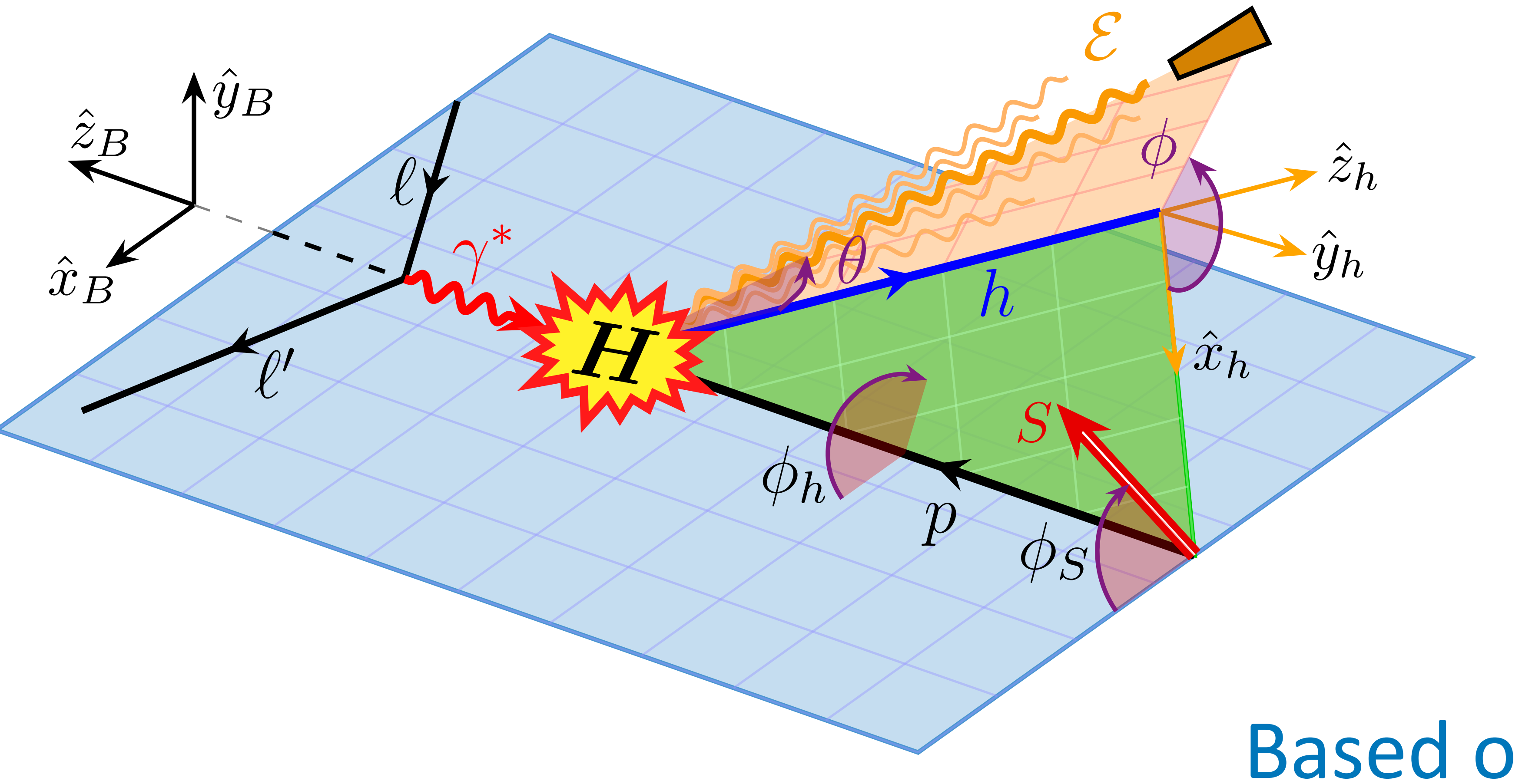
(Collins-Type) Fragmentation Energy Correlator

Shu-Tao Zhang

Peking University



Based on: arXiv 2509.18892 (10.1007/JHEP02(2026)244)

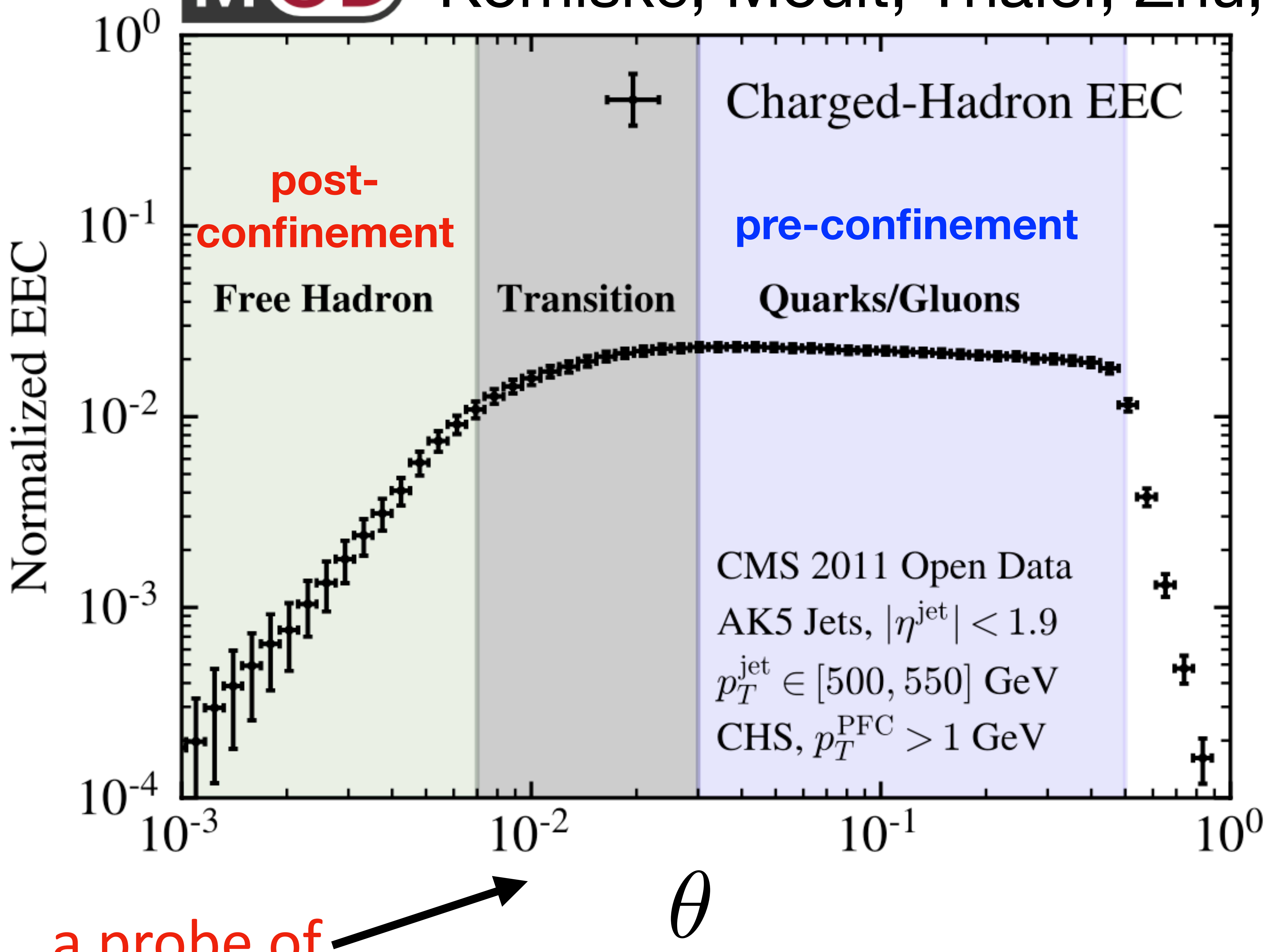
in Collaboration with Qing-Hong Cao, Zhite Yu, Chien-Peng Yuan, Hua Xing Zhu,

and on-going work + Xiaohui Liu, Feng Yuan, in progress

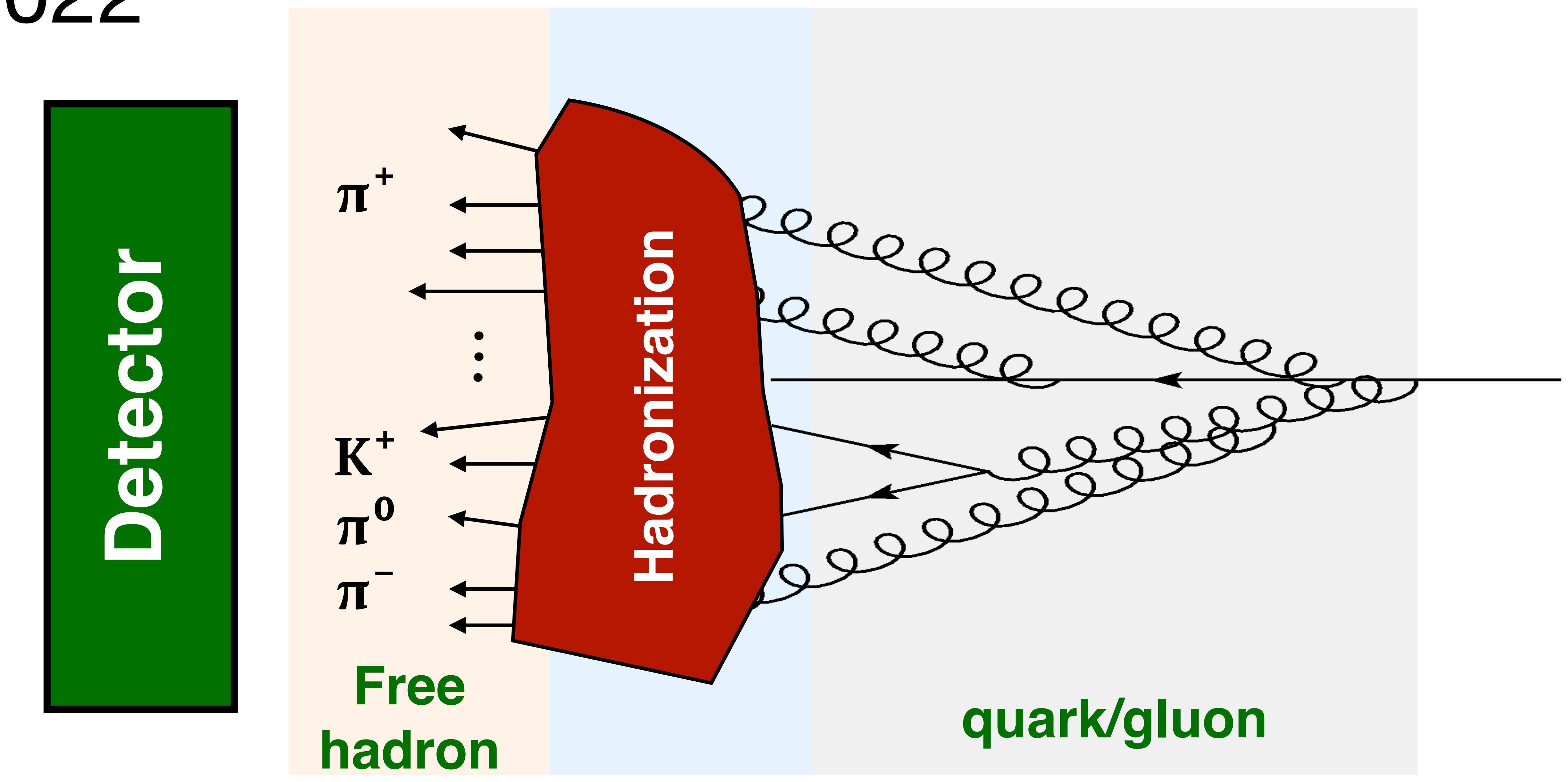
XXIII Annual Workshop on SCET, KIAS, March. 2-5, 2026

Energy Correlators as a novel way to probe scale

Komiske, Moult, Thaler, Zhu, 2022



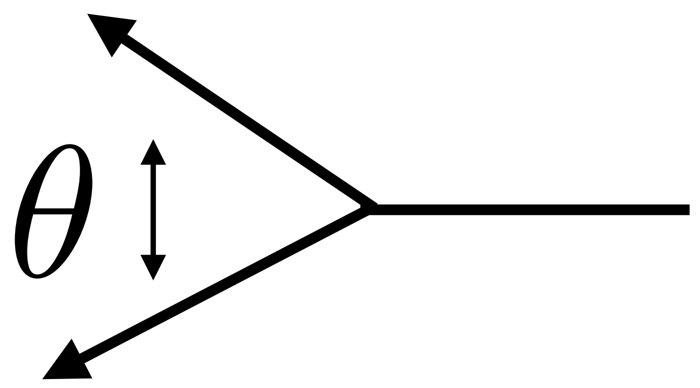
a probe of "deconfinement" scale



low energy — high energy

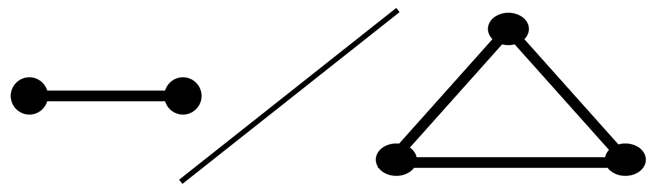
late time — early time

$$\Delta p_T \sim \sqrt{E_1 E_2 \theta}$$



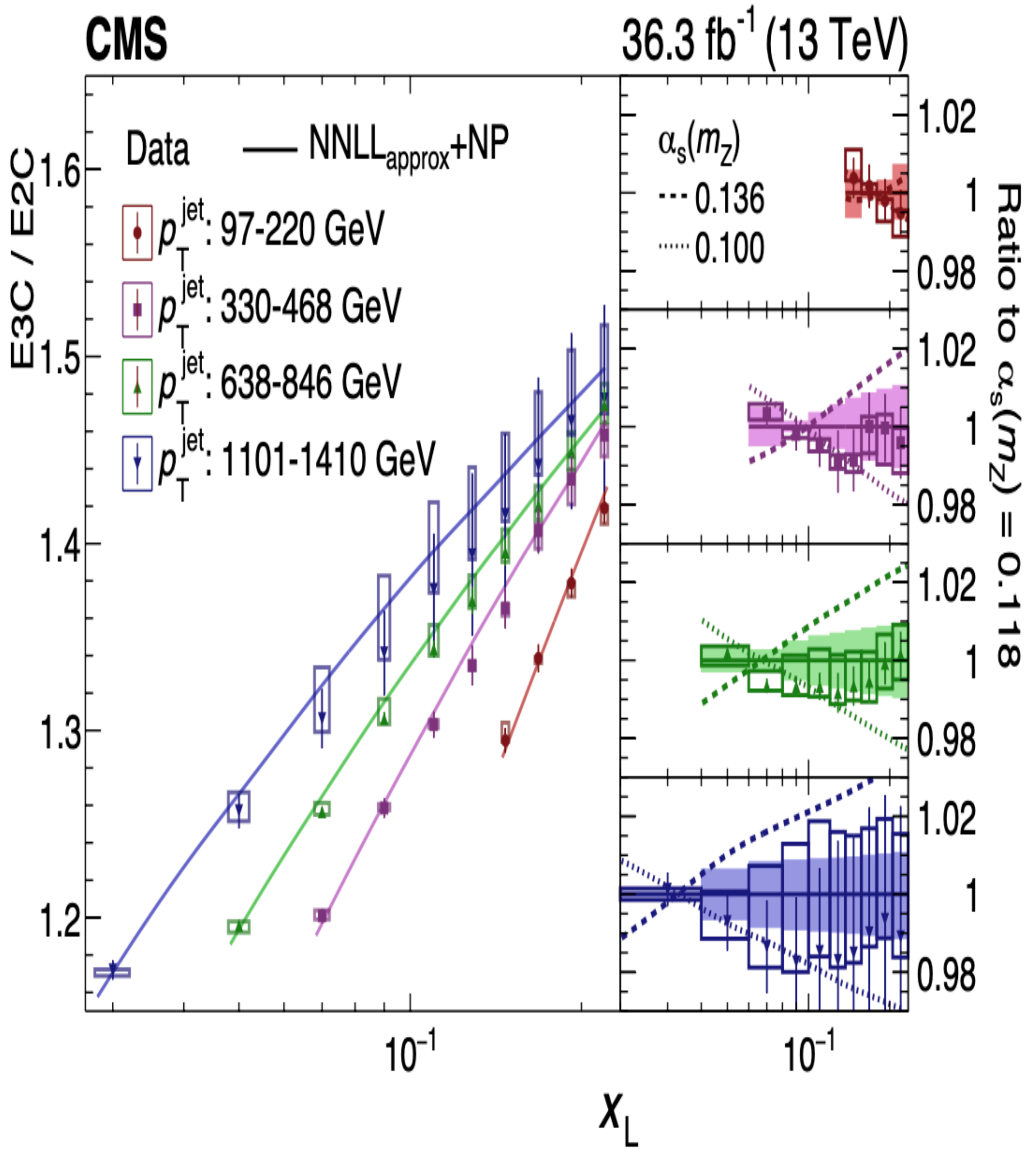
Recent applications

strong coupling measurement

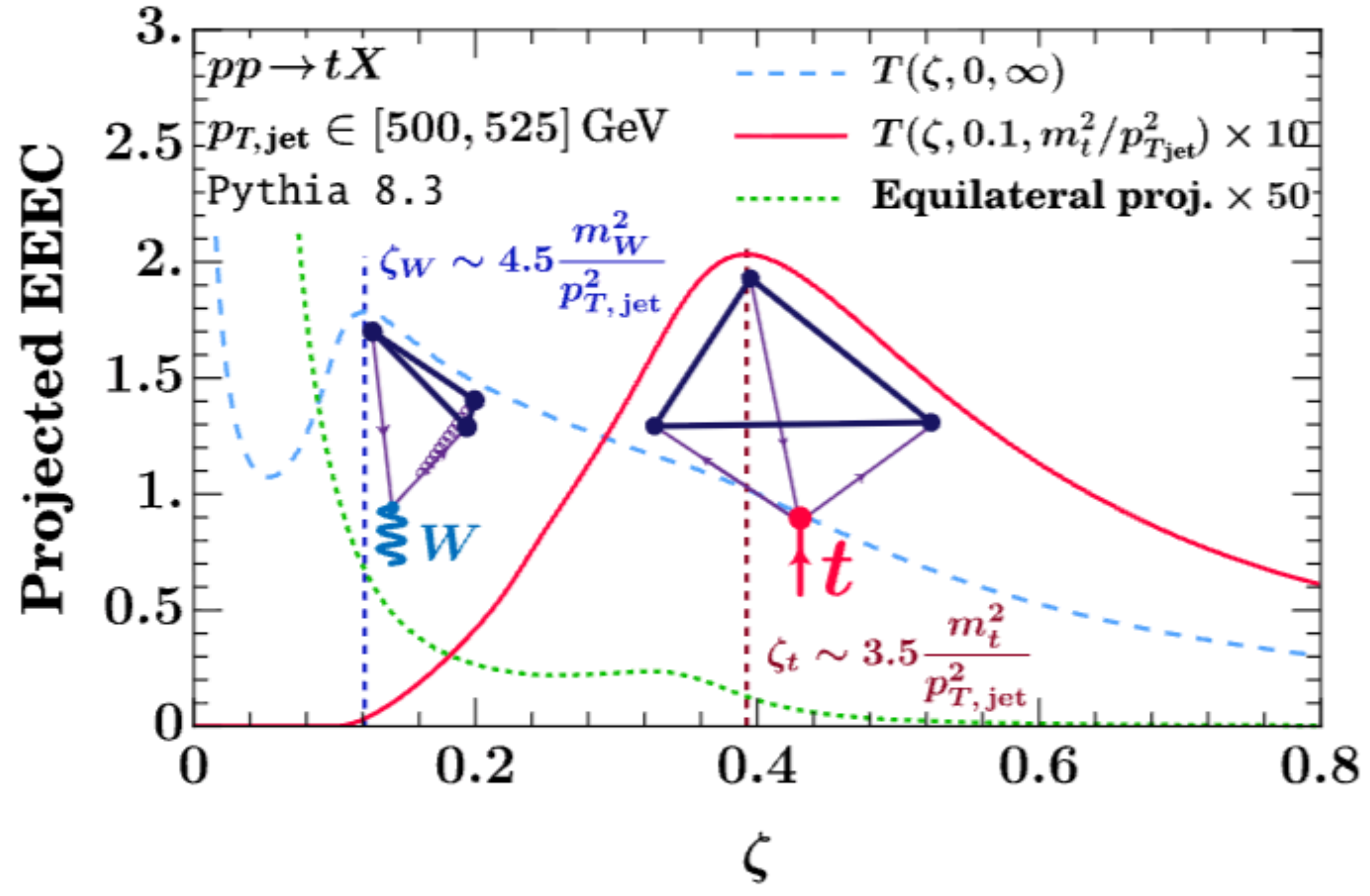


$\sim \theta^{\gamma_3(\alpha_s)-\gamma_4(\alpha_s)}$

Chen, et.al., 2004.11381, 2307.07510

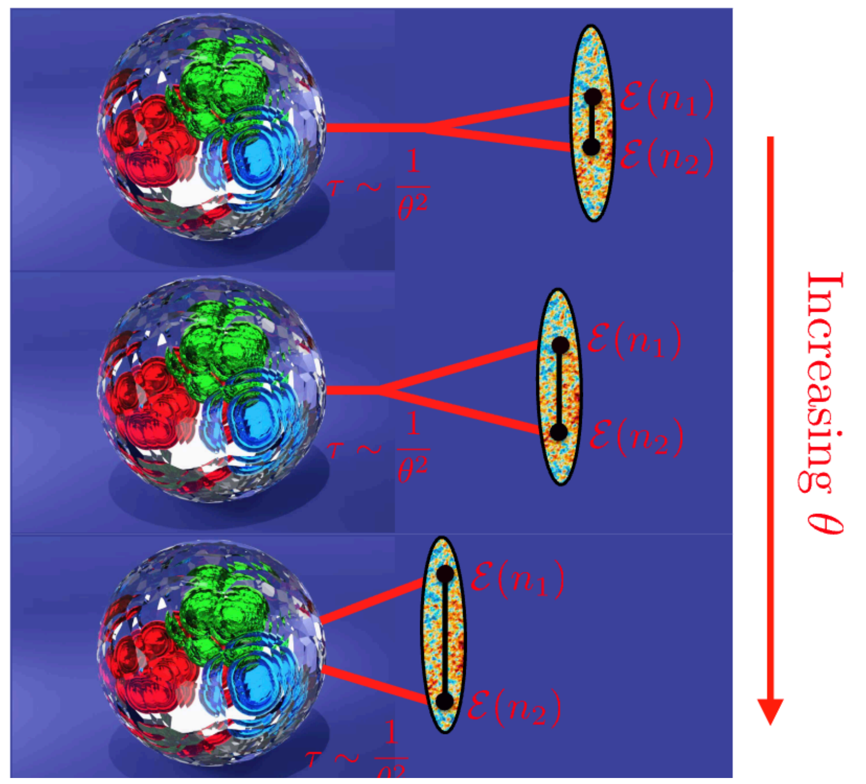


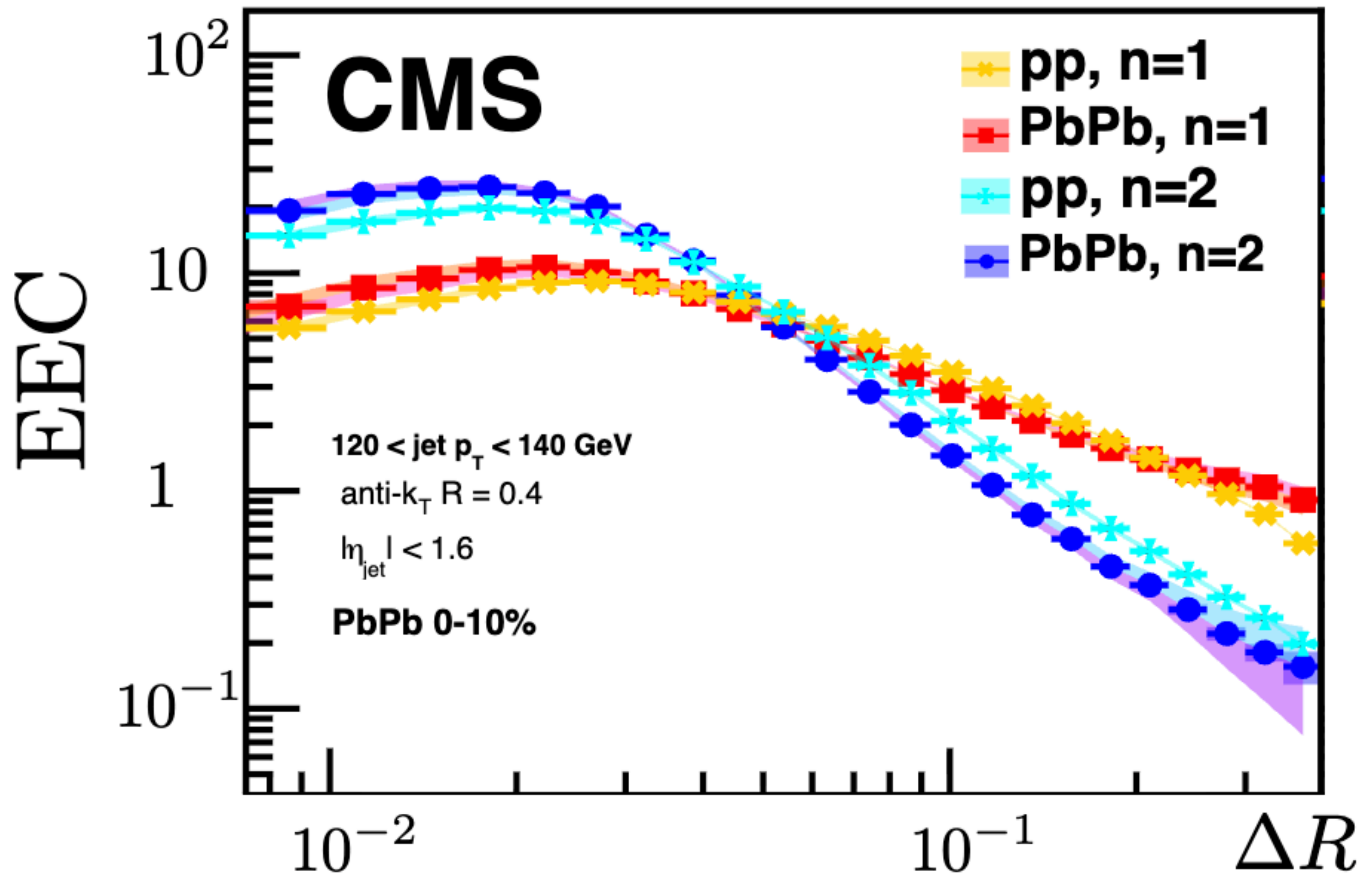
top quark mass determination



Holguin, et.al., 2201.08393

Properties of QGP





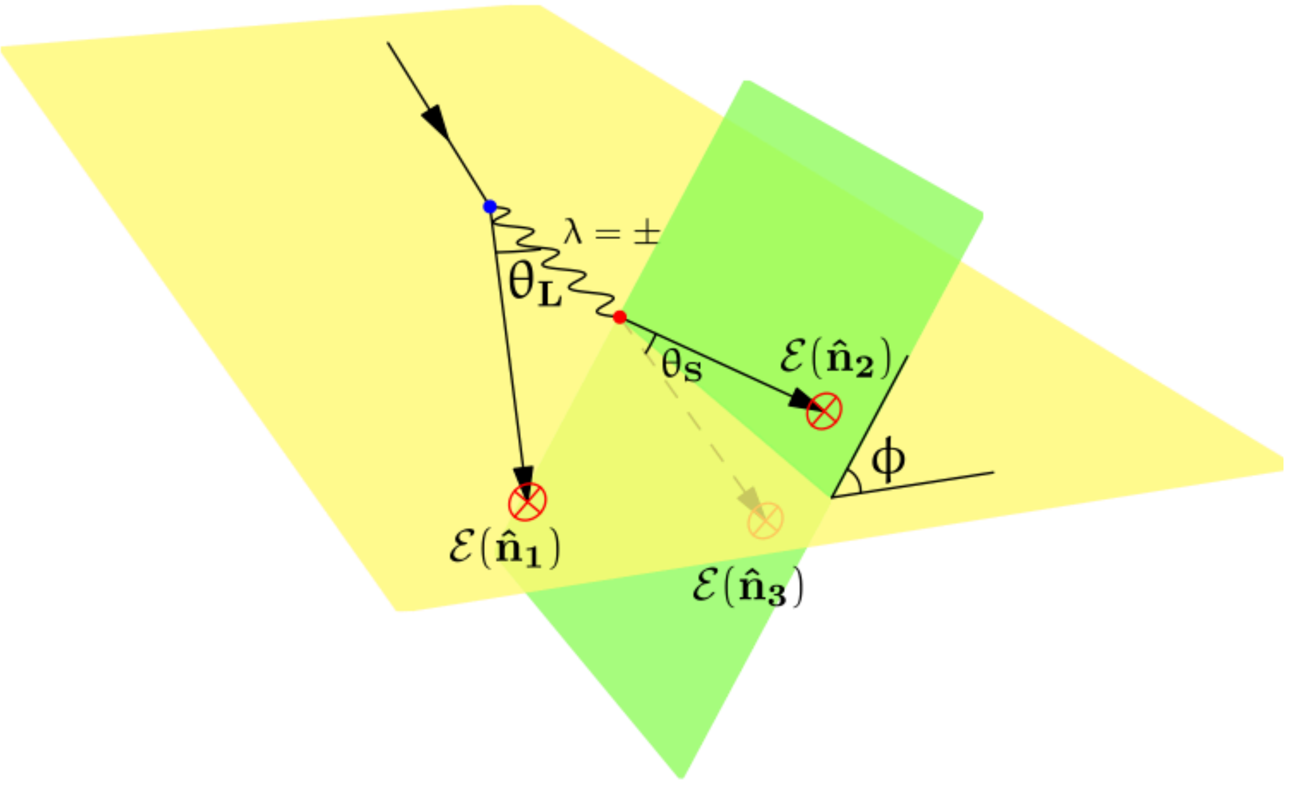
Andres, et.al., 2209.11236

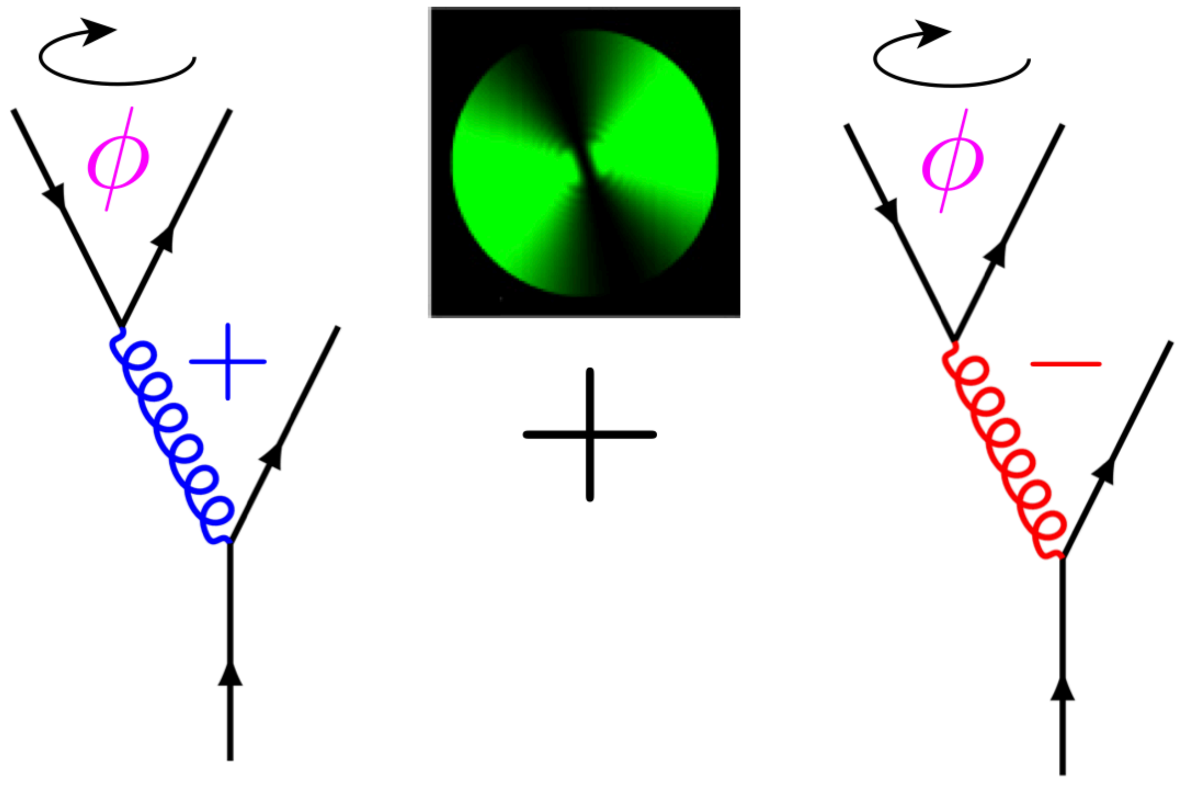
Spin and EECs

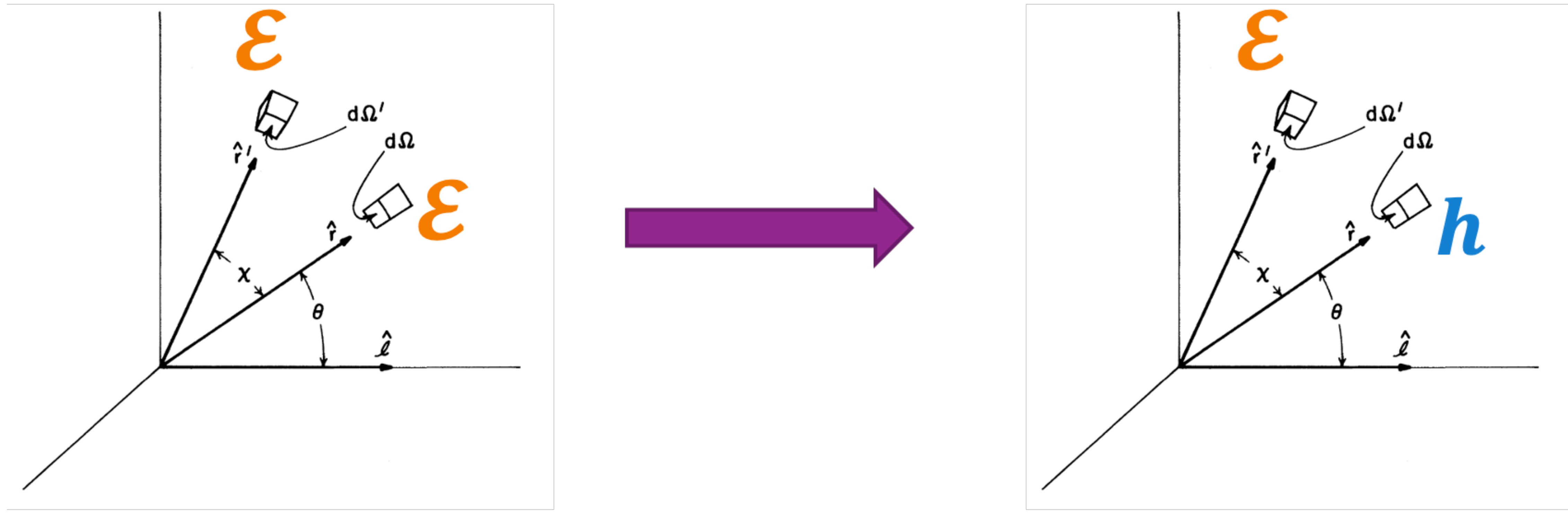
There have been some recent works on using EEC to probe spin physics

To probe $\sin(n\phi)$ and $\cos(n\phi)$ with odd n, one must break the symmetry between the two detectors

Minimal way: introduce a hadron tag





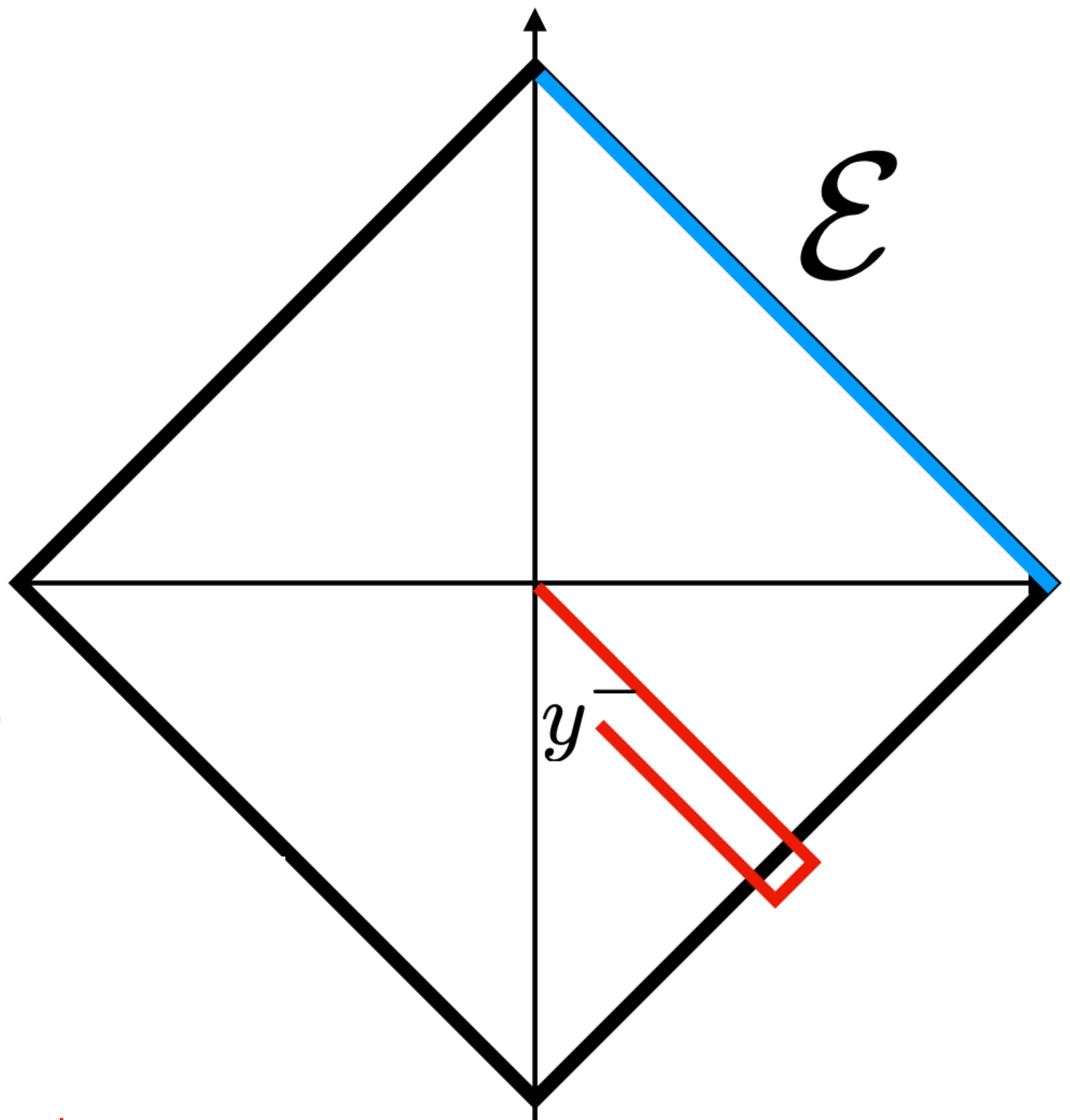


2011.02492
see also: 2308.10942, 2310.15159, 2509.14960, 2509.17596,

Some other modifications: 2308.00746, 2508.19404…

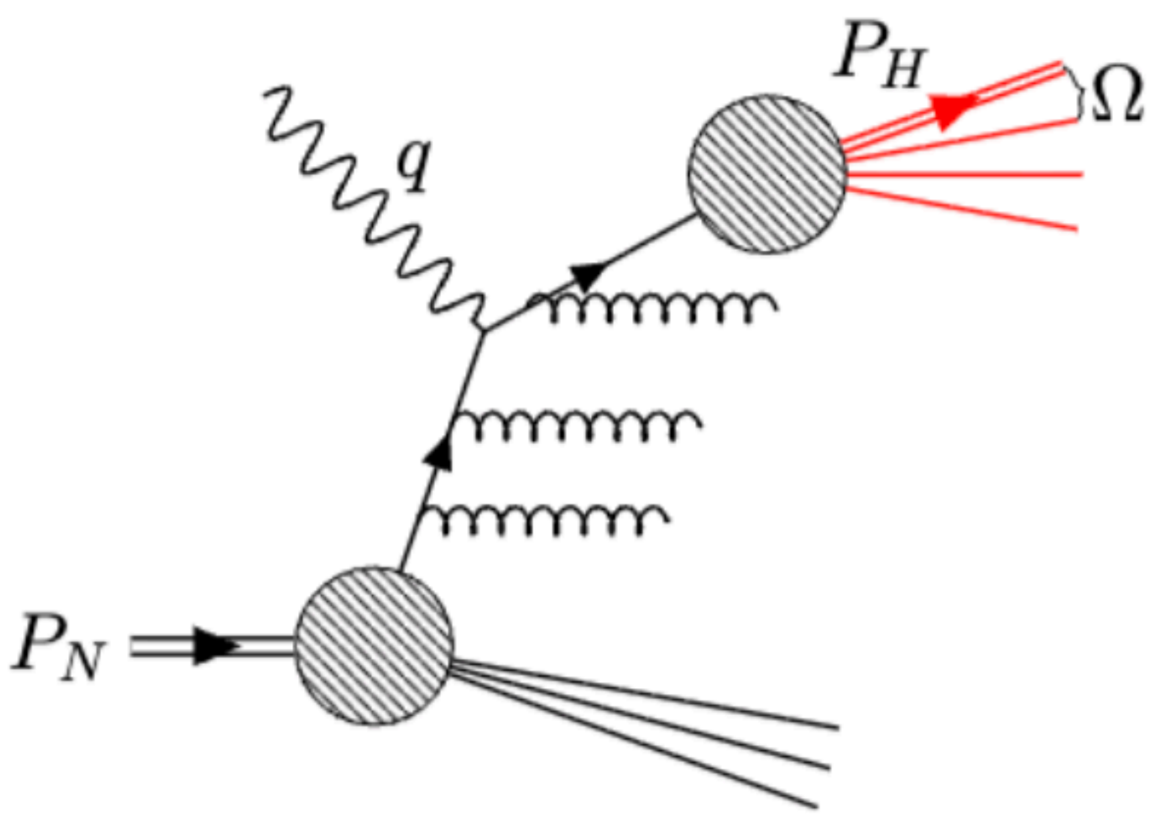
The fragmentation energy correlators

$$\mathcal{D}^{[\Gamma]}_{h/q,1}(z,\boldsymbol{n};p_h) = \frac{z}{2N_c}\sum_X \int \frac{dy^-}{2\pi} e^{ip_h^+ y^-/z}\,\mathrm{Tr}[\,\Gamma\langle 0|W(\infty,y^-;w)\,\psi(y^-)\,\mathcal{E}(\boldsymbol{n})\,|\,h,X;\ \mathrm{out}\,\rangle$$
$$\times\langle h,X;\ \mathrm{out}\,|\bar{\psi}(0)W^\dagger(\infty,0;w)|0\rangle]\,,$$



energy flow operator

$$\mathcal{E}(\boldsymbol{n})\,|X;\mathrm{out}\rangle = \sum_{i\in X} E_i\,\delta^{(2)}(\boldsymbol{n}-\boldsymbol{n}_i)|X;\mathrm{out}\rangle$$



Boost invariant definition:

$$\Lambda_F = m_h\,e^{y_h-\eta}$$

Large Λ_F, perturbative

Collinear limit:

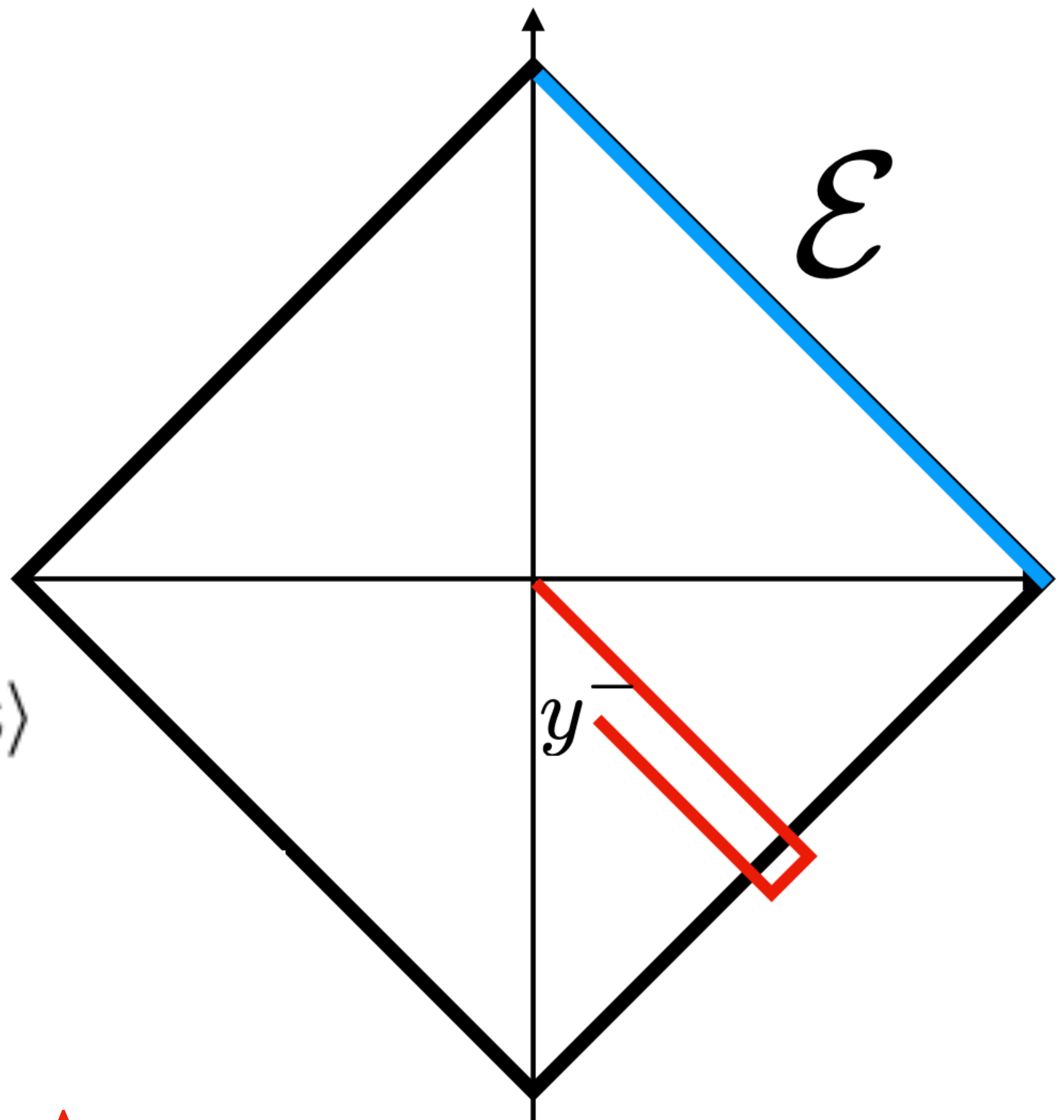
$$\Lambda_F \simeq E_h\theta$$

Small Λ_F, nonperturbative

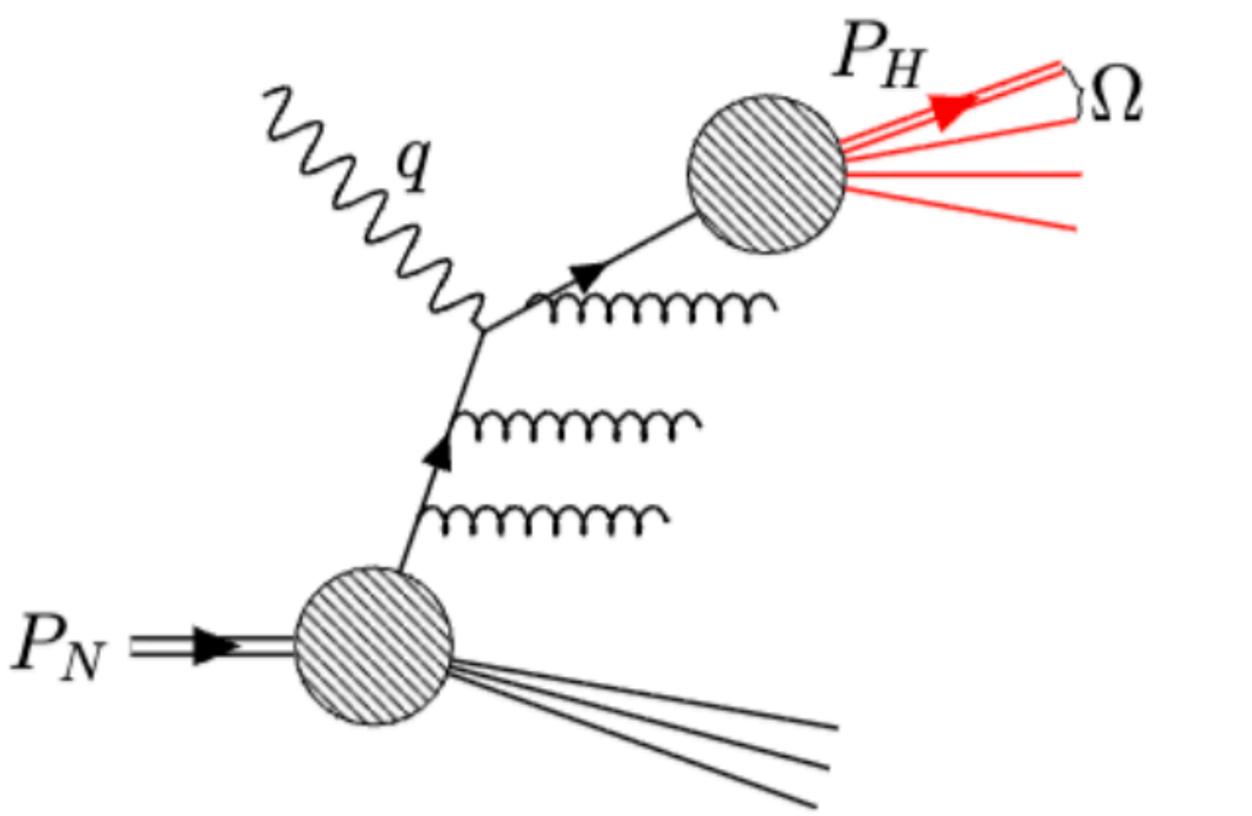
Probe radiation pattern around h

The fragmentation energy correlators

$$\mathcal{D}^{[\Gamma]}_{h/q,1}(z,\Lambda_F,\phi) = \frac{z}{2N_c}\sum_X \int \frac{dy^-}{2\pi p_h^+} e^{ip_h^+ y^-/z}\, \mathrm{Tr}\Big[\Gamma\langle 0|\, W(\infty,y^-;w)\,\psi(y^-)$$
$$\times\, \mathcal{E}_L(\eta,\phi)\,|h,X;\mathrm{out}\rangle\langle h,X;\mathrm{out}|\,\bar\psi(0)\,W^\dagger(\infty,0;w)\,|0\rangle\Big]$$



energy flow operator $\mathcal{E}_L(\eta,\phi)|X;\mathrm{out}\rangle = \sum_{i\in X} p_i^+\,\delta(\eta-\eta_i)\delta(\phi-\phi_i)|X;\mathrm{out}\rangle$



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Small Λ_F, nonperturbative

Probe radiation pattern around h

A physical payoff: scale scan for chiral-odd (chSB sensitive) probe

Collins effect and chiral symmetry breaking

Fragmentation of Transversely Polarized Quarks Probed in Transverse Momentum Distributions (TMD FF)

John Collins

Physics Department, Penn State University,
University Park, PA 16802, U.S.A.

The new fragmentation function is sensitive to the coupling of the fragmentation process to (spontaneous) chiral symmetry breaking.

Collins, hep-ph/9208213

Collins effect is sensitive to chSB

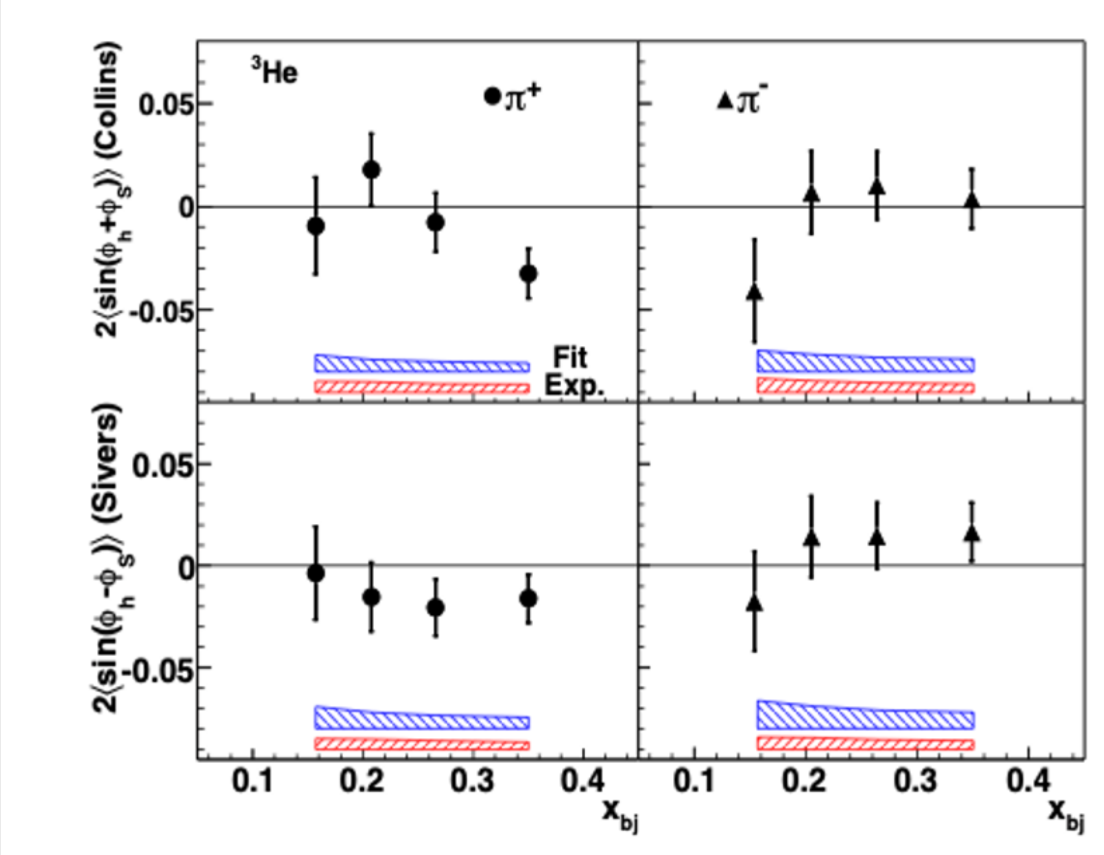
This definition is easily generalized to give the transverse spin dependence $\Delta\hat{D}$ of the distribution of hadrons in a polarized quark of transverse spin $s_\perp^\mu$:

$$\Delta\hat{D}_{H/a}(z, k_\perp, s_\perp) \equiv$$

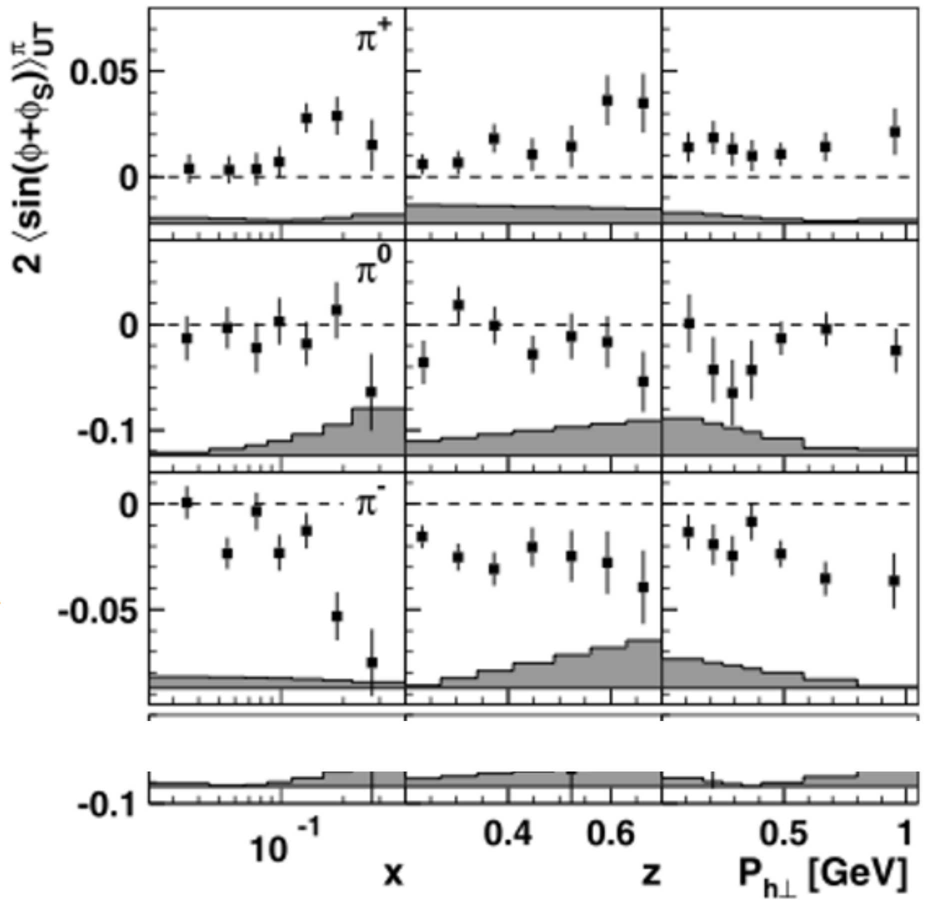
$$\sum_X \int \frac{dy^- d^2y_\perp}{12(2\pi)^3} e^{ik^+y^- - ik_\perp\cdot y_\perp} \,\mathrm{tr}\, \gamma^+\gamma_5\,\gamma_\perp\cdot s_\perp \langle 0|\psi_a(0,y^-,y_\perp)|HX\rangle\langle HX|\bar{\psi}_a(0)|0\rangle.$$

Projector for transverse spin is chiral odd

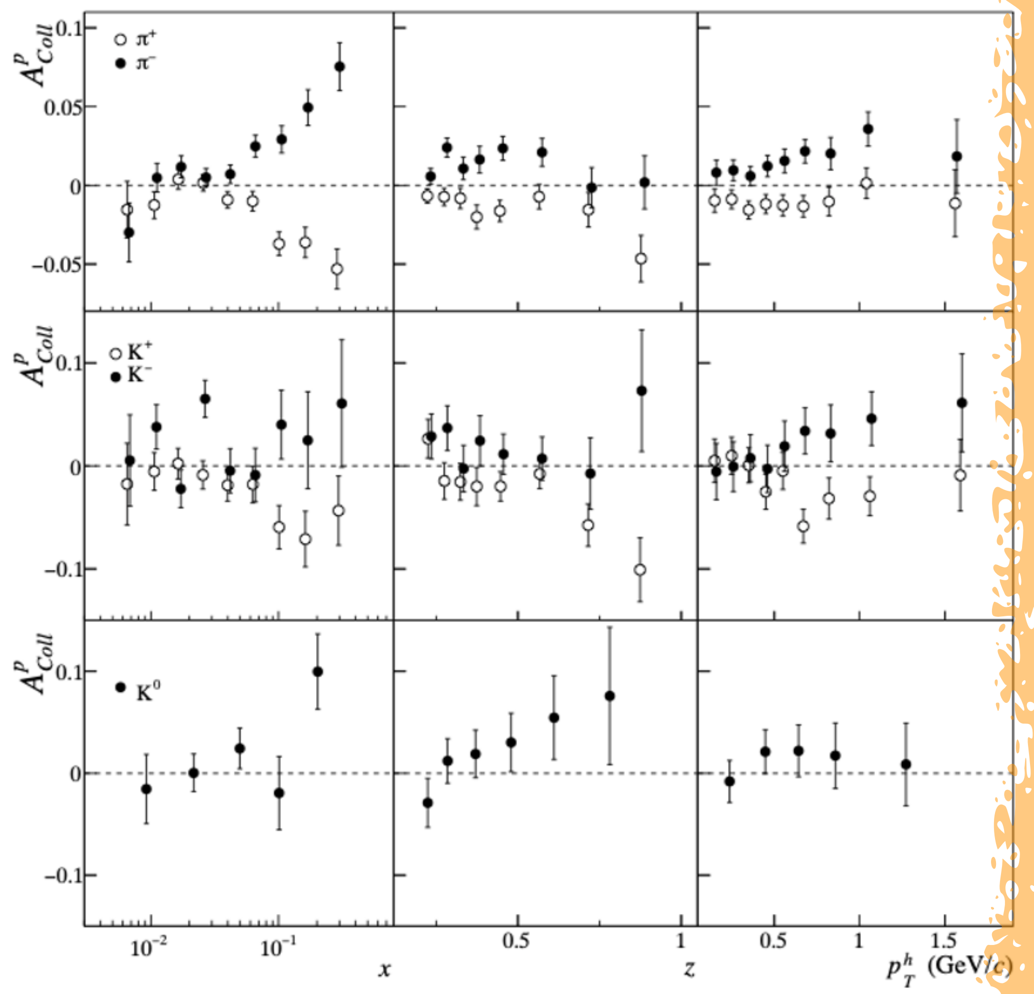
Experiment results



JLab, PRL.107.072003



HERMES,PLB.2010.08.012

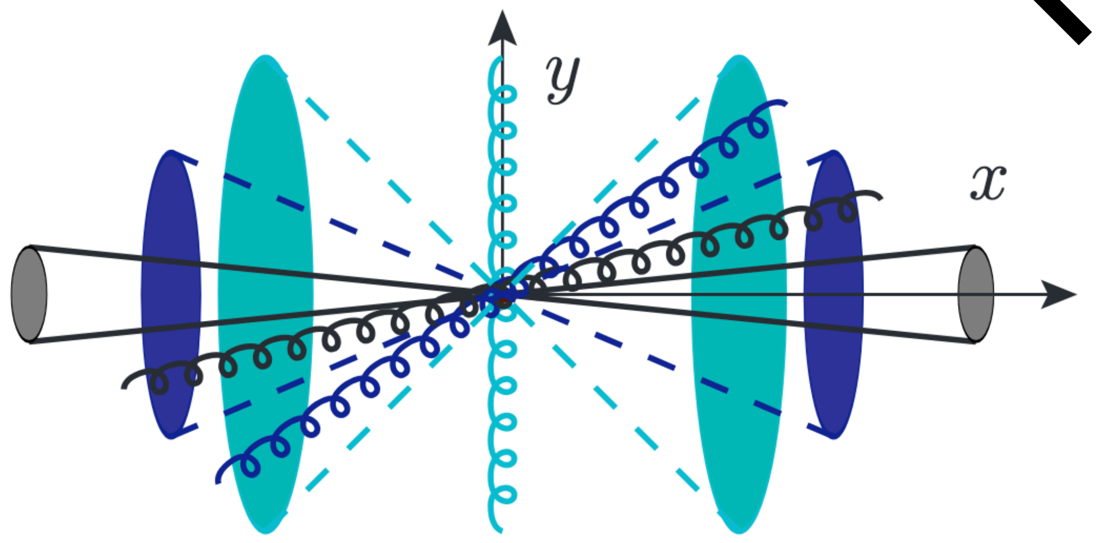


COMPASS, PLB.2015.03.056

What's different?

TMD FF

- TMD factorization
- Sudakov suppression
- Soft gluon contaminate the azimuth distribution



$$S_g \sim \frac{1}{k_J \cdot k_g} \propto \frac{1}{\cosh(y_g - y_J) - \cos(\phi_g - \phi_J)}$$
$$= c_0 + c_1 \cos(\phi_g - \phi_J) + c_2 \cos(2(\phi_g - \phi_J)) + \ldots$$

Hatta et al.
PRL.126.142001
PRD.104.054037

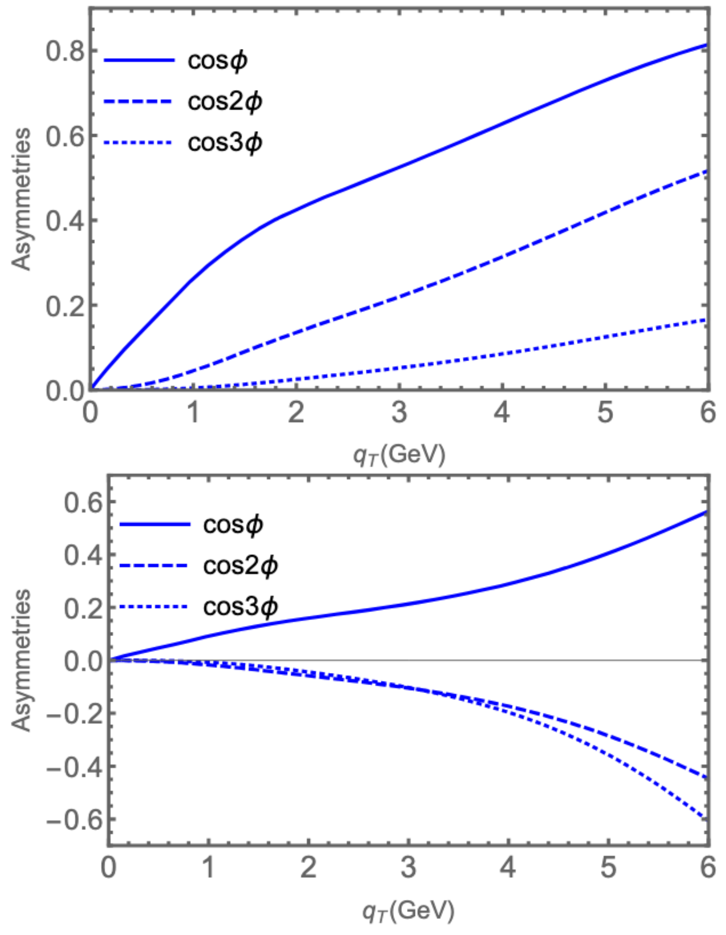


FIG. 4. Azimuthal asymmetries in lepton-jet production in ep collisions at $\sqrt{s}$=140 GeV, $P_\perp = 20$ GeV, $y_l = 1.5$, $Q = 25$ GeV, g_Λ =0.1GeV with different jet cone sizes $R = 0.4$ (top panel) and $R = 1.0$ (bottom panel).

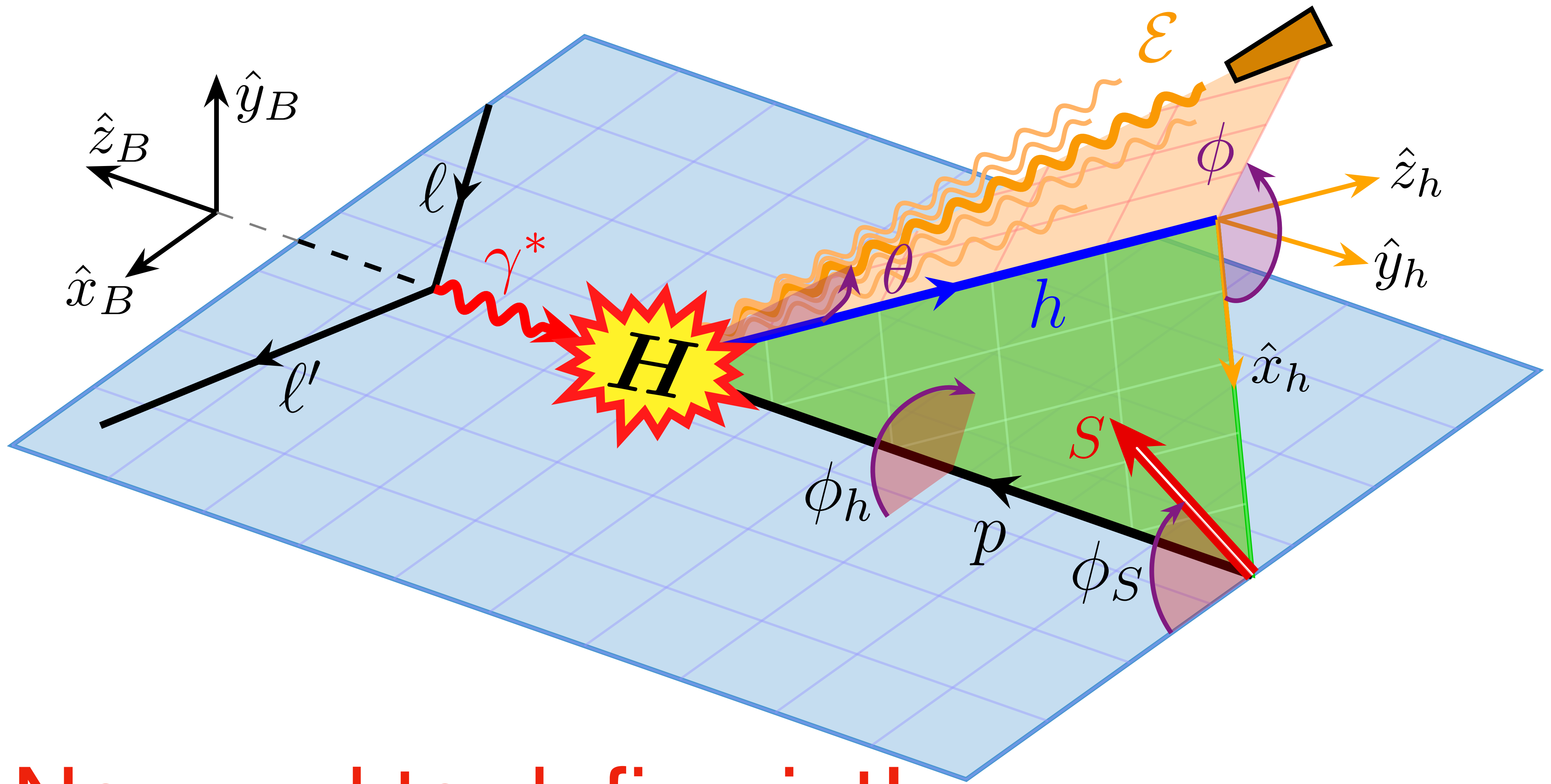
FEC

- Collinear factorization
- No Sudakov double log
- Energy weight: soft insensitive

1905.01310

2004.11381

FECs in SIDIS



No need to define jet!

$$\overline{|\mathcal{M}_\mathcal{E}|^2} \simeq \sum_{a,b} \int \frac{d\xi_1}{\xi_1} \int \frac{d\xi_2}{\xi_2^2} \Big\{ \mathcal{D}_{1,h/b}(\xi_2, \Lambda_F, \mu) \Big[f_{a/p}(\xi_1,\mu)\, C_{ab}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}; \frac{\boldsymbol{p}_{hT}}{Q}, \frac{Q^2}{\mu^2}\right) + P_N g_{a/p}(\xi_1,\mu)\, \Delta C_{ab}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}; \frac{\boldsymbol{p}_{hT}}{Q}, \frac{Q^2}{\mu^2}\right)\Big] + \mathcal{H}^{\perp}_{1,h/b}(\xi_2, \Lambda_F, \mu)\, h_{a/p}(\xi_1,\mu) \sum_{i,j=1}^{2} \frac{(\hat{p}_h \times \boldsymbol{n}_T)^i}{|\boldsymbol{n}_T|} T^{ij}_{ab}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}; \frac{\boldsymbol{p}_{hT}}{Q}, \frac{Q^2}{\mu^2}\right) s_T^j \Big\} + \mathcal{O}(\Lambda_F/Q, \Lambda_F/p_{hT}).$$

$$\mathcal{D}^{[\Gamma]}_{h/q,1}(z,\Lambda_F,\phi) = \frac{z}{2N_c} \sum_X \int \frac{dy^-}{2\pi p_h^+} e^{ip_h^+ y^-/z}\, \mathrm{Tr}\Big[\Gamma \langle 0|\, W(\infty, y^-; w)\, \psi(y^-) \times \mathcal{E}_L(\eta,\phi)\, |h, X; \mathrm{out}\rangle\langle h, X; \mathrm{out}|\, \bar{\psi}(0)\, W^\dagger(\infty, 0; w)\, |0\rangle\Big]$$

$$\mathcal{D}^{[\gamma^+\gamma^i\gamma_5/2]}_{h/q,1}(z,\Lambda_F,\phi) = (-\sin\phi, \cos\phi)^i\, \mathcal{H}^{\perp}_{1,h/q}(z,\Lambda_F,\phi)$$

s_T: initial transverse spin

T^{ij}: spin transfer matrix

h: transversity

$\mathscr{H}^{\perp}$: Collins-Type FEC

Pure collinear factorization formula
No Sudakov logs!

Decomposition of polarization functions

The master formula:

$$\left[\frac{y^2}{z(1-\varepsilon)}\,\frac{\alpha_e^2}{(8\pi^2Q^2)^2}\right]^{-1}\frac{d\Sigma}{dx\,dQ^2\,d\phi_S\,dz\,dp_{hT}^2\,d\phi_h\,d\eta\,d\phi}$$

$$= \mathcal{C}_{F_{UU,T}}[f,\mathcal{D}] + \varepsilon\,\mathcal{C}_{F_{UU,L}}[f,\mathcal{D}]$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\,\mathcal{C}_{F_{UU}^{(1)}}[f,\mathcal{D}]\cos\phi_h + \varepsilon\,\mathcal{C}_{F_{UU}^{(2)}}[f,\mathcal{D}]\cos(2\phi_h)$$

→ pQCD distortion to TMD

$$+\,s_T\left\{\left[-\mathcal{C}_{F_{UT,T}}\left[h,\mathcal{H}^{\perp}\right] - \varepsilon\,\mathcal{C}_{F_{UT,L}}\left[h,\mathcal{H}^{\perp}\right]\right]\sin\left(\phi-(\phi_S-\phi_h)\right)\right.$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\left[\mathcal{C}_{F_{UT}^{(1+)}}\left[h,\mathcal{H}^{\perp}\right]\sin\left(\phi-(\phi_S-2\phi_h)\right) + \mathcal{C}_{F_{UT}^{(1-)}}\left[h,\mathcal{H}^{\perp}\right]\sin\left(\phi-\phi_S\right)\right]$$

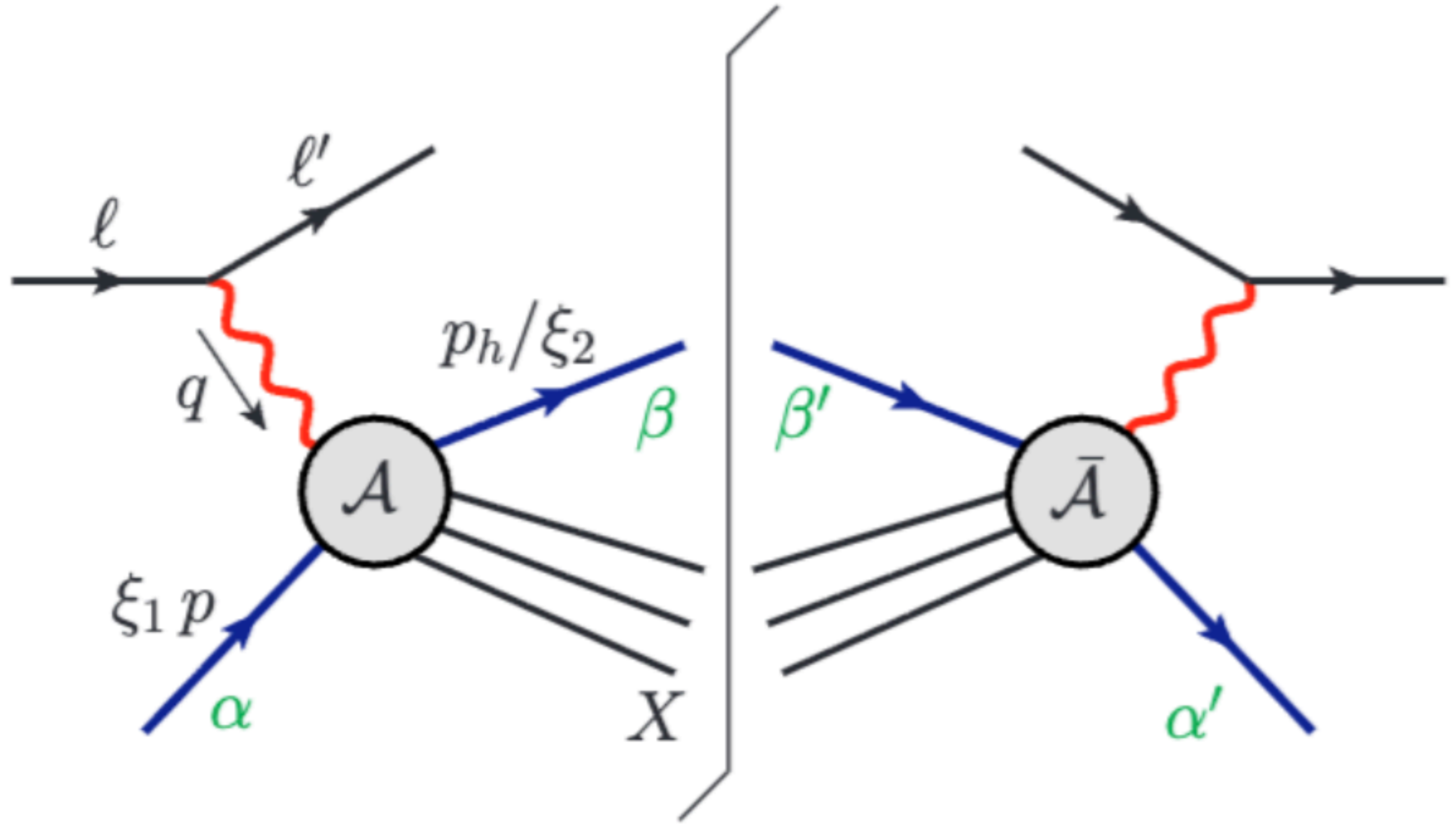
$$\left.+\varepsilon\left[\mathcal{C}_{F_{UT}^{(2+)}}\left[h,\mathcal{H}^{\perp}\right]\sin\left(\phi-(\phi_S-3\phi_h)\right) + \mathcal{C}_{F_{UT}^{(2-)}}\left[h,\mathcal{H}^{\perp}\right]\sin\left(\phi-(\phi_S+\phi_h)\right)\right]\right\}$$

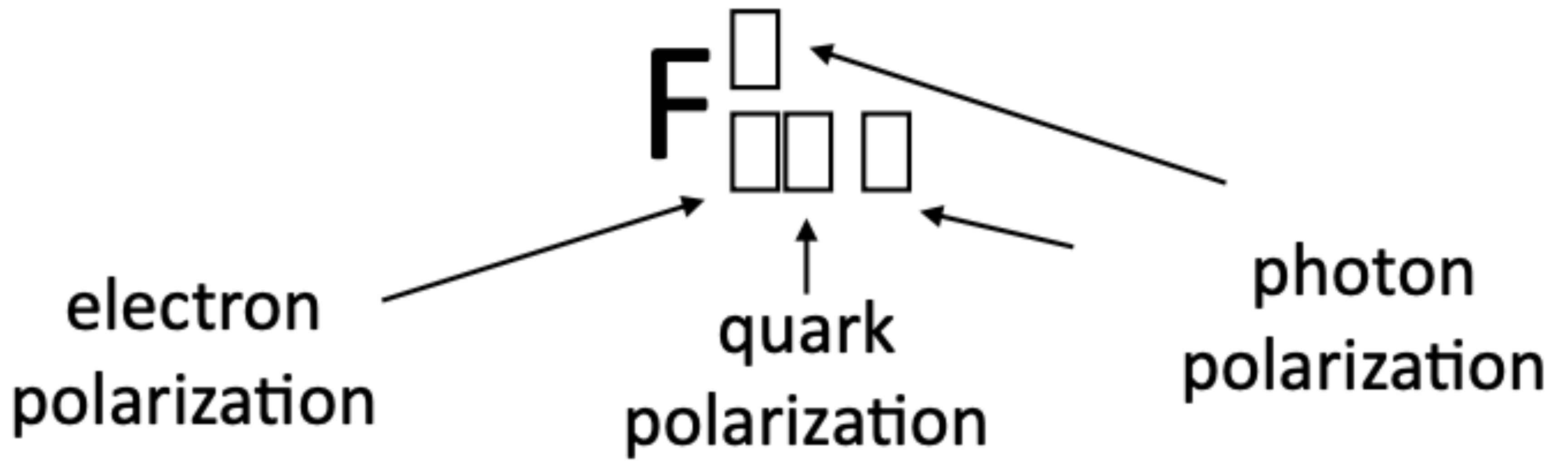
+ longitudinal polarization, power corrections, etc

$$\mathcal{C}_F[f,\mathcal{D}] = \mathcal{C}_F[f,\mathcal{D}]\left(Q^2,x,z,p_{hT}^2;\Lambda_F\right)$$

hard factor

$$\equiv \int\frac{d\xi_1}{\xi_1}\int\frac{d\xi_2}{\xi_2^2}\,f_{q/p}(\xi_1)\,\mathcal{D}_{1,h/q}(\xi_2,\Lambda_F)\,F\left(Q^2,x/\xi_1,z/\xi_2,p_{hT}^2/\xi_2^2\right)$$





Modulation effects generated by non-perturbative coupling of transversity and Collins

p_{hT}-Integrated Asymmetry in SIDIS

We show P_{hT} integrated asymmetry below to improve statistics

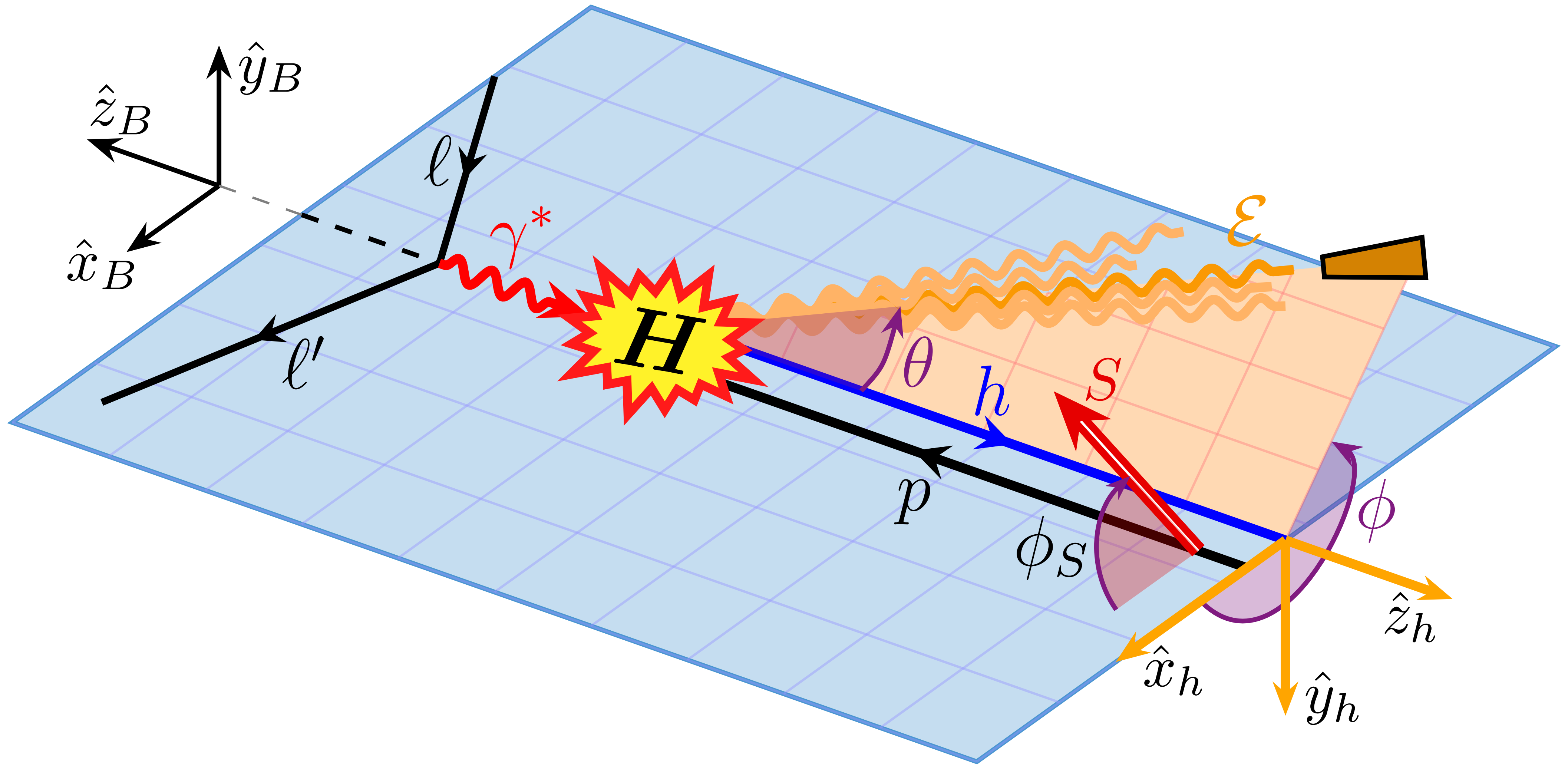
p_{hT}-inclusive cross section:

LO implementation

$$\frac{d\Sigma}{dx\, dQ^2\, d\phi_S\, dz\, d\Lambda_F^2\, d\phi}$$

$$= \sum_q \frac{y^2 \alpha_e^2 e_q^2}{(1-\varepsilon)Q^4} \Big(f_{q/p}(x) \mathscr{D}_{1,h/q}(z, \Lambda_F^2)$$

$$- \varepsilon h_{q/p}(x) \mathscr{H}^{\perp}_{1,h/q}(z, \Lambda_F^2) \sin(\phi - \phi_S) \Big)$$



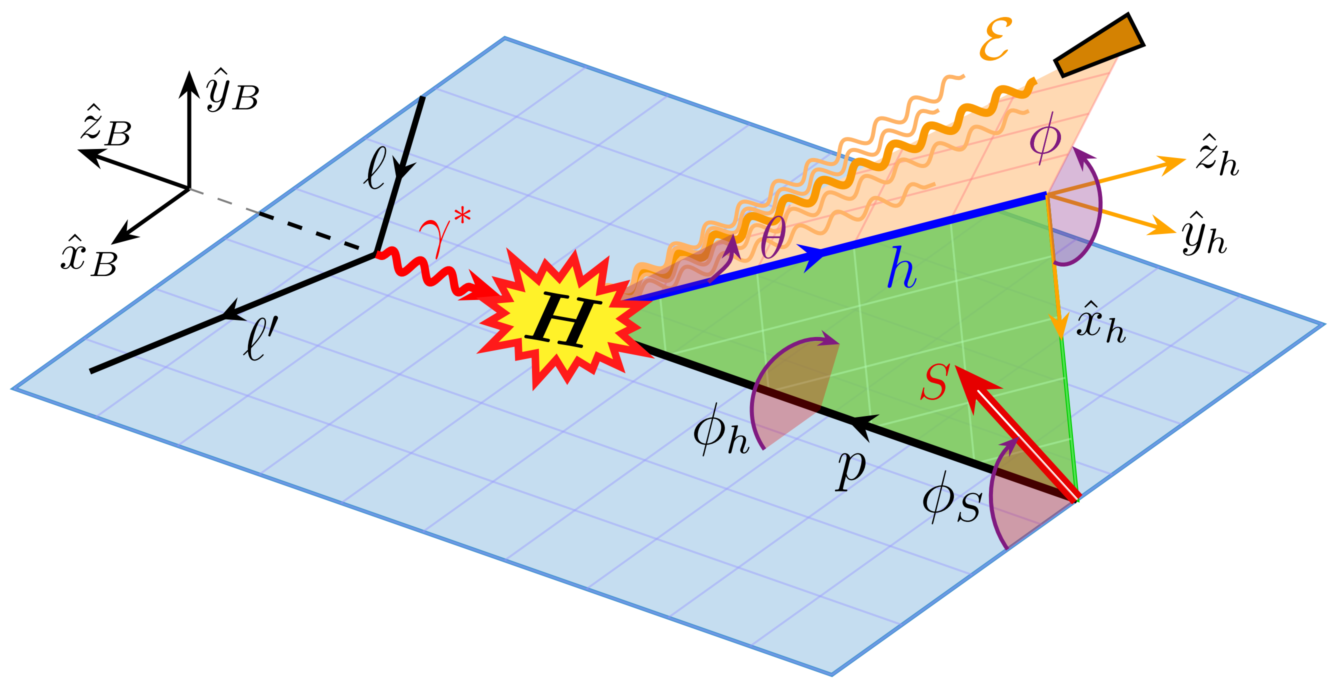
end-point regulation!

$$\varepsilon = \frac{1-y}{1-y+\frac{y^2}{2}}$$

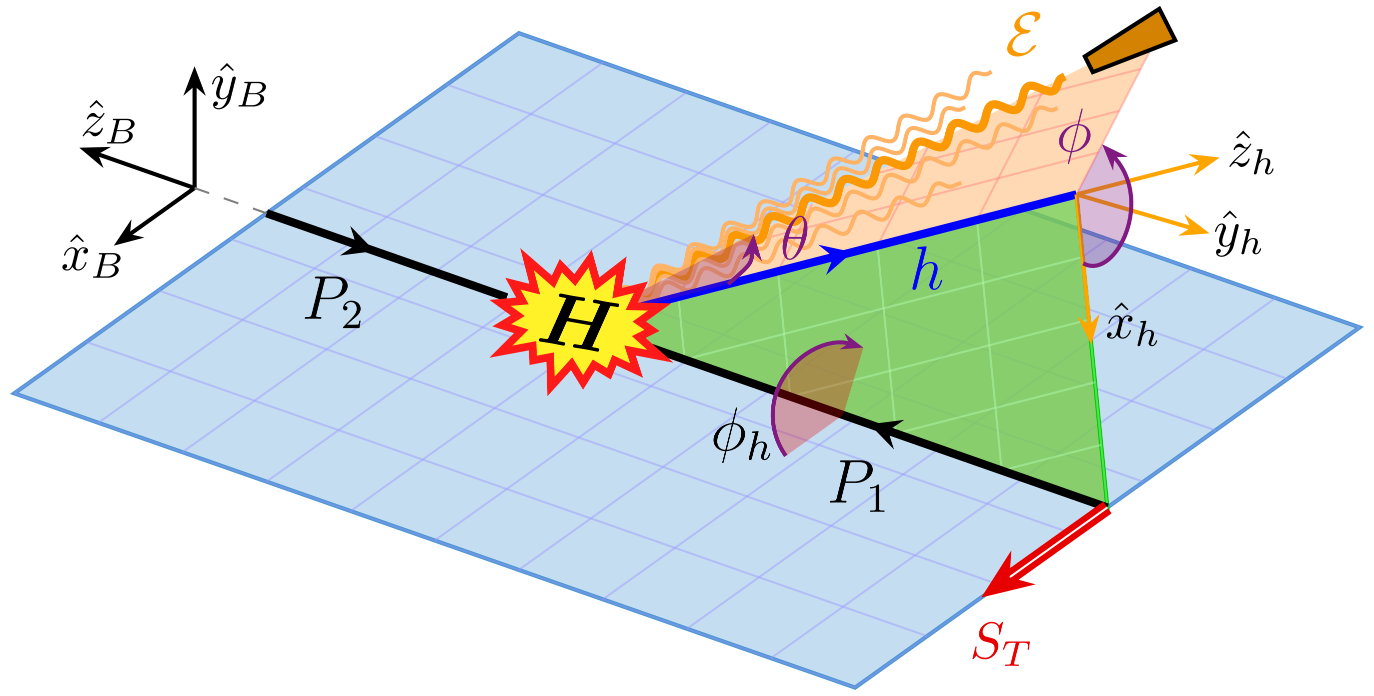
Virtual photon depolarization parameter

Factorization in different processes

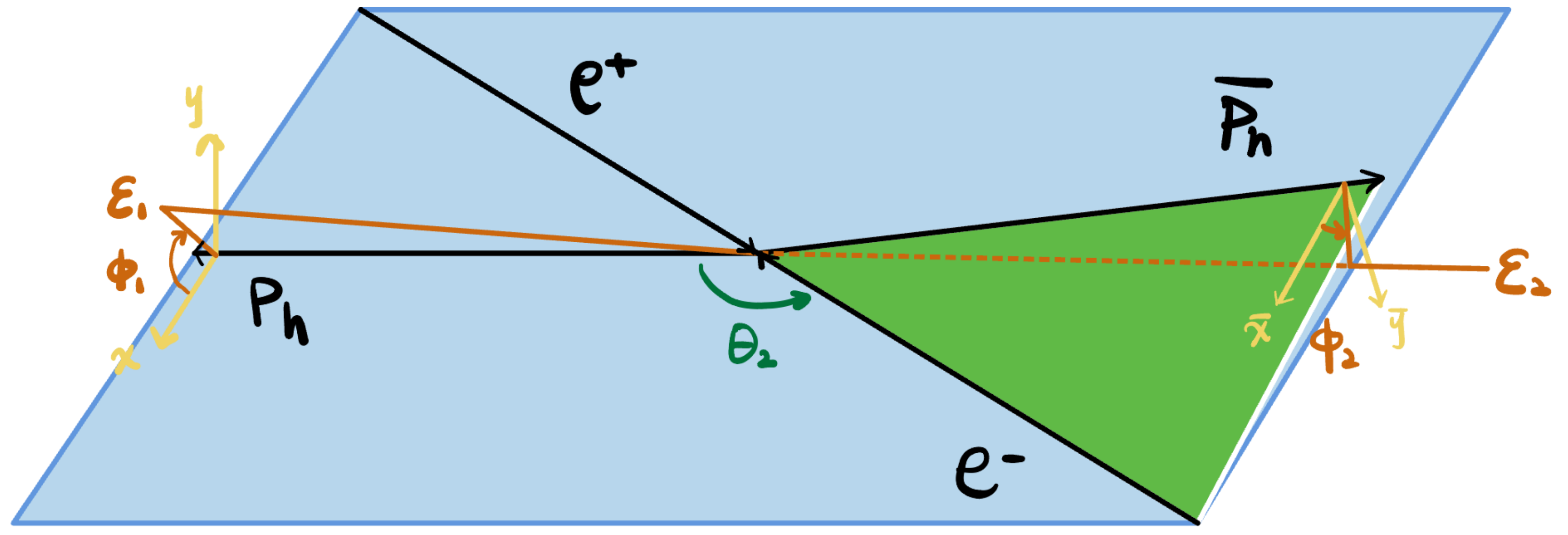
SIDIS



$p^{\uparrow}p$



e^+e^-



PDFs Hard Coeff. FEC

$$|M_{\mathscr{E}}|^2 \simeq \sum_{a,b} \int \frac{d\xi_1}{\xi_1} \int \frac{d\xi_2}{\xi_2^2} \Bigg\{ \mathscr{D}_{1,h/b}(\xi_2, \Lambda_F^2) \left[f_{a/p}(\xi_1) C_{ab}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}; \frac{\boldsymbol{p}_{hT}}{Q}\right) + P_N g_{a/p}(\xi_1, \mu) \Delta C_{ab}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}; \frac{\boldsymbol{p}_{hT}}{Q}\right)\right] + \mathscr{H}^{\perp}_{1,h/b}(\xi_2, \Lambda_F^2) h_{a/p}(\xi_1) \sum_{i,j=1}^{2} \left(\frac{(\hat{\boldsymbol{p}}_h \times \boldsymbol{n}_T)^i}{|\boldsymbol{n}_T|} T^{ij}_{ab}\left(\frac{x}{\xi_1}, \frac{z}{\xi_2}; \frac{\boldsymbol{p}_{hT}}{Q}\right) s_T^j \right) \Bigg\}$$

$$\left|\overline{M_{\mathscr{E}}}\right|^2 \simeq \frac{2\pi}{s} \sum_{a,b,c} \int \frac{dx_b}{x_b^2} f_{b/p}(x_b, \mu) \Bigg\{ \int \frac{dx_a}{x_a^2} f_{a/p}(x_a, \mu) \frac{1}{z} \mathscr{D}_{h/c}(z, \Lambda_F^2, \mu) C_{ab\to c}(\hat{s}, \hat{t}) + \int \frac{dx_a}{x_a^2} h_{a/p}(x_a, \mu) \frac{1}{z} \mathscr{H}^{\perp}_{h/c}(z, \Lambda_F^2, \mu) \sum_{i,j=1}^{2} \frac{(\hat{p}_h \times \boldsymbol{n}_T)^i}{|\boldsymbol{n}_T|} T^{ij}_{ab\to c}(\hat{s}, \hat{t}, \phi_h) S_T^j \Bigg\},$$

Two FEC are needed: double helicity flip

$$\left|\overline{M_{\mathscr{E}}}\right|^2 \simeq \sum_{a,b} \int \frac{d\xi_1}{\xi_1^2} \int \frac{d\xi_2}{\xi_2^2} \Bigg\{ \mathscr{D}_{h_1/a}(\xi_1, \Lambda_{F_1}) \mathscr{D}_{h_2/b}(\xi_2, \Lambda_{F_2}) C_{ab}\left(\frac{z}{\xi_1}, \frac{\bar{z}}{\xi_2}, y, Q\right) + \mathscr{H}_{h_1/a}(\xi_1, \Lambda_{F_1}) \mathscr{H}_{h_2/b}(\xi_2, \Lambda_{F_2}) g^i \bar{g}^j T^{ij}_{ab}\left(\frac{z}{\xi_1}, \frac{\bar{z}}{\xi_2}, y, Q\right) \Bigg\},$$

where $g^i = \frac{(\hat{\boldsymbol{P}}_h \times \boldsymbol{n}_{T1})^i}{|\boldsymbol{n}_{T1}|}$ and $\bar{g}^i = \frac{(\hat{\bar{\boldsymbol{P}}}_h \times \boldsymbol{n}_{T2})^i}{|\boldsymbol{n}_{T2}|}$, $\quad y = \frac{1 + \cos\theta_2}{2}$

To expose the scaling behavior, we redefine for the following:

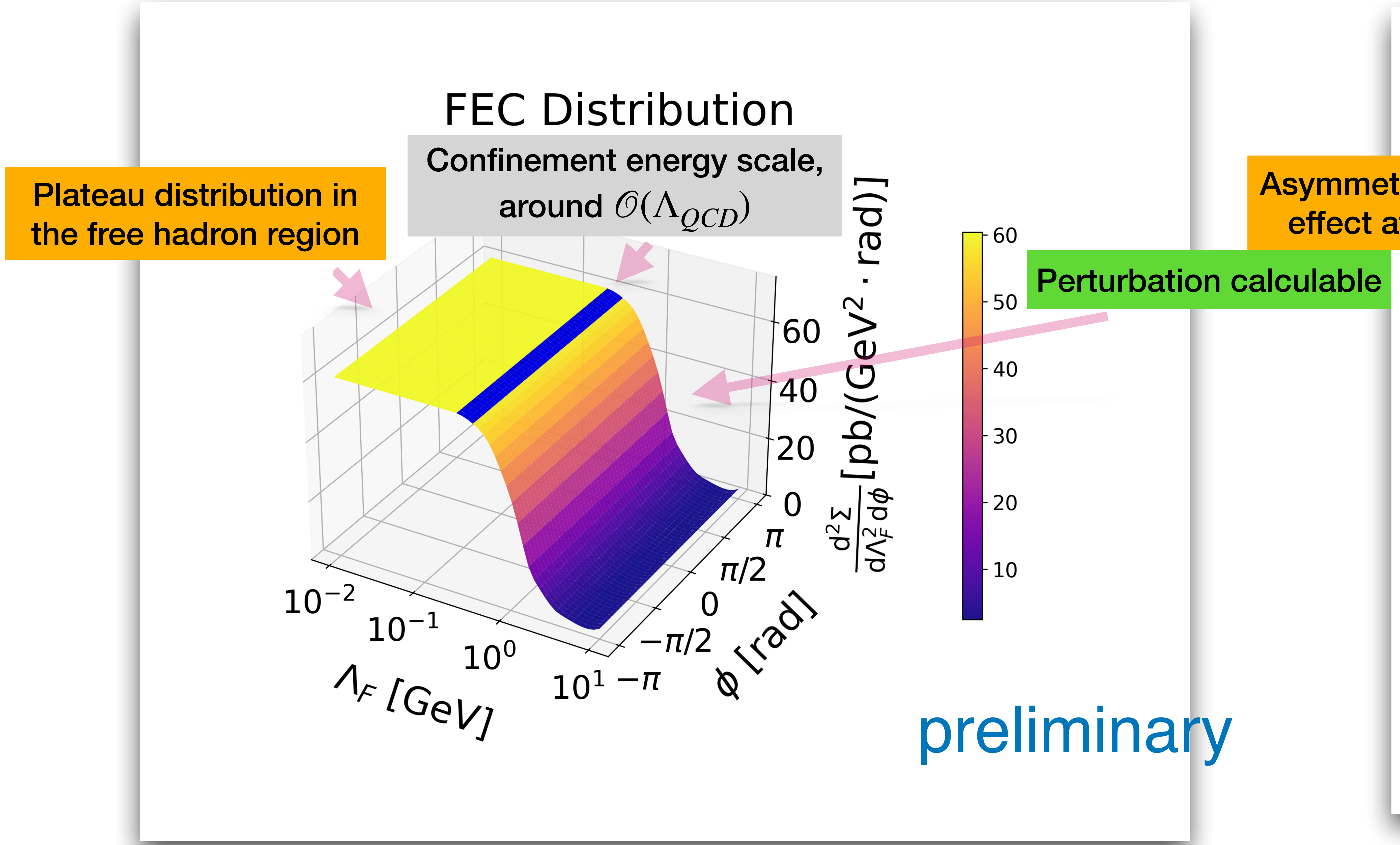
$$\mathscr{D}(\mathscr{H}^{\perp})(z, \Lambda_F) \to \mathscr{D}(\mathscr{H}^{\perp})(z, \Lambda_F^2) = \frac{\mathscr{D}(\mathscr{H}^{\perp})(z, \Lambda_F)}{2\Lambda_F^2}, \quad \frac{d\Sigma}{d\eta} \to \frac{d\Sigma}{d\Lambda_F^2}$$

s_T: initial transverse spin

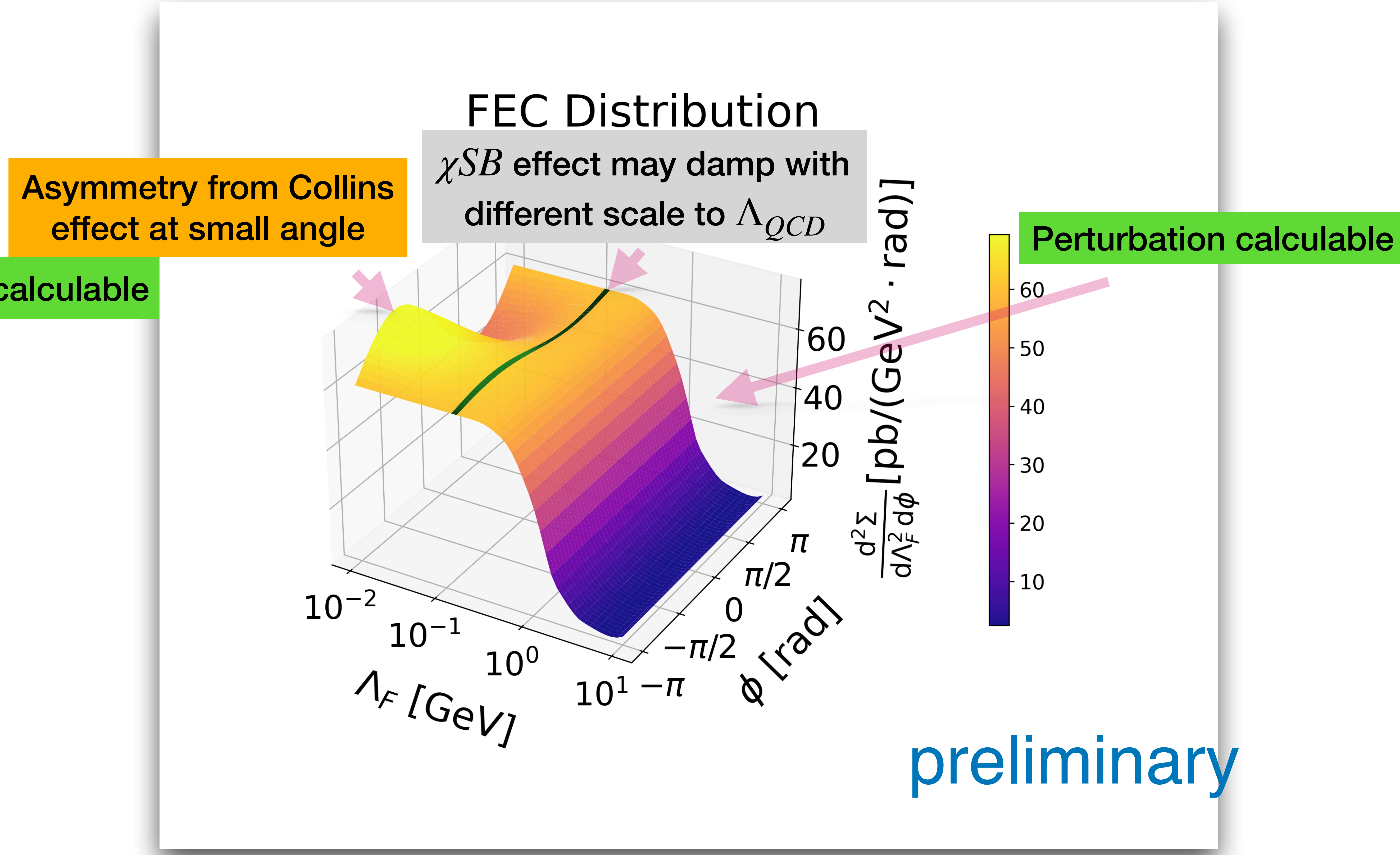
T^{ij}: spin transfer matrix

Physical expectation in Λ_F scan

Unpolarized only



With Collins effects



Interesting to compare the confinement scale and χSB scale

Phenomenology predictions

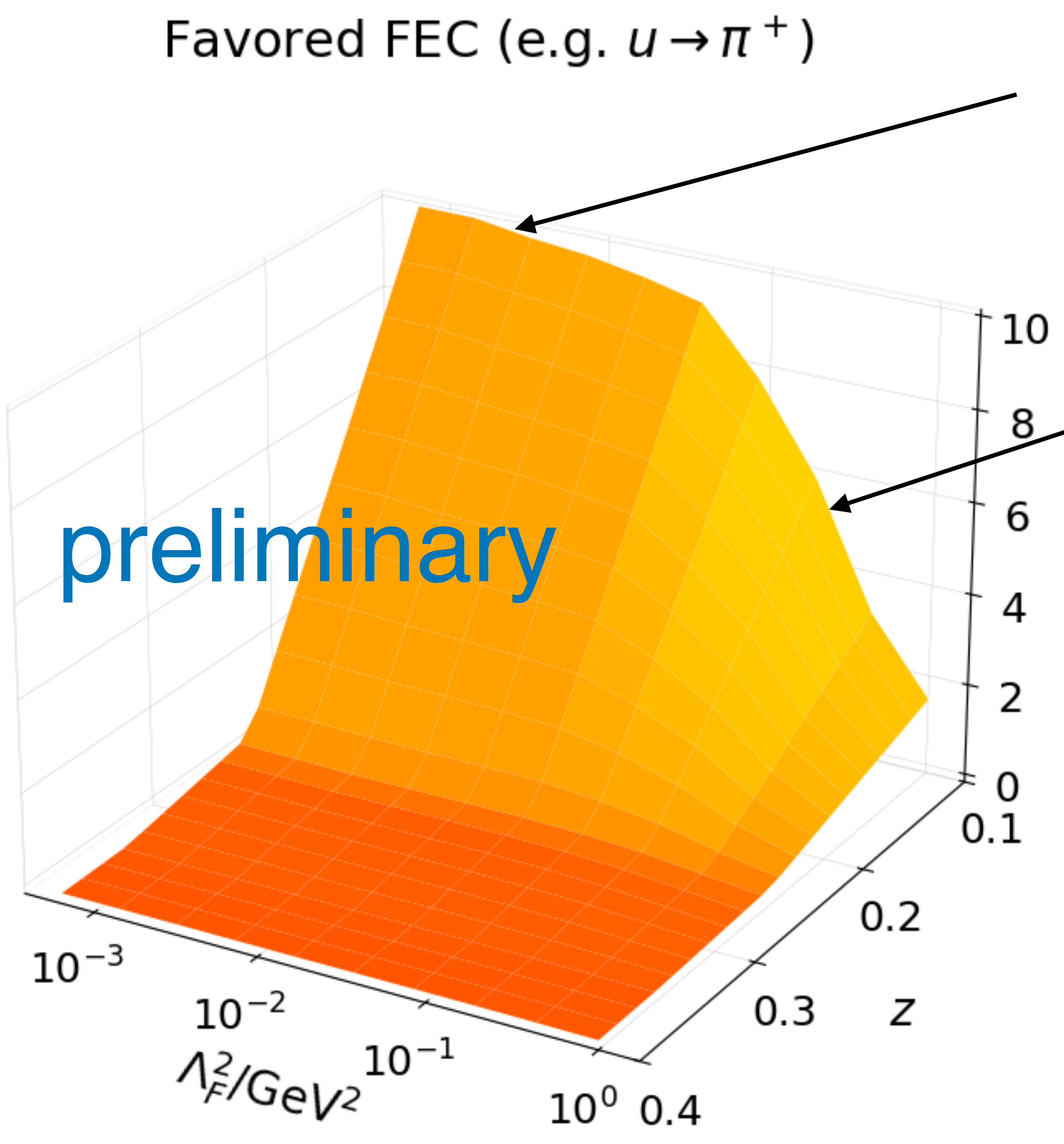
Extract the LO-FEC

Unpolarized FEC extracted from e^+e^- simulation in Pythia8.316

Fited Unpolarized FEC Scaling at small Λ_F^2

No ϕ modulation for unpolarized contribution

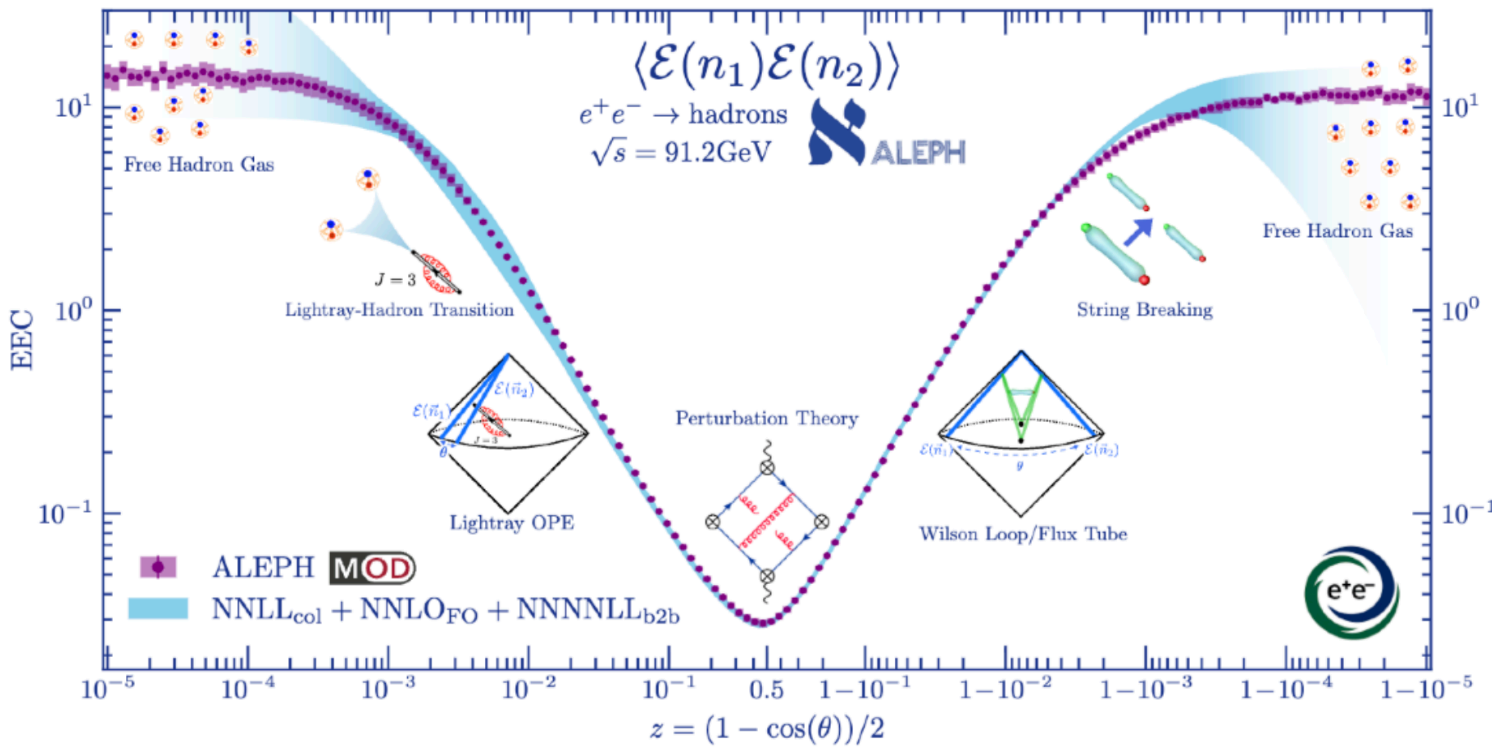
For the Collins FEC we can not fit from Pythia



post-confinement

pre-confinement

compared with e^+e^-



Sum rule with TMD Collins function

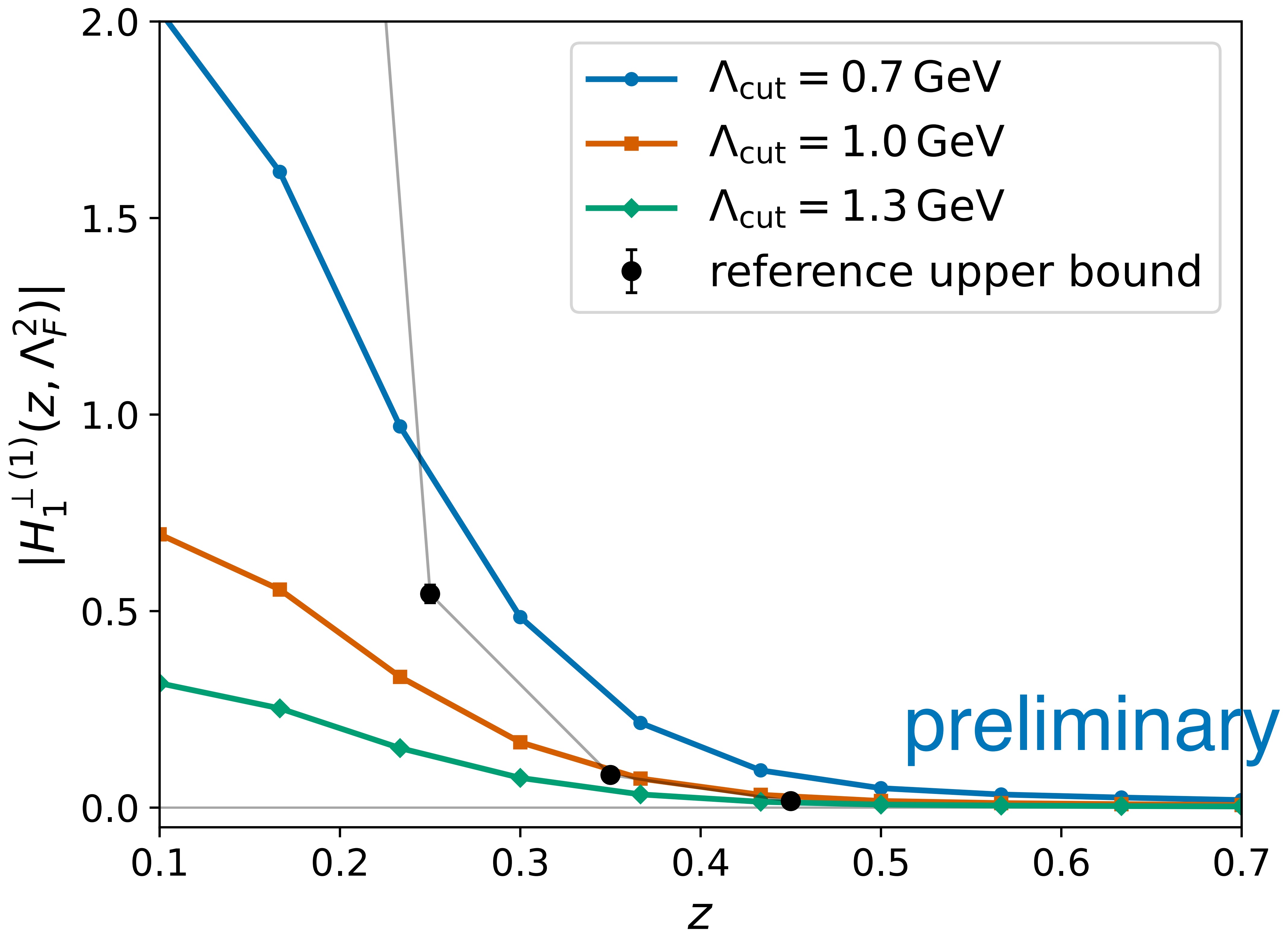
Liu, Zhu, 2403.08874

$$\int d\Lambda_F^2 \, \Lambda_F \, \mathscr{H}^{\perp}_{1,h/q}(z, \Lambda_F^2) = -z^2 \int \frac{dk_T^2}{2M_h} k_T^2 \, \boxed{H_1^{\perp}(z, z^2 k_T^2)}$$

$$= -M_h H(z)$$

JAM3D, 2205.00999

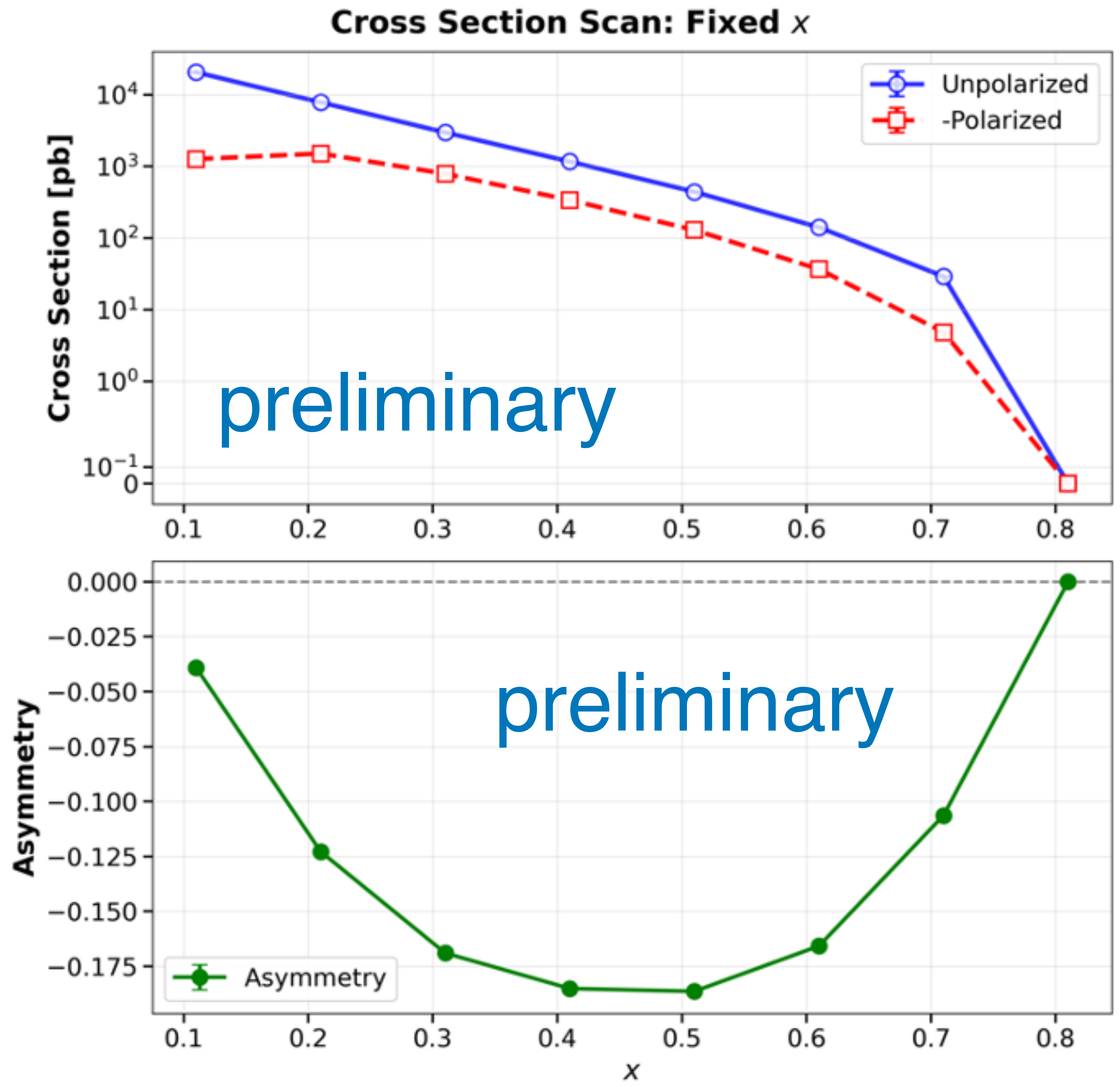
Choose an upper bound Λ^2_{cut} to the l.h.s., and we assume $\mathscr{H}^{\perp}_{1,h/q}(z, \Lambda_F^2)$ has plain distribution in Λ_F^2



JLab kinematics for SIDIS

- Take π^+ as case study, the asymmetry after integration over $(x, Q^2, \phi_S, \Lambda_F^2)$, it is about 0.05, which is a strong signal.

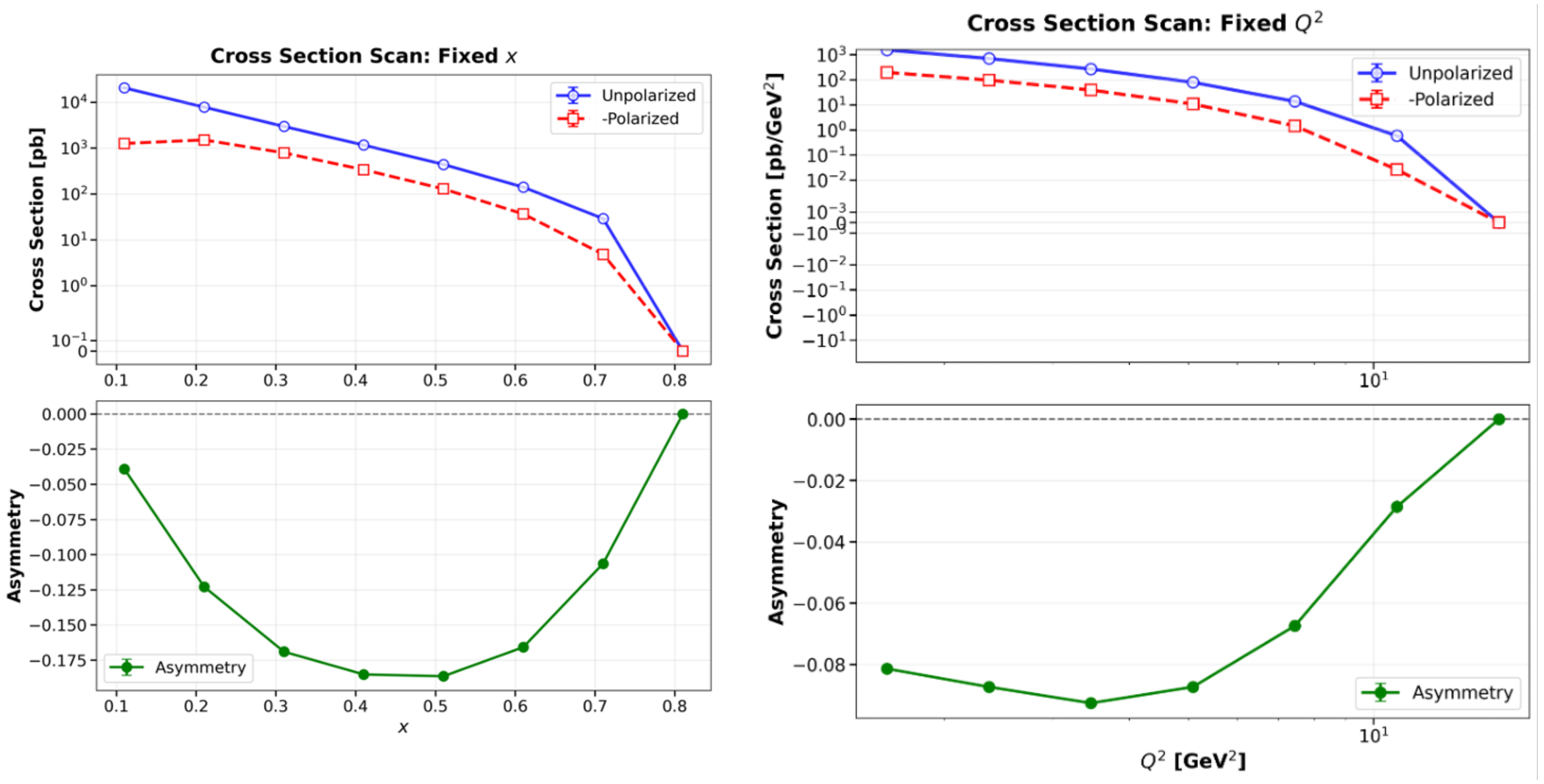
The kinematic cut
$x \in (0.1,1)$,
$z \in (0.1,0.9)$,
$E_h > 1.25$ GeV,
$Q^2 > 1$ GeV^2,
$W^2 > 4$ GeV^2,
$\Lambda_F^2 < 1$ GeV^2.



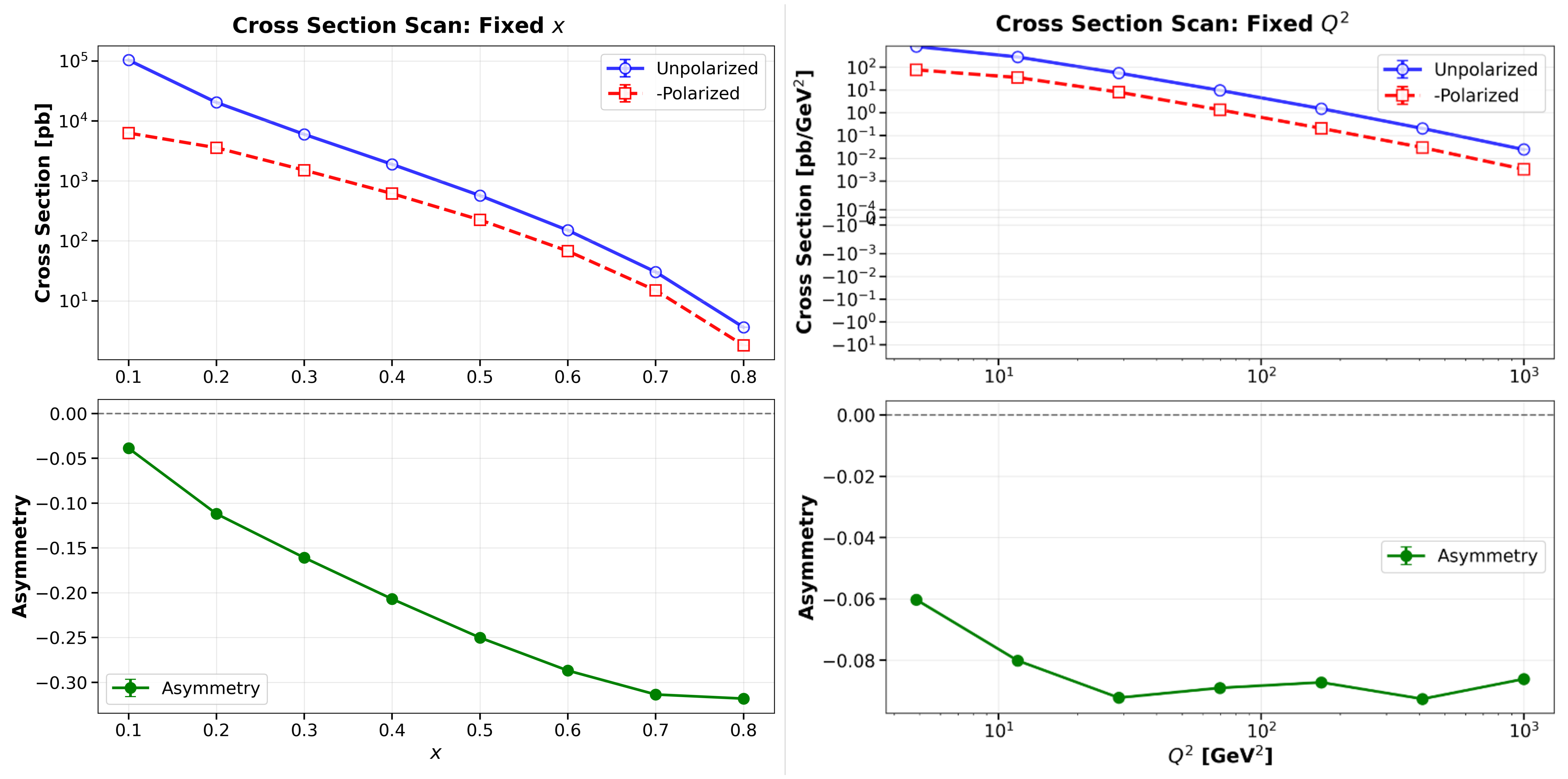
- Differential over Bjorken x, one can expect larger asymmetry.
- Data ready for plotting the figure.
- SoLID will have 100 times more luminosity than CLAS12.

Future EIC

JLab



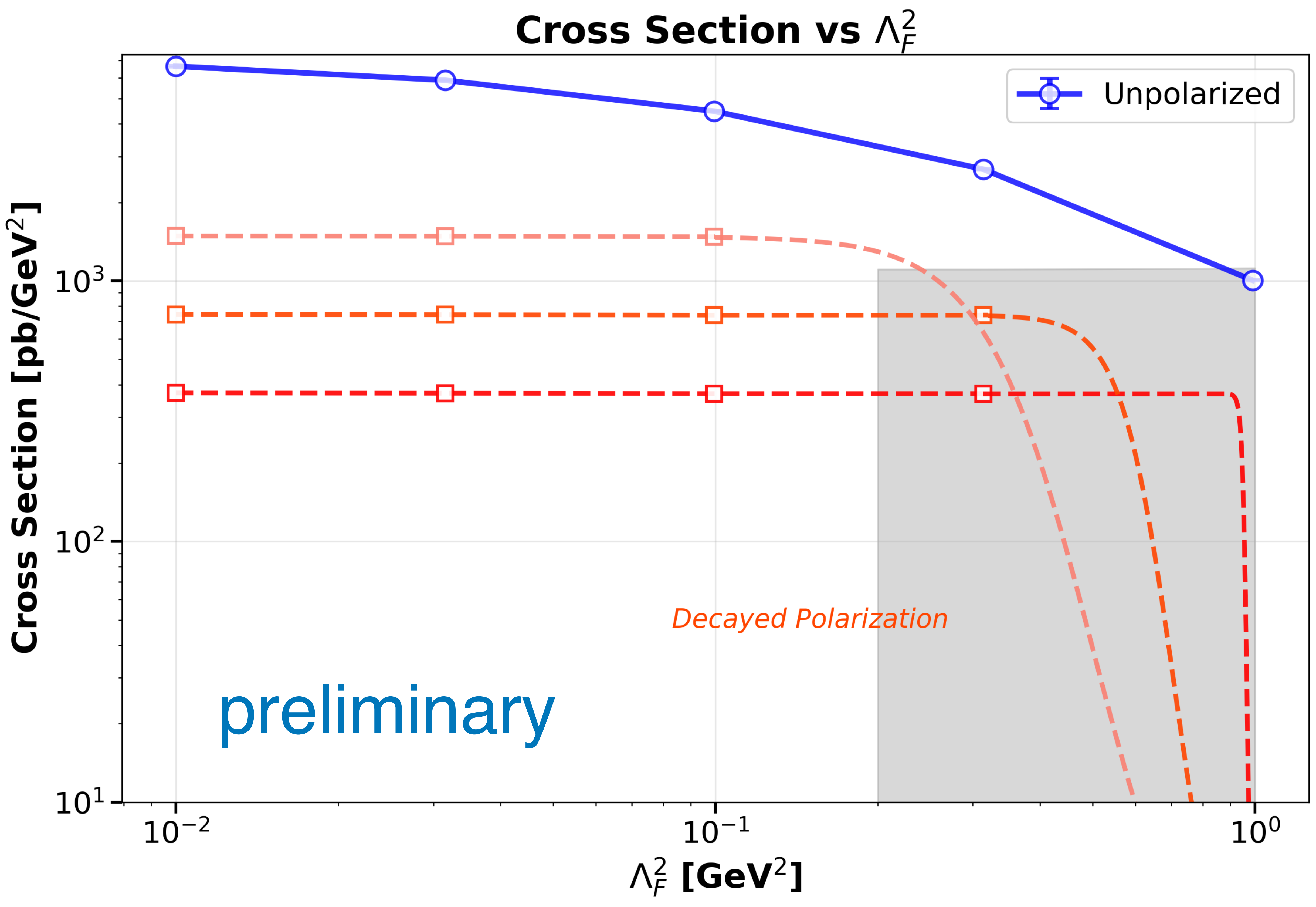
EIC

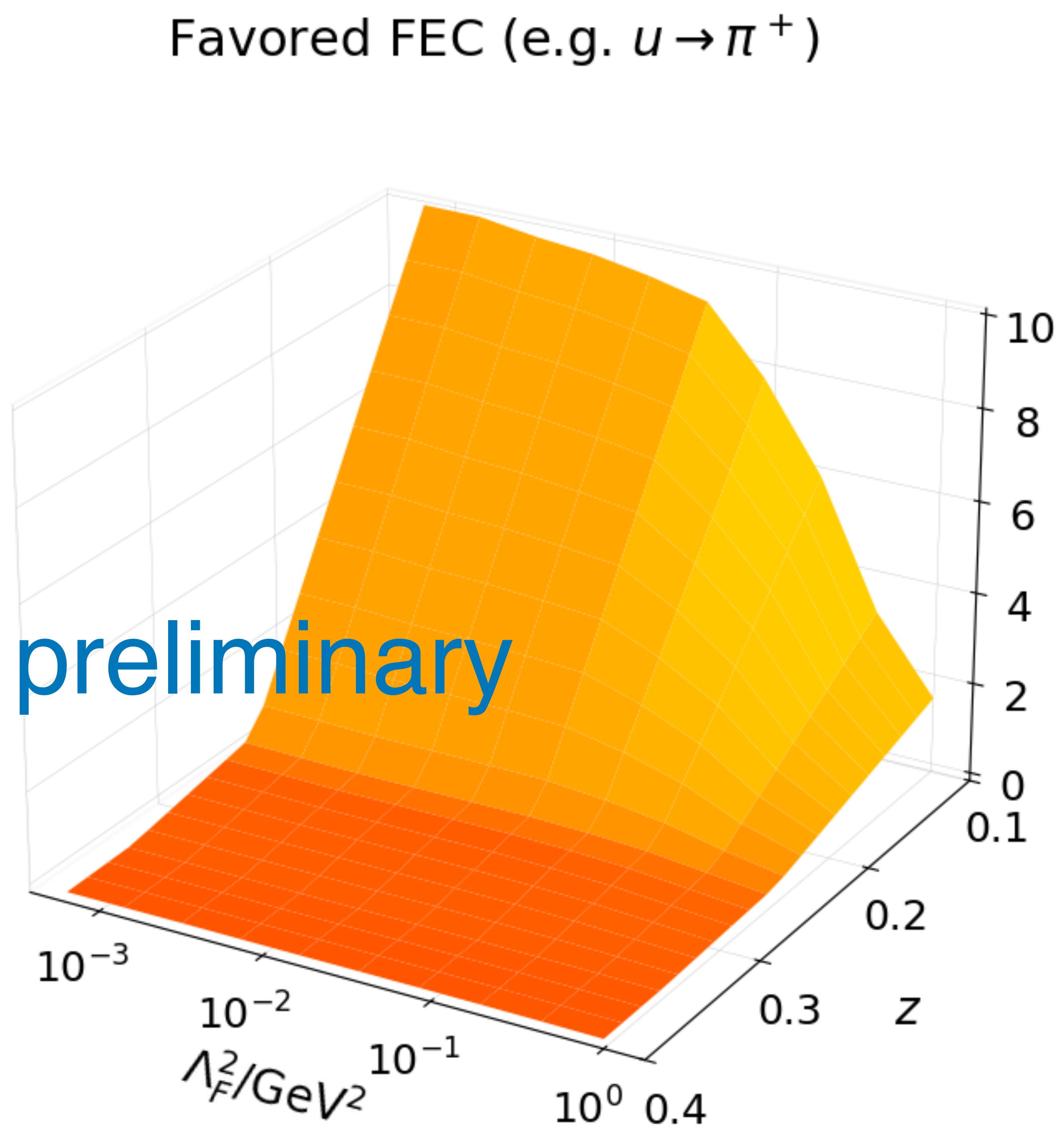


Wide coverage on kinematic region: x and Q^2

The scale at which Collins effects appear

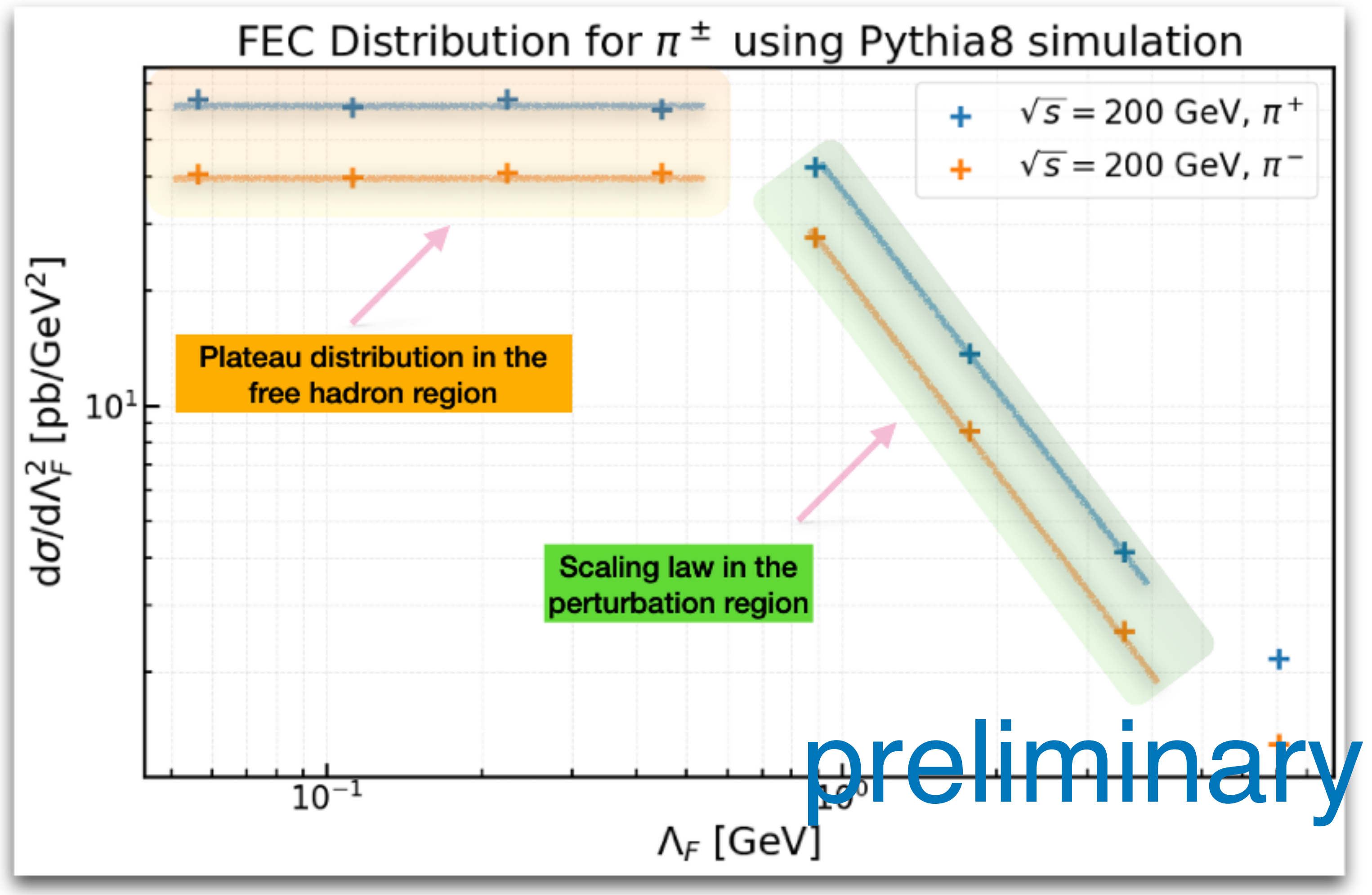
We need experiment input to tell us the behavior at the tail of Λ_F scan.

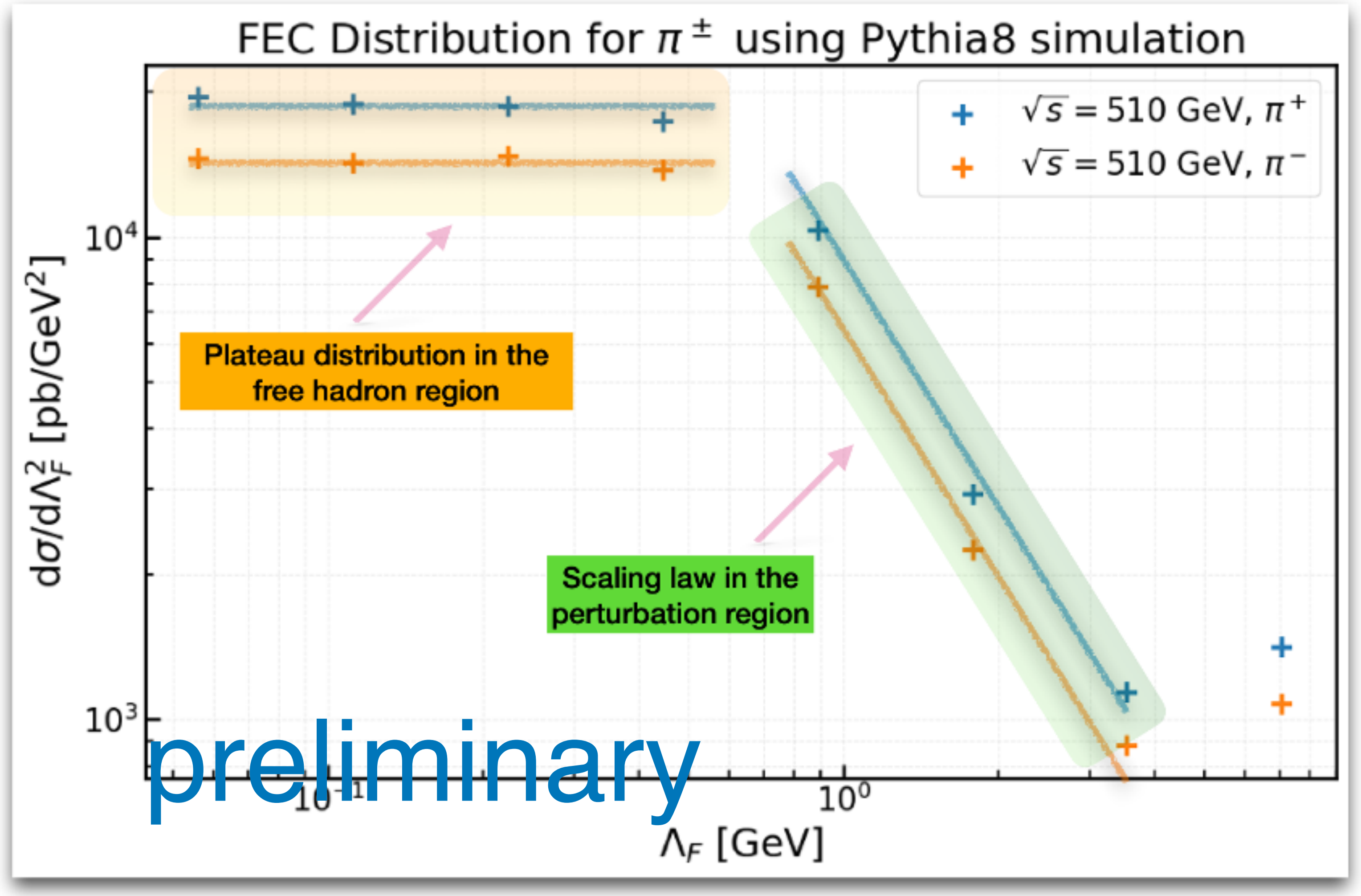




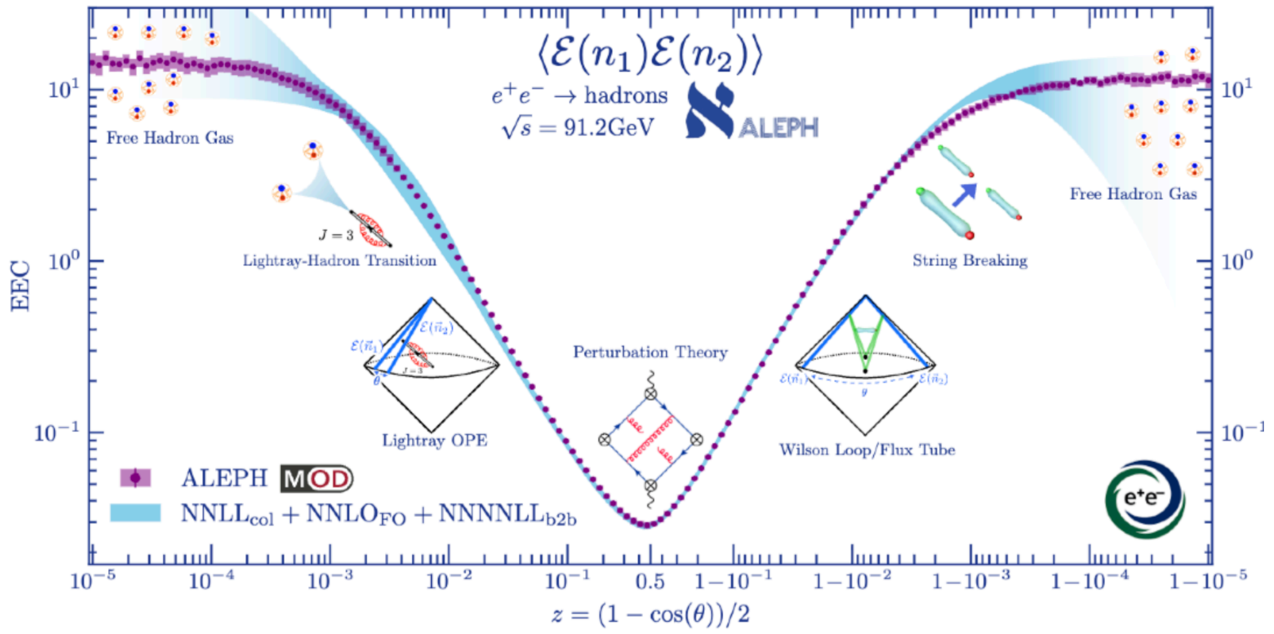
Asymmetry in pp collision (RHIC)

Pythia data for Unp. FEC





compared with e^+e^-



Theoretical calculation for asymmetry

kinematic cut

$\eta \in (2.0, 4.0)$

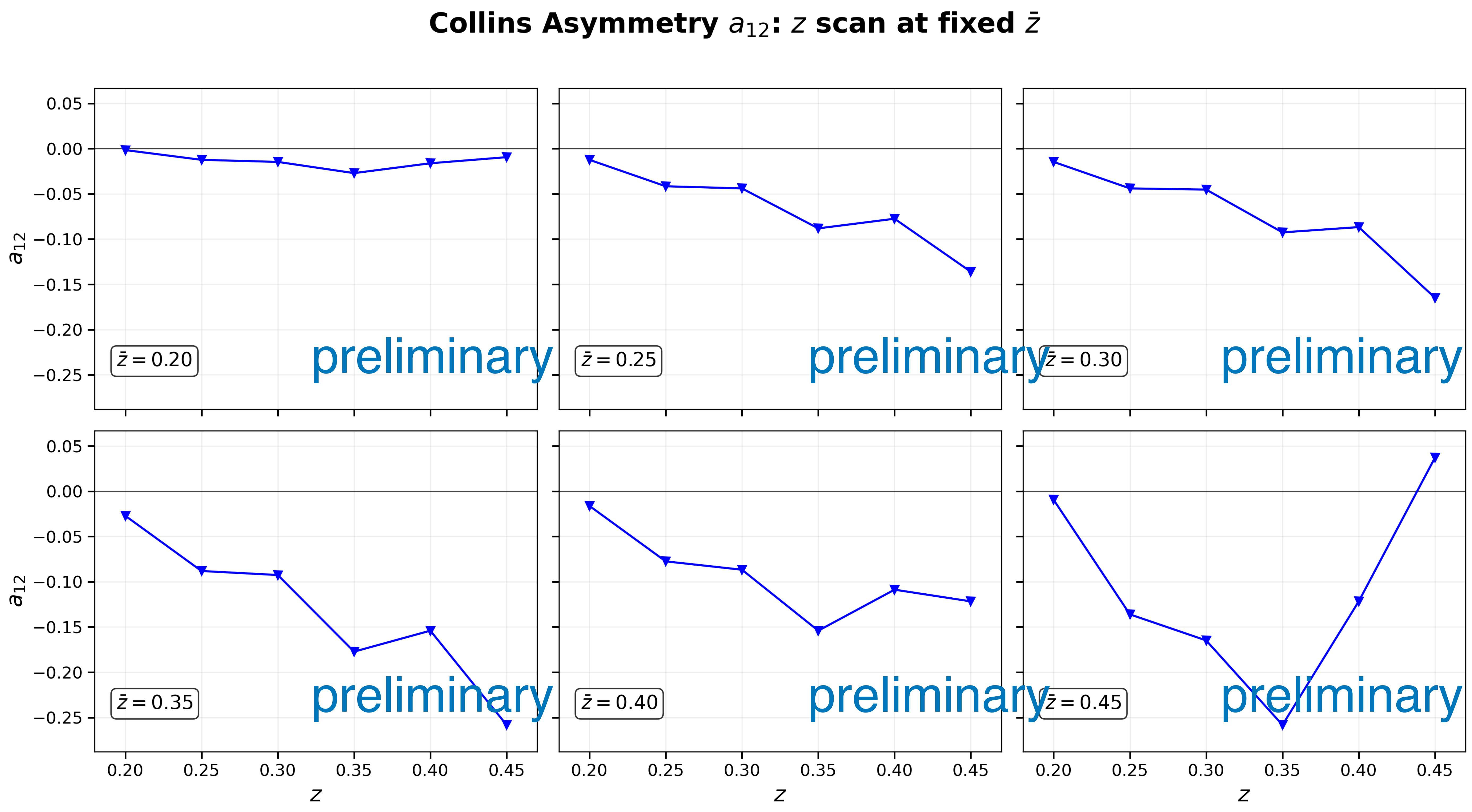
$p_{hT} > 5.0$GeV

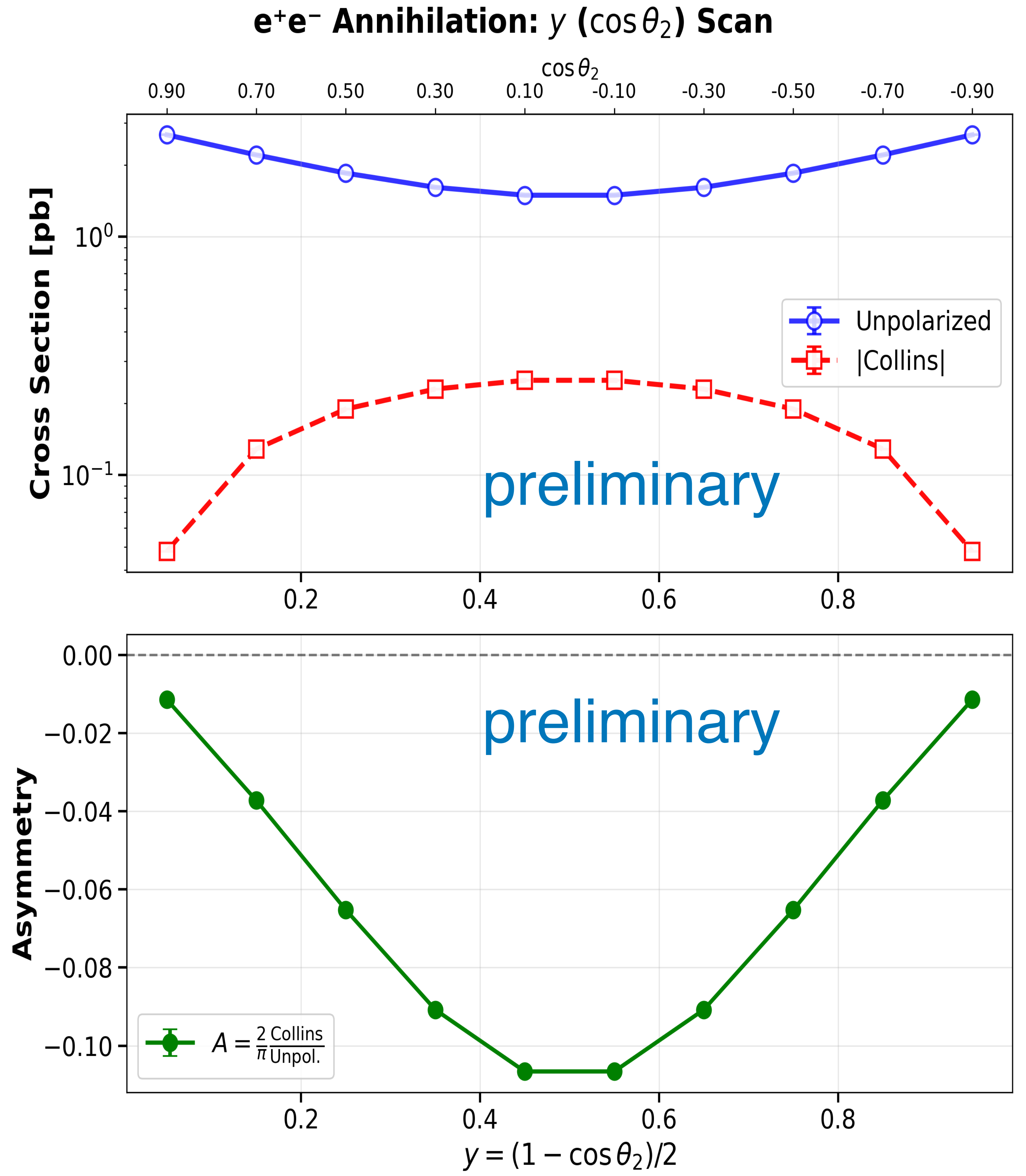
$P_{T,jet} > 13$GeV

$\Lambda_F^2 < 1$GeV^2

Asymmetry	π^+	π^-
$\sqrt{s} = 200$GeV	-0.17	-0.09
$\sqrt{s} = 510$GeV	0.25	0.10

Asymmetry in e^+e^- collision (Belle)





Transverse spin: hadrons fly perpendicular to the beam.

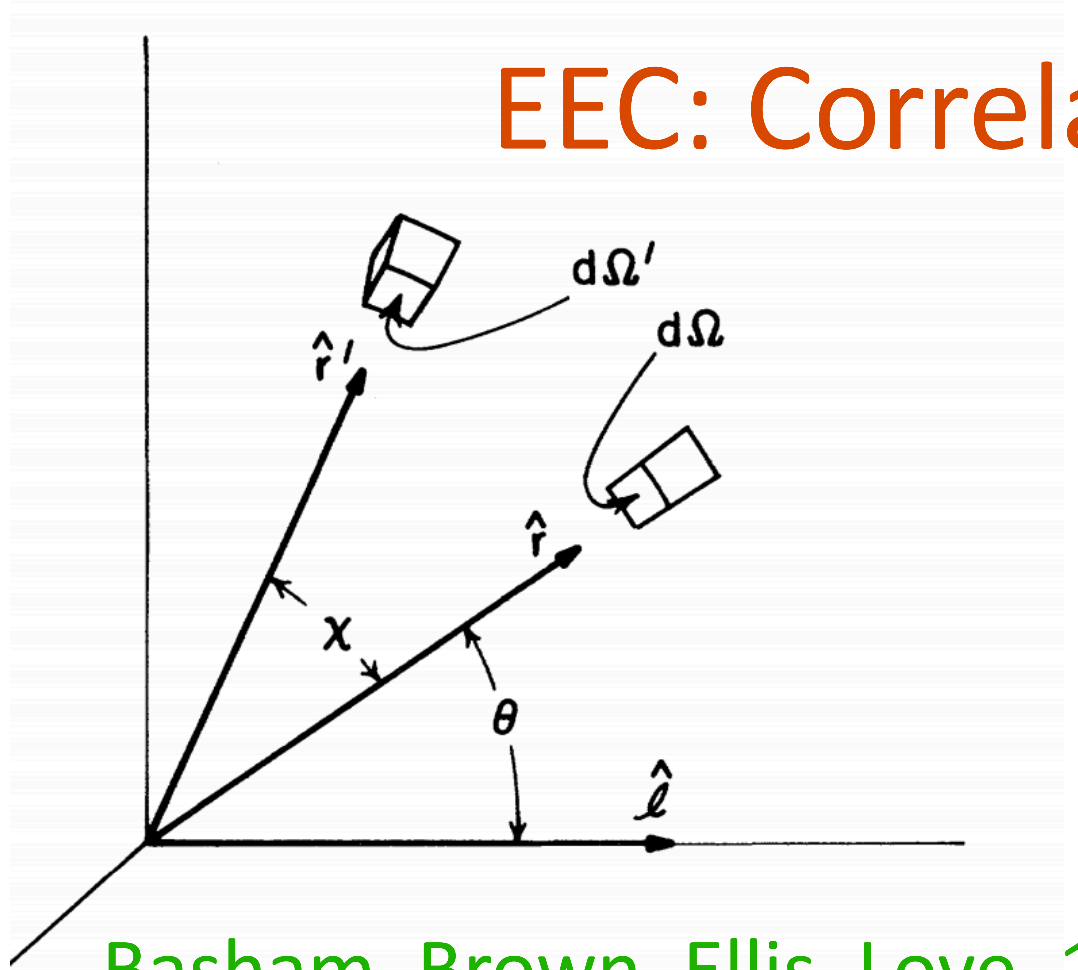
Summary

- Energy correlators provides a new way to probe different physics scales
- Fragmentation energy correlators as a new observable probing spin physics
  - Theoretically clean: pure collinear, no Sudakove log, no jet algorithm
  - Additional handle on the emergence scale of Collins effects => related to chiral symmetry breaking scale?
  - Size unknown, estimated from previous TMD measurement by sum rule
- Measurable signals across different experiments

Backup slide

What is Energy Correlator

EEC: Correlation of energy deposition between two detector at an angle χ

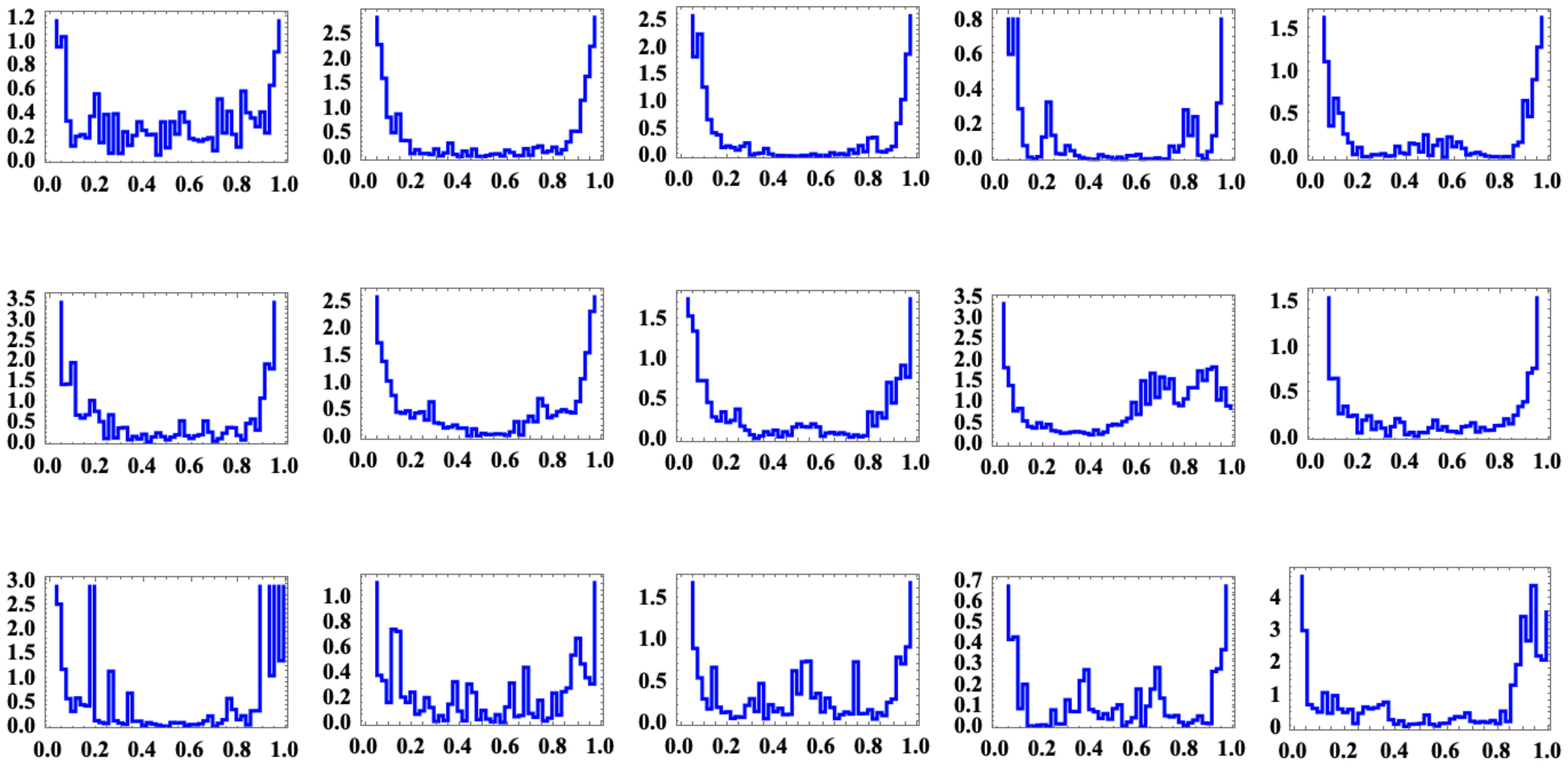


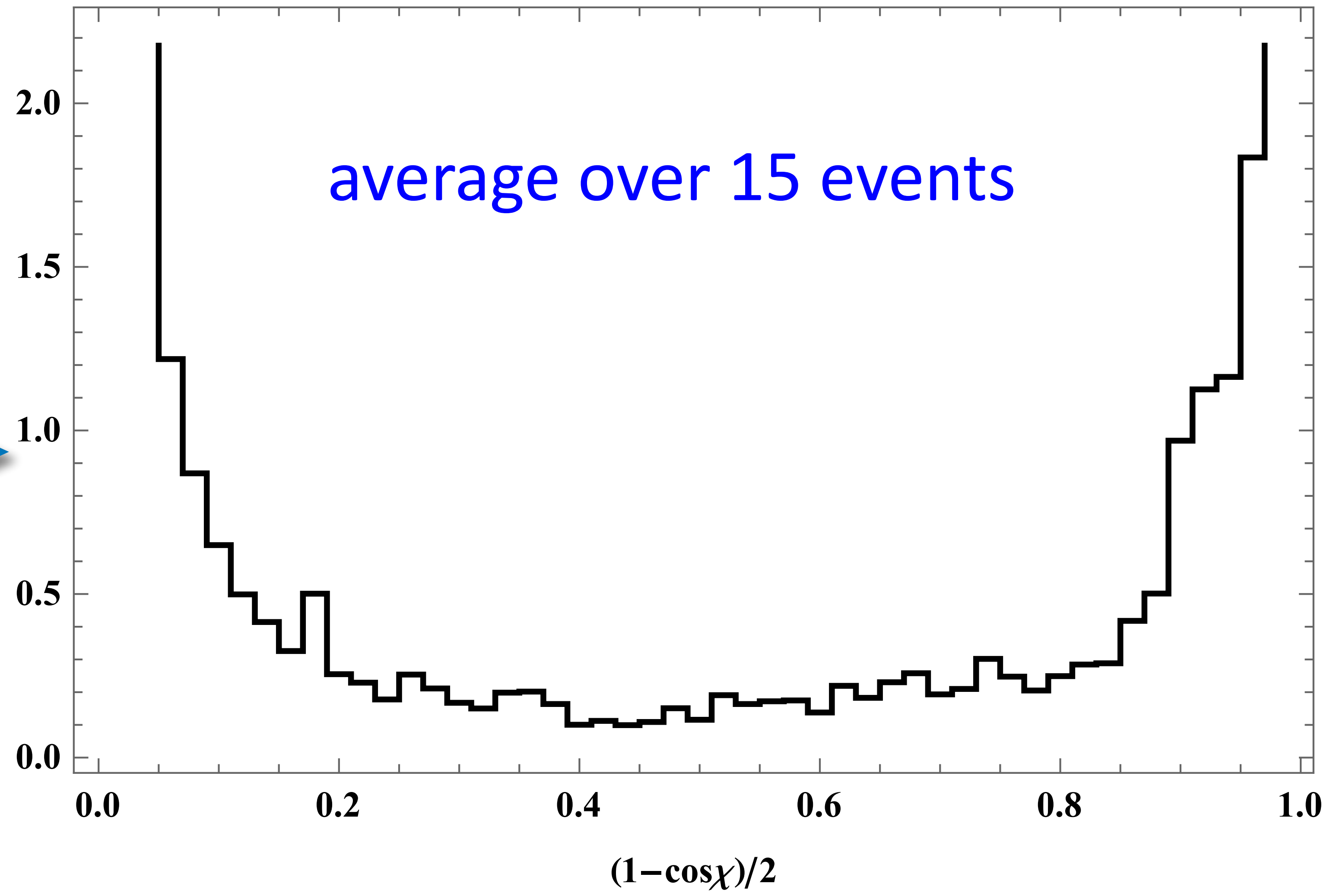
Basham, Brown, Ellis, Love, 1978

Weighted cross section

$$\mathrm{EEC}(\chi) = \frac{1}{N}\sum_{\text{events}}^{N}\sum_{i,j}^{N_{\text{particles}}}\frac{E_iE_j}{E_{\text{tot}}^2}\left(\frac{1}{\Delta\chi}\int_{\chi-\Delta\frac{\chi}{2}}^{\chi+\Delta\frac{\chi}{2}}\delta(\chi'-\chi_{ij})d\chi'\right)$$

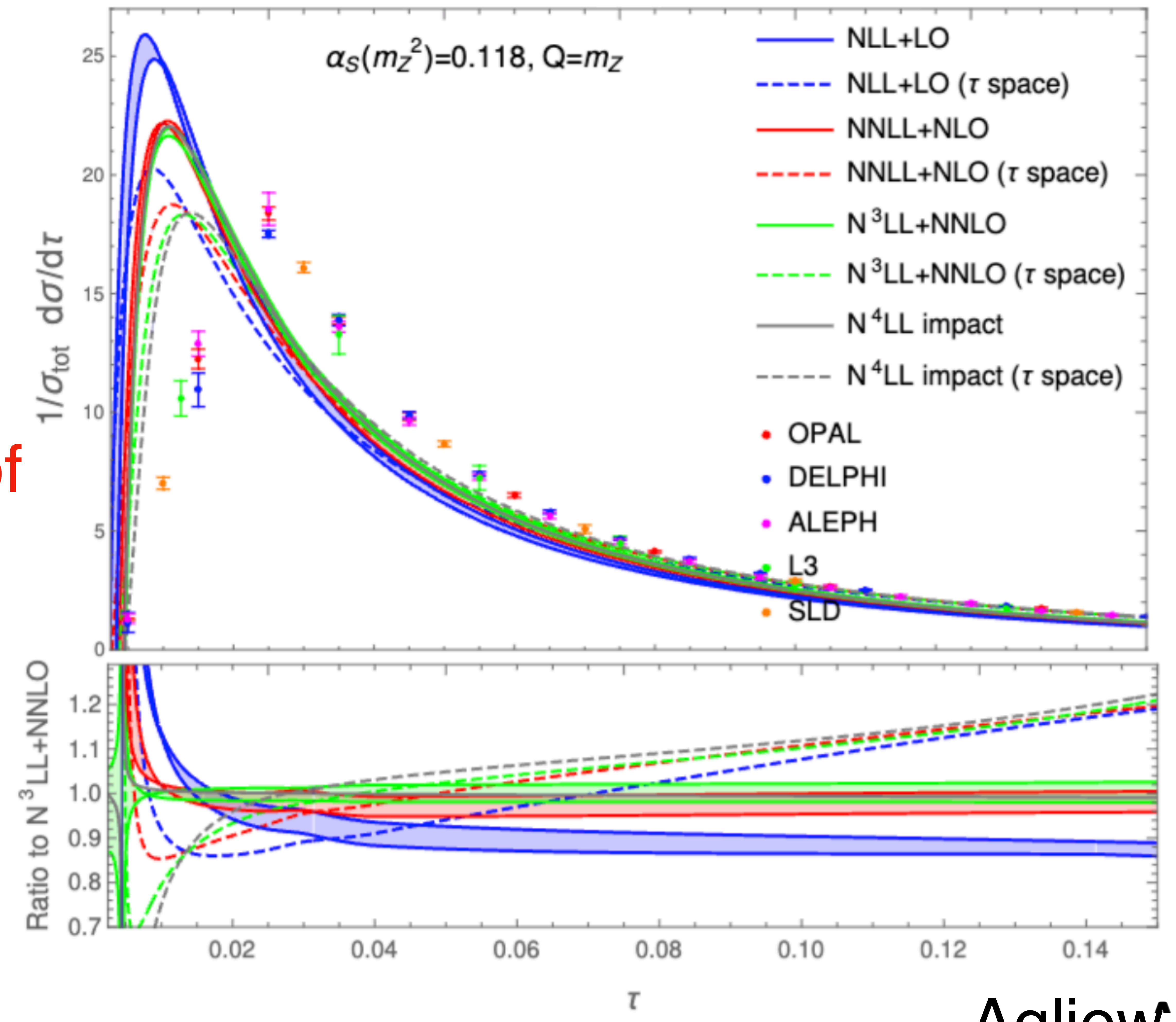
Measurement on a single event gives a function





Example of Sudakov logs

Sudakov peak perturbatively generated



No sharp scale for on-set of non-perturbative effects

Agliew et.al., 2502.01570

Experiments results on di-hadron Collins effect

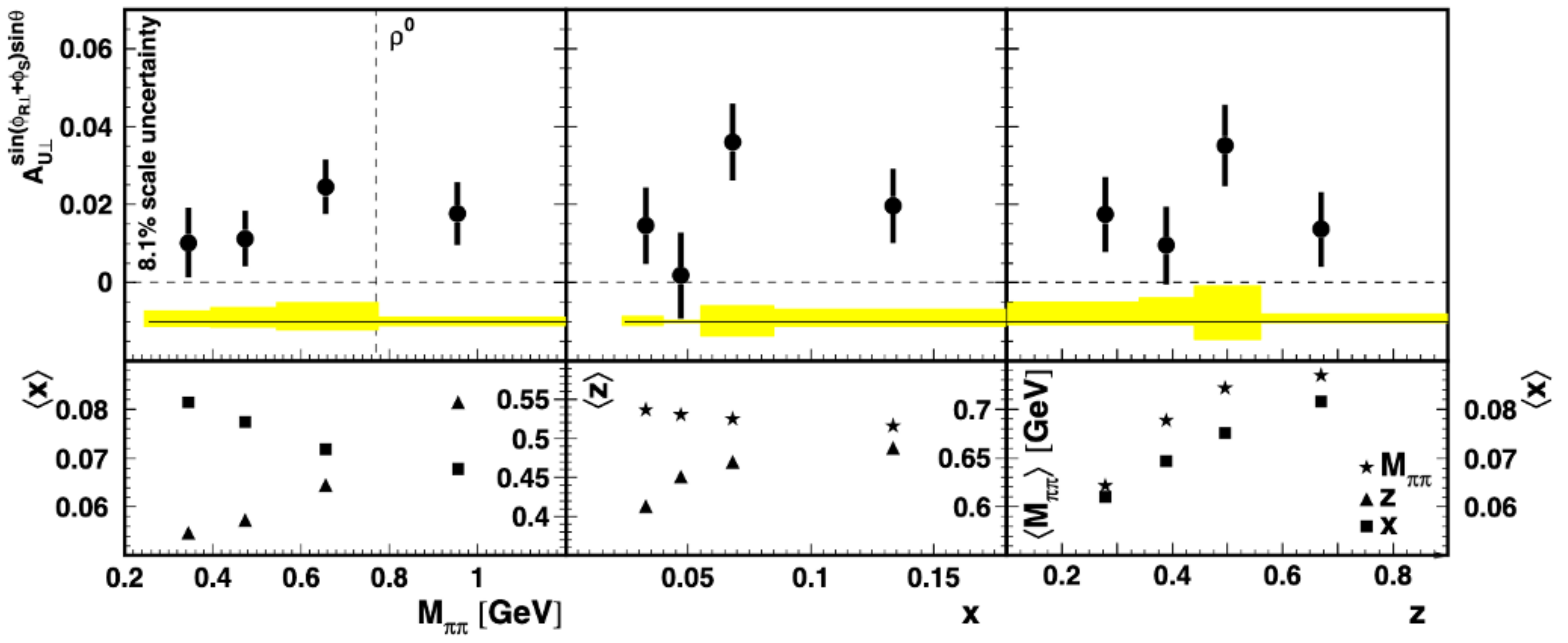


Figure 2: The top panels show $A_{U\perp}^{\sin(\phi_{R\perp}+\phi_S)\sin\theta}$ versus $M_{\pi\pi}$, x, and z. The bottom panels show the average values of the variables that were integrated over. For the dependence on x and z, $M_{\pi\pi}$ was constrained to the range 0.5 GeV $< M_{\pi\pi} <$ 1.0 GeV, where the signal is expected to be largest. The error bars show the statistical uncertainty. A scale uncertainty of 8.1% arises from the uncertainty in the target polarization. Other contributions to the systematic uncertainty are summed in quadrature and represented by the asymmetric error band.

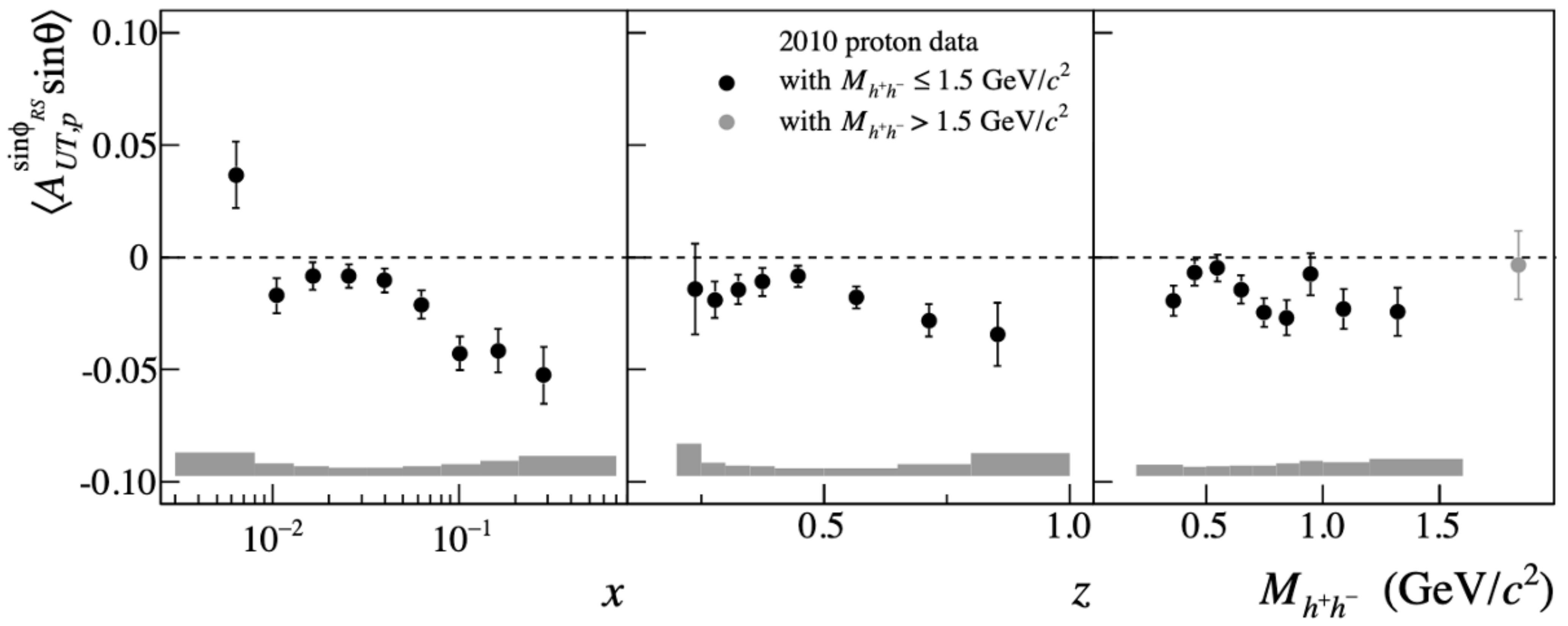


Fig. 3: Proton asymmetry, integrated over the angle θ, as a function of x, z and $M_{h^+h^-}$, for the data taken with the proton (NH_3) target in the year 2010. The grey bands indicate the systematic uncertainties. The last bin in $M_{h^+h^-}$ contains events which were removed from the sample used for results shown as a function of x and z.

HERMES,JHEP 06 (2008) 017

COMPASS, PLB.2014.06.080

Decomposition of polarization functions

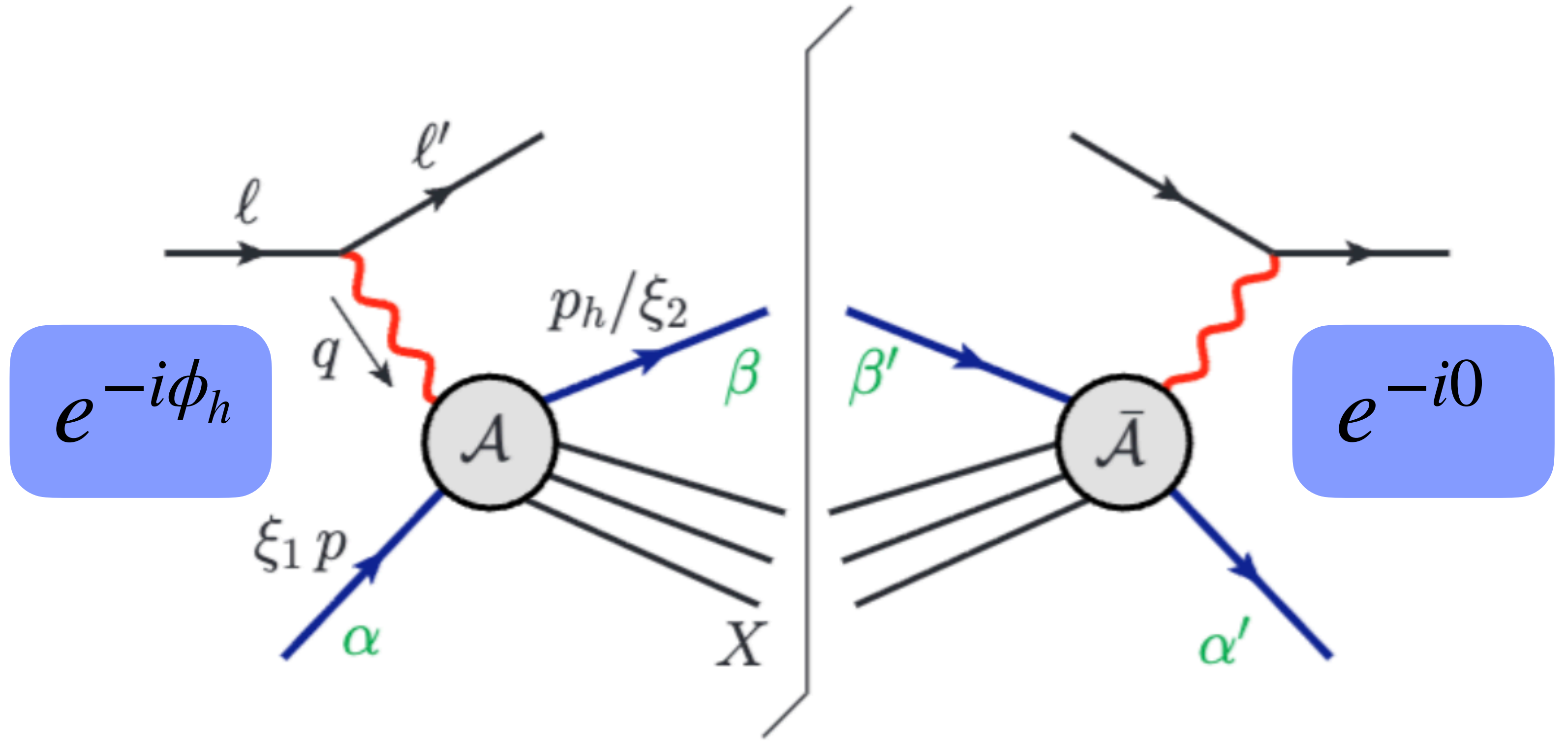
The master formula:

$$\left[\frac{y^2}{z(1-\varepsilon)}\frac{\alpha_e^2}{(8\pi^2 Q^2)^2}\right]^{-1}\frac{d\Sigma}{dx\,dQ^2\,d\phi_S\,dz\,dp_{hT}^2\,d\phi_h\,d\eta\,d\phi}$$
$$= \mathcal{C}_{F_{UU,T}}[f,\mathcal{D}] + \varepsilon\,\mathcal{C}_{F_{UU,L}}[f,\mathcal{D}]$$
$$+ s_T\left\{\left[-\mathcal{C}_{F_{UT,T}}\left[h,\mathcal{H}^\perp\right] - \varepsilon\,\mathcal{C}_{F_{UT,L}}\left[h,\mathcal{H}^\perp\right]\right]\sin\left(\phi-(\phi_S-\phi_h)\right)\right.$$
$$+ \sqrt{2\varepsilon(1+\varepsilon)}\left[\mathcal{C}_{F_{UT}^{(1+)}}\left[h,\mathcal{H}^\perp\right]\sin\left(\phi-(\phi_S-2\phi_h)\right) + \mathcal{C}_{F_{UT}^{(1-)}}\left[h,\mathcal{H}^\perp\right]\sin\left(\phi-\phi_S\right)\right]$$
$$\left.+\varepsilon\left[\mathcal{C}_{F_{UT}^{(2+)}}\left[h,\mathcal{H}^\perp\right]\sin\left(\phi-(\phi_S-3\phi_h)\right) + \mathcal{C}_{F_{UT}^{(2-)}}\left[h,\mathcal{H}^\perp\right]\sin\left(\phi-(\phi_S+\phi_h)\right)\right]\right\}$$

+ longitudinal polarization, power corrections, etc

Diagonal photon density matrix, transverse polarized quark

$$\begin{array}{c} \\ + \\ 0 \\ - \end{array}\begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 1 & \sqrt{\varepsilon(1+\varepsilon)} & -\varepsilon \\ \sqrt{\varepsilon(1+\varepsilon)} & 2\varepsilon & -\sqrt{\varepsilon(1+\varepsilon)} \\ -\varepsilon & -\sqrt{\varepsilon(1+\varepsilon)} & 1 \end{pmatrix}\end{array}$$



Decomposition of polarization functions

The master formula:

$$\left[\frac{y^2}{z(1-\varepsilon)}\frac{\alpha_e^2}{(8\pi^2 Q^2)^2}\right]^{-1}\frac{d\Sigma}{dx\,dQ^2\,d\phi_S\,dz\,dp_{hT}^2\,d\phi_h\,d\eta\,d\phi}$$

$$= \mathcal{C}_{F_{UU,T}}[f,\mathcal{D}] + \varepsilon\mathcal{C}_{F_{UU,L}}[f,\mathcal{D}]$$

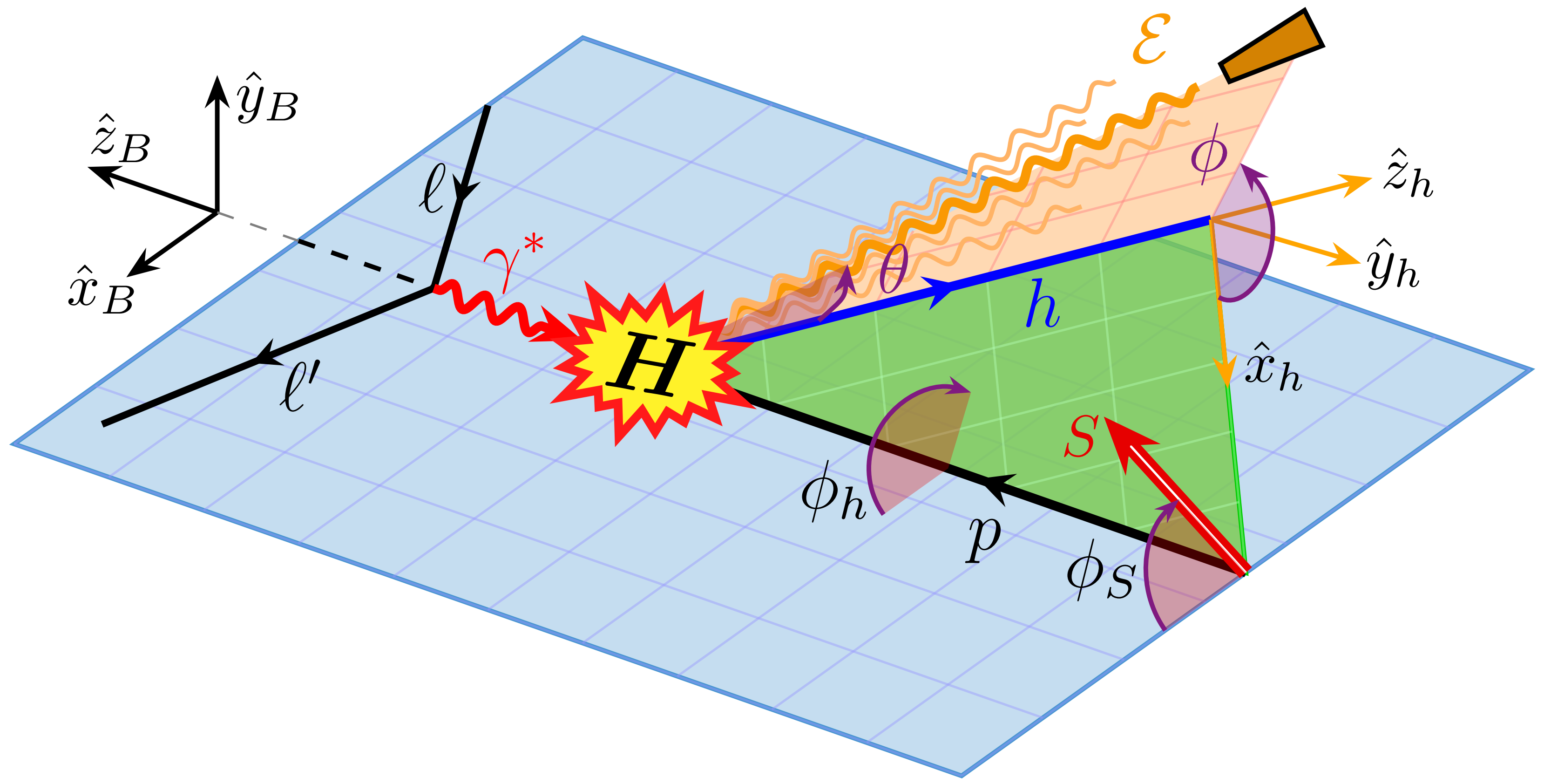
$$+\sqrt{2\varepsilon(1+\varepsilon)}\,\mathcal{C}_{F_{UU}^{(1)}}[f,\mathcal{D}]\cos\phi_h + \varepsilon\,\mathcal{C}_{F_{UU}^{(2)}}[f,\mathcal{D}]\cos(2\phi_h)$$

$$+ s_T\left\{\left[-\mathcal{C}_{F_{UT,T}}\left[h,\mathcal{H}^{\perp}\right] - \varepsilon\mathcal{C}_{F_{UT,L}}\left[h,\mathcal{H}^{\perp}\right]\right]\sin\left(\phi-(\phi_S-\phi_h)\right)\right.$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\left[\mathcal{C}_{F_{UT}^{(1+)}}\left[h,\mathcal{H}^{\perp}\right]\sin\left(\phi-(\phi_S-2\phi_h)\right)+\mathcal{C}_{F_{UT}^{(1-)}}\left[h,\mathcal{H}^{\perp}\right]\sin\left(\phi-\phi_S\right)\right]$$

$$\left.+\varepsilon\left[\mathcal{C}_{F_{UT}^{(2+)}}\left[h,\mathcal{H}^{\perp}\right]\sin\left(\phi-(\phi_S-3\phi_h)\right)+\mathcal{C}_{F_{UT}^{(2-)}}\left[h,\mathcal{H}^{\perp}\right]\sin\left(\phi-(\phi_S+\phi_h)\right)\right]\right\}$$

+ longitudinal polarization, power corrections, etc



Multi-dimensional distribution simplified by integrated over hadron pT

$$\int_0^{2\pi} d\phi_h \sin(\cdots \pm n\phi) = 0$$

Naively only $C_{F_{UT}^{(1-)}}$ survives

But $C_{F_{UT}^{(2-)}}$ also survives and gives the leading contribution

LO contribution to Collins effect

The master formula:

$$\left[\frac{y^2}{z(1-\varepsilon)}\frac{\alpha_e^2}{(8\pi^2Q^2)^2}\right]^{-1}\frac{d\Sigma}{dx\,dQ^2 d\phi_S\, dz\, dp_{hT}^2 d\phi_h\, d\eta\, d\phi}$$
$$= \mathcal{C}_{F_{UU,T}}[f,\mathcal{D}] + \varepsilon\mathcal{C}_{F_{UU,L}}[f,\mathcal{D}]$$

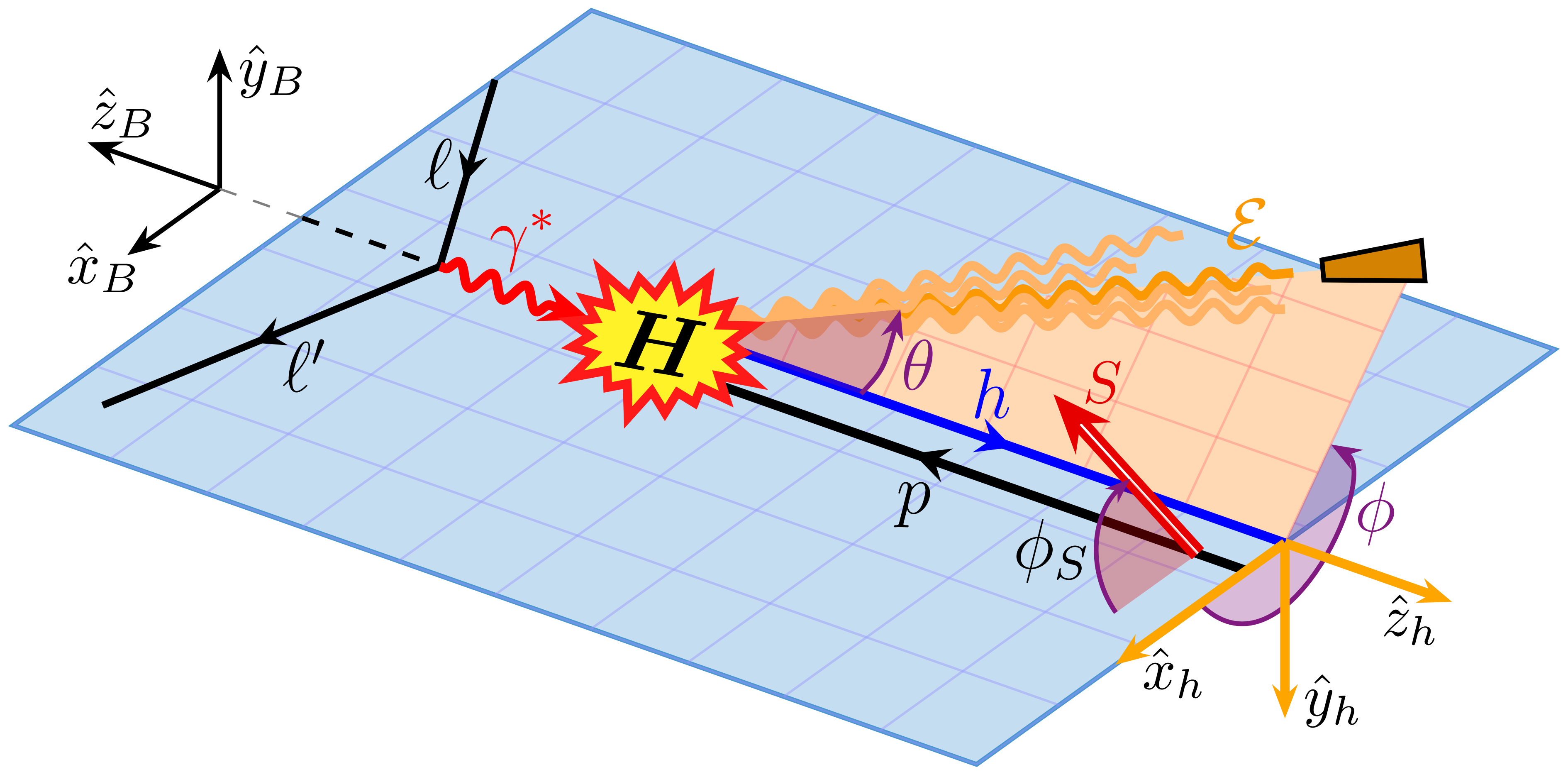
$$+ \sqrt{2\varepsilon(1+\varepsilon)}\,\mathcal{C}_{F_{UU}^{(1)}}[f,\mathcal{D}]\cos\phi_h + \varepsilon\,\mathcal{C}_{F_{UU}^{(2)}}[f,\mathcal{D}]\cos(2\phi_h)$$

$$+ s_T\left\{\left[-\mathcal{C}_{F_{UT,T}}\left[h,\mathcal{H}^\perp\right] - \varepsilon\mathcal{C}_{F_{UT,L}}\left[h,\mathcal{H}^\perp\right]\right]\sin\left(\phi-(\phi_S-\phi_h)\right)\right.$$

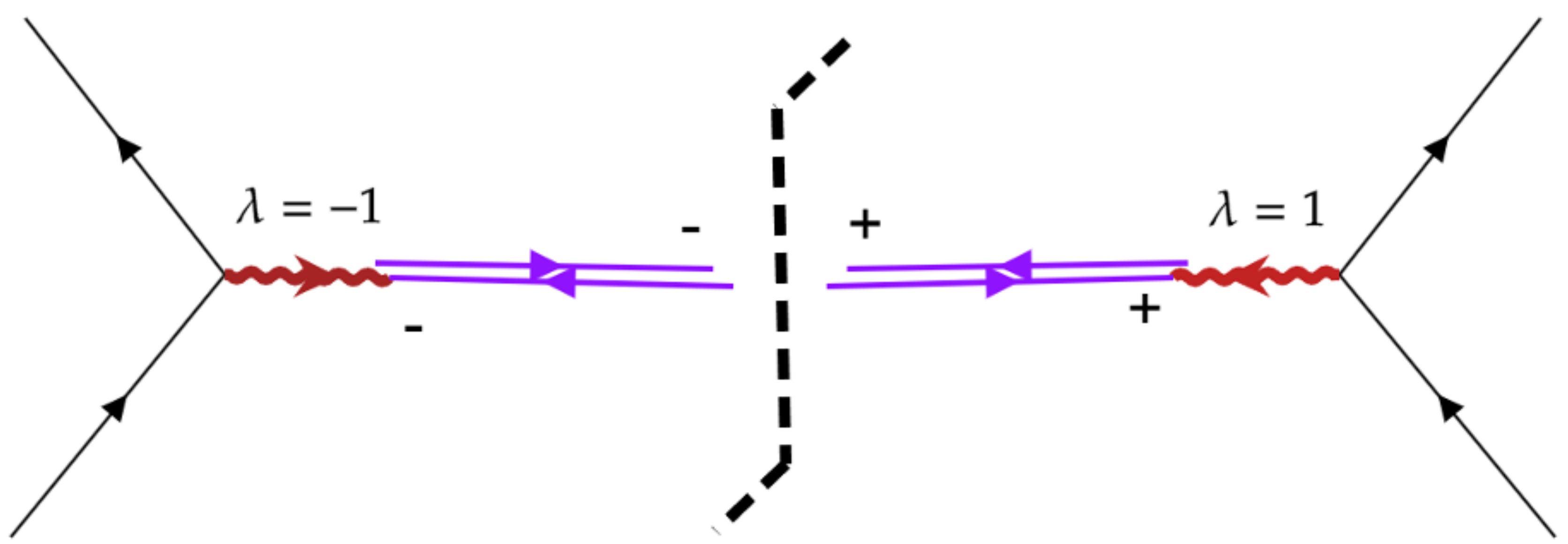
$$+ \sqrt{2\varepsilon(1+\varepsilon)}\left[\mathcal{C}_{F_{UT}^{(1+)}}\left[h,\mathcal{H}^\perp\right]\sin\left(\phi-(\phi_S-2\phi_h)\right) + \mathcal{C}_{F_{UT}^{(1-)}}\left[h,\mathcal{H}^\perp\right]\sin\left(\phi-\phi_S\right)\right]$$

$$\left. +\varepsilon\left[\mathcal{C}_{F_{UT}^{(2+)}}\left[h,\mathcal{H}^\perp\right]\sin\left(\phi-(\phi_S-3\phi_h)\right) + \mathcal{C}_{F_{UT}^{(2-)}}\left[h,\mathcal{H}^\perp\right]\sin\left(\phi-(\phi_S+\phi_h)\right)\right]\right\}$$

+ longitudinal polarization, power corrections, etc

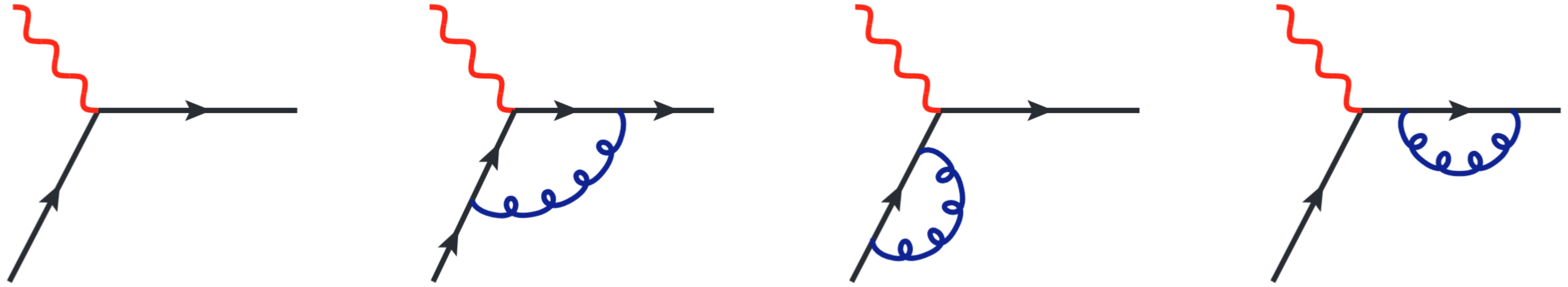


Only transverse polarized terms allowed by Leading Order kinematics

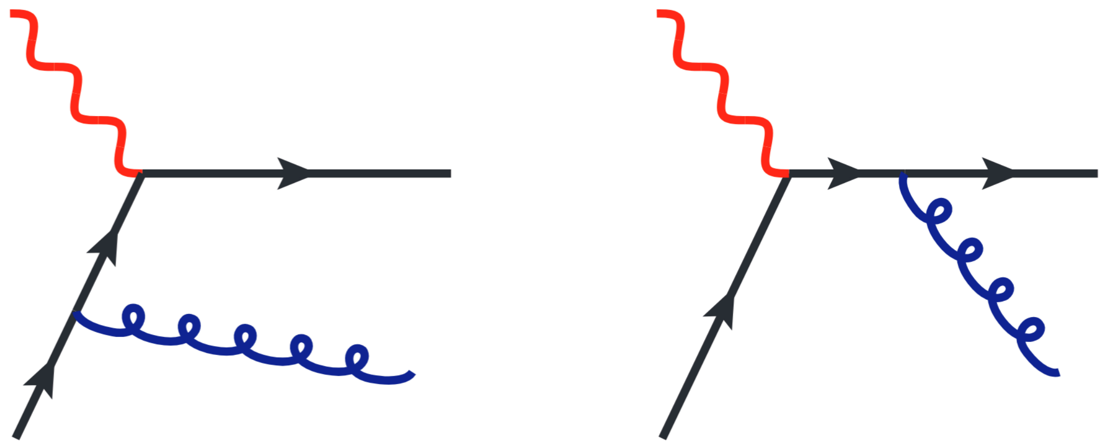


NLO consistency: end-point treatment

Tree + virtual



Real



Term that proportional to $\delta(p_T)$

$$\int d^2p_T = \frac{1}{2}\int dp_T^2 \int d\phi_h$$

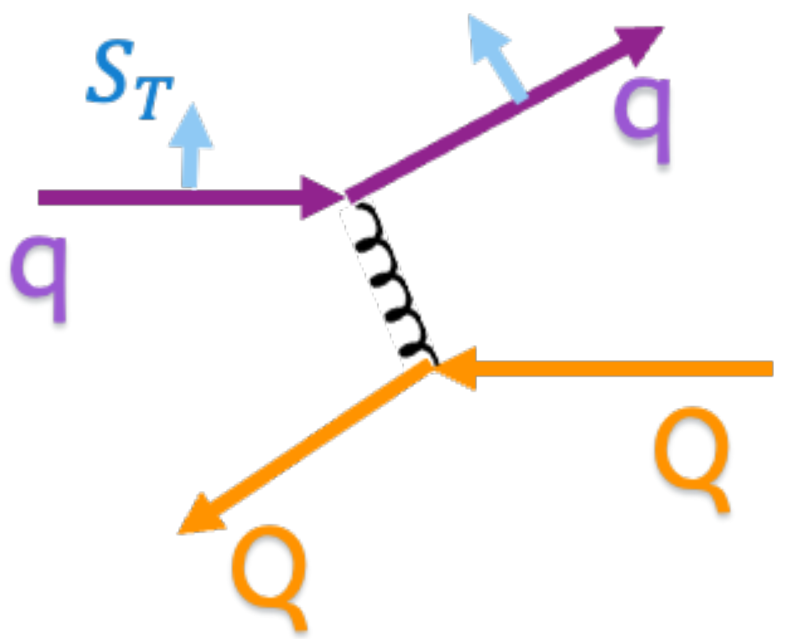
$$\int d^2p_T f(p_T^2, \phi_h) = \frac{1}{2}\int dp_T^2 \int d\phi_h \{f(p_T^2, \phi_h) - f_{\text{singular}}(p_T^2, \phi_h)\}$$

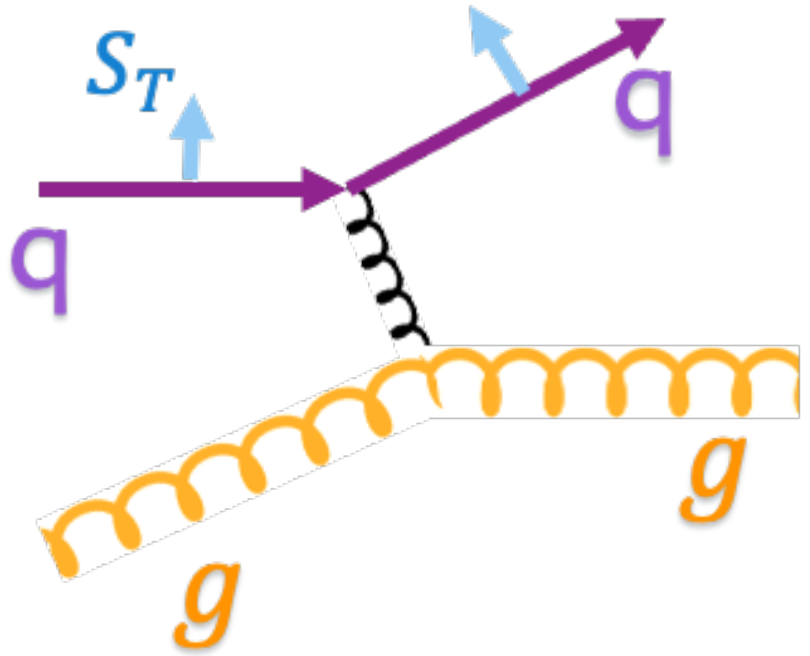
$$+\frac{1}{2}\int dp_T^2 \int d\phi_h f_{\text{singular}}(p_T^2, \phi_h = \pi)$$

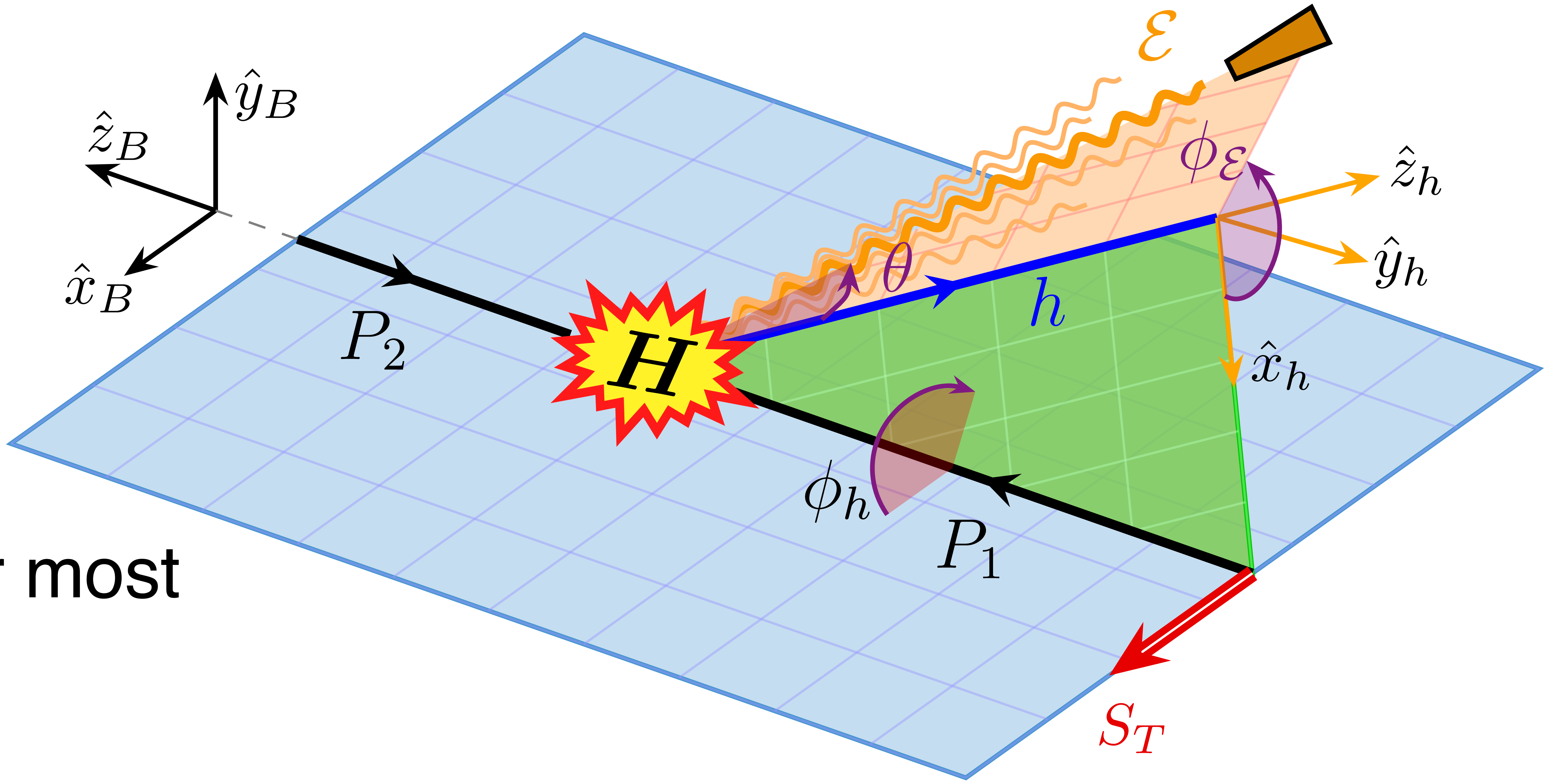
At $p_T^2 = 0$, ϕ_h is not defined

Solution: similar philosophy to the plus distribution:

t channel enhancement in pp collision







Forward region contribute to the spin transfer most

$$T_{qQ\to qQ} = -\frac{8\hat{s}\hat{u}}{9\hat{t}^2}, \qquad T_{q\bar{q}\to q\bar{q}} = \frac{8}{27},$$

$$T_{qq\to qq} = \frac{8\hat{s}(\hat{t}-3\hat{u})}{27\hat{t}^2}, \qquad T_{qg\to qg} = \frac{2}{9}\left(4+\frac{9\hat{s}(\hat{s}+\hat{t})}{\hat{t}^2}\right).$$

$$T_{q\bar{q}\to q\bar{q}} = \frac{8\hat{u}(2\hat{t}+3\hat{u})}{27\hat{t}^2},$$

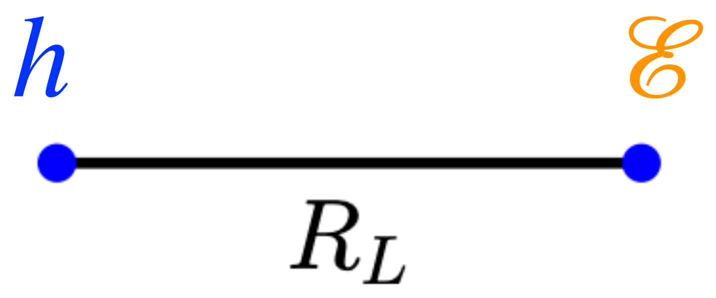
with $T^{ij}_{ab} = T_{ab}R_z(-\phi)\,.$

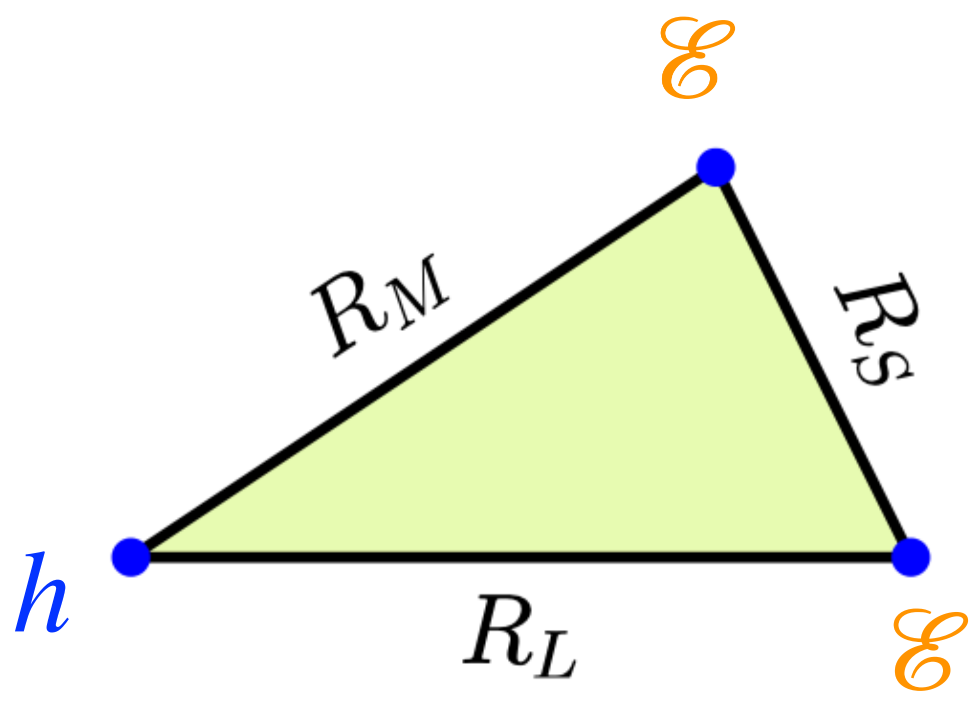
FE^{n}C and CMB

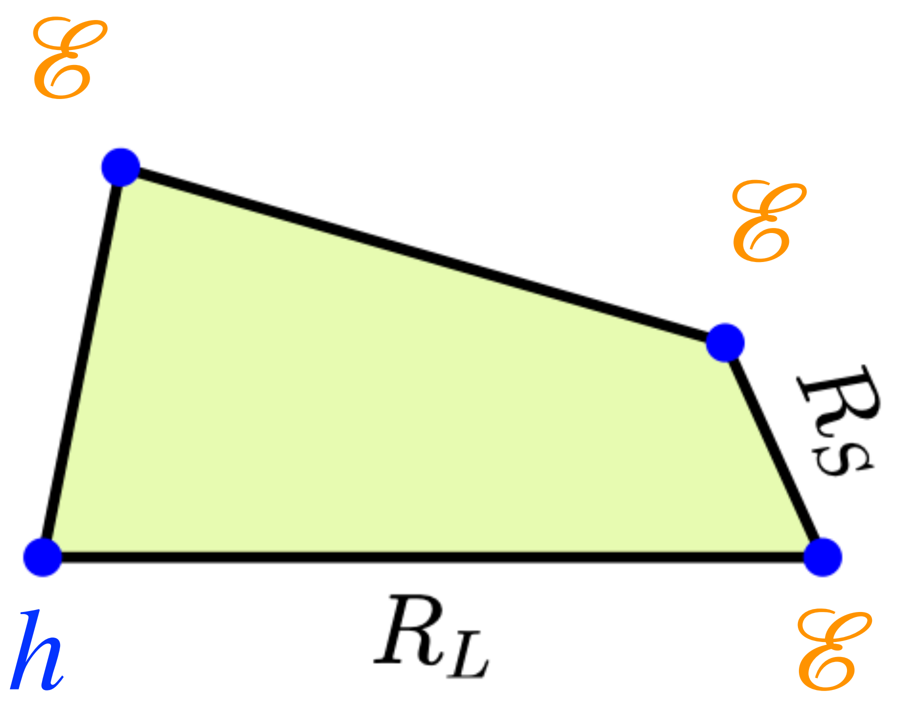
$$\mathcal{D}^{[\Gamma]}_{h/q,1}(z,\boldsymbol{n};p_h) = \frac{z}{2N_c}\sum_X \int \frac{dy^-}{2\pi} e^{ip_h^+ y^-/z}\, \mathrm{Tr}\big[\,\Gamma\langle 0|W(\infty,y^-;w)\,\psi(y^-)\,\mathcal{E}(\boldsymbol{n})\mid h,X;\ \text{out}\,\rangle$$
$$\times\langle h,X;\ \text{out}\,|\bar{\psi}(0)W^\dagger(\infty,0;w)|0\rangle\big]\,,$$

Extend to FE^{n}C

$$\mathcal{D}^{[\Gamma]}_{h/q,N}(z,\{\boldsymbol{n}_i\};p_h) = \frac{z}{2N_c}\sum_X \int \frac{dy^-}{2\pi} e^{ip_h^+ y^-/z}\ \mathrm{Tr}\Big[\Gamma\,\langle 0|\,W(\infty,y^-;w)\,\psi(y^-)$$
$$\times\,\mathcal{E}(\boldsymbol{n}_1)\cdots\mathcal{E}(\boldsymbol{n}_N)|h,X;\text{out}\rangle\langle h,X;\text{out}|\,\bar{\psi}(0)\,W^\dagger(\infty,0;w)\,|0\rangle\Big]$$







…

2201.07800

Power spectra probe background dynamics (H, ε, …)

$$\langle \zeta_{k_1}\zeta_{k_2}\rangle = (2\pi)^3 P_\zeta(k)\,\delta^3(k_1+k_2)\quad,\quad P_\zeta(k)\propto k^{n-4}$$

– *but, many different models, can produce similar power spectra*

Higher-order correlations can distinguish different models

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle = (2\pi)^3 B_\zeta(k_1,k_2,k_3)\,\delta^3(k_1+k_2+k_3)$$

– *non-Gaussianity ← non-linearity ← interactions = physics+gravity*

1004.0818