High Precision heavy boson jet substructure with Energy Correlators

(2601.20923)

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DESY.

erc
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Outline

- Motivation
- Exploiting symmetries of detector operators
- Boosting rest frame EEC distribution
- SCET derivation of the boosted Sudakov

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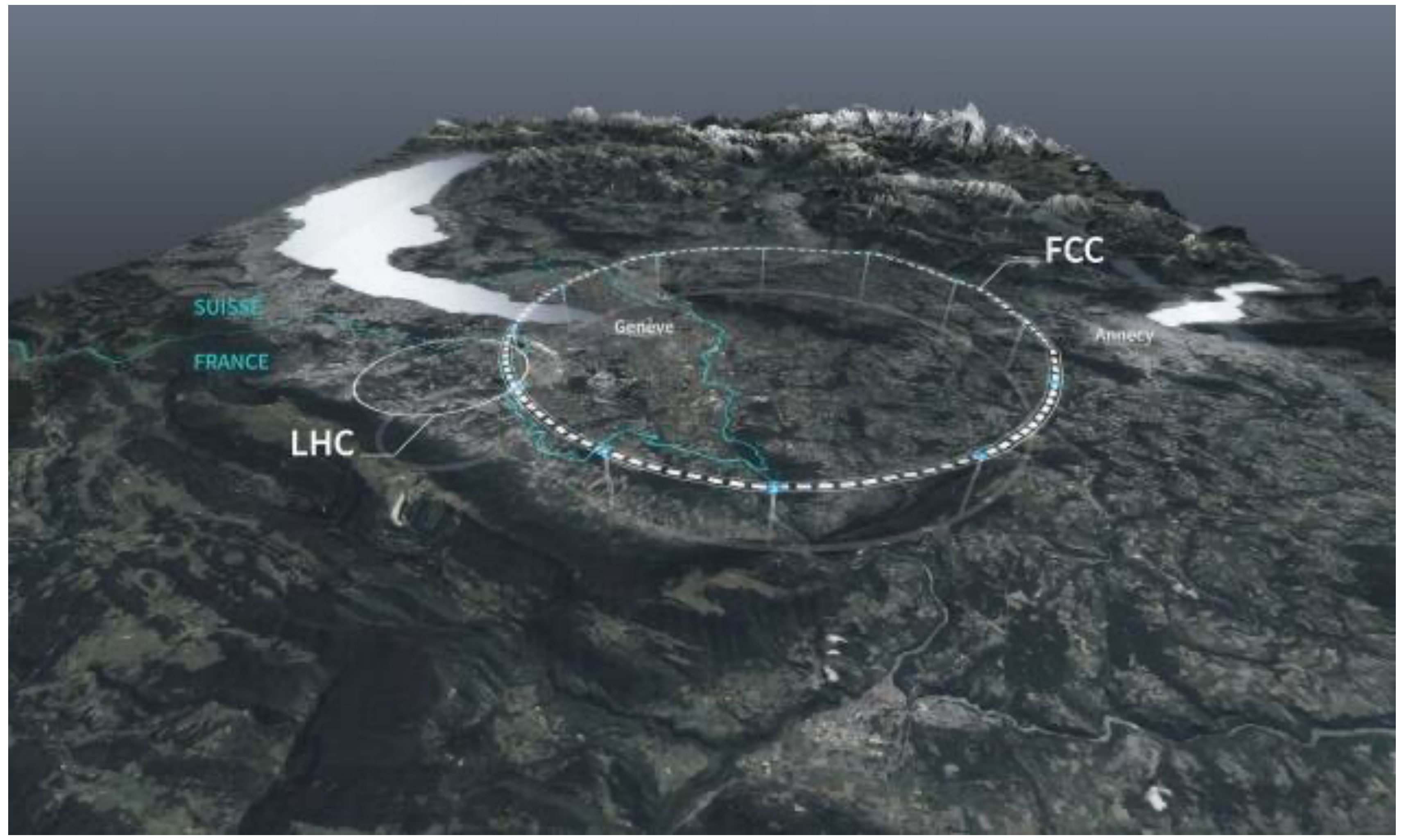
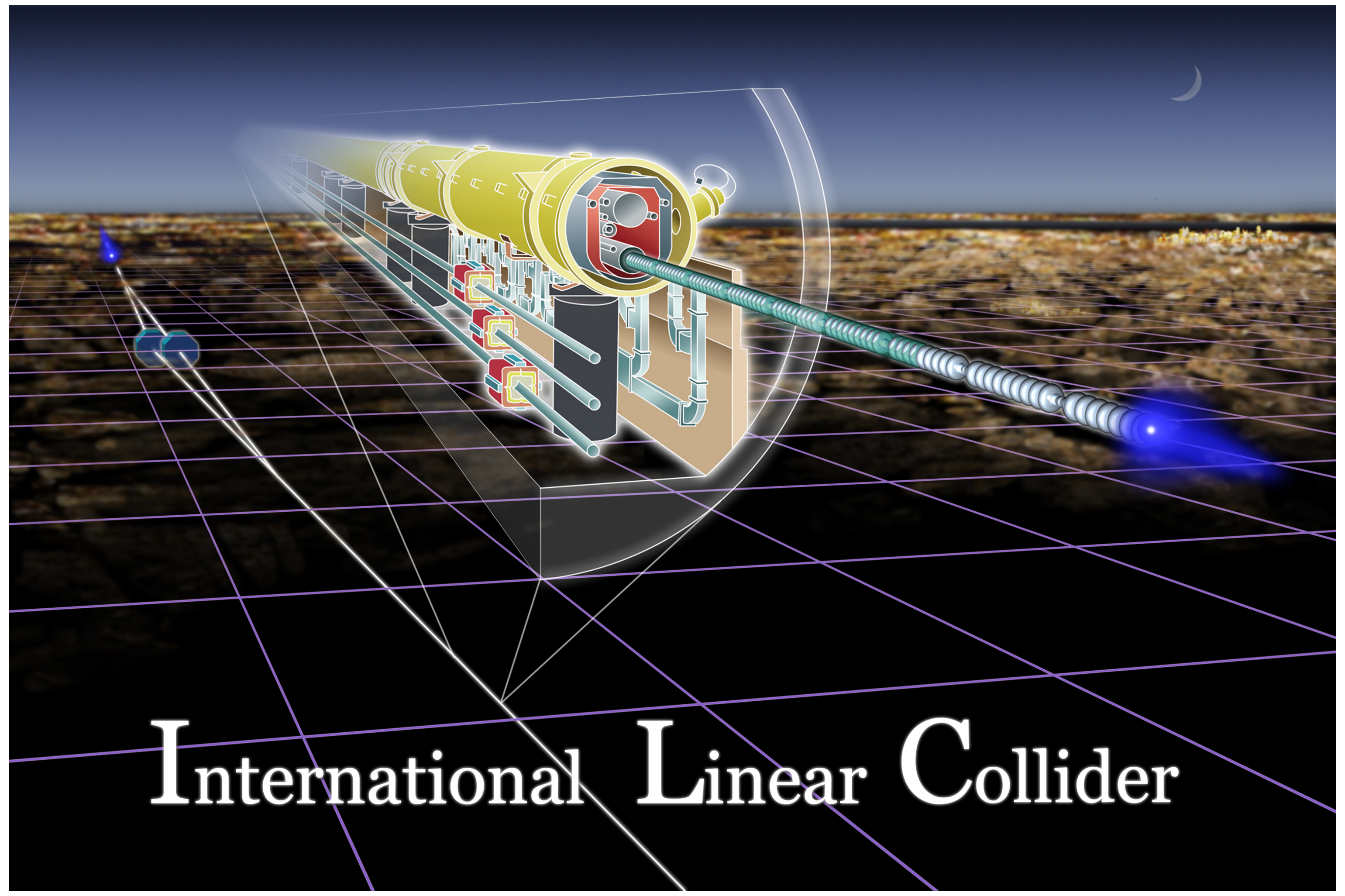
Future collider: Our only hope

“An electron-positron Higgs factory is the highest-priority next collider. For the longer term, the European particle physics community has the ambition to operate a proton-proton collider at the highest achievable energy.”

— 2020 UPDATE OF THE EUROPEAN STRATEGY FOR PARTICLE PHYSICS





Precision is key for future colliders

- Proposal to measure $\sin^2\theta_W$ and $\alpha_{\rm em}$ at FCC-ee using forward backward asymmetry $A_{\rm FB}^{\mu\mu}$ and the ratio between the number of electrons and the number of muons produced at a fixed angle θ, $\mathscr{R}_{e^-/\mu^-}(\theta)$

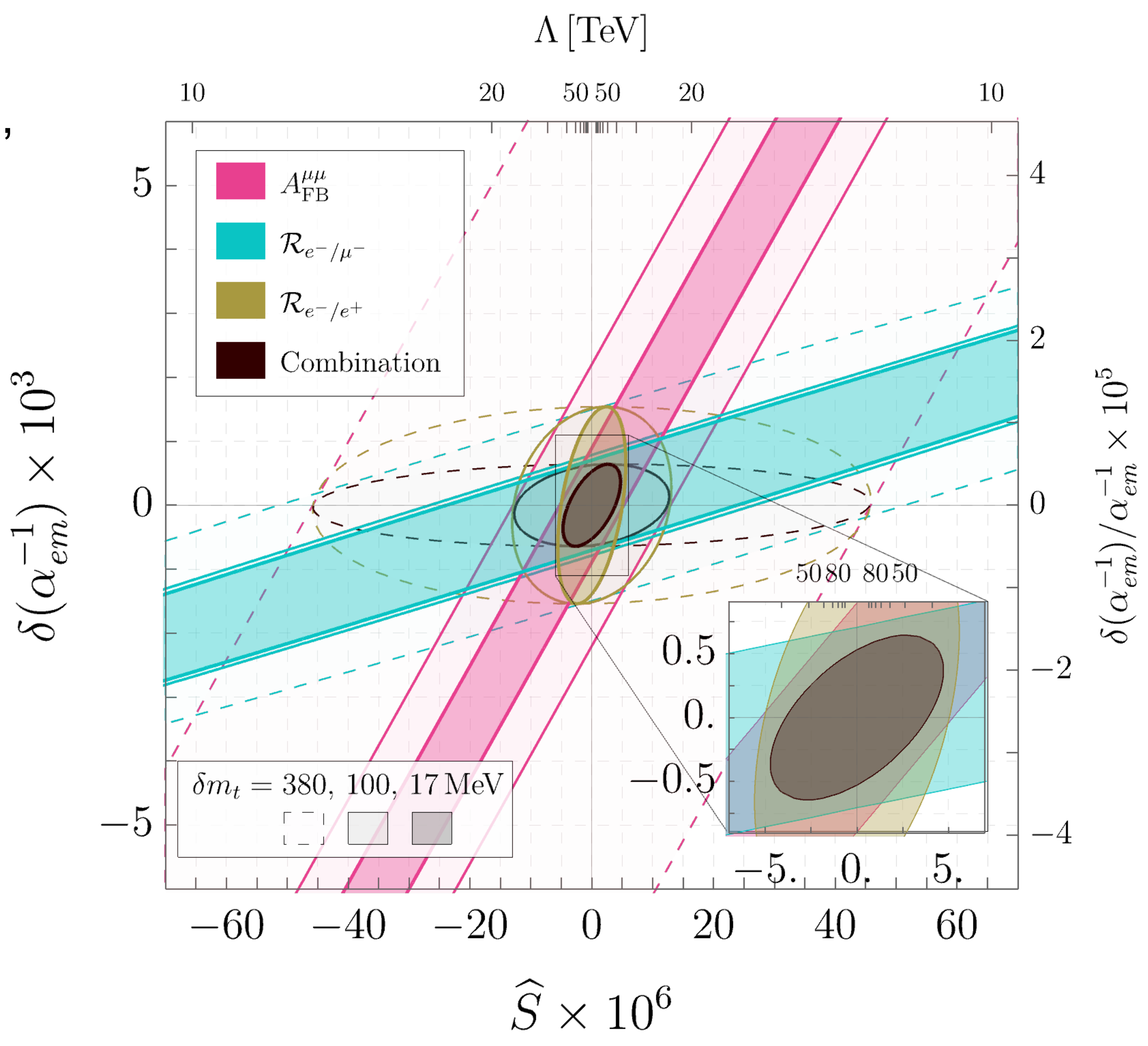
- BSM electroweak symmetry breaking at a scale Λ leave an imprint on $\sin^2\theta_W$: $\Lambda = m_W\,\widehat{S}^{-1}$

$$\frac{\delta \sin^2\theta_W^{\rm eff}}{\sin^2\theta_W^{\rm eff}} \Big/ 10^{-5} \simeq -\frac{\delta(\alpha_{\rm em}^{-1})}{10^{-3}} - \frac{\delta m_t}{65\,{\rm MeV}} + \frac{\widehat{S}}{5\times10^{-6}}$$

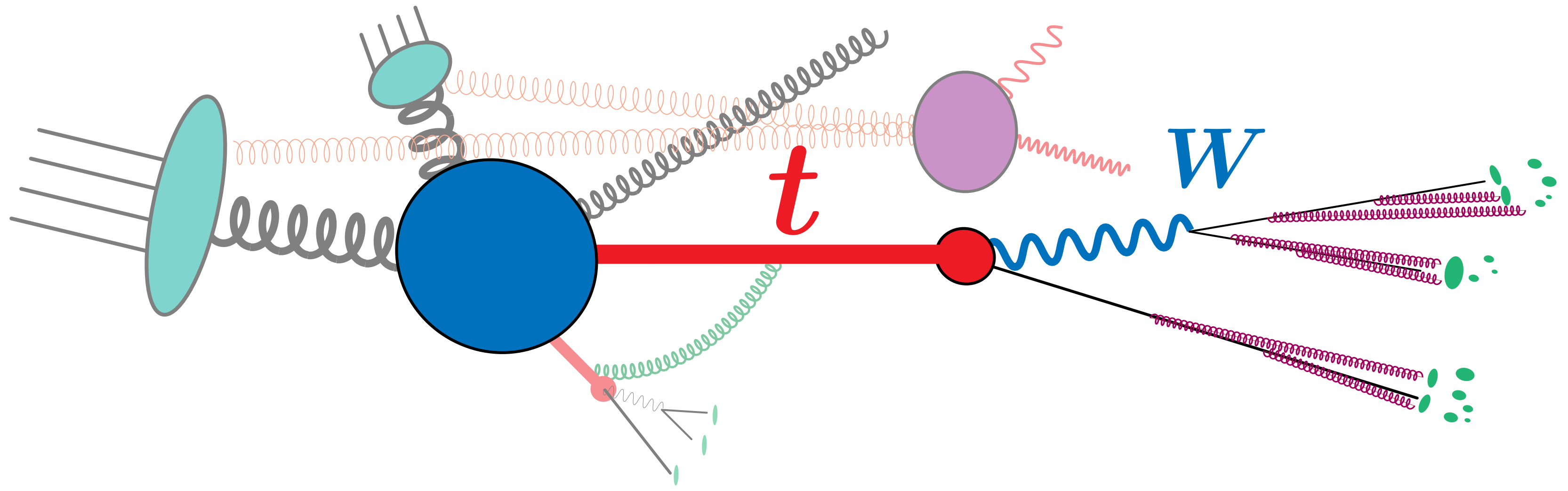
- The main bottleneck is the top mass uncertainty:
  - With $\delta m_t \sim 380\,$MeV expect sensitivity at FCCee only up to 10 TeV

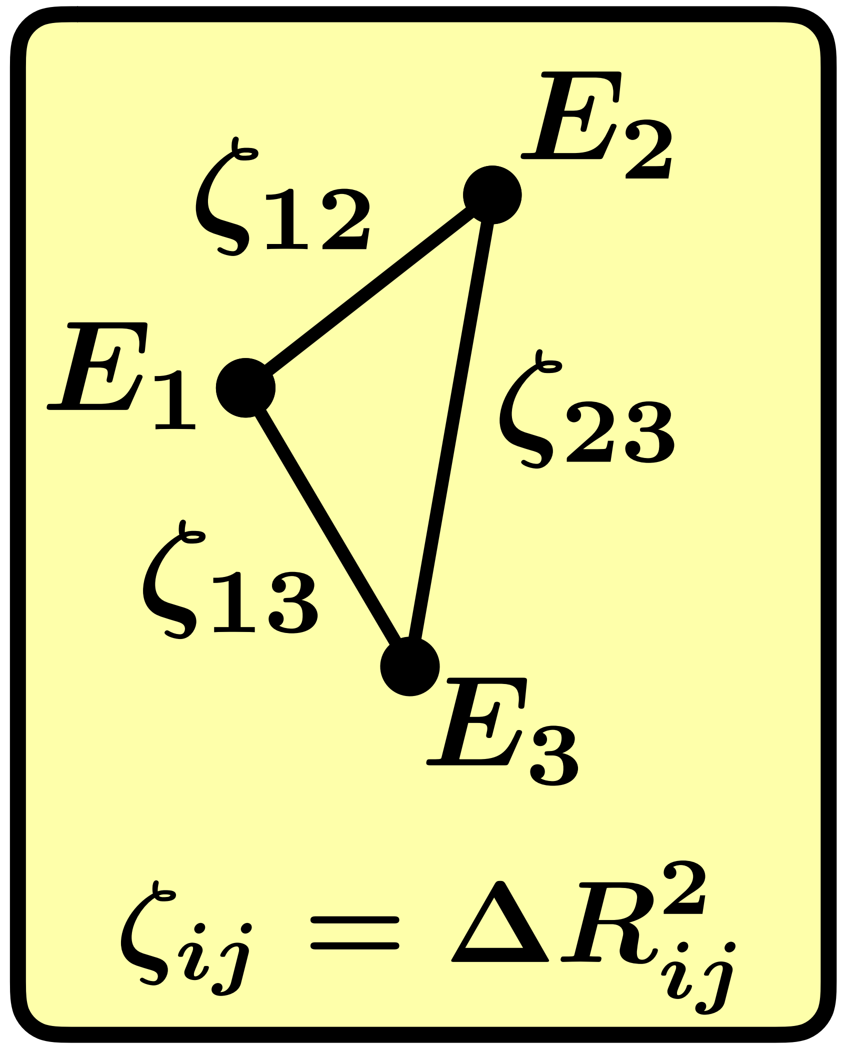
- We must bring down the top quark mass uncertainty using the HL-LHC data

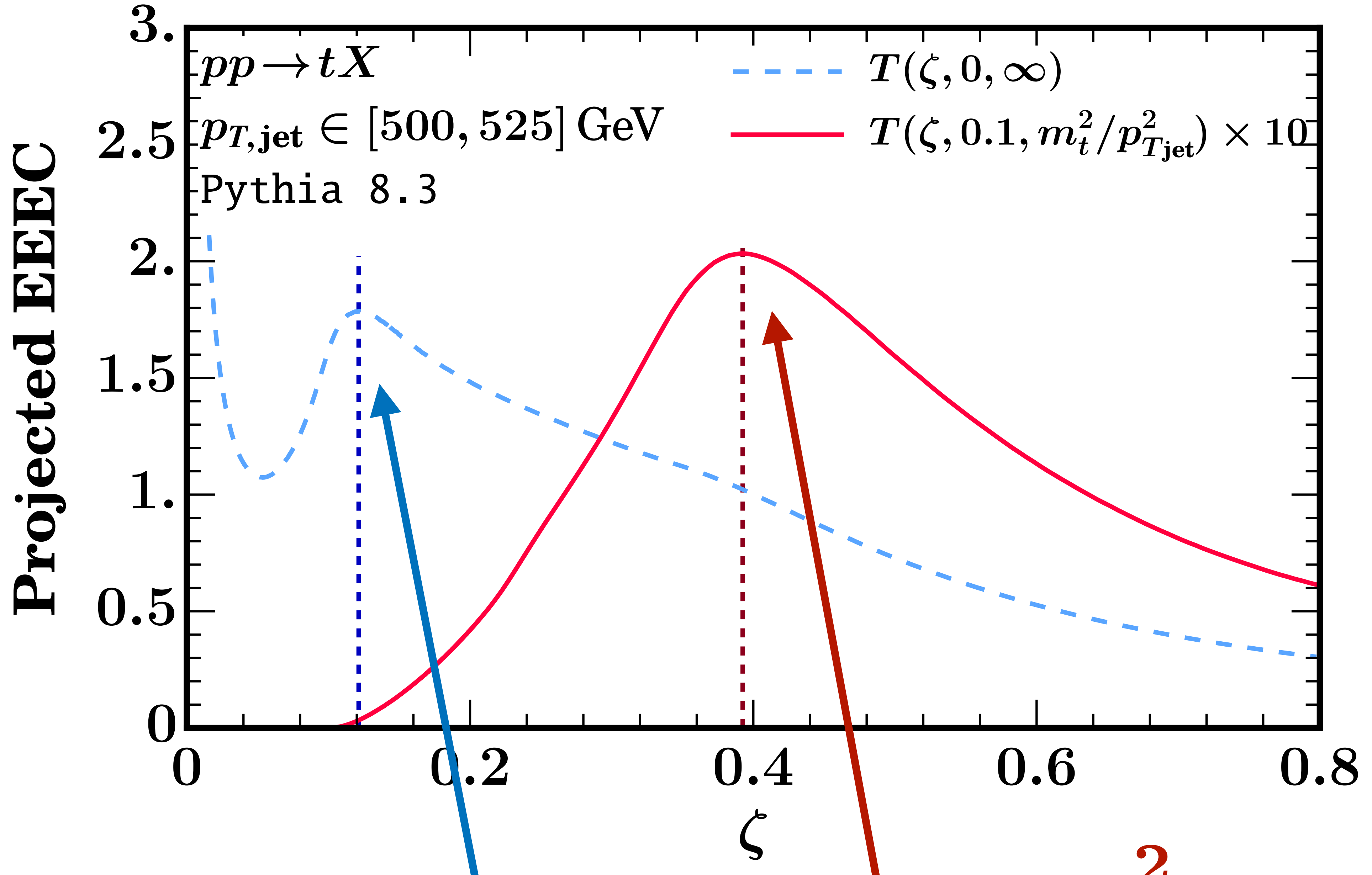
M. Riembau 2501.05508



The Standard Candle approach in nutshell







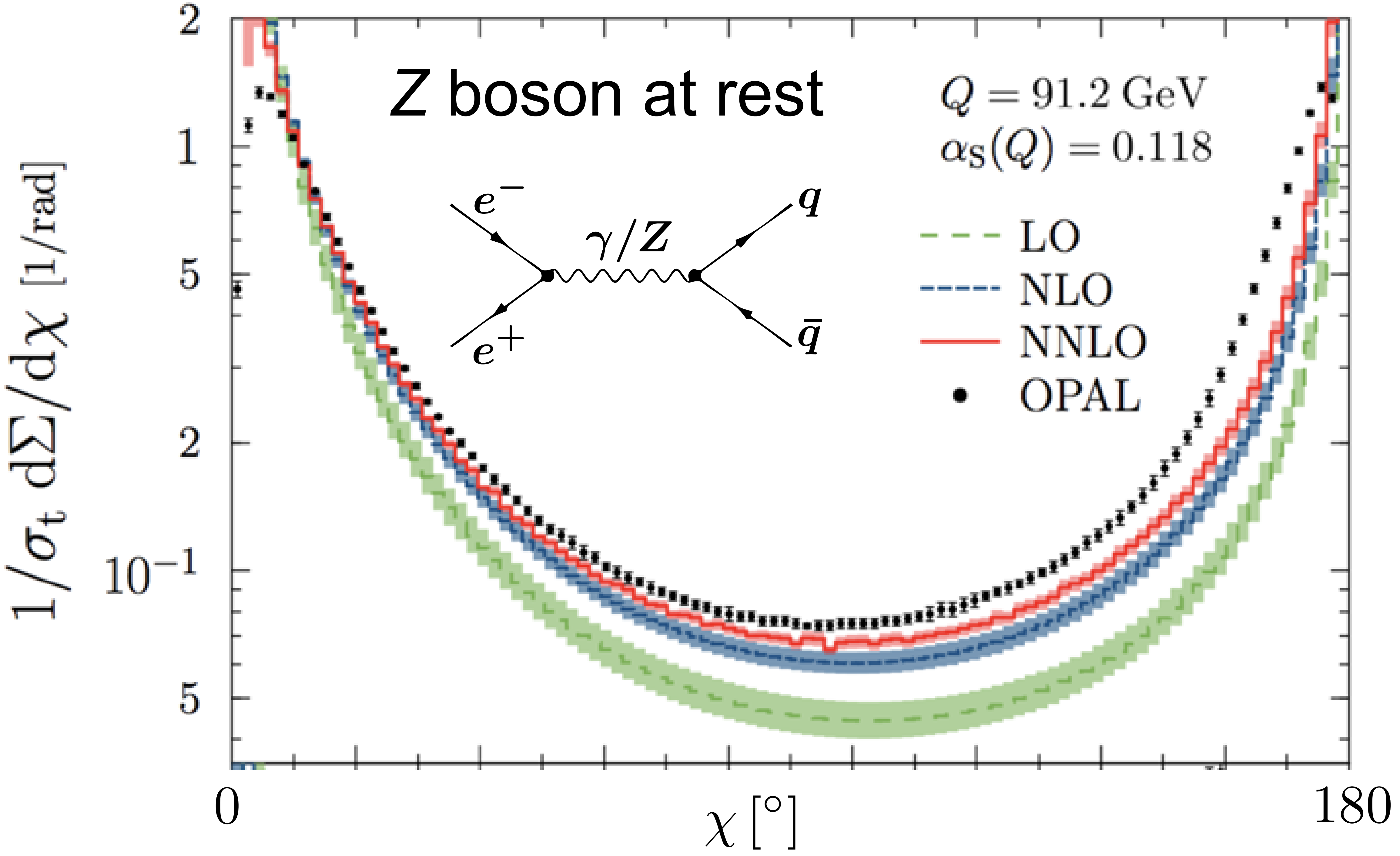
$$\zeta_W \propto \frac{m_W^2}{p_{T,\text{jet}}^2} \qquad \zeta_t \propto \frac{m_t^2}{p_{T,\text{jet}}^2}$$

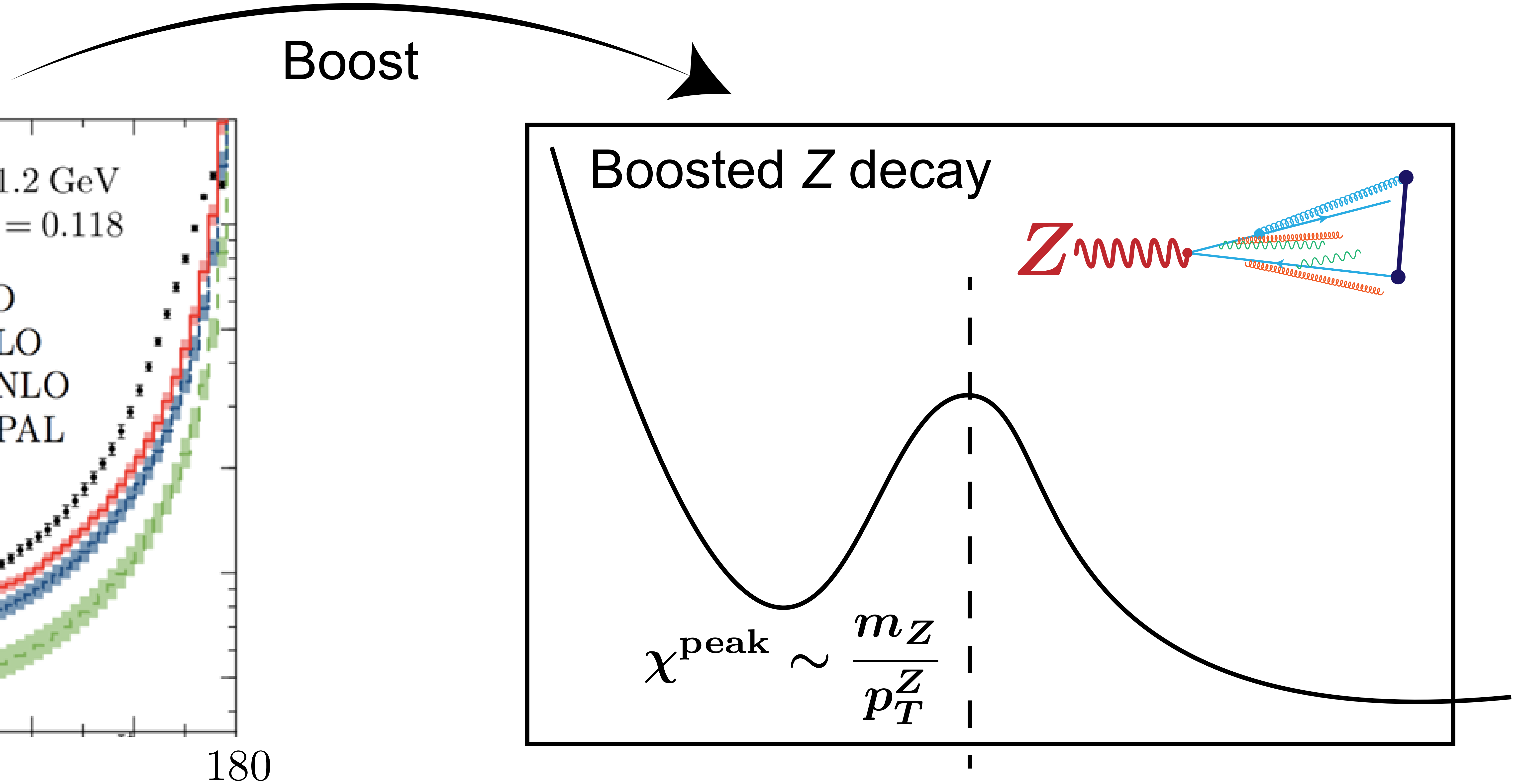
Holguin, Moult, AP, Procura, Schöfbeck, Schwarz 2023-24

$$m_t \propto m_W \sqrt{\frac{\zeta_t}{\zeta_W}}$$

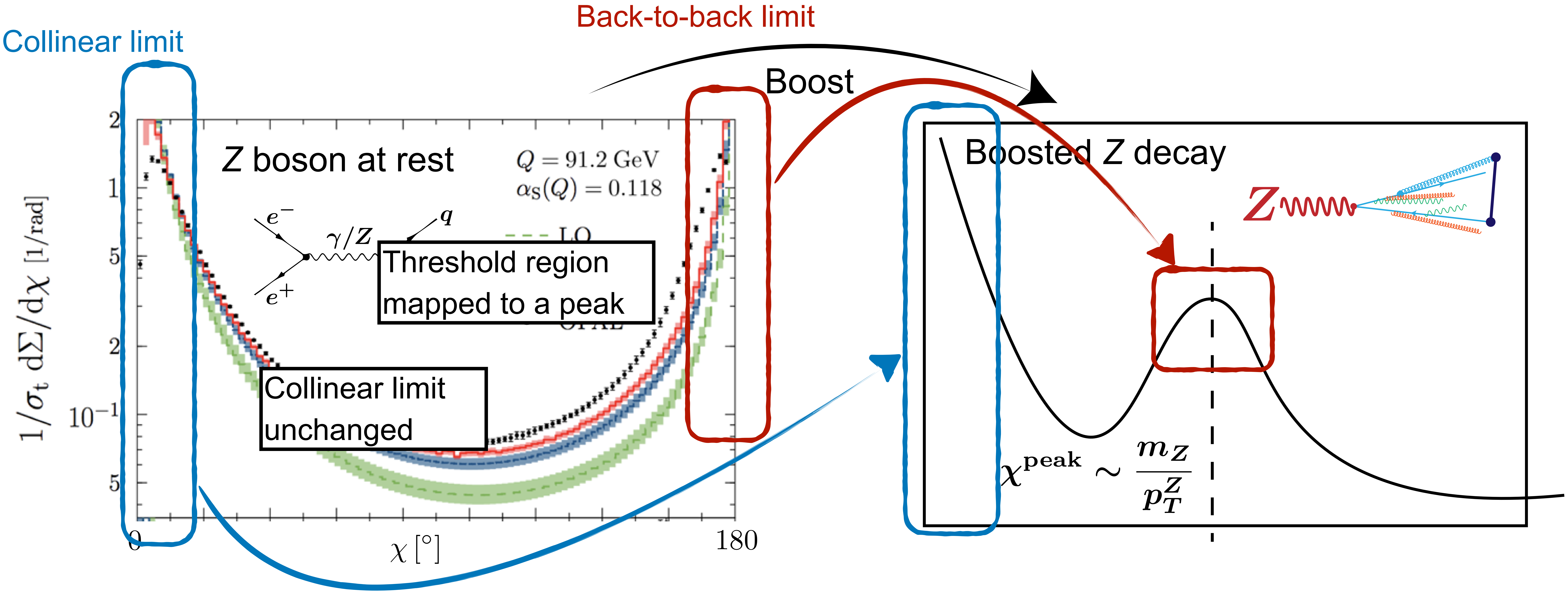
- Remove the shared energy scale
- Calibrate M_{top} using the W mass : $m_W = 80.377 \pm 0.012\,\text{GeV}$
- Exploit the W inside the top jets as a standard candle

EEC on boosted heavy resonances





EEC on boosted heavy resonances



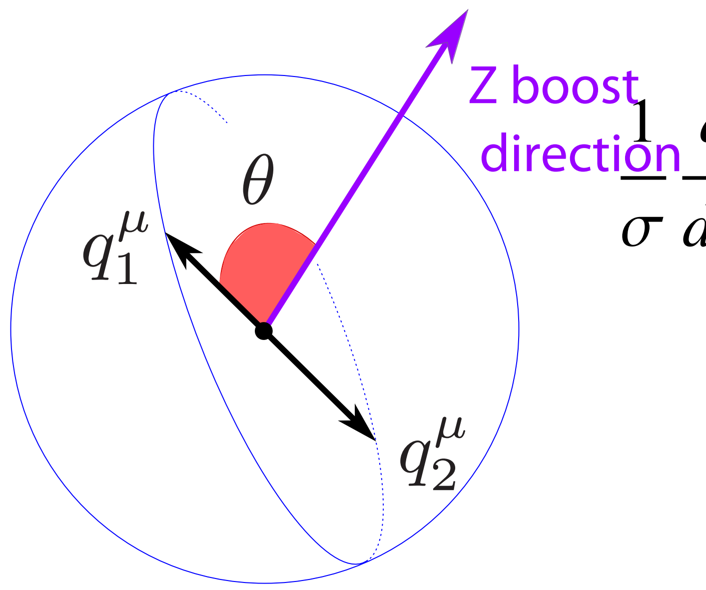
Goal of this work is to make this intuition concrete and initiate study of boosted heavy boson jet substructure with EECs.

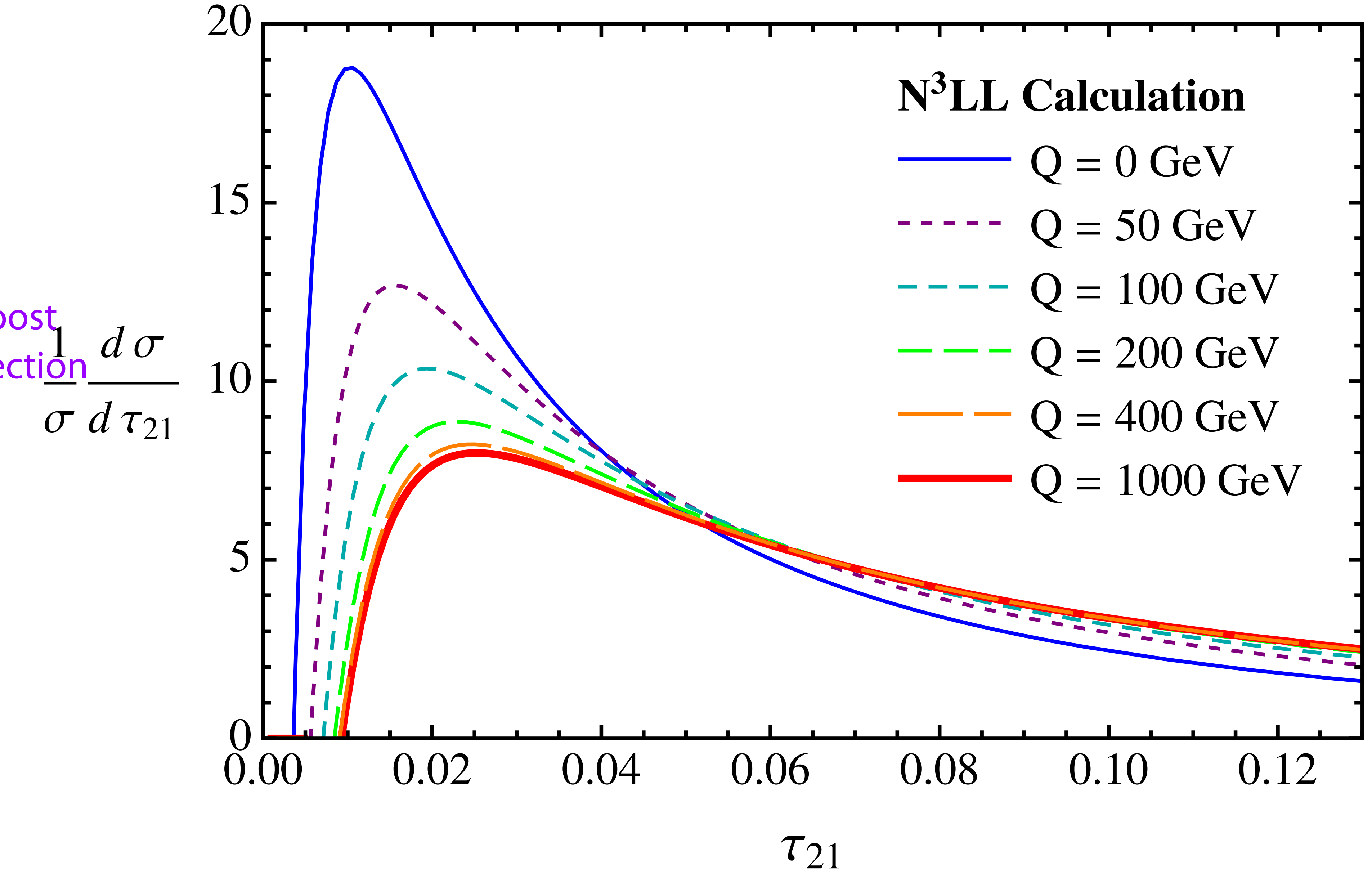
Also see A. Gao, K. Lee, X. Y. Zhang '26

Boosted event shapes

- Our work is inspired by Feige, Schwartz, Stewart, Thaler 1204.3898
- Two-jettiness measured on boosted Z jets from rest frame distribution:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_{21}} = H\int\frac{\mathrm{d}\cos\theta}{2}\int \mathrm{d}s_1\mathrm{d}s_2\mathrm{d}k_1\mathrm{d}k_2 \times S\big(k_1,k_2,\{n_i\},\mu\big)J(s_1,\mu)J(s_2,\mu) \times \delta\bigg(\tau_{21}-\frac{1}{P_Z^+}\Big(k_1+k_2+\frac{s_1}{2E_1}+\frac{s_2}{2E_2}\Big)\bigg)$$

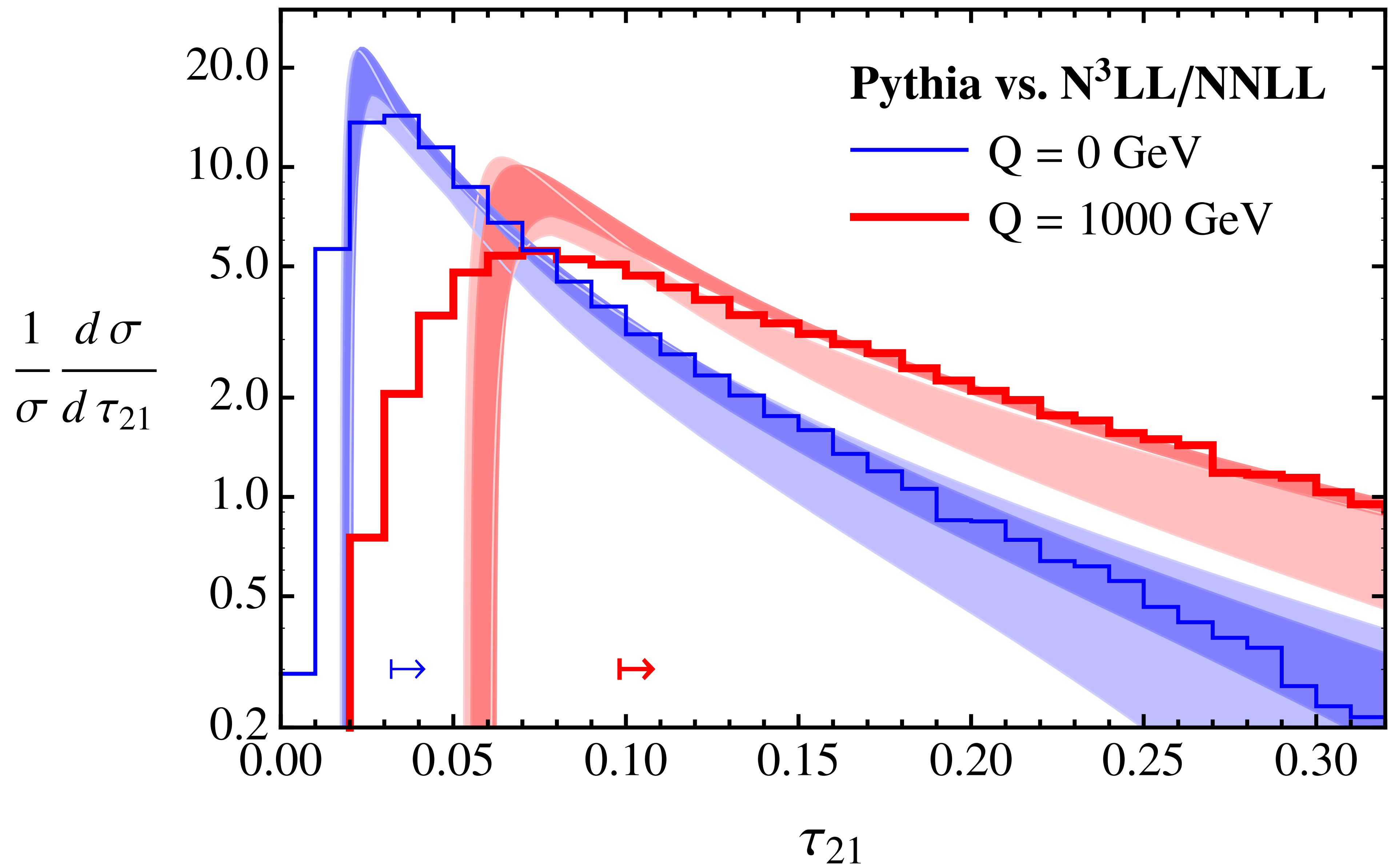




- Key ideas: universality of collinear physics + relation of the soft function to the hemisphere 2-jettiness soft function:

$$S(k_1,k_2,n_1\cdot n_2,\mu,\Lambda) = \beta_\theta^2\, S\left(\beta_\theta k_1,\beta_\theta k_2,2,\mu,\Lambda\right) = S_{\mathrm{hemi}}\left(k_1,k_2,\mu/\beta_\theta,\Lambda/\beta_\theta\right)\,,$$

$$\beta_\theta = \sqrt{\frac{2}{n_1\cdot n_2}} = \frac{\sqrt{m_Z^2+Q^2\sin^2\theta}}{m_Z},$$



- We will see that EEC admits such a boosting property for *the entire distribution*

Energy-Energy correlator

Consider boosted Zs in the $e^+e^- \to ZZ \to \text{hadrons} + \ell^+\ell^-$ process

$$\frac{d\sigma^{ee}_{\text{EEC}}(\Gamma)}{d^4q\, d\theta} = \frac{L_{\text{incl}}(q^2, P_{e^+}\cdot q, P_{e^-}\cdot q, \Gamma)}{2E^2_{\text{cm}}} \frac{1}{\left[q^2 - M_Z^2\right]^2 + \left[\Gamma_Z M_Z\right]^2} \frac{H^{ee}_{\text{EEC}}(q,\theta)}{q_0^2}.$$

q: Z momentum

weighted by energy

$$H^{ee}_{\text{EEC}}(q,\theta) = \left(\frac{q_\mu q_\nu}{M_Z^2} - g_{\mu\nu}\right) \sum\!\!\!\!\!\!\int_{X_n} \sum_{ij\in X_n} E_i E_j\, \delta(\theta - \theta_{ij}) \int d^4x\, e^{iq\cdot x} \langle 0|J_Z^{\dagger\mu}(x)|X_n\rangle\langle X_n|J_Z^\nu(0)|0\rangle,$$

Current describing hadronic Z decay

Project on unpolarized Z decay

The LHC observable: $pp \to Z + X$:

$$\frac{d\sigma_{\text{EEC}}}{d^4q\, d\chi} = \frac{W_{\text{incl}}(q^2, P_a\cdot q, P_b\cdot q)}{2E^2_{\text{cm}}} \frac{1}{\left[q^2 - M_Z^2\right]^2 + \left[\Gamma_Z M_Z\right]^2} \frac{H_{\text{EEC}}(q,\chi)}{q_T^2}.$$

For pp change $n_1 \cdot n_2 \to \chi = \Delta R$ and $E_iE_j \to p_{T_i}p_{T_j}$

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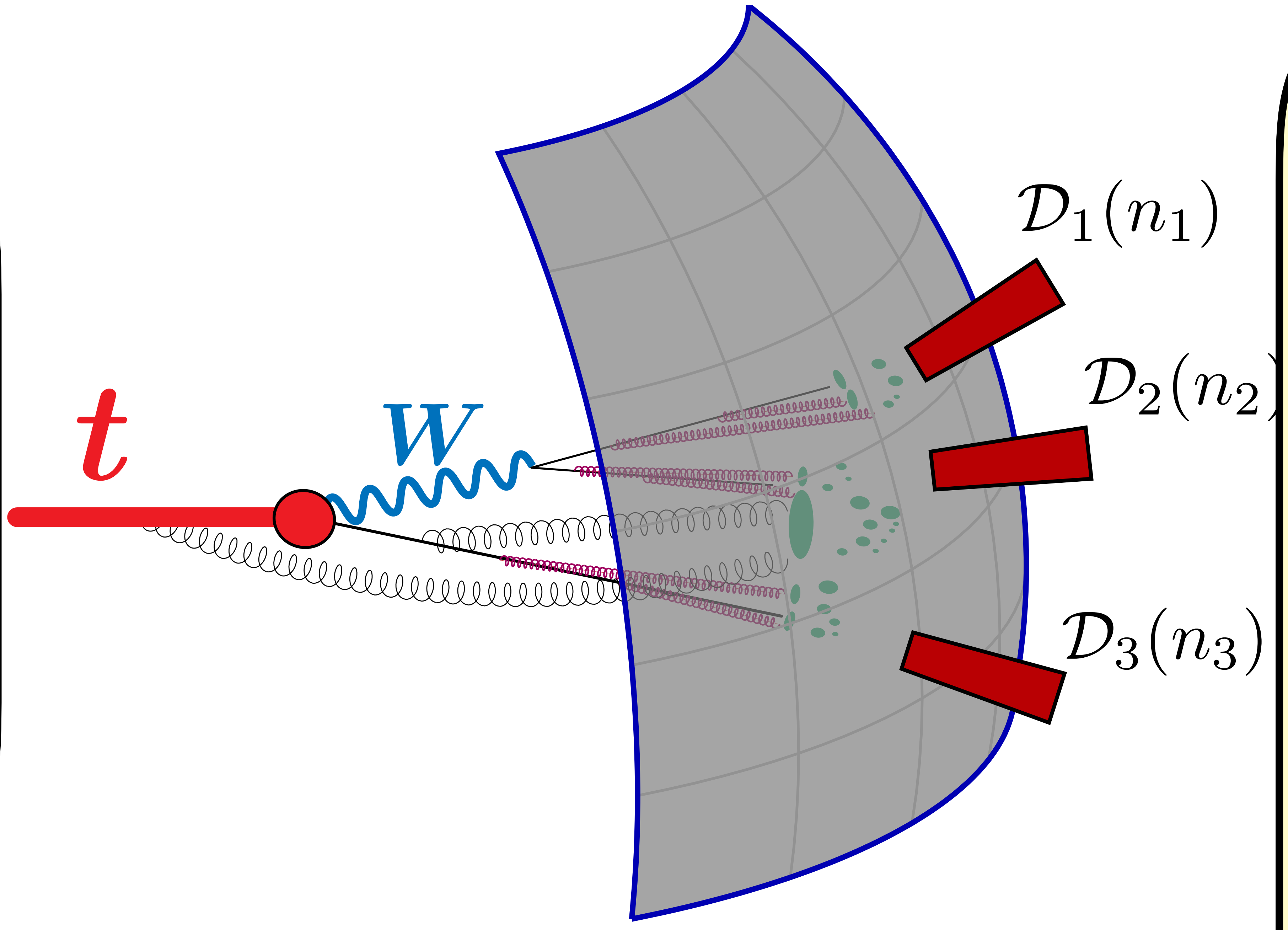
A new approach to boosted jet substructure

The bulk perspective :

- Compute scattering amplitudes and observables as a function of particle momenta
- Integrate over the phase space to get the cross section



The screen perspective :

- Consider observables that can be thought of as assigning a weight factor $w(X)$
- Represent the weights using operators living at celestial infinity → "Detector operators" $\mathscr{D}_i(n_i)$
- Express observables as correlation functions of sources and detector operators

q: Momentum of the resonance decaying into X

$$\frac{\mathrm{d}\sigma(q)}{\mathrm{d}w} = \sum_X (2\pi)^4 \delta^{(4)}(q - k_X)\delta(\hat{w}(X) - w)\left|\mathcal{M}_{\mathcal{O}\to X}\right|^2$$

differential in some observable w

Some local operator or current that couples the resonance to hadronic stuff

$$\mathrm{d}\sigma_w \equiv \sum_X (2\pi)^4 \delta^{(4)}(q - k_X)\hat{w}(X)\left|\mathcal{M}_{\mathcal{O}\to X}\right|^2$$

weighted by w

$$= \int \mathrm{d}^4x\, e^{iq\cdot x}\langle 0|\mathcal{O}^\dagger(x)\hat{w}(X)\mathcal{O}(0)|0\rangle$$

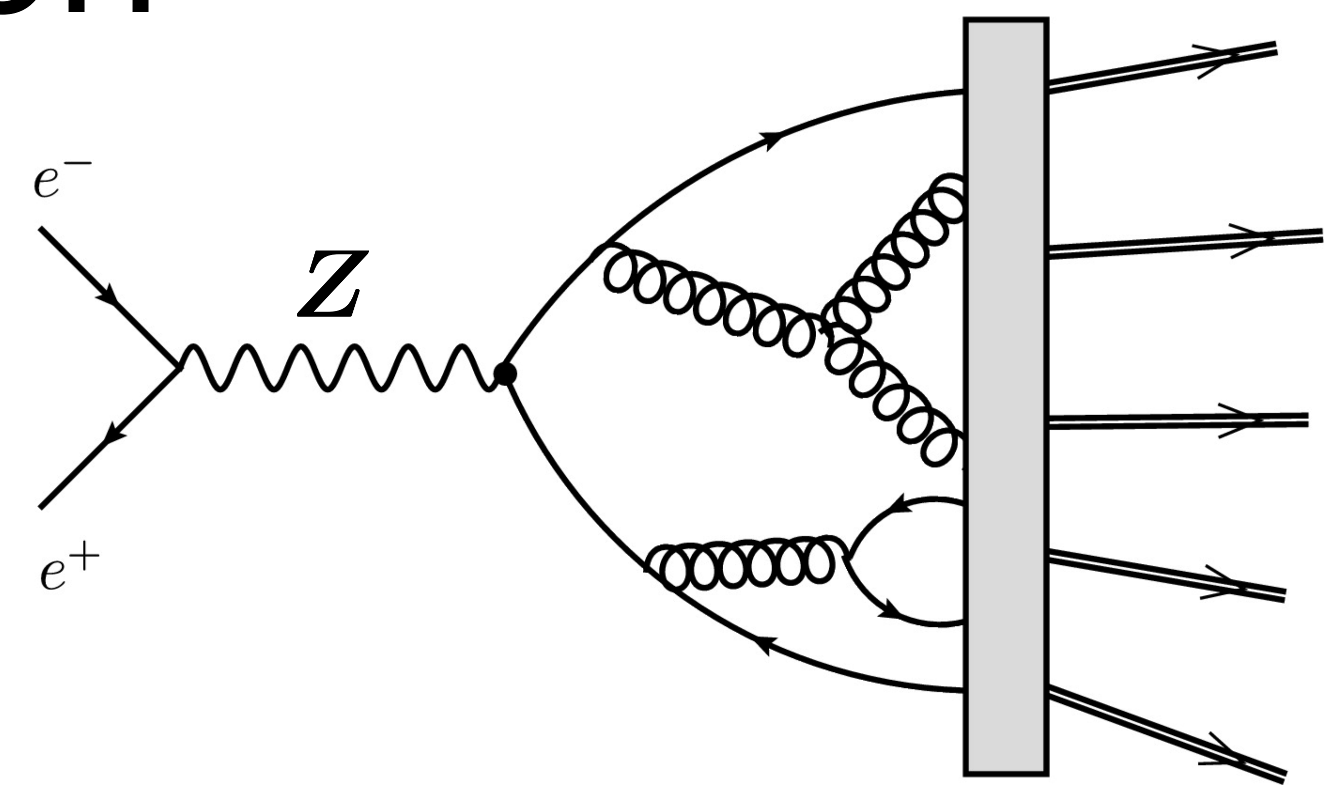
Compose the weights in terms of detector operators

$$\hat{w} \sim \int \mathcal{D}_1(n_1)\mathcal{D}_2(n_2)\ldots\mathcal{D}_N(n_N)$$

Belitsky, et al. Nuclear Physics B 884 (2014) 206–256

Observable as a correlation function

- The energy weighted hadronic tensor H_{EEC} also describes $e^+e^- \to Z \to q\bar{q}$ process
- Rephrase as a correlation function:



$$H^{ee}_{\text{EEC}}(q,\theta) = \left(\frac{q_\mu q_\nu}{M_Z^2} - g_{\mu\nu}\right) \int \mathrm{d}^4x\, e^{\mathrm{i}q\cdot x} \langle 0 | J_Z^{\dagger\mu}(x) \sum_{X_n} |X_n\rangle\langle X_n| \sum_{ij\in X_n} E_i E_j\; J_Z^\nu(0)|0\rangle\; \delta(\theta - \theta_{ij}),$$

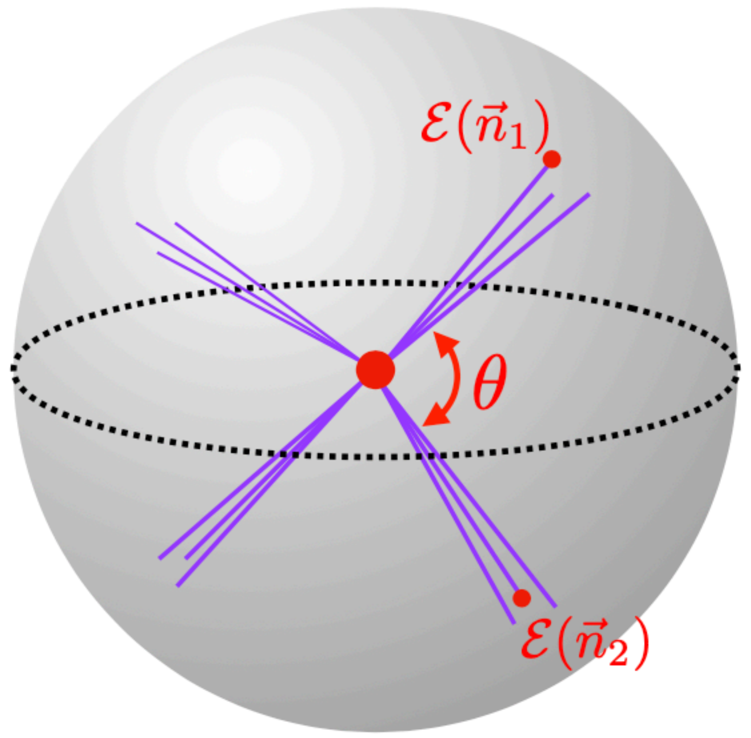
$$= \left(\frac{q_\mu q_\nu}{M_Z^2} - g_{\mu\nu}\right) \int \mathrm{d}^2\boldsymbol{n}_1 \mathrm{d}^2\boldsymbol{n}_2 \int \mathrm{d}^4x\, e^{\mathrm{i}q\cdot x} \langle 0 | J_Z^{\dagger\mu}(x) \mathcal{E}(n_1)\mathcal{E}(n_2) J_Z^\nu(0)|0\rangle\; \delta\left(\theta - \theta_{12}\right)$$

Represent the EEC using energy flow operators

$$\boldsymbol{\mathcal{E}(n)|X_n\rangle = \sum_{i\in X_n} E_i \delta^{(2)}(n - n_i)|X_n\rangle}$$

This is an example of a detector operator

n : the direction where this "detector" is placed



Defining the detector operators (free theory)

- For a free massless scalar,

S. C. Huot, M. Kologlu, P. Kravchuk, D. Meltzer, D. Simmons-Duffin 2209.00008

$$H \propto \int \mathrm{d}^3 \boldsymbol{p}\, a^\dagger(\boldsymbol{p}) a(\boldsymbol{p}) = \int \mathrm{d}^2 \boldsymbol{n} \int_0^\infty \mathrm{d}E\, E^2\, a^\dagger(E\boldsymbol{n}) a(E\boldsymbol{n}) \equiv \int \mathrm{d}^2 \boldsymbol{n}\, \mathcal{E}(\boldsymbol{n})$$

- One can generalize this to define an operator measuring E^{J-1} flux:

$$\mathcal{E}(n)|X_n\rangle = \sum_{i \in X_n} E_i \delta^{(2)}(n - n_i)|X_n\rangle$$

$$\mathcal{E}_J(n) \propto \int_0^\infty \mathrm{d}E\, E^J a^\dagger(En) a(En)$$

- Define $n^\mu = (1, \vec{n})$. Under Lorentz boost along the direction $\vec{n}$, we have $n \to \rho n$. The powers of ρ under boost define the collinear spin J_L:

$$\mathcal{E}_J(\rho n) = \rho^{-J-1} \mathcal{E}_J(n)$$

- For $\mathscr{E}_J(\vec{n})$ we have mass dimension, $-\Delta_L = J - 1$, collinear spin, $J_L = -1 - J$

- For energy flow ($J = 2$) we have $J_L = -3$

Detector operators in interacting theory

- Instead of using creation-annihilation operators, define the detectors in terms of light-ray operators:

Spin of the local operator

Kravchuk, Simmons-Duffin 2018

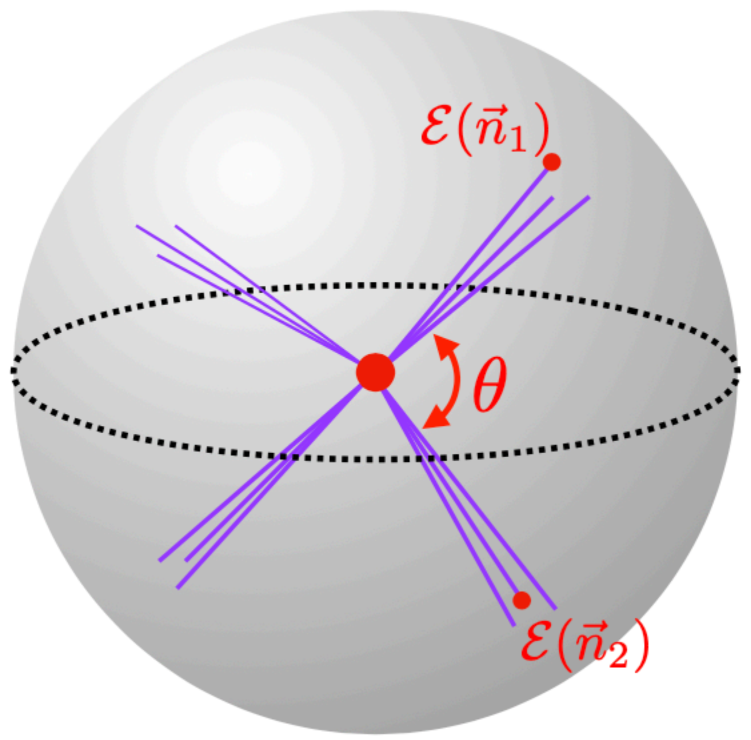
$$\mathcal{O}_J(x,\bar{n}) \sim \phi(x)\partial^{\mu_1}\partial^{\mu_2}\ldots\partial^{\mu_J}\phi(x)\bar{n}_{\mu_1}\bar{n}_{\mu_2}\ldots\bar{n}_{\mu_J}$$

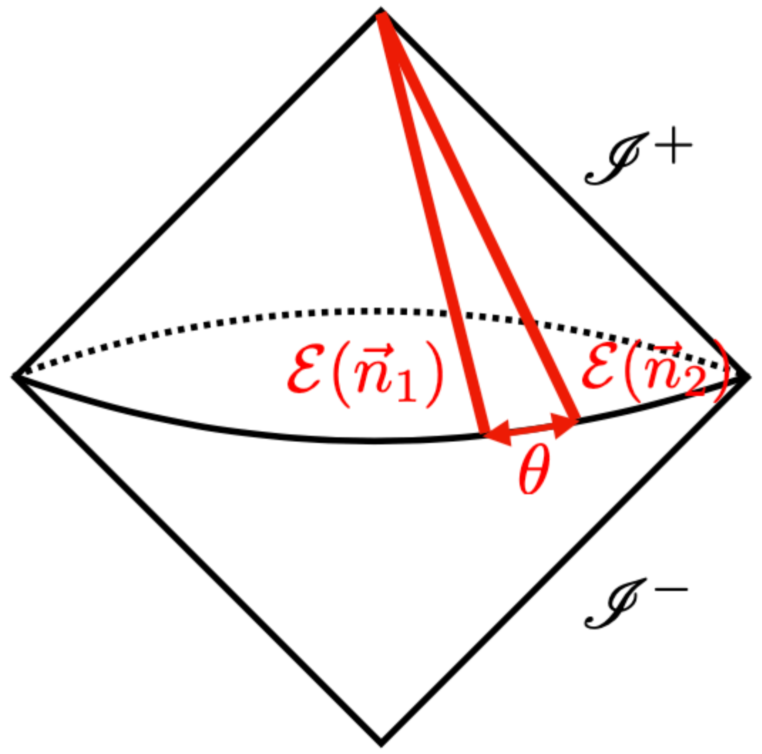
Contract Lorentz indicies with $\bar{n}^\mu$

Integral of a local operator $\mathcal{O}_J$ along null ray

$$\mathcal{E}_J(n) = 2\mathbf{L}[\mathcal{O}_J](\infty, n)$$

The starting point of the integration
(Here at spatial infinity)





For energy flow operator $J = 2$ this is equivalent to:

$$\mathcal{E}(n) = \lim_{\bar{n}\cdot x\to\infty}\left(\frac{x\cdot\bar{n}}{n\cdot\bar{n}}\right)^2\int_{-\infty}^{\infty} \mathrm{d}(x\cdot n)\ \frac{\bar{n}_\mu\bar{n}_\nu}{(n\cdot\bar{n})^2}T^{\mu\nu}(x),$$

Energy momentum tensor

- For $J \neq 2$ interactions renormalize the detector dimension Δ_L: $\Delta_L = \Delta_{L,0}(J_L) + \gamma_L(J_L)$
- The detector spin J_L remains exact nonperturbatively

S. C. Huot, M. Kologlu, P. Kravchuk, D. Meltzer, D. Simmons-Duffin 2209.00008

Why this fancy rigmarole?

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- Correlation functions have nice theory properties:
  - Not IR divergent like scattering amplitudes
  - Easier to capture their symmetry properties
  - Can give new insights completely missed out in the bulk picture

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle_q \equiv \frac{1}{\sigma_0}\left(\frac{q_\mu q_\nu}{M_Z^2} - g_{\mu\nu}\right)\int \mathrm{d}^4x\, e^{\mathrm{i}q\cdot x}\, \langle 0 \big| J_Z^{\dagger\mu}(x)\, \mathcal{E}(n_1)\mathcal{E}(n_2)\, J_Z^\nu(0)\big| 0\rangle$$

- Recall the re-parameterization symmetry of the operator: $\boldsymbol{\mathcal{E}(\rho n) = \rho^{-3}\mathcal{E}(n)}$

- **Constraints the correlator to a unique form:**

Dimensionless EEC Shape function with no collinear spin

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle_q = \frac{q^2 \mathcal{F}_{\mathcal{E}\mathcal{E}}\left(\bar{z},\ M_Z^2/q^2,\ \Lambda_{\mathrm{QCD}}^2/q^2\right)}{4\pi^2 (n_1\cdot n_2)^3},$$

Power fixed by the dimensions

Form fixed by the collinear spin of $\mathscr{E}(n)$

A unique Lorentz invariant combination of q^μ, n_1^μ, and n_2^μ invariant under rescaling $n_i \to \rho_i n_i$:

$$\bar{z} = \frac{q^2\, n_1\cdot n_2}{2\, q\cdot n_1\ q\cdot n_2},$$

(In a CFT): Belitsky, et al. Nuclear Physics B 884 (2014) 206–256

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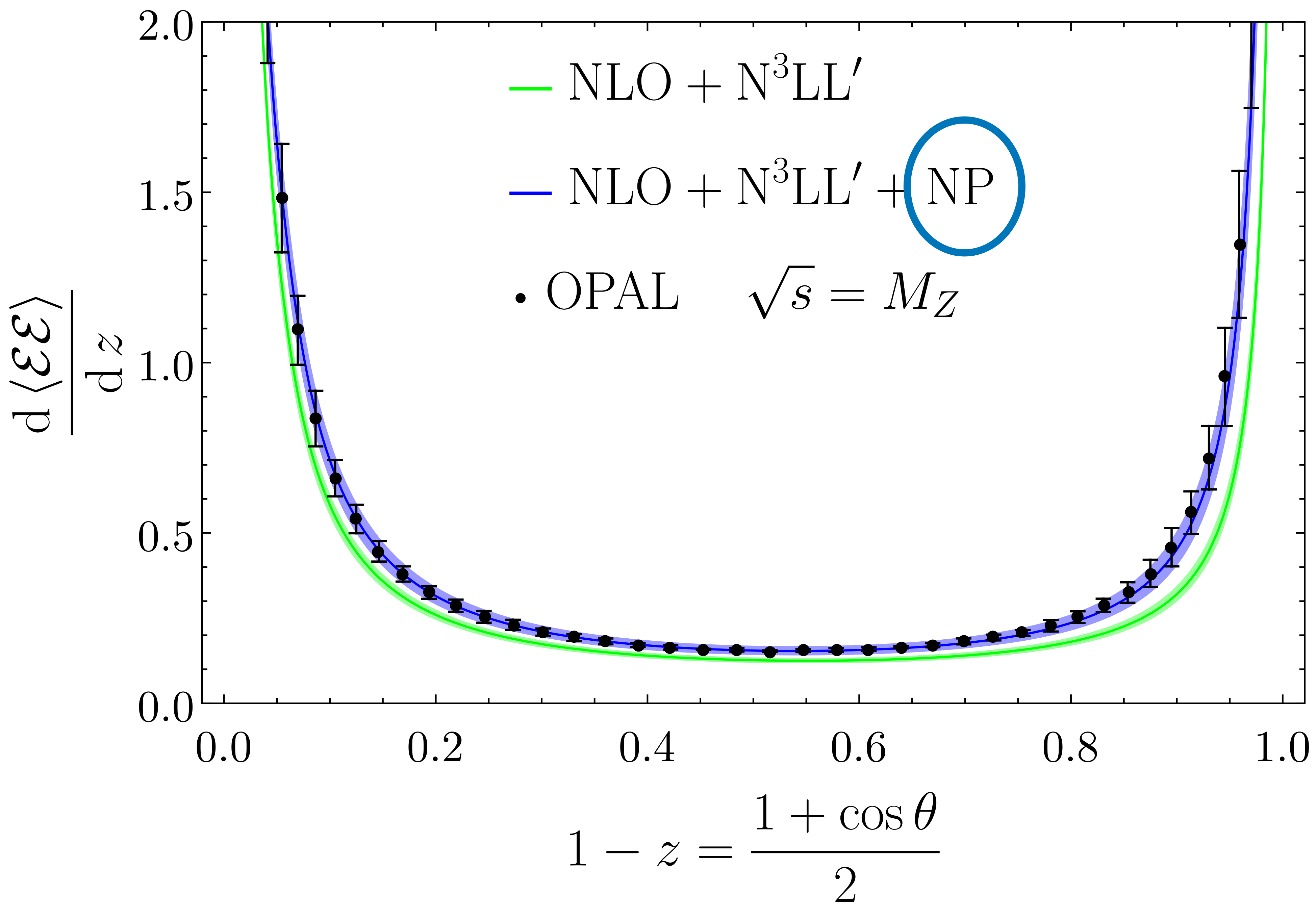
EEC from boosting

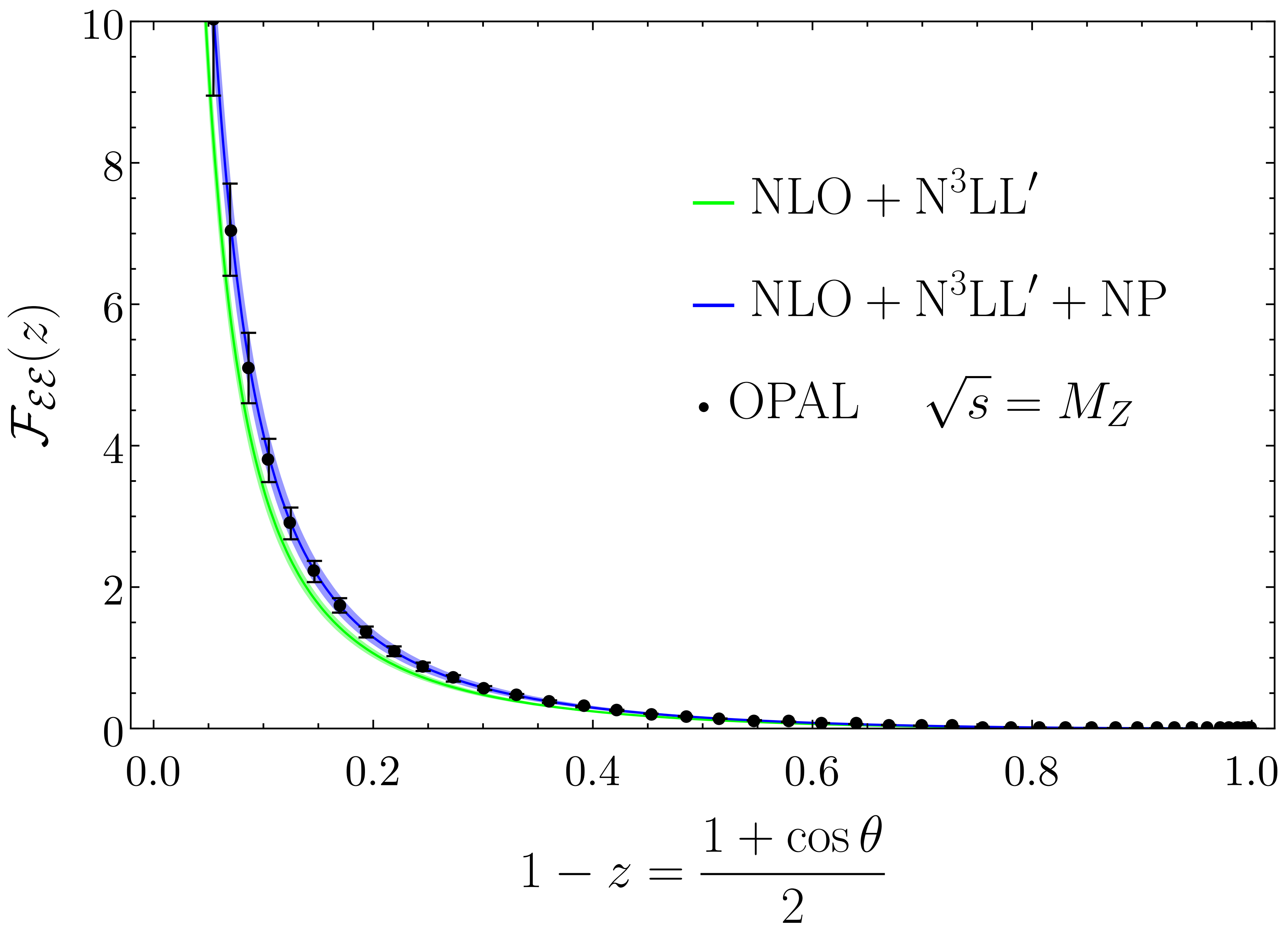
Holguin, Moult, AP Procura, Sule 2601.20923

- In the rest frame of the Z we have $\bar{z} = z = (1 - \cos\theta)/2$

- Use high precision calculation of EEC in the Z boson rest frame to get the EEC shape function

$$\mathcal{F}_{\mathcal{E}\mathcal{E}}\left(z, \frac{\Lambda^2_{\mathrm{QCD}}}{M_Z^2}\right) = \frac{8z^3}{M_Z^2} \int \mathrm{d}^2\boldsymbol{n}_1 \mathrm{d}^2\boldsymbol{n}_2 \, \langle \mathcal{E}(n_1)\mathcal{E}(n_2) \rangle_{Z\text{-pole}} \, \delta(z - n_1 \cdot n_2/2)$$





Nonperturbative corrections: We include the linear Ω_1 effect in the fixed order region and use the dispersive model in the back-to-back region.

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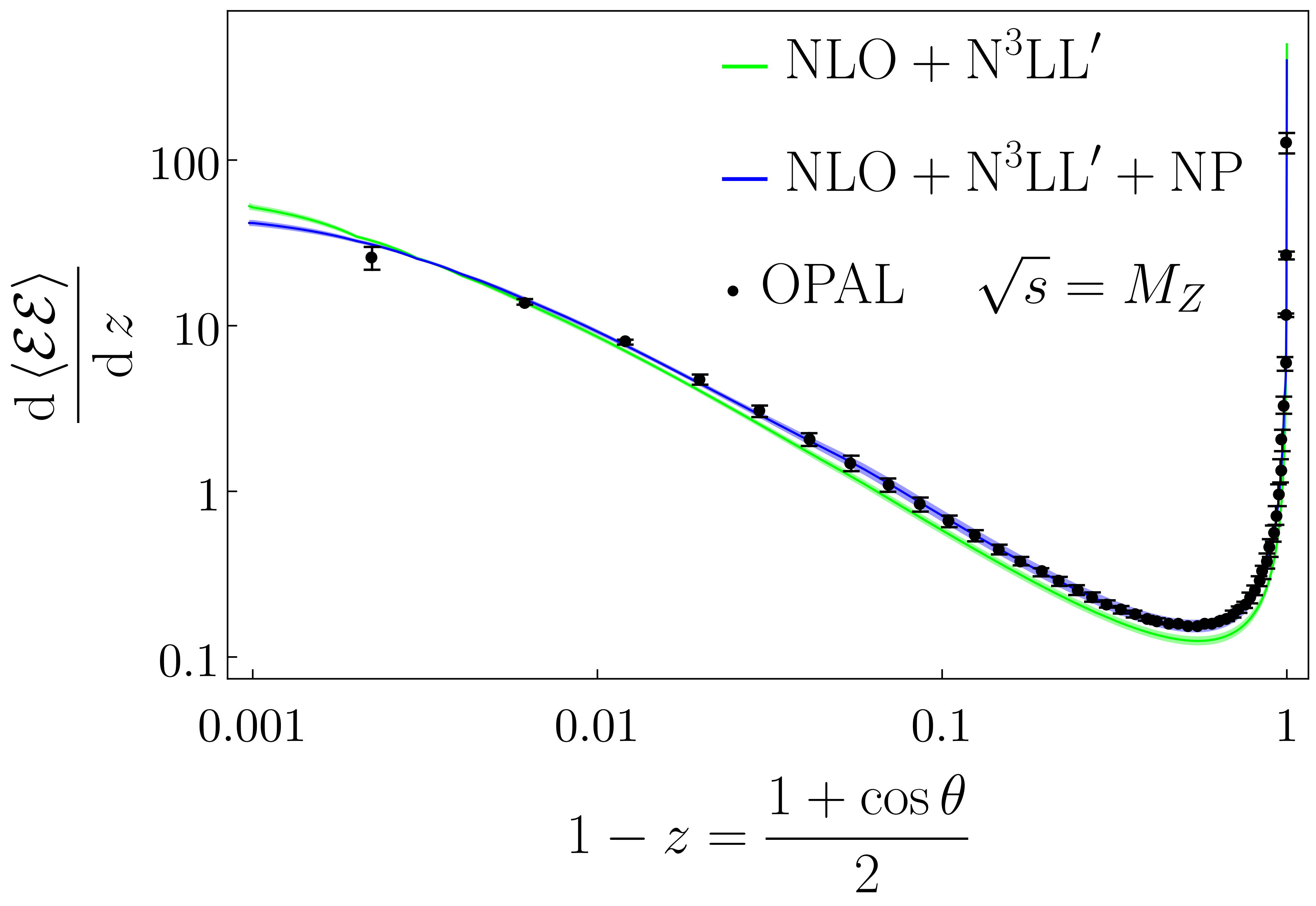

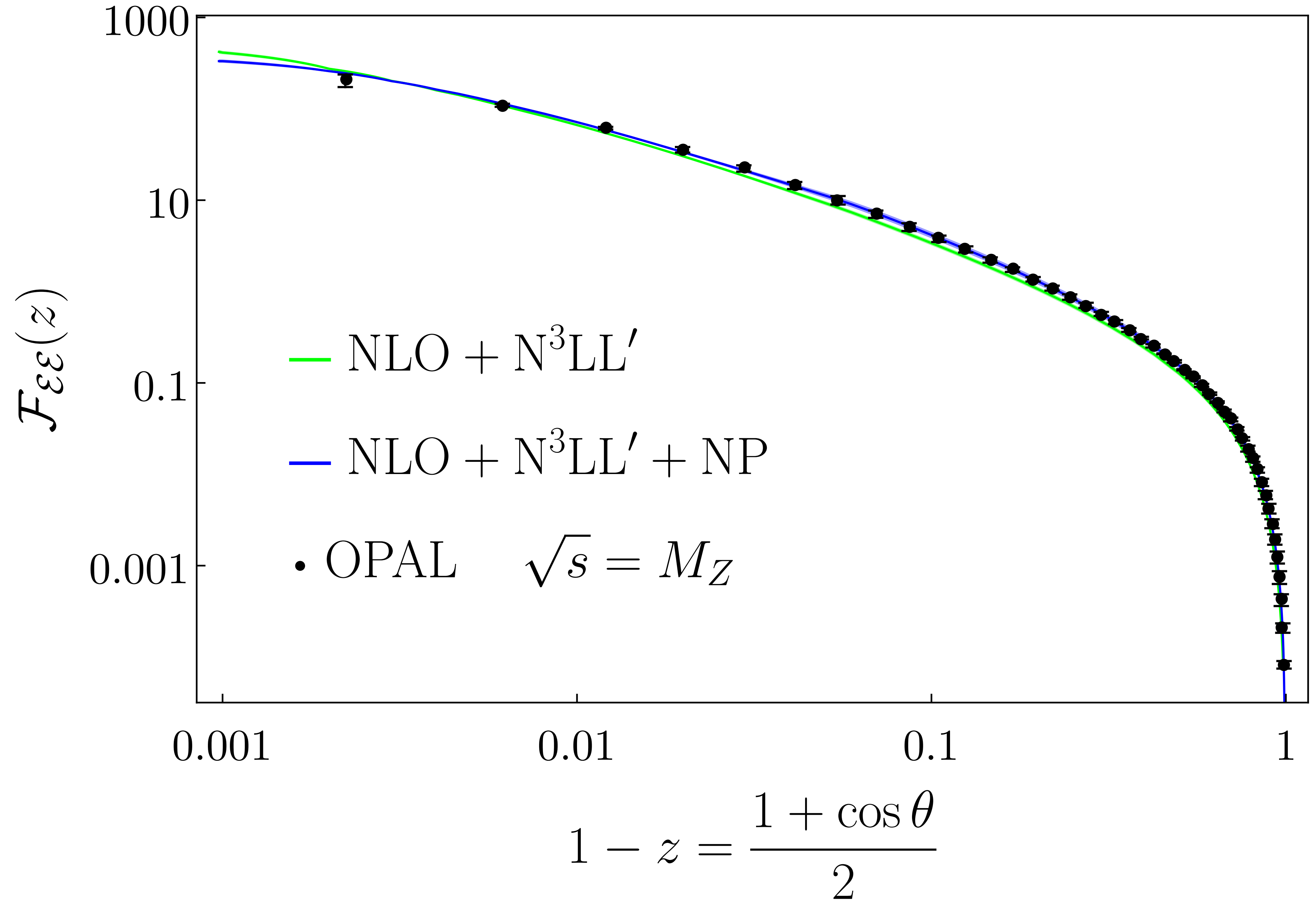

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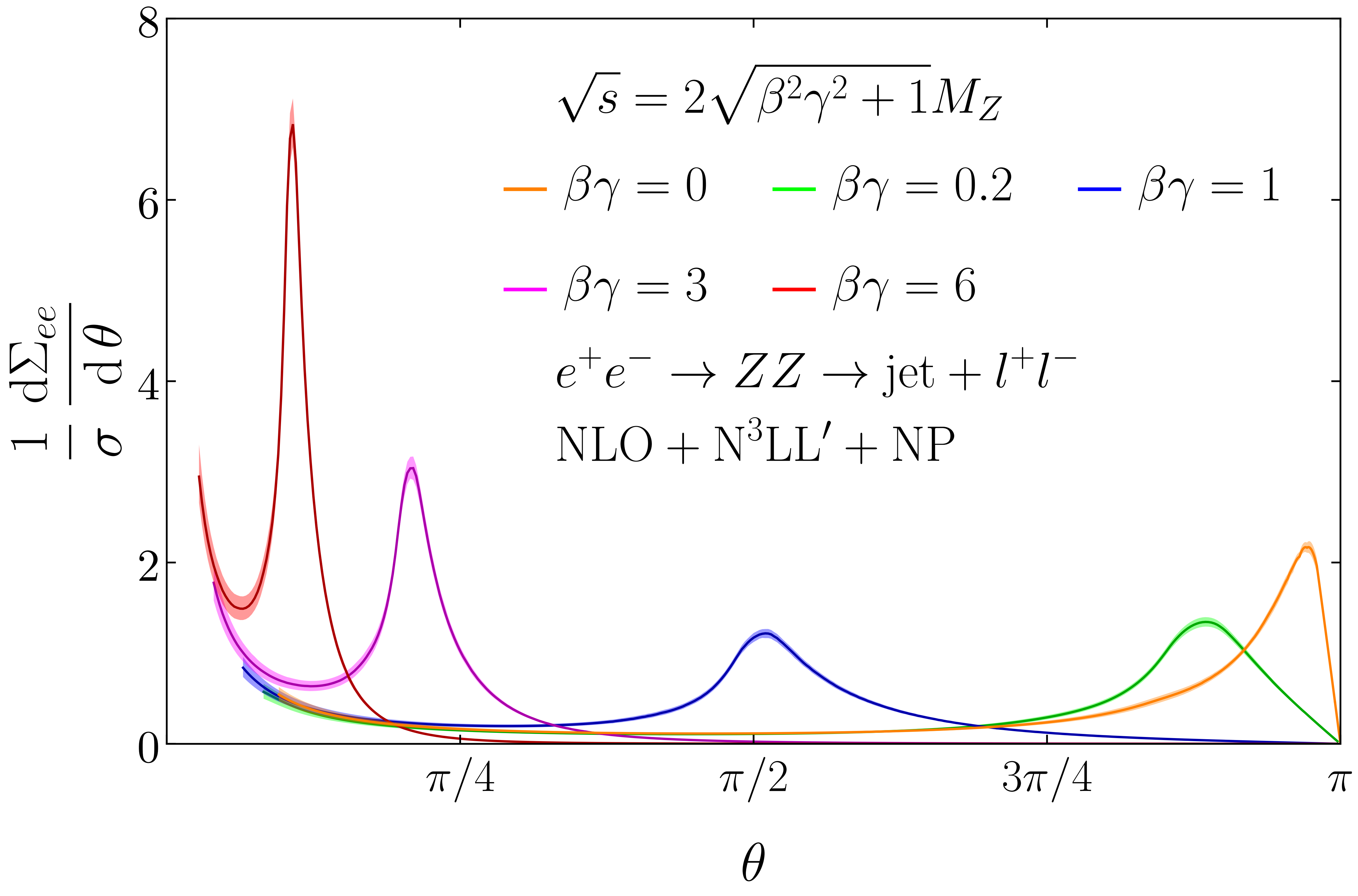
EEC from boosting

Holguin, Moult, AP Procura, Sule 2601.20923

- Numerically compute the EEC for the boosted frame by modifying the argument of the EEC shape function

$$\frac{d\Sigma_{ee}(\Gamma)}{d\theta} = N \int d^2\boldsymbol{n} \int d^2\boldsymbol{n}_1 d^2\boldsymbol{n}_2 \, \frac{L_{\rm incl}(\boldsymbol{n}) \, M_Z^2}{4\pi^2 q_0^2 \, (n_1 \cdot n_2)^3} \, \mathcal{F}_{\mathcal{E}\mathcal{E}}\left(\frac{M_Z^2 \, n_1 \cdot n_2}{2\, q \cdot n_1 \, q \cdot n_2}\right) \, \delta(\theta - \theta_{12}) \, ,$$



- Compute this for $ee \to ZZ \to \text{hadrons} + \ell^+\ell^-$ for $E_{\rm cm} = 2Q = 2q^0$ and $q^2 = M_Z^2$

- Average over the Z boson direction $\vec{n}$ relative to the rest frame

- Peak location determined solely by the boost: $z_{\rm peak} \approx 1/(\beta\gamma)^2 = \left(M_Z/p_T^Z\right)^2$

EEC from boosting

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$$\frac{\mathrm{d}\Sigma_{ee}(\Gamma)}{\mathrm{d}\theta} = N \int \mathrm{d}^2\boldsymbol{n} \int \mathrm{d}^2\boldsymbol{n}_1 \mathrm{d}^2\boldsymbol{n}_2 \; \frac{L_{\mathrm{incl}}(\boldsymbol{n})\; M_Z^2}{4\pi^2 q_0^2\; (n_1 \cdot n_2)^3} \; \mathcal{F}_{\mathcal{E}\mathcal{E}}\left(\frac{M_Z^2\; n_1 \cdot n_2}{2\, q \cdot n_1\;\, q \cdot n_2}\right) \; \delta\left(\theta - \theta_{12}\right) \,,$$

- Restrict EEC only on jet constituents in simulations

- Boosted OPAL data agrees well with simulations. Jet radius effects only visible after $z > 0.06$

EEC from boosting

- Computation for pp collisions require integrating over a p_T^Z bin. Extract $\mathcal{N}(p_T^Z, \eta_Z)$ from Herwig.

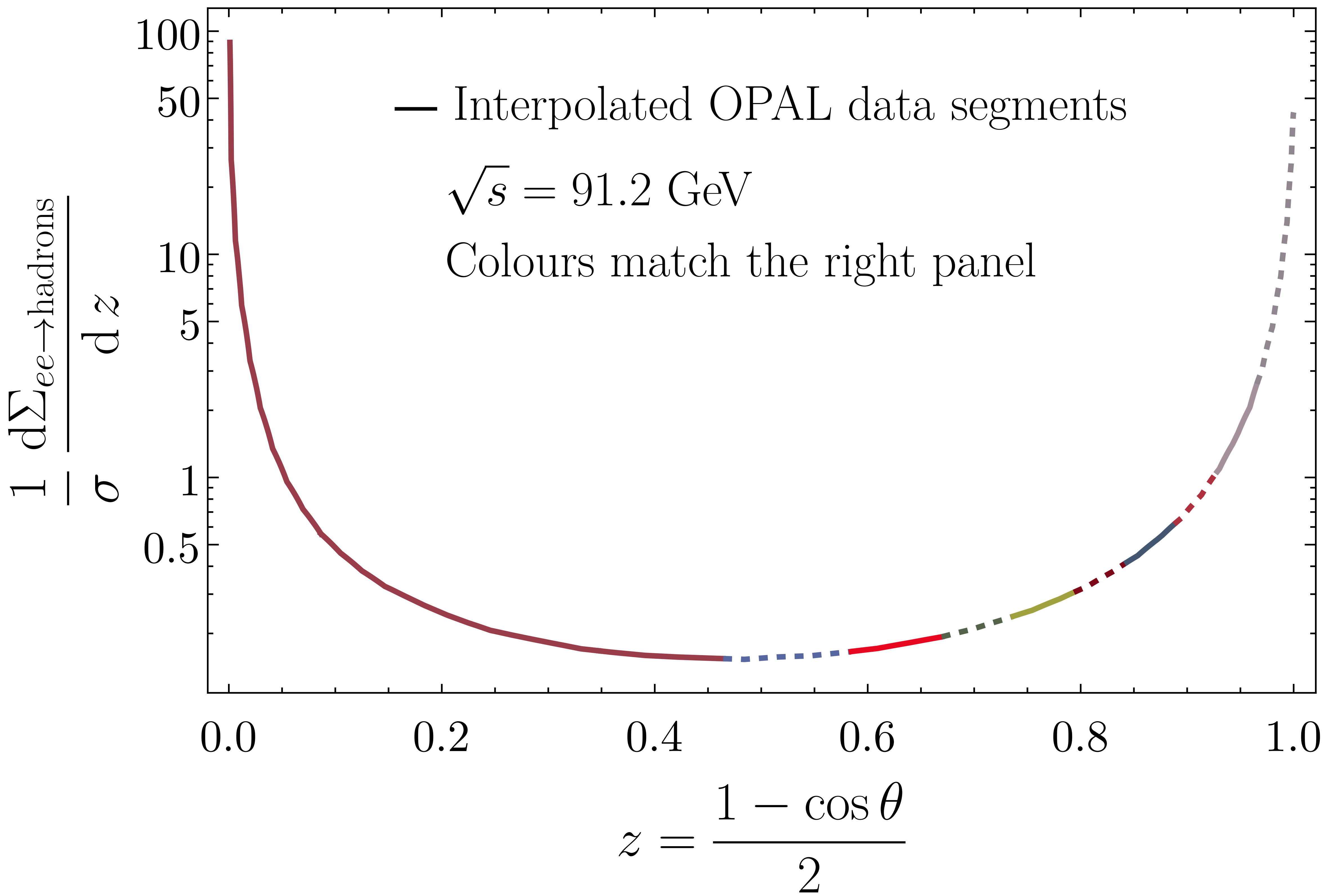
$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\chi\,\mathrm{d}p_T^Z\,\mathrm{d}\eta_Z} = \mathcal{N}(p_T^Z, \eta_Z) \int \frac{\mathrm{d}\phi_Z}{2\pi} \int \mathrm{d}^2\boldsymbol{n}_1 \mathrm{d}^2\boldsymbol{n}_2 \; \frac{M_Z^2}{4\pi^2 (p_T^Z)^2 \; (n_1\cdot n_2)^3}\, \mathcal{F}_{\mathcal{E}\mathcal{E}}\left(\frac{M_Z^2\; n_1\cdot n_2}{2\,p_Z\cdot n_1\; p_Z\cdot n_2}\right)\; \delta\left(\chi-\chi_{12}\right)\;,$$

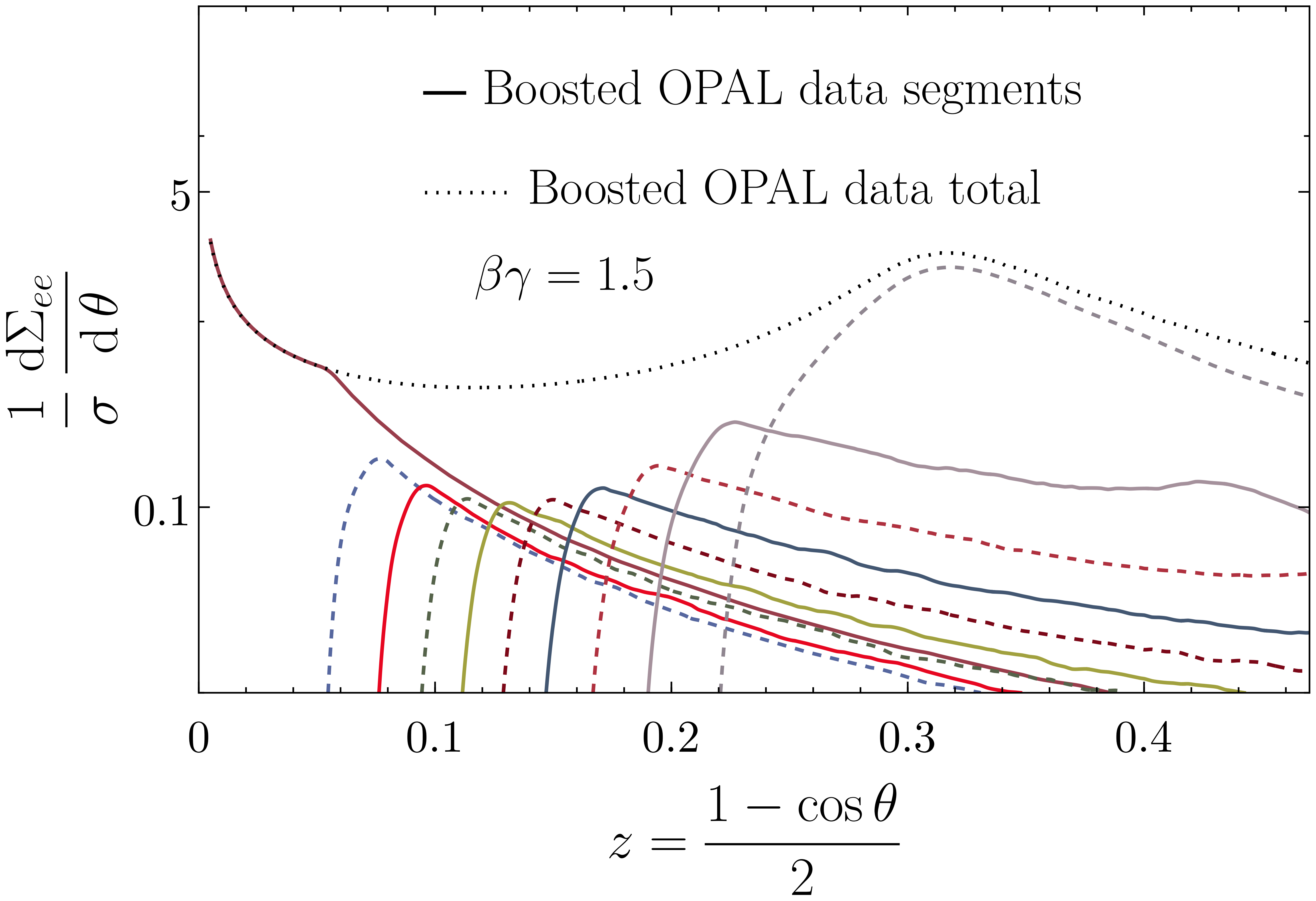
- Model fake Z jets background as a power law: $\dfrac{\mathrm{d}\Sigma_{\text{Fake-}Z}}{\mathrm{d}\chi\,\mathrm{d}p_T^Z\,\mathrm{d}\eta_Z} \sim \dfrac{1}{\chi}$

- Ignore power corrections from underlying event ($\sim R^2$) and momentum lost by the jet ($\sim M_Z/(p_T^Z R)$)

Migration of EEC features with boosting

- Peak location: $\frac{q^2\, n_1 \cdot n_2}{2\, q \cdot n_1\, q \cdot n_2} \approx 1 \quad \Rightarrow \quad z_{\text{peak}} \approx 1/\gamma^2$
- Boosting the Z boson compresses features from their rest-frame angular positions to smaller angles
- Dominant contribution of each feature at $z_{\text{feature}} \approx z_{\text{feature}}^{\text{rest frame}}/\gamma^2$ but exhibits a substantial tail due to integration over allowed boosts.





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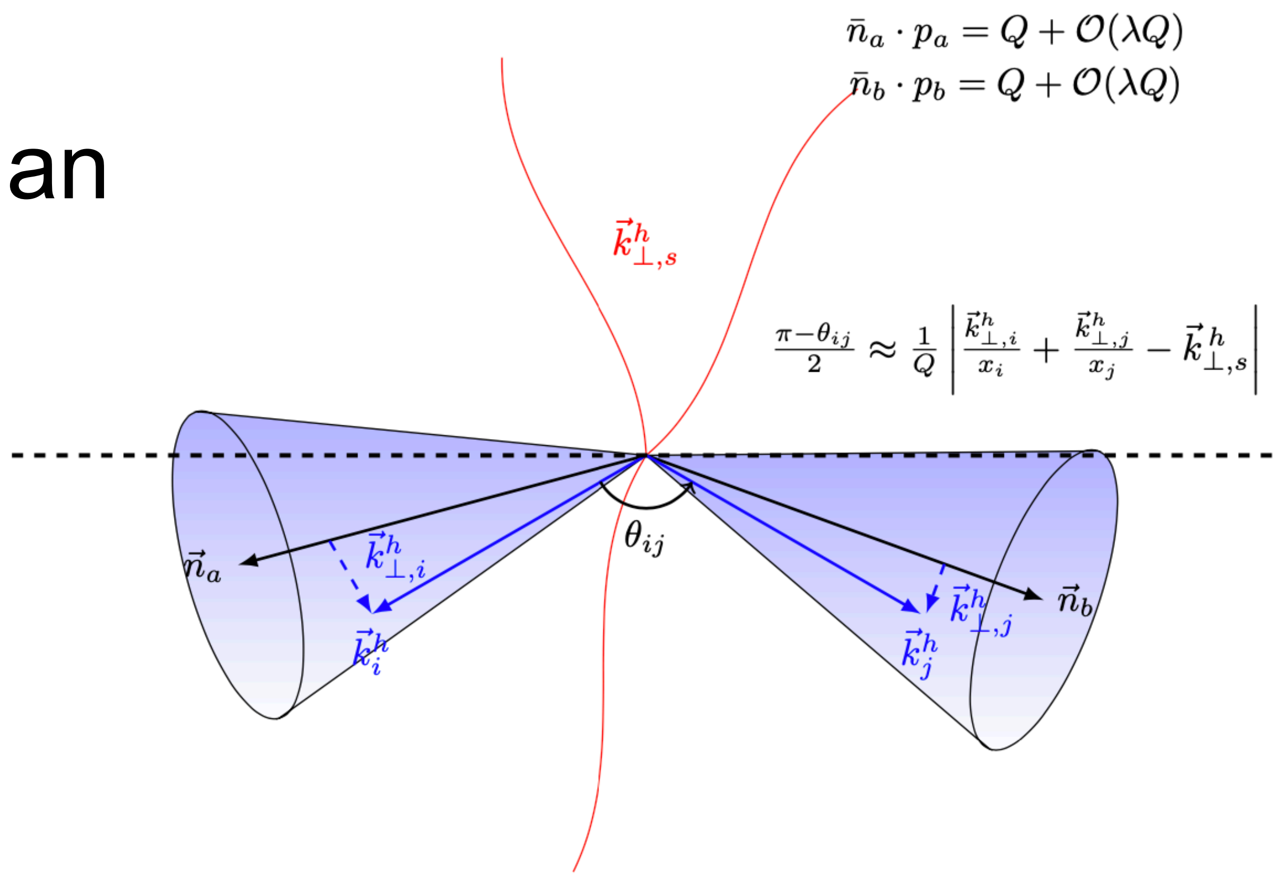
Explicit derivation of boosted EEC Sudakov

Moult, Zhu 2018



- The factorization for back-to-back EEC Sudakov was derived by Moult and Zhu, 2018.
- Their derivation is somewhat hard-wired for rest-frame kinematics. Here we present an alternate derivation working explicitly with lab frame kinematics making the role of the Z boost manifest.
- Expect a more straightforward generalization to the boosted top case.
- Key idea: phrase EEC on boosted Z jets in terms of di-hadron distribution:

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle_q = \sum_{h_1,h_2\in\text{jet}} \int \mathrm{d}\Phi_{h_1,h_2} \left(\prod_{i=1}^{2} E_{h_i}\delta^{(2)}(\boldsymbol{n}_i - \boldsymbol{n}_{h_i})\right) \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma(q)}{\mathrm{d}\Phi_{h_1,h_2}}\,.$$

$$\frac{\mathrm{d}\sigma(q)}{\mathrm{d}\Phi_{h_1,h_2}} = \left(\frac{q_\mu q_\nu}{M_Z^2} - g_{\mu\nu}\right) \sum\!\!\!\!\!\!\int_{X_n} (2\pi)^d \delta^{(d)}\left(P_{X_n} + p_{h_1} + p_{h_2} - q\right)$$
$$\times \langle 0|J_Z^{\dagger\mu}(0)|X_n, h_1, h_2\rangle\langle h_1, h_2, X_n|J_Z^\nu(0)|0\rangle\,.$$

- The distribution factorizes in the Sudakov region when $X_n \to X_{n_1} + X_{n_2} + X_{cs}$

Born configurations of Z decay

- At the born level, Z decays into $q(p_a)\bar{q}(p_b)$

$$p_a = E_a n_a^\mu\,, \qquad p_b = E_b n_b^\mu\,, \qquad n_{a,b} \equiv (1, \boldsymbol{n}_{a,b})\,.$$

- Work with rescaled reference vectors:

$$\tilde{n}_a^\mu = \frac{1}{\sqrt{z_{ab}}}\big(1, \boldsymbol{n}_a\big)\,, \qquad \tilde{n}_b^\mu = \frac{1}{\sqrt{z_{ab}}}\big(1, \boldsymbol{n}_b\big)\,,$$

- The born level partons and the Z carry momenta:

$$p_a^\mu = \big(0, \omega_a, \mathbf{0}_\perp\big)_{(ab)}\,, \qquad p_b^\mu = \big(\omega_b, 0, \mathbf{0}_\perp\big)_{(ab)}\,, \qquad q^\mu = \big(\omega_a, \omega_b, \mathbf{0}_\perp\big)_{(ab)}\,,$$

$$\omega_a = q\cdot\tilde{n}_b = \frac{Q - P_Z \boldsymbol{n}\cdot\boldsymbol{n}_b}{\sqrt{z_{ab}}}\,, \qquad \omega_b = q\cdot\tilde{n}_a = \frac{Q - P_Z \boldsymbol{n}\cdot\boldsymbol{n}_a}{\sqrt{z_{ab}}}\,.$$

- In this configuration the cross ratio is pinned to its maximum value:

$$\bar{z}_{ab} \equiv \frac{M_Z^2 n_a\cdot n_b}{2\, q\cdot n_a\, q\cdot n_b} = \frac{M_Z^2}{\omega_a\omega_b} = 1$$

Formation of the Sudakov feature

- To describe the situation beyond leading order, work with "hadron frame decomposition”

$$\tilde{n}_1^\mu = \frac{1}{\sqrt{z_{12}}}\big(1, \boldsymbol{n}_1\big)\,, \qquad \tilde{n}_2^\mu = \frac{1}{\sqrt{z_{12}}}\big(1, \boldsymbol{n}_2\big)\,, \qquad z_{12} = \frac{n_1 \cdot n_2}{2} = \frac{1 - \boldsymbol{n}_1 \cdot \boldsymbol{n}_2}{2}\,.$$

- The detected hadrons are exactly aligned along these directions:

$$p_{h_1}^\mu = \sqrt{z_{12}} E_{h_1} \tilde{n}_1^\mu\,, \qquad p_{h_2}^\mu = \sqrt{z_{12}} E_{h_2} \tilde{n}_2^\mu\,,$$

- But, the Z boson acquires a transverse component fixed by deviation of $\bar{z}_{12}$ from 1:

$$q^\mu = \underbrace{q \cdot \tilde{n}_2}_{\omega_2} \frac{\tilde{n}_1^\mu}{2} + \underbrace{q \cdot \tilde{n}_1}_{\omega_1} \frac{\tilde{n}_2^\mu}{2} + q_\perp^\mu = \big(\omega_2, \omega_1, q^\mu_{\perp(12)}\big)_{(12)}\,, \quad \boldsymbol{q}^2_{\perp(12)} = M_Z^2\big(1 - \bar{z}_{12}\big)/\bar{z}_{12}\,.$$

- The Z boson recoils in the transverse plane against the collinear and soft radiation which is all that is allowed in the Sudakov region:

$$q^\mu_{\perp(12)} = P^\mu_{X_a\perp(12)} + P^\mu_{X_b\perp(12)} + P^\mu_{X_s\perp(12)} \sim \lambda M_Z\,,$$

$$P_{X_a} \sim M_Z\big(\lambda^2, 1, \lambda\big)_{(12)}\,, \quad P_{X_b} \sim M_Z\big(1, \lambda^2, \lambda\big)_{(12)}\,, \quad P_{X_s} \sim M_Z\big(\lambda, \lambda, \lambda\big)_{(12)}\,.$$

Consistency with the EEC shape function

- The di-hadron distribution neatly factorizes in the hadron frame decomposition:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma(q)}{\mathrm{d}\Phi_{h_1,h_2}} = \frac{16(2\pi)^d\zeta_1\zeta_2}{M_Z^{(d-2)}\Omega_{d-2}r_\epsilon}\sum_f \frac{|a_f|^2+|v_f|^2}{\sum_{f'}(|a_{f'}|^2+|v_{f'}|^2)}\,\mathcal{H}_f^{(0)}(M_Z^2)$$

$$\times\int \mathrm{d}^{d-2}b_\perp\, e^{+ib_\perp\cdot q_{\perp(12)}}\Big[\mathcal{D}_{h_1/f}(\zeta_1,b_\perp)\mathcal{D}_{h_2/\bar{f}}(\zeta_2,b_\perp)S_{n_1n_2}(b_\perp)+(n_1,f\leftrightarrow n_2,\bar{f})\Big]\,,$$

$b_\perp$ conjugate to the hadron-frame $q_{\perp(12)}$

TMDFF (w/o $\sqrt{S_\perp}$)

q_T soft function

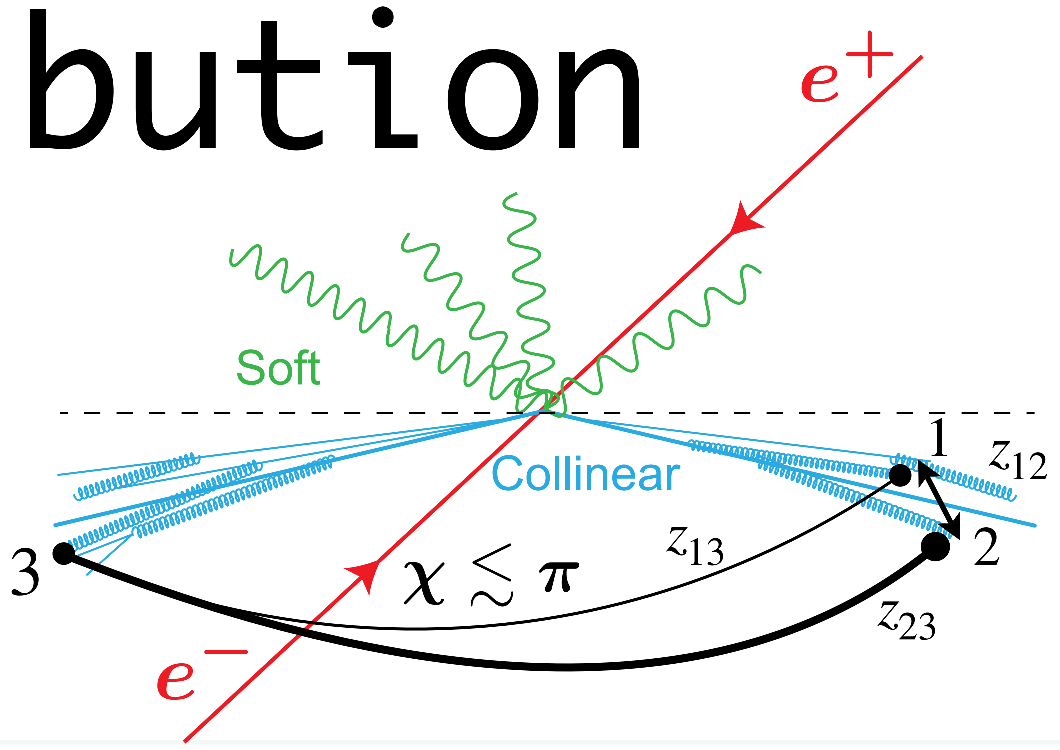
- Integrating over hadron energies, and inclusively summing over hadrons, we can extract the shape function and see that it's explicitly a function of $\bar{z}_{12}$:

$$\mathcal{F}_{\mathcal{E}\mathcal{E}}\left(\bar{z}_{12},\frac{\Lambda_{\mathrm{QCD}}^2}{M_Z^2}\right) = \frac{4^\epsilon}{r_\epsilon}M_Z^{2-2\epsilon}\sum_f \frac{|a_f|^2+|v_f|^2}{\sum_{f'}(|a_{f'}|^2+|v_{f'}|^2)}\,\mathcal{H}_f^{(0)}(M_Z^2)$$

$$\times\int_0^\infty \mathrm{d}b_T\, b_T^{1-2\epsilon}\,J_0\Big(M_Z b_T\sqrt{1-\bar{z}_{12}}\Big)\Big[J^f_{\mathrm{EEC}}(b_T)J^{\bar{f}}_{\mathrm{EEC}}(b_T)S_{n_1n_2}(b_T)\Big]\,.$$

Holguin, Lee, Moult, AP, Procura [2603.xxxxx]

Next steps: Squeezed trihadron distribution



Squeezed EEEC region: $\sqrt{z_{12}} \sim \sqrt{1-z_{13}} \sim \sqrt{1-z_{23}} \sim \lambda \ll 1$

NEW!

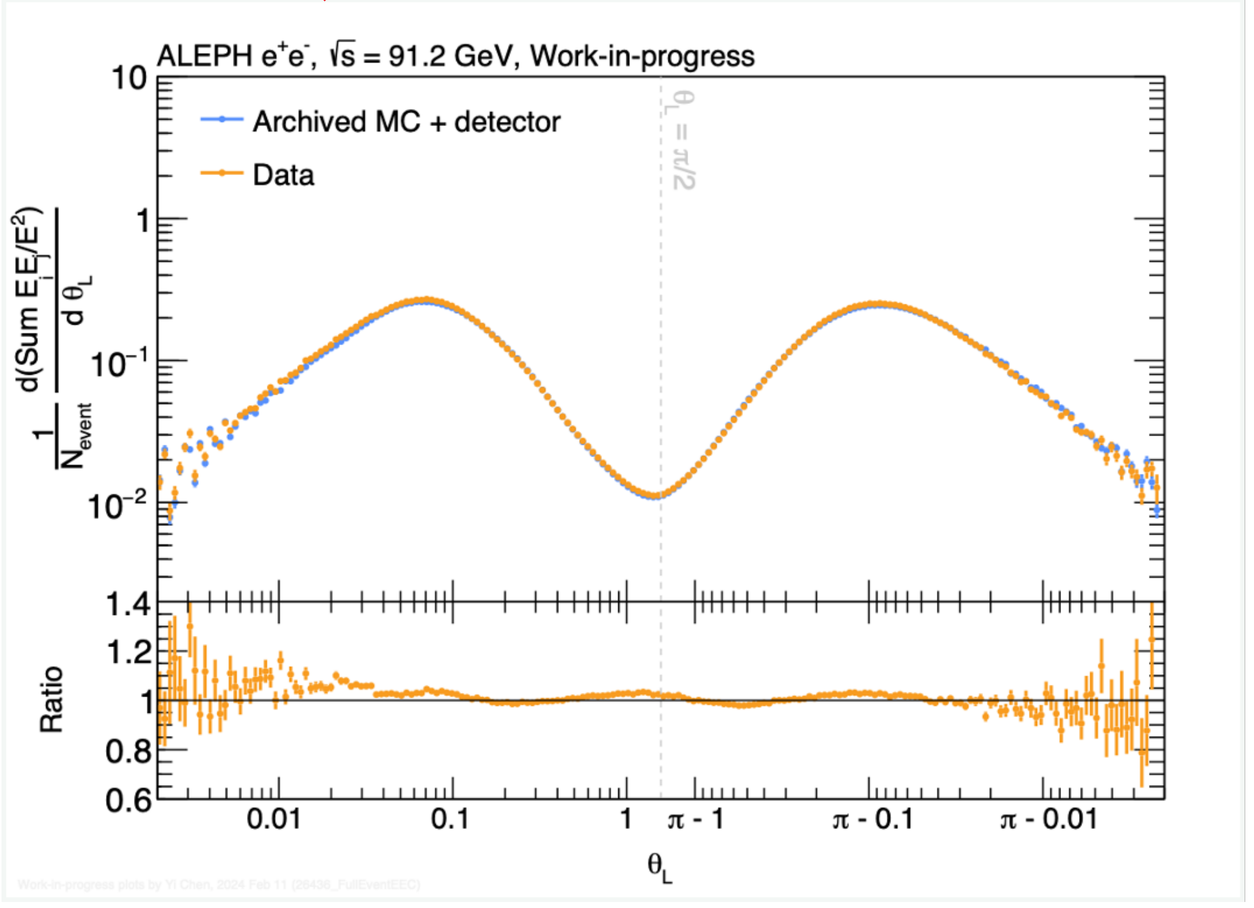
We derive a new factorization formula:

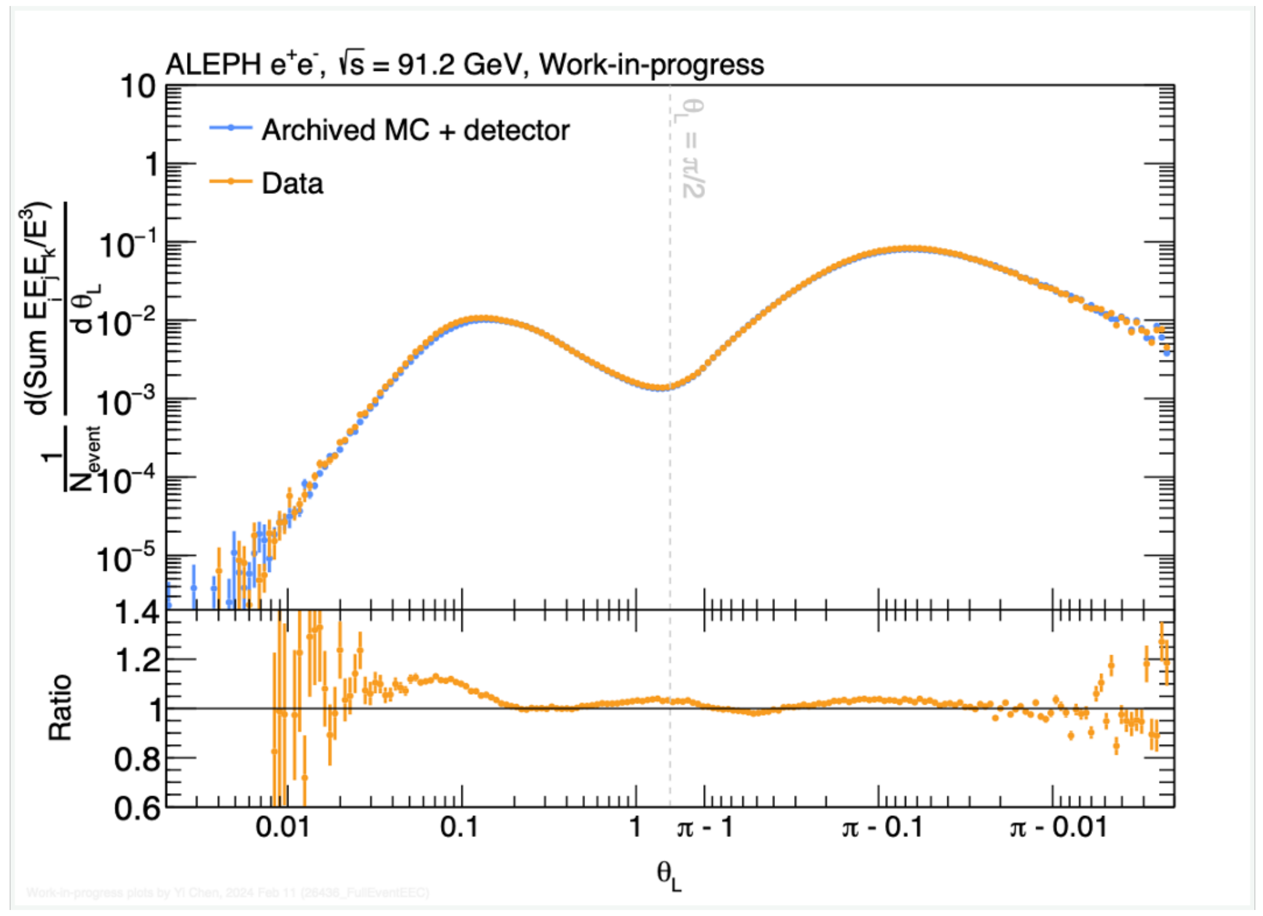
$$\frac{1}{\sigma_0}\frac{d\sigma_{\rm EEEC}}{dz_{13}dz_{12}dz_{23}} = \frac{1}{8}\int d^2\boldsymbol{q}_T\,\delta\!\left(1-z_{13}-\frac{\boldsymbol{q}_T^2}{Q^2}\right)\int đ^2\boldsymbol{b}_T\, e^{-i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}$$
$$\times\sum_f H_f(Q,\mu)J^f_{\rm EEEC}(Qb_T,\{z_{ij}\},L_b,L_\nu)J^{\bar f}_{\rm EEC}(L_b,L_\nu)S_\perp(b_T,\mu,\nu)$$

The squeezed EEEC jet function involves dihadron TMD + contact term:

$$J^{f(1)}_{\rm EEEC} \equiv J^{(1)}_{f(12)}(b_TQ,\{z_{ij}\},\epsilon)$$
$$+\,\delta(z_{12})\delta(z_{23}-z_{13})\int_0^1 dz\, z^d\left[\mathcal{D}^{(1)}_{g/q}(z,b_\perp,\epsilon)+\mathcal{D}^{(1)}_{q/q}(z,b_\perp,\epsilon)\right]$$

$$\mathcal{D}_{i/j}(z,b_\perp,\epsilon) \sim \frac{1}{z^2}$$





Key takeaways:

- Rapidity divergence only in the contact term
- Non-trivial cancellation of IR poles

$$\int_0^1 dz\, z^d\left[\mathcal{D}^{(1)}_{g/q}(z,b_\perp)+\mathcal{D}^{(1)}_{q/q}(z,b_\perp)\right]$$
$$= C_F\left(\frac{3}{\epsilon}+3L_b-\frac{4\pi^2}{3}+10\right)+2C_FL_b\left(\frac{3}{2}-2\ln\frac{Q}{\nu}\right)$$

$$J^{(1)}_{f(12)} = C_F\left(-\frac{3}{\epsilon}-3\ln\frac{\mu^2}{Q^2}-\frac{37}{3}+2F^{(0)}_{x_2,z_{12}}(b_TQ)\right)\delta(z_{12})\delta(z_{23}-z_{13})$$
$$+\,C_F\left[\frac{F_{x_2}(b_TQ\sqrt{z_{12}},0)}{z_{12}}\right]_+\frac{\Theta(z_{12})}{\pi}\int d\Omega^{(2)}_{d-2}\,\delta_{z_{23}}$$

Conclusions

- Presented the first comprehensive theoretical study of EEC measurements on the hadronic decays of boosted heavy neutral particles.
- EEC on boosted Z decays can be computed directly from measurements of the EEC performed in e+e- collisions at the Z pole
- Complemented this symmetry-based picture with an explicit SCET factorization of the EEC in the laboratory frame, starting directly from the di-hadron distribution in boosted Z jets.

Thank you!

Backup slides

Nonperturbative corrections

Korchemsky, Sterman 1998; Schindler, Stewart, Sun 2023; Lee, AP, Stewart, Sun 2024

- In the fixed order region we have

$$\frac{\mathrm{d}\Sigma^{z\sim 0.5}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z} = \frac{\mathrm{d}\Sigma^{\mathrm{pert.}}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z} + \frac{1}{\left[z^{3/2}(1-z)^{3/2}\right]_+}\frac{\Omega}{Q} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right),$$

- In the back-to-back region, one needs resummation. Matched distribution at parton-level:

$$\frac{\mathrm{d}\Sigma^{\mathrm{NLO+N^3LL'}}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z} = \frac{\mathrm{d}\Sigma^{\mathrm{NLO}}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z} + \frac{\mathrm{d}\Sigma^{\mathrm{N^3LL'\,b2b}}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z} - \left.\frac{\mathrm{d}\Sigma^{\mathrm{NLO}}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z}\right|_{z\to 1},$$

- Include nonperturbative corrections in the $z \to 1$ region via the dispersive model: Dokshitzer, Marchesini, Webber 1999

$$\frac{\mathrm{d}\Sigma^{\mathrm{NLO+N^3LL'+NP}}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z} = f_{\mathrm{NP}}(z)\,\frac{\mathrm{d}\Sigma^{\mathrm{NLO+N^3LL'}}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z}.$$

$$f_{\mathrm{NP}}(z) = (1-g(z,a,b))\,\frac{\mathrm{d}\Sigma^{z\sim 0.5}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z}\Bigg/\frac{\mathrm{d}\Sigma^{\mathrm{NLO}}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z} + g(z,a,b)\,\frac{\mathrm{d}\Sigma^{\mathrm{Dokshitzer}\ et\ al.}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z}\Bigg/\frac{\mathrm{d}\Sigma^{\mathrm{NLL\,b2b}}_{ee\to\mathrm{hadrons}}}{\mathrm{d}z}.$$

- $g(z,a,b)$ smoothly interpolates for $a = 0.93 < z < b = 0.998$ and preserves the sum rule