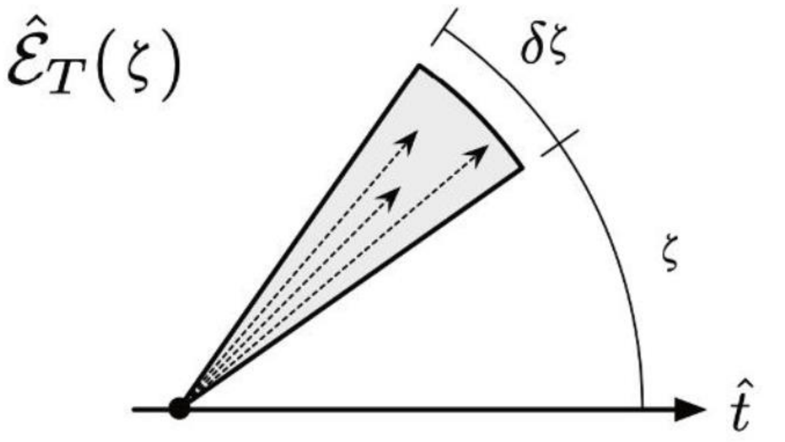


$\sim \frac{\alpha_s \Omega'_1(\zeta_1)}{Q} \sim \frac{\alpha_s \Lambda_{QCD}}{Q}$

What is $\Omega_n(\zeta_1, \dots, \zeta_n)$?

$$\Omega_n(\zeta_1, \dots, \zeta_n) = \langle 0 | Y_{\bar{n}} Y_n^\dagger \mathcal{E}_{\mathrm{T}}(\zeta_1) \dots \mathcal{E}_{\mathrm{T}}(\zeta_n) Y_{\bar{n}} Y_n^\dagger | 0 \rangle$$

Wilson Line $Y_n(0) = P \exp\left(i\, g \int_0^\infty ds\;\; n \cdot A_{us}(ns)\right)$



Transverse Energy Flow Operator $\mathcal{E}_T(\zeta_1)|N\rangle = \sum_{i \in N} |k_i^\perp| \delta(\zeta - \zeta_i) |N\rangle$

Nonperturbative radiation only

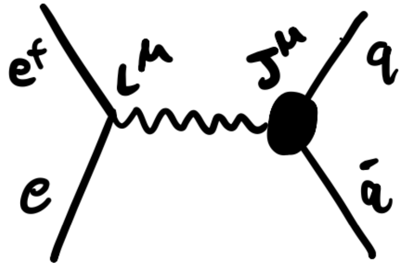
How do these appear from event shapes?

Event Shape SCET I Momentum Sectors

Collinear	$p_n \sim Q(\lambda^2, 1, \lambda)$
Anti-collinear	$p_{\bar{n}} \sim Q(1, \lambda^2, \lambda)$
Ultra-soft	$p_{us} \sim Q(\lambda^2, \lambda^2, \lambda^2)$
Nonperturbative	$p_{NP} \sim (\Lambda_{\mathrm{QCD}}, \Lambda_{\mathrm{QCD}}, \Lambda_{\mathrm{QCD}})$

For event shapes, $\lambda^2 \sim \tau$

Event shape factorization from SCET



$$\frac{d\sigma}{de} = \frac{1}{2Q^2} \sum_N \left| \langle N | J^\mu | 0 \rangle L_\mu \right|^2 (2\pi)^4 \delta^4(Q - p_N) \delta\big(e - e(N)\big)$$

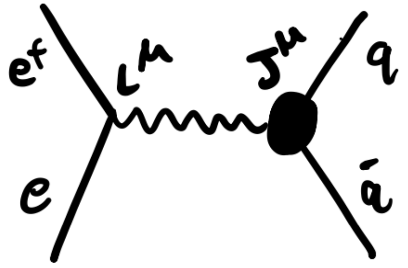
ultrasoft gluons decouple from collinear fields

$$\frac{1}{\sigma_0} \frac{d\sigma}{de} = \int de_J \; \sigma_J(e_J) S(e - e_J)$$

Soft function: $S_e(e) = \frac{1}{N_C} \mathrm{Tr} \sum_X \left| \langle X | Y_n Y_{\bar{n}}^\dagger | 0 \rangle \right|^2 \delta\big(Qe - Qe(X)\big)$

Ultrasoft + nonperturbative

Event shape factorization from SCET



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Ultrasoft + nonperturbative

Separate perturbative and nonperturbative

$$\frac{1}{\sigma_0}\frac{d\sigma}{de} = \int de_J \frac{d\hat{\sigma}}{de}\left(e - \frac{e_J}{Q}\right) F_e(e_j) + \cdots$$

Shape function: $F_e(e) = \frac{1}{N_C}\mathrm{Tr}\ \sum_{X_{NP}} \left|\langle X_{NP}\middle|Y_n Y_{\bar{n}}^\dagger\middle|0\rangle\right|^2 \delta\big(Qe - Qe(X_{NP})\big)$

nonperturbative

Where's Ω_n: What's e?

A generic event shape e has the form, with η as *pseudorapidity*

$$\hat{e}|X\rangle = e(X)|X\rangle = \frac{1}{Q}\int d\eta\ f_e(\eta)\mathcal{E}_T(\eta)|X\rangle$$

Recall, $\zeta = \frac{1-\cos\theta}{2}$. For massless particles, $\eta = -\ln\tan\frac{\theta}{2}$.

Example: thrust $f(\eta) = e^{-|\eta|} \rightarrow \sum_i\left(p_i^{\perp}e^{|-\eta_i|}\right) = \sum_i(|\vec{p}_i| - |\vec{p}_i \cdot \hat{t}|)$

Where's Ω_n

Recall:

1. $e|X\rangle = \frac{1}{Q}\int d\eta\, f(\eta)\mathcal{E}_T(\eta)|X\rangle$

2. $S_e(e) = \frac{1}{N_C} Tr\ \sum_{X_\mu} \left|\langle X_\mu|Y_n Y_{\bar{n}}^\dagger|0\rangle\right|^2 \delta\left(e - e\left(X_\mu\right)\right)$ (Ultrasoft + nonperturbative)

We take a tree-level OPE of $S_e(e)$, or equivalently, of $F_e\left(e_j\right)$ (nonperturbative)

$$\delta(e) - \delta'(e)\frac{1}{Q}\int d\eta\, f(\eta)\Omega_1(\eta) + \frac{\delta''(e)}{2}\frac{1}{Q^2}\int d\eta d\eta' f(\eta)f(\eta')\ \Omega_2(\eta,\eta') + \ \dots$$

Where we now have the nonperturbative matrix elements $\Omega_n(\zeta_1, \dots, \zeta_n) = \langle 0|Y_{\bar{n}}Y_n^\dagger \mathcal{E}_\mathrm{T}(\zeta_1) \dots \mathcal{E}_\mathrm{T}(\zeta_n) Y_{\bar{n}}Y_n^\dagger|0\rangle$!

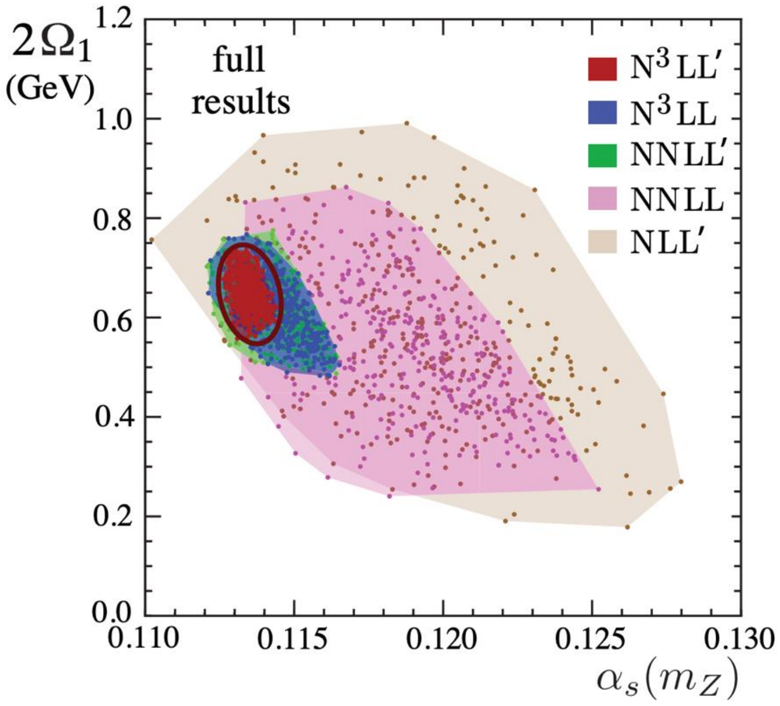
Ω_1 can be extracted from event shape observables

There exists a boost symmetry such that Ω_1 is independent of angle [Lee and Sterman '06]

$U(\Lambda(\eta'))\mathcal{E}_T(\eta)U(\Lambda(\eta'))^\dagger = \mathcal{E}_T(\eta+\eta')$, and Wilson lines are boost independent

$$\frac{1}{Q}\int d\eta\, f(\eta)\langle 0|Y_{\bar{n}}Y_n^\dagger \mathcal{E}_T(\eta) Y_{\bar{n}}Y_n^\dagger|0\rangle$$
$$= \frac{1}{Q}\left(\int d\,\eta\, f(\eta)\right)\langle 0|Y_{\bar{n}}Y_n^\dagger \mathcal{E}_T(0) Y_{\bar{n}}Y_n^\dagger|0\rangle$$

Therefore, $\Omega_1(\eta) = \langle 0|Y_{\bar{n}}Y_n^\dagger \mathcal{E}_T(0) Y_{\bar{n}}Y_n^\dagger|0\rangle = \Omega_1(0)$ is just a number!



[Abbate, Fickinger, Hoang, Mateu, and Stewart 2010]

$\Omega_n(\zeta_1, \ldots, \zeta_n), n > 1$ cannot be directly extracted from event shapes

Information about $\mathrm{M}_2 = \frac{1}{Q^2} \int d\eta d\eta' \langle 0 | Y_{\bar{n}} Y_n^\dagger f(\eta) f(\eta') \, \mathcal{E}_\mathrm{T}(\eta) \, \mathcal{E}_\mathrm{T}(\eta') Y_{\bar{n}} Y_n^\dagger | 0 \rangle$ can be extracted by taking event shape moments, but these are hard to disentangle from perturbative corrections

[$\mathrm{M}_2 + O(\alpha_s) \, \mathrm{M}_1 = \ (0.74 \pm 0.09 \pm 0.11 \ \mathrm{Ge}V)^2$ Abbate et al. 2012]

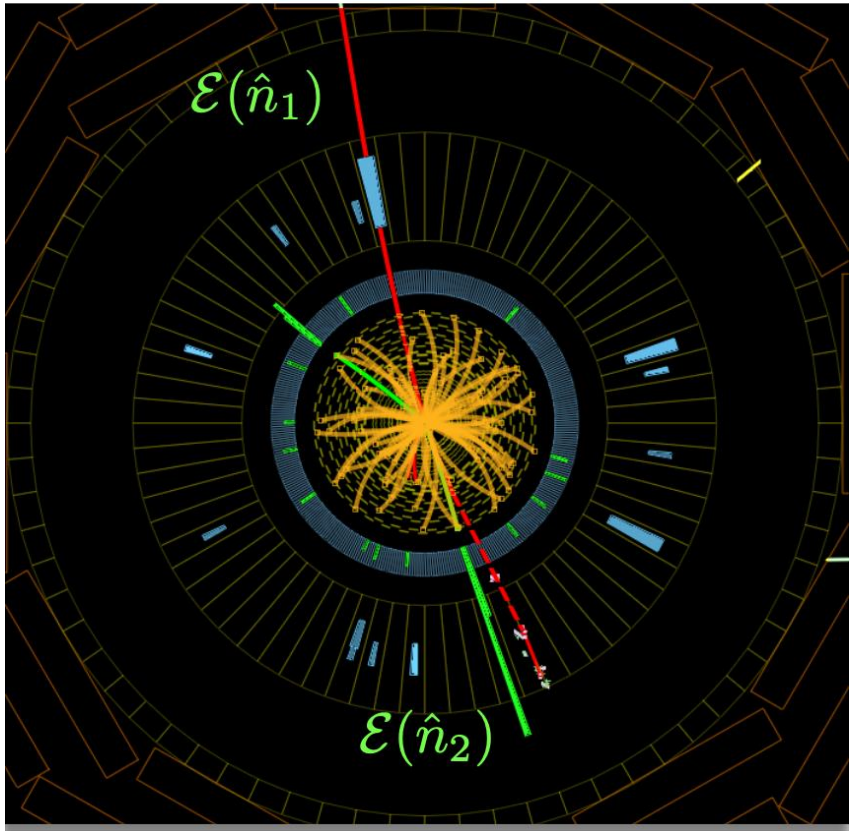
However, Ω_2 cannot – boost symmetry is insufficient to move Ω_2 outside the the η integrals:

$$\frac{1}{Q^2} \int d\eta d\eta' \langle 0 | Y_{\bar{n}} Y_n^\dagger f(\eta) f(\eta') \, \mathcal{E}_\mathrm{T}(\eta) \, \mathcal{E}_\mathrm{T}(\eta') Y_{\bar{n}} Y_n^\dagger | 0 \rangle = \frac{1}{Q^2} \int d\eta d\eta' \langle 0 | \, Y_{\bar{n}} Y_n^\dagger \, f(\eta + \eta') f(\eta') \, \mathcal{E}_\mathrm{T}(\eta) \, \mathcal{E}_\mathrm{T}(0) Y_{\bar{n}} Y_n^\dagger | 0 \rangle$$

How to get Ω_2? Through $\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}$!

$$\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}$$

The EEC

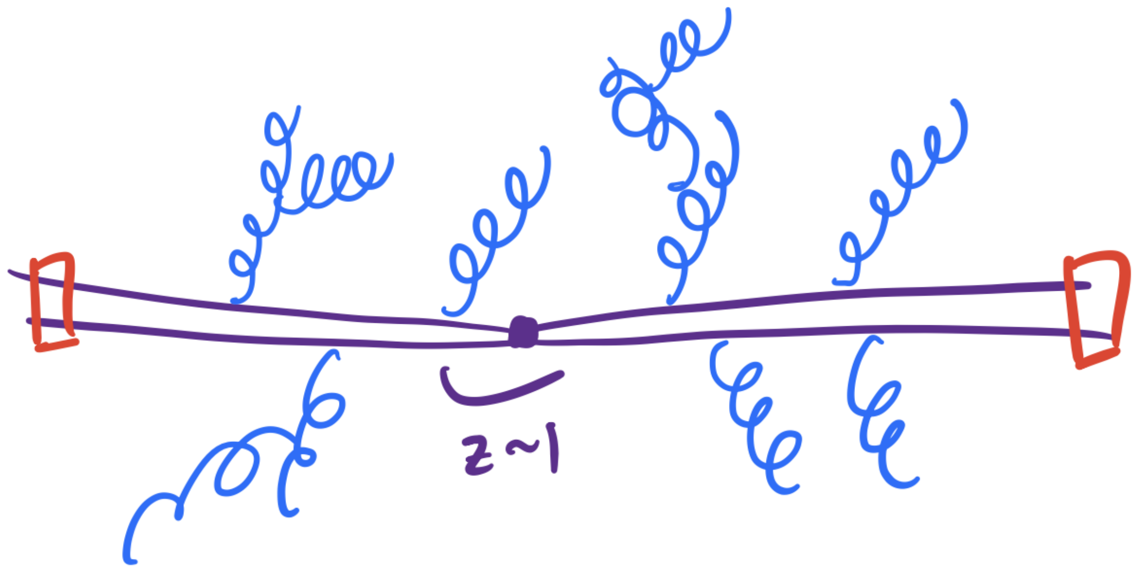


$$\mathrm{EEC}(z) = \sum_{\{i,j\}} \int d\sigma\,(e^+e^- \to\ i\ j\ X\ )\ \delta\big(z\ - z_{ij}\big)\,\frac{E_i E_j}{Q^2}$$

$z_{ij} = \frac{1 - \cos\theta_{ij}}{2}$ (just like ζ)

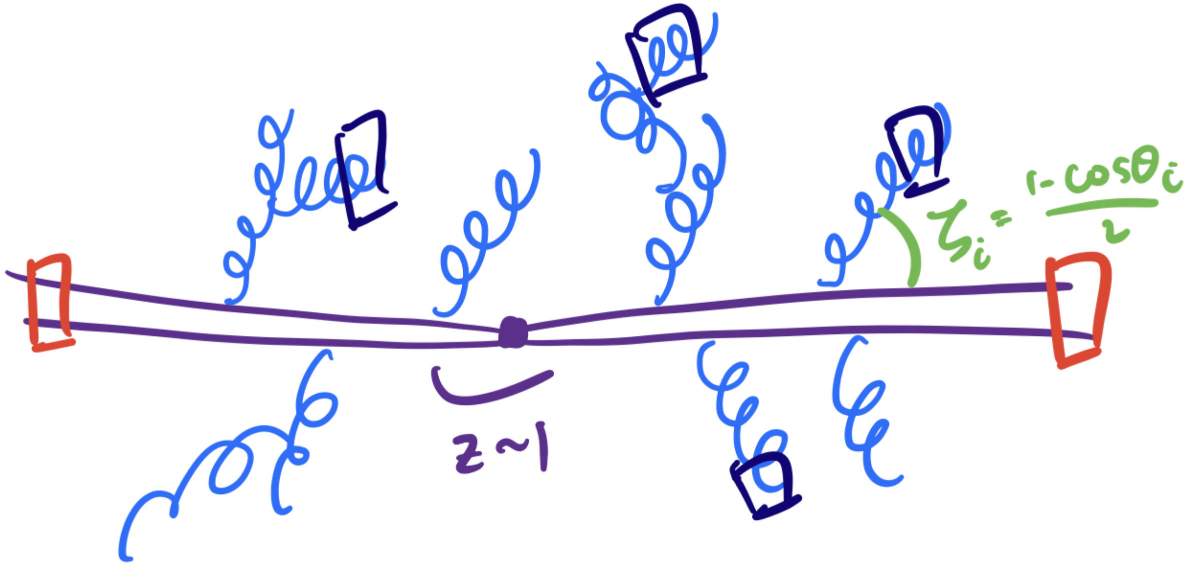
The Back-to-Back EEC

$$\mathrm{EEC_{b2b}}(\mathrm{z}) = \mathrm{EEC}(\mathrm{z})\,[1 + O(1 - z)]$$



$\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}(\mathrm{z}, \{\zeta_1, \ldots, \zeta_n\}) = \mathrm{EEC_{b2b}}(\mathrm{z})$ with n additional measurements

$\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}$

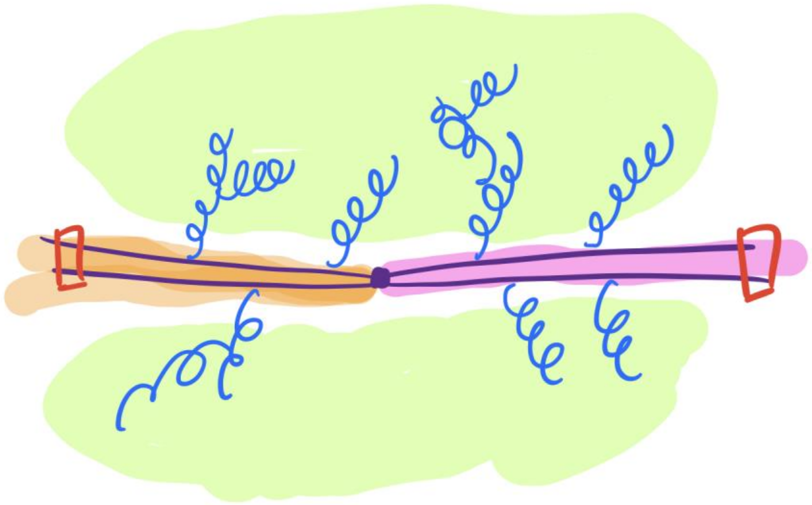


$$\frac{d\Sigma}{dz\, d\zeta_1 \ldots d\zeta_n} = \sum_{\{i,j,k_1 \ldots k_n\}} \int d\sigma \left(e^+ e^- \to ij\, X\right) \delta\left(z - z_{ij}\right) \frac{E_i E_j}{Q^2} \times \prod_{l,m} \delta\left(\zeta_l - \zeta_{k_m}\right) \frac{E_l}{Q}$$

ζ_i are far from 0 and 1

The Back To Back EEC, Factorized

Soft radiation is in SCET II, $\lambda \sim Q\sqrt{1-z}$



$p_s \sim Q(\lambda, \lambda, \lambda)$

$p_{NP} \sim (\Lambda_{\mathrm{QCD}}, \Lambda_{\mathrm{QCD}}, \Lambda_{\mathrm{QCD}})$

$$\frac{d\sigma}{dz} = \frac{1}{2}\int d^2k_\perp \int \frac{d^2b_\perp}{(2\pi)^2} e^{-ib_\perp \cdot k_\perp} H(Q,\mu)\, J^q_{EEC}(b_\perp,\mu,\nu)\, J^{\bar{q}}_{EEC}(b_\perp,\mu,\nu) S_{EEC}(b_\perp,\mu,\nu)\,\delta\left(1 - z - \frac{k_\perp^2}{Q^2}\right)$$

[Moult and Zhu 2019]

$$S_{EEC}(b_T) = \int d^2k_{\perp,s}\, \langle 0|Y_{\bar{n}}(0)Y_n^\dagger(0)\delta^2(k_{\perp,s} - \hat{P})Y_n(0)Y_{\bar{n}}^\dagger(0)|0\rangle e^{i\, b_\perp \cdot k_{\perp,s}}$$

$\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}$, factorized

We demonstrate $\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}$ obeys the same factorization structure into hard, jet, and soft functions.

The soft function is transverse energy-weighted and contains factors of $\frac{1}{4[(1-\zeta_i)\,\zeta_i]^{3/2}}$

$$\frac{d\sigma}{dz\, d\zeta_1 \ldots d\zeta_n} = \frac{1}{2}\int d^2k_\perp \int \frac{d^2b_\perp}{(2\pi)^2} e^{-ib_\perp\cdot k_\perp} H(Q,\mu)\, J^q_{EEC}(b_\perp,\mu,\nu)\, J^{\bar{q}}_{EEC}(b_\perp,\mu,\nu) S^n_{EEC}(b_\perp,\mu,\nu,\{\zeta_i\})\,\delta\left(1-z-\frac{k_\perp^2}{Q^2}\right)$$

$$S^n_{EEC}(b_T) = \frac{1}{Q^n}\int d^2k_{\perp,s} \prod_i \frac{1}{4[(1-\zeta_i)\,\zeta_i]^{3/2}} \langle 0|Y_{\bar{n}}(0)Y_n^\dagger(0) \prod_i \mathcal{E}_{\mathrm{T}}(\eta(\zeta_i))\, \delta^2\big(k_{\perp,s} - \hat{P}\big) Y_n(0) Y_{\bar{n}}^\dagger(0)|0\rangle e^{i\, b_\perp\cdot k_{\perp,s}}$$

The soft function

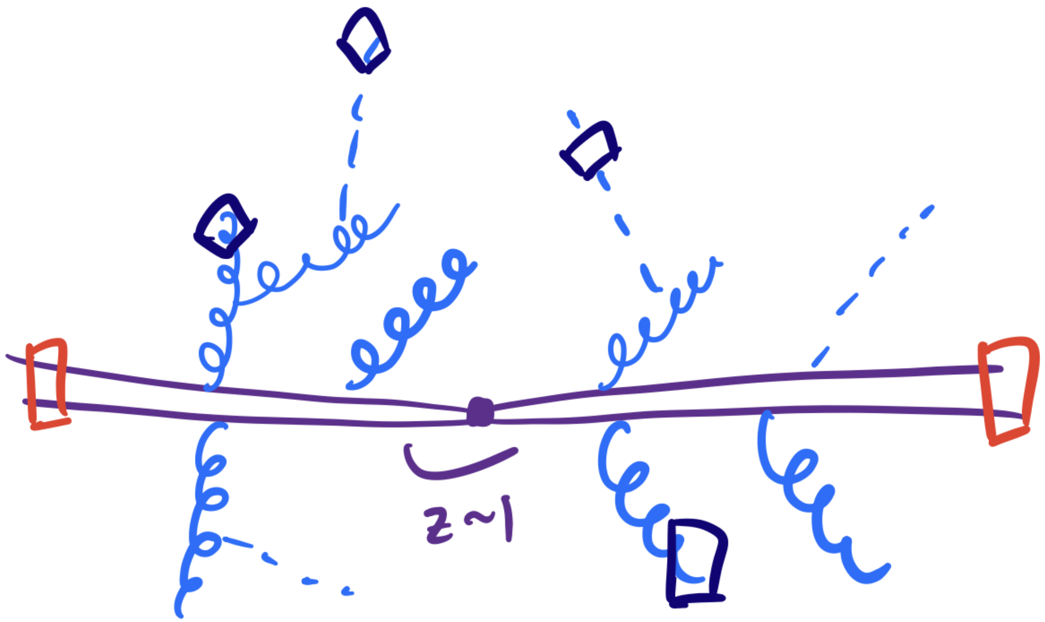
$$S^1(\chi, b_\perp) = \sum_X \int d^2k_{\perp,s} \langle 0|Y_{\bar{n}}(0)Y_n^\dagger(0)|X\rangle\langle X|\delta^2\big(k_{\perp,s} - \hat{P}\big)Y_n(0)Y_{\bar{n}}^\dagger(0)|0\rangle\, e^{i\, b_\perp\cdot k_{\perp,s}}$$

$$\times \sum_{i\in X}\frac{E_i}{Q}\delta\left(\zeta - \frac{1-\cos\theta_i}{2}\right)$$

$$= \frac{1}{Q}\int d^2k_{\perp,s}\, \frac{1}{4[(1-\zeta)\zeta]^{3/2}} \langle 0|Y_{\bar{n}}(0)Y_n(0)\mathcal{E}_{\mathrm{T}}(\eta(\zeta))\delta^2\big(k_{\perp,s} - \hat{P}\big)Y_n(0)Y_{\bar{n}}(0)|0\rangle e^{i\, b_\perp\cdot k_{\perp,s}}$$

$\mathrm{EEC}_{\mathrm{b2b}}^{\mathrm{n}}(z, \{\zeta_i\})$ OPE

Let's expand the soft function, letting $\mathcal{E}_{\mathrm{T}} = \mathcal{E}_{\mathrm{T,pert}} + \mathcal{E}_{\mathrm{T,non\ pert}}$, in $1 \gg \alpha_s \sim \frac{\Omega_1}{Q\sqrt{1-z}}$

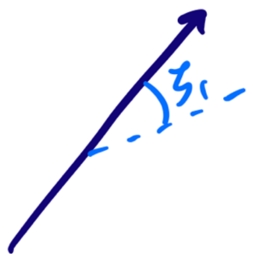


$$S^1_{EEC}(b_\perp,\zeta) = \frac{1}{Q}\frac{1}{4[(1-\zeta)\zeta]^{3/2}}\int d^2k_{\perp,s}\ \langle 0|Y_{\bar{n}}(0)Y_n^\dagger(0)\left(\mathcal{E}_{\mathrm{T},P}\big(\eta(\zeta)\big) + \mathcal{E}_{\mathrm{T},NP}\big(\eta(\zeta)\big)\right)\delta^2\big(k_{\perp,s} - \hat{P}\big)Y_n(0)Y_{\bar{n}}^\dagger(0)|0\rangle e^{i\,b_\perp\cdot k_{\perp,s}}$$

OPE Part 1: Non-Perturbative

The non-perturbative soft function has no $b_\perp$ dependence and can be expressed via the Ω_n

$$S_{NP}^{1}(\zeta, b_\perp) = \frac{1}{Q}\int d^2k_{\perp,s}\, \frac{1}{4[(1-\zeta)\zeta]^{3/2}} \langle 0|Y_{\bar{n}}(0)Y_n^\dagger(0)\mathcal{E}_{\mathrm{T},NP}\big(\eta(\zeta)\big)\, \delta^2\big(k_{\perp,s} - \hat{P}\big)\, Y_n(0)Y_{\bar{n}}^\dagger(0)|0\rangle e^{i\, b_\perp \cdot k_{\perp,s}}$$



$$\sim \frac{\Omega_1(\zeta_1)}{Q} \sim \frac{\Lambda_{QCD}}{Q}$$

$$\hat{P} \sim O(\Lambda_{QCD}) \ll O\big(Q\sqrt{1-z}\big) \sim k_{\perp,s}$$

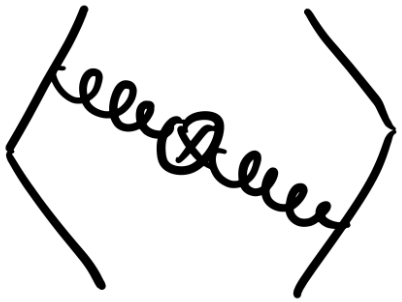
$$\delta^2\big(k_{\perp,s} - \hat{P}\big) \sim \delta^2(k_{\perp,s})$$

Net non-perturbative contribution $S_{NP}^{1}(\zeta, b_\perp) \sim \frac{\Omega_1(\zeta)}{Q}$

$$\left(S_{NP}^{n}(\{\zeta_1 \ldots \zeta_n\}, b_\perp) \sim \frac{\Omega_n(\{\zeta_i\})}{Q^n}\right)$$

OPE Part 2 - Perturbative

Jet function contribution: $\frac{d\sigma}{dz} \sim \delta(1-z) \sim \frac{1}{1-z}$



S^1 contribution $\sim \frac{1}{b_\perp}$

Fourier transform via $\delta\left(1 - z - \frac{k_\perp^2}{Q^2}\right) e^{-ib_\perp \cdot k_\perp}$: $\frac{1}{b_\perp} \sim \sqrt{1-z}$

$$\left(S_P^n(\{\zeta_1 \dots \zeta_n\}, b_\perp) \sim \frac{1}{b_\perp^n} \sim \left(\sqrt{1-z}\right)^n \right)$$

Overall perturbative z dependence to $\mathrm{EEC}^1_{\mathrm{b2b}}$: $\frac{d\sigma}{dz\, d\zeta} \sim \frac{1}{\sqrt{1-z}}$

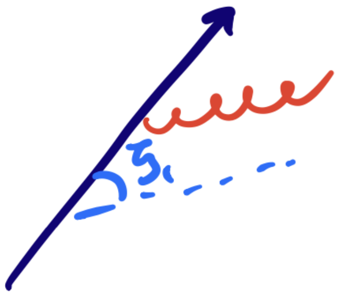
OPE Part 3 – Mixed Terms

Now let's consider $n = 2$

$$S^2(\zeta_1, \zeta_2, b_\perp) \sim \langle 0|Y_{\bar{n}}(0)Y_n(0) \ldots \mathcal{E}_{\mathrm{T}}(\zeta_1)\mathcal{E}_{\mathrm{T}}(\zeta_2) \ldots Y_n(0)Y_{\bar{n}}^\dagger(0)|0\rangle$$

$$S^2_{mixed}(\zeta_1, \zeta_2, b_\perp) \sim \langle 0|Y_{\bar{n}}(0)Y_n(0) \ldots \left(\mathcal{E}_{\mathrm{T,P}}(\zeta_1)\mathcal{E}_{\mathrm{T,NP}}(\zeta_2) + \mathcal{E}_{\mathrm{T,NP}}(\zeta_1)\mathcal{E}_{\mathrm{T,P}}(\zeta_2)\right) \ldots Y_n(0)Y_{\bar{n}}^\dagger(0)|0\rangle$$

$$S^2_{mixed}(\zeta_1, \zeta_2, , b_\perp) \sim \frac{\Omega_1'(\zeta)\alpha_s}{\sqrt{1-z}}$$



$$\sim \frac{\alpha_s \Omega'_1(\zeta_1)}{Q} \sim \frac{\alpha_s \Lambda_{QCD}}{Q}$$

Additional z-dependence

- $\mathrm{EEC}^{0}_{\mathrm{b2b}}$ has additional $\log(1-z)$ factors
- $\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}$ *also* has additional $\log(1-z)$ factors
- $\frac{\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}}{\mathrm{EEC}^{0}_{\mathrm{b2b}}}$? We can check that the soft functions have the same anomalous

  dimension: **NO** additional $\log(1-z)$ factors (at least up to NNLL)

Some OPEs

$$\frac{\mathrm{EEC}^1_{\mathrm{b2b}}(z,\zeta)}{\mathrm{EEC}^0_{\mathrm{b2b}}(z)\sqrt{1-z}} \sim \frac{1}{[\zeta(1-\zeta)]^{\frac{3}{2}}}\frac{1}{Q}\left(\frac{\Omega_1}{\sqrt{1-z}} + O(Q\alpha_s)\right)$$

$$\frac{\mathrm{EEC}^2_{\mathrm{b2b}}(z,\zeta_1,\zeta_2)}{\mathrm{EEC}^0_{\mathrm{b2b}}(z)(1-z)} \sim \frac{1}{[\zeta_1(1-\zeta_1)]^{\frac{3}{2}}}\frac{1}{[\zeta_2(1-\zeta_2)]^{\frac{3}{2}}}\frac{1}{Q^2}\left(\frac{\Omega_2(\zeta_1,\zeta_2)}{\left(\sqrt{1-z}\right)^2} + O\left((Q\alpha_s)^2\right) + \frac{\Omega_1'(\zeta_i)\alpha_s Q}{\sqrt{1-z}}\right)$$

Higher Order Jet Function Contribution

- Are there additional contributions?

$$\mathrm{EEC(z)} = \mathrm{EEC_{pert}(z)} + \frac{1}{2\left(z(1-z)\right)^{3/2}} \frac{\Omega_1}{Q}$$

- In the usual EEC (including the back-to-back region) without soft measurements, we also get Ω_1!

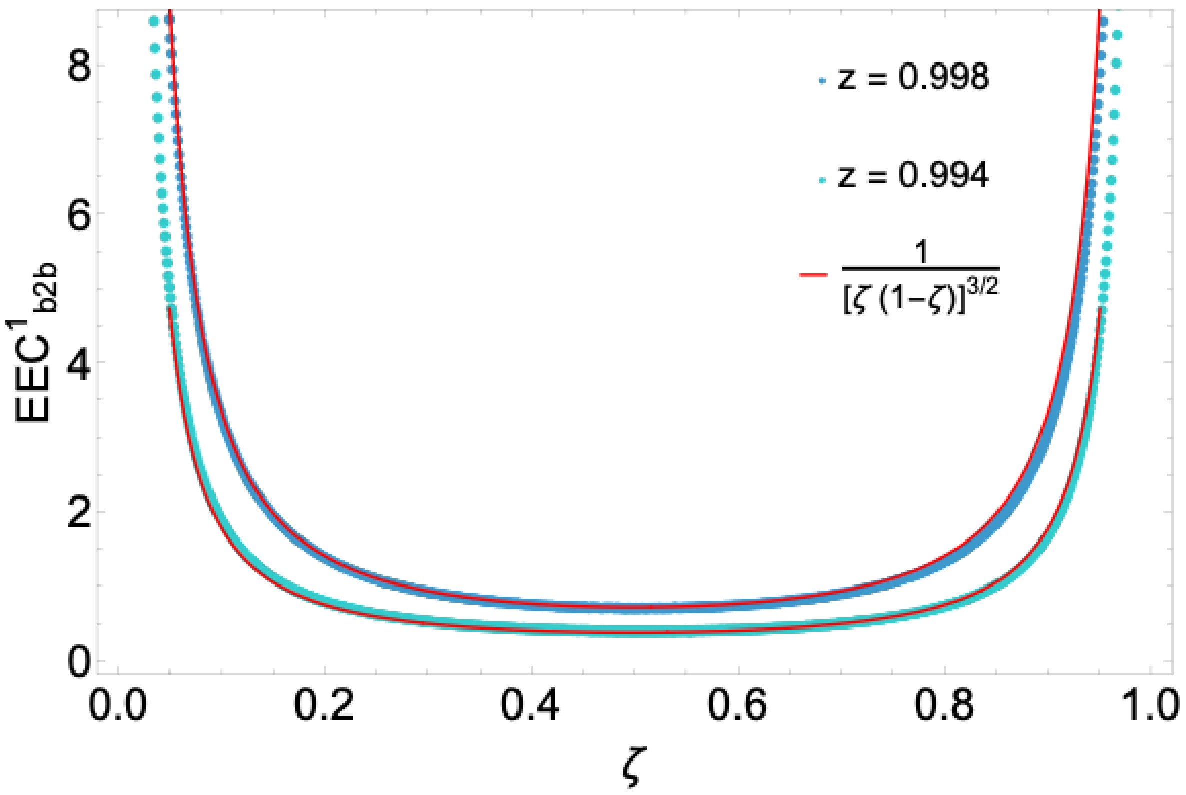
- This comes from the jet function, which can be verified via a renormalon calculation:

$$J^q J^{\bar{q}} \sim 1 + b_T \Omega_1 \sim 1 + \frac{\Omega_1}{Q\sqrt{1-z}}$$ (suppressed in power counting!)

Comparison With Pythia

Pythia, ζ-dependence

$\mathrm{EEC}^{1}_{\mathrm{b2b}}(z,\zeta)$ has a $\frac{1}{[\zeta(1-\zeta)]^{\frac{3}{2}}}$ dependence for all z



Extracting Ω_1

Now let's get Ω_1 from the OPE of $\mathrm{EEC}^1_{\mathrm{b2b}}$

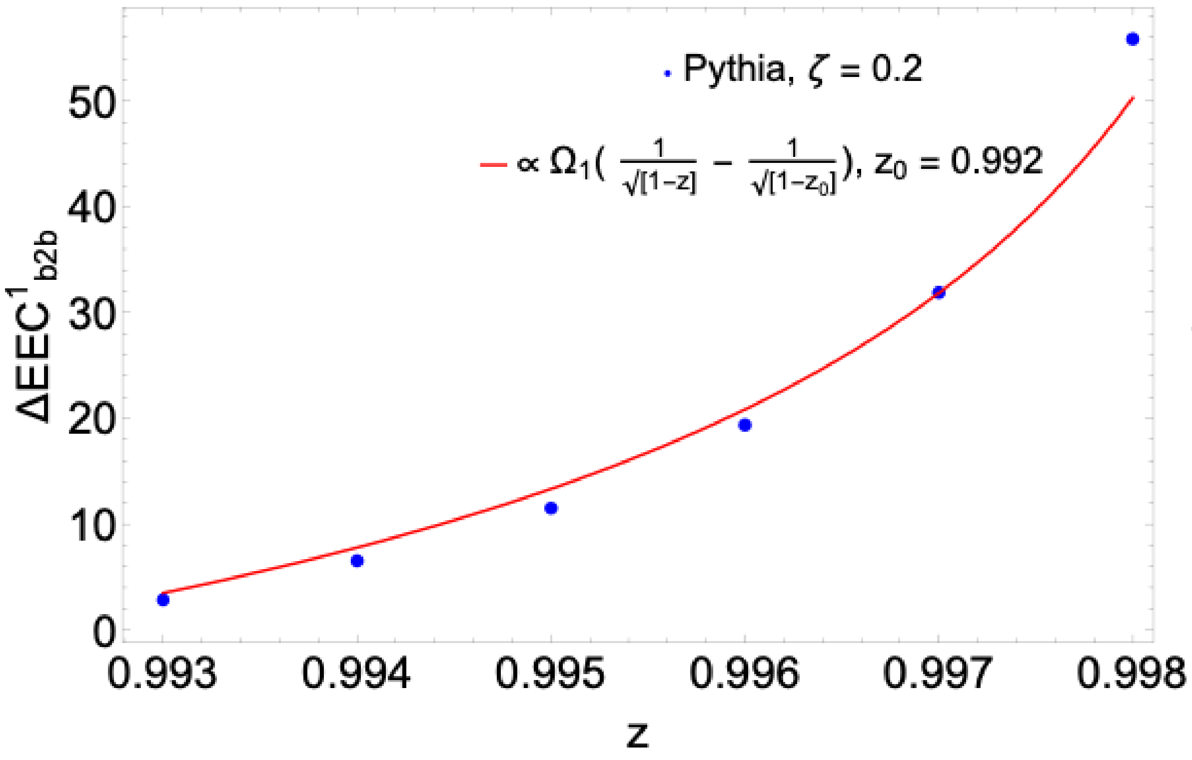
$$\mathrm{O}(z_1, \chi_1) = \frac{\mathrm{EEC}^1_{\mathrm{b2b}}(z_1, \zeta_1)}{\mathrm{EEC}^0_{\mathrm{b2b}}(z_1)\sqrt{1 - z_1}} \sim \frac{1}{[\zeta_1(1-\zeta_1)]^{\frac{3}{2}}} \frac{1}{Q} \left(\frac{\Omega_1}{\sqrt{1 - z_1}} + O(\alpha_s Q) \right)$$

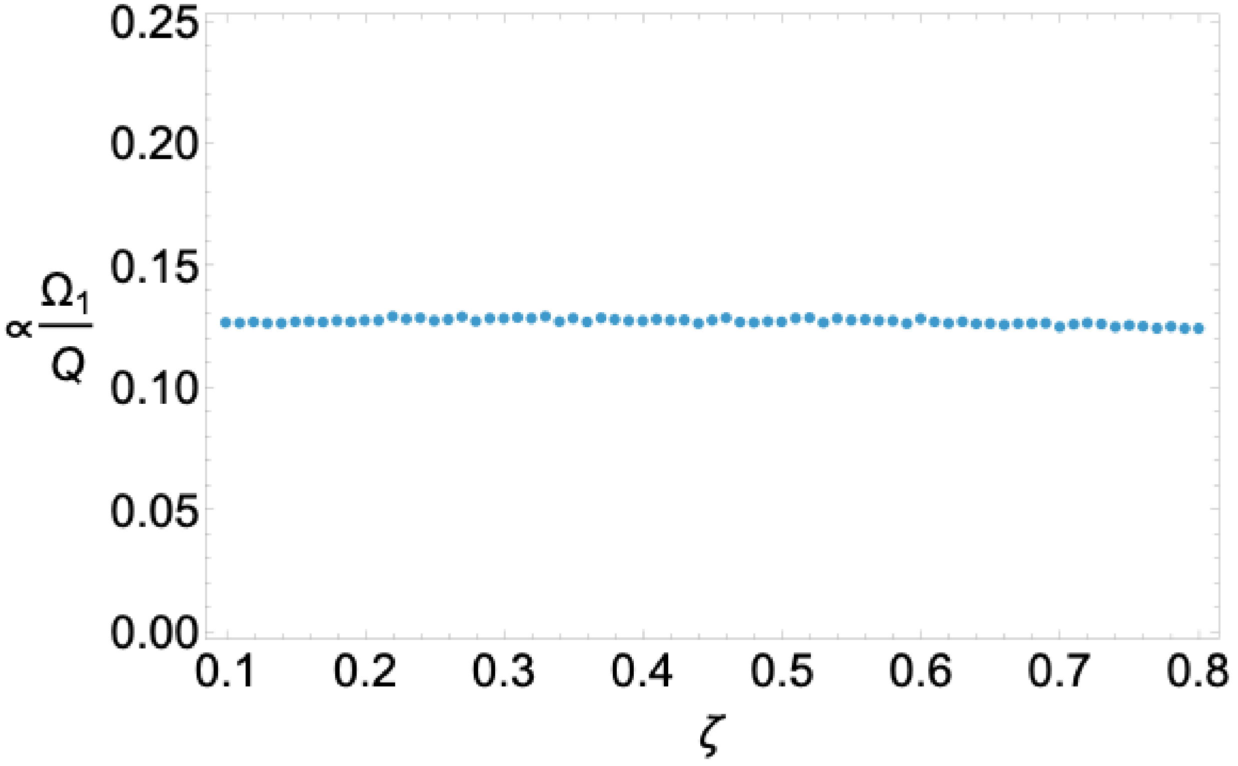
$$\mathrm{O}(z_1, \zeta_1) - \mathrm{O}(z_2, \zeta_1) \sim \frac{\Omega_1}{Q} \left(\frac{1}{\sqrt{1 - z_1}} - \frac{1}{\sqrt{1 - z_2}} \right)$$

No degeneracy with α_s!

Ω_1 from Pythia

Ω_1 extraction is independent of ζ, as predicted by the boost symmetry





Extracting Ω_2

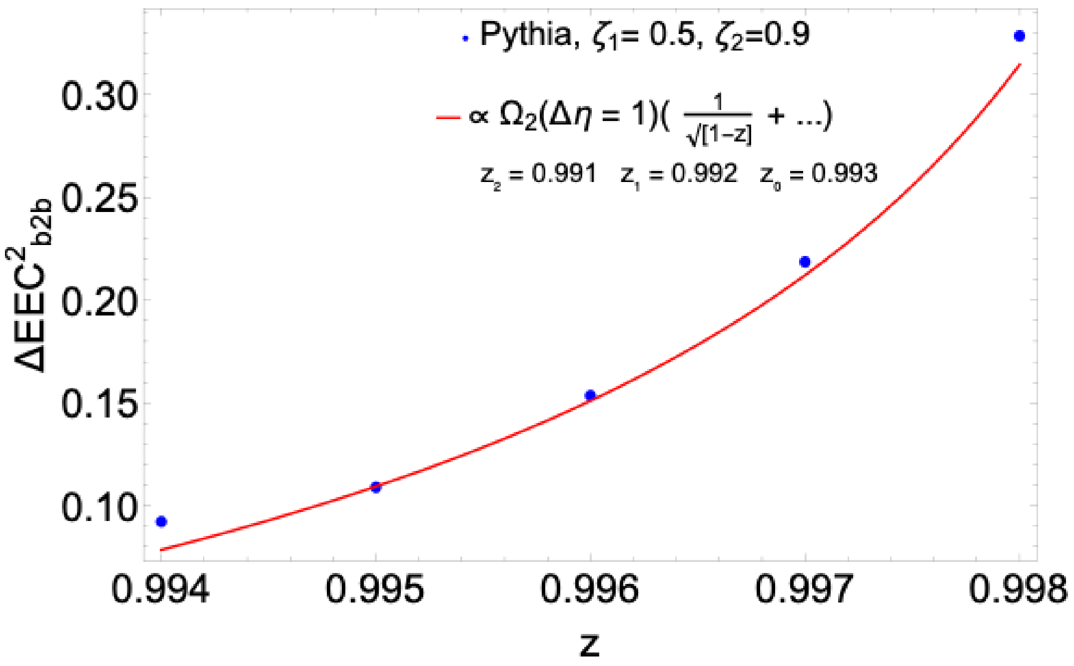
$$O(z_1,\zeta_1,\zeta_2) = \frac{\mathrm{EEC}^1_{\mathrm{b2b}}(z_1,\zeta_1,\zeta_2)}{\mathrm{EEC}^0_{\mathrm{b2b}}(z_1)(1-z_1)} \sim \frac{1}{[\zeta_1(1-\zeta_1)]^{\frac{3}{2}}}\frac{1}{[\zeta_2(1-\zeta_2)]^{\frac{3}{2}}}\frac{1}{Q^2}\left(\frac{\Omega_2(\zeta_1,\zeta_2)}{\left(\sqrt{1-z_1}\right)^2} + O((\alpha_s Q)^2) + \frac{\Omega_1'(\zeta_i)\alpha_s Q}{\sqrt{1-z_1}}\right)$$

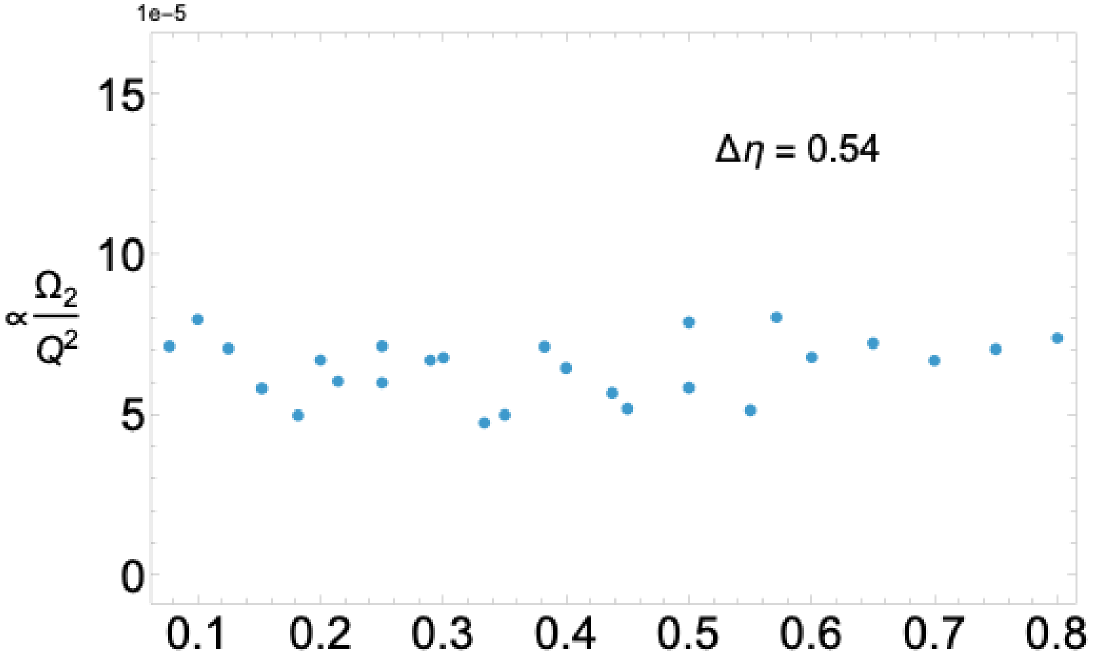
A couple of subtractions of $O(z_i,\zeta_1,\zeta_2)$ to isolate Ω_2 $\rightarrow F(O,z_1,z_2,z_3,z_4,\zeta_1,\zeta_2)$

$$F(O,z_1,z_2,z_3,z_4,\zeta_1,\zeta_2) \sim \frac{\Omega_2(\zeta_1,\zeta_2)}{Q^2}\left(\frac{1}{\sqrt{1-z_1}} + \frac{1}{\sqrt{1-z_2}} - \frac{1}{\sqrt{1-z_3}} - \frac{1}{\sqrt{1-z_4}}\right)$$

Ω_2 from Pythia

$\Omega_2(\zeta_1, \zeta_2)$ extraction is independent of $\Delta\eta = \log\frac{1-\zeta_1}{\zeta_1}\frac{\zeta_2}{1-\zeta_2}$





Conclusions

- Matrix elements for nonperturbative $\Omega_n(\zeta_1 \ldots \zeta_n)$ for event shape observables can be seen from SCET factorization
- We generalize the SCET factorization for $\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}$ to jet functions and an energy-weighted soft function
- Factorization of $\mathrm{EEC}^{\mathrm{n}}_{\mathrm{b2b}}$ allows us to derive an OPE linear in Ω_n, which is verified by Pythia