# Born-Projected Leptons in Drell–Yan Final-State

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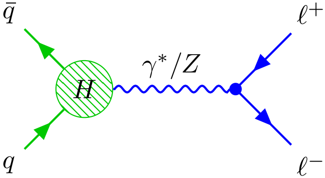
$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} = \frac{1}{2s} \sum_{V,V'} W^{\mu\nu}_{VV'}(q)\, L_{VV'\mu\nu}(q)$$

- $W^{\mu\nu}$: Hadronic tensor for $pp \to V/V' + X \to \cdots$
- $L_{\mu\nu}$: Leptonic tensor for $V/V' \to \ell^+\ell^-$




All QCD dynamics in $W^{\mu\nu}$. $L^{\mu\nu}$ tree level (2-body).

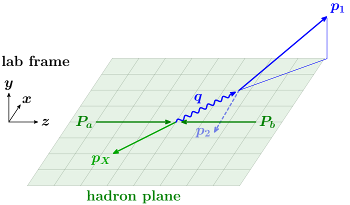
Initial-Final factorisation enables to resum QCD corrections to $W^{\mu\nu}$ to $\mathrm{N^3LO + N^4LL}$.

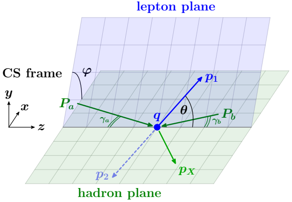
# Angular Decomposition in Vector Boson Decays

For a **2-body decay** of a vector boson, the differential cross section can be decomposed as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\cos(\theta)\,\mathrm{d}\varphi} = \frac{3}{16\pi}\frac{\mathrm{d}\sigma}{\mathrm{d}^4q}\sum_{i=-1}^{7} A_i g_i(\theta,\varphi)$$

- $A_i$: Angular coefficients.
- $\theta$ and $\varphi$: spherical coordinates of $p_1$ in the **Collins-Soper frame**.
- Used by experimentalists to study boson polarization and to extract the weak mixing angle.

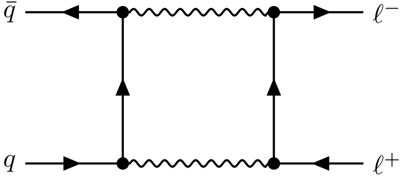


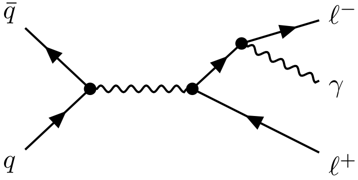




[J.C. Collins & D.E. Soper, Phys.Rev.D 16 (1977) 2219]

# Including EW/QED corrections

- Initial-state radiation (ISR).
- Virtual contributions, including initial-final interference.
- Final-state radiation (FSR).

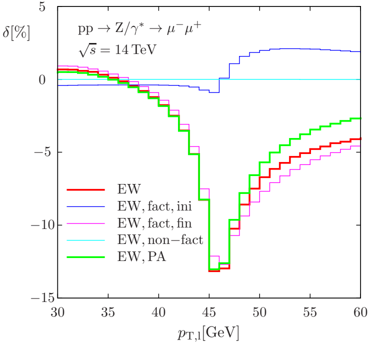



ISR resummation analogous to QCD:

[L. Cieri, G. Ferrera & G. Sborlini, JHEP 08 (2018) 165]
[G. Billis, F. Tackmann & J. Tablert, JHEP 03 (2020) 182]
[L. Buonocore, L. Rottoli & P. Torrielli, JHEP 07 (2024) 193]
[A. Autieri, S. Camarda, L. Cieri, G. Ferrera & G. Sborlini, arxiv:2511.07324]
and more!

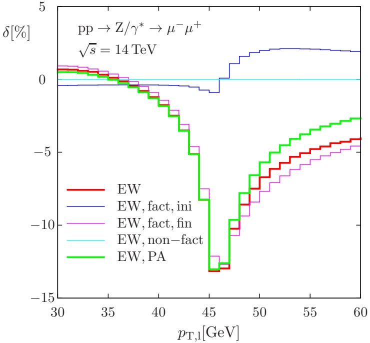
# Electroweak Corrections in Drell–Yan

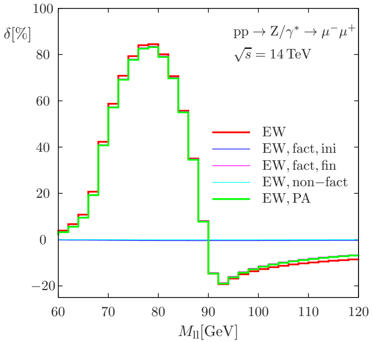



[S. Dittmaier et al., Nucl. Phys. B 885 (2014) 318]

- Initial-final interference **breaks factorisation**.
- Contributions: factorisable initial, factorisable final, non-factorisable.
- Systematic treatment via the *pole approximation*.
- Non-factorisable corrections are negligible.
- Factorisable EW contributions to $L^{\mu\nu}$ are the dominant contributions.

# Electroweak Corrections in Drell–Yan




[S. Dittmaier et al., Nucl. Phys. B 885 (2014) 318]

## Why FSR matters

- Shifts the reconstructed dilepton invariant mass.
- Strongly affects the cross section near the $Z$ pole.
- $q_T \neq p_T^{\ell\ell}$. We resum the first, but measure the latter.

# FSR and Angular Observables

## Reminder: Angular decomposition requirements

- Leptonic tensor at $\mathbf{LO}$.
- Two-particle final state.
- Angular variables defined in the **Collins–Soper frame**.

## Effect of QED radiation

- Final-state photon emission: $\ell^+\ell^-\gamma$.
- Final state no longer two-body.
- Leptons are not emitted back-to-back.
- How to even define the angular variables $\theta, \varphi$?

Some generalisations to the angular coefficients have been used:

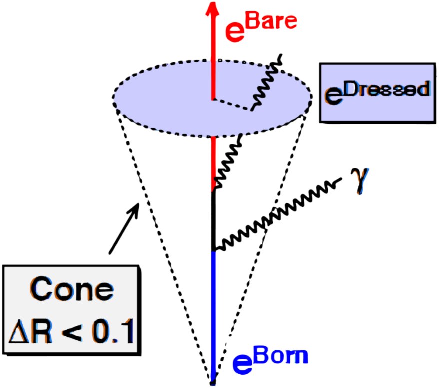
[R. Frederix & T. Vitos, Eur. Phys. J. C 80 (2020) 939]

[M. Pellen, R. Poncelet, A. Popescu & T. Vitos, Eur. Phys. J. C 82 (2022) 693]

# Lepton Definitions

In the presence of **final-state radiation**, leptons can be defined in several ways:

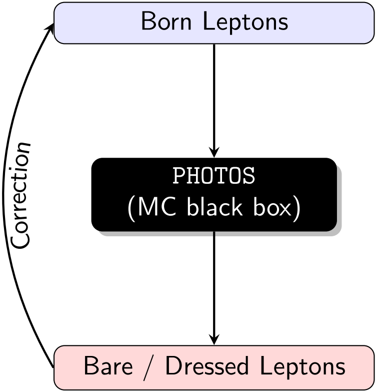
- **Bare leptons**: the charged lepton itself, ignoring any photons radiated.
- **Dressed leptons**: the lepton combined with photons in a small cone ($\Delta R < 0.1$).
- **Born leptons**: the theoretical lepton before any QED radiation.



# Monte Carlo Born Leptons

Born leptons are typically reconstructed using Monte Carlo tools: PHOTOS.

- PHOTOS simulates QED photon emission.
- **MC-Born lepton** are inferred by "undoing" this radiation.
- Used to recover LO-like kinematics.
- The procedure is a black box.
- **How to make them theoretically well-defined?**



# Born-Projected Leptons

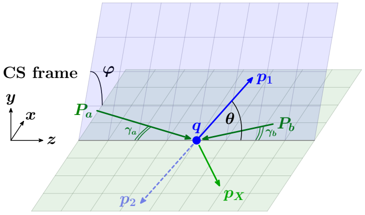
Decay with FSR: $V(q) \;\rightarrow\; \ell^+(p_1)\ell^-(p_2) \;+\; \gamma(k)$

**Final-state radiation**

- Bare leptons satisfy:

$$q^\mu = p_1^\mu + p_2^\mu + k^\mu, \qquad k^\mu \neq 0$$

- Leptons are not back-to-back in the Collins–Soper frame.
- Angular variables are ill-defined.




**Born-projected leptons**

- Define projected momenta: $P_1^\mu$, $P_2^\mu$
- Enforce:

$$q^\mu = P_1^\mu + P_2^\mu,$$

$$P_i^\mu \rightarrow p_i^\mu \text{ as } k^\mu \rightarrow 0$$

- Defines a map from $\ell^+\ell^-\gamma$ to an effective **two-body** phase space.

[M. Ebert, J. Michel, I. Stewart & F. Tackmann, JHEP 04 (2021) 102]

# Projected Leptonic Tensor

The full multi-body leptonic system is projected onto effective two-body kinematics:

$$F^{\mu\nu}_{VV'}(P_1, P_2) = (2\pi)^2 \int L^{\mu\nu}_{VV'}(\Phi_L)\, \delta^{(4)}\big(P_1 - \hat{P}_1(\Phi_L)\big)\; \mathrm{d}\Phi_L$$

Here $\hat{P}_1(\Phi_L)$ defines the Born momentum in terms of the full leptonic phase space.

Using Lorentz covariance and the orthogonality relation $q_\mu F^{\mu\nu} = 0$:

$$F^{\mu\nu}(P_1, P_2) = \mathcal{F}^{\mu\nu}_{+}(\theta,\varphi)\, F_+(q^2) + \mathcal{F}^{\mu\nu}_{0}(\theta,\varphi)\, F_0(q^2) + \mathcal{F}^{\mu\nu}_{-}(\theta,\varphi)\, F_-(q^2)$$

The scalar $F_i$ coefficients **generalise** the $\mathrm{LO}$ leptonic ones:

$$F_\pm = L_\pm\,(1 + \mathcal{O}(\alpha_{\mathrm{em}}))\,, \qquad F_0 = \mathcal{O}(\alpha_{\mathrm{em}})$$

[M. Ebert, J. Michel, I. Stewart & F. Tackmann, JHEP 04 (2021) 102]

# Generalised Angular Decomposition

Using Born-projected leptons $P_1$ and $P_2$, the final state is effectively a two-body decay. The angular decomposition is therefore recovered.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\;\mathrm{d}\cos\theta\;\mathrm{d}\varphi} = \frac{3}{16\pi}\frac{\mathrm{d}\sigma}{\mathrm{d}^4q}\sum_{i=-1}^{7}\tilde{A}_i\,g_i(\theta,\varphi)$$

**Generalised angular coefficients (including EW corrections)**

$$\tilde{A}_0 = \frac{F_+W_0 + F_0W_{\mathrm{inc}}}{\left(F_+ + \frac{3}{2}F_0\right)W_{\mathrm{inc}}}, \qquad \tilde{A}_{i>0} = \frac{F_{\pm(i)}W_i}{\left(F_+ + \frac{3}{2}F_0\right)W_{\mathrm{inc}}}$$

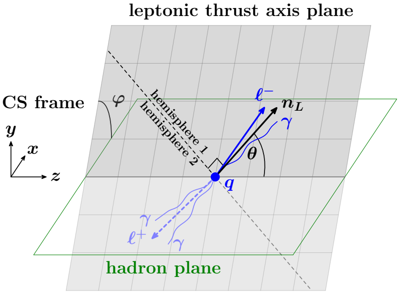
These coefficients reduce to the $\mathrm{LO}$ ones for $k \to 0$.

Unlike other generalisations, they retain a **clear physical interpretation**.

[M. Ebert, J. Michel, I. Stewart & F. Tackmann, JHEP 04 (2021) 102]

# Born-Projections via the Thrust Axis

Any **IR−safe** projection $\hat{P}_1^\mu(\Phi_L)$ defines a valid Born-projected lepton. One choice is to cluster the FSR using the **thrust axis**:




$$\hat{P}_1^\mu(\Phi_L) = \frac{Q}{2}\,(t^\mu + n_L^\mu)\,,$$
$$Q = \sqrt{q^2},$$
$$t^\mu = \frac{1}{Q} q^\mu,$$
$$\vec{n}_L(\Phi_L) = \pm \arg\max_{\vec{n}\in S^2} \sum_{i\in L} |\vec{n}\cdot\vec{p}_i|$$

- Choose the sign such that in the absence of radiation $\hat{P}_1(\Phi_L) = p_1$.
- Provides a **concrete definition** of Born leptons, enabling **calculable** theoretical predictions.

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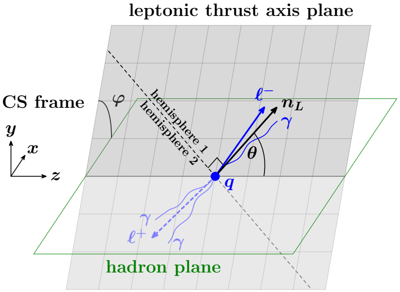



At NLO :

$$\vec{n}_L = \begin{cases} \hat{p}_1 & \text{if } |\vec{p}_1| > |\vec{p}_2| \\ -\hat{p}_2 & \text{if } |\vec{p}_2| > |\vec{p}_1| \end{cases}$$

- Choose the sign such that in the absence of radiation $\hat{P}_1(\Phi_L) = p_1$.
- Provides a **concrete definition** of Born leptons, enabling **calculable** theoretical predictions.

[M. Ebert, J. Michel, I. Stewart & F. Tackmann, JHEP 04 (2021) 102]

# Born Leptons: Theory vs Experiment

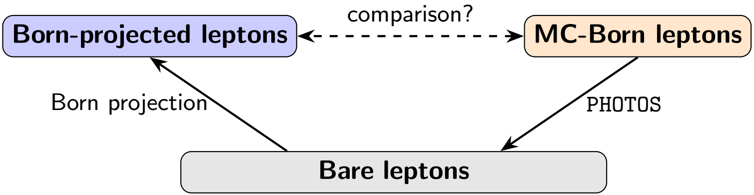
Comparing Born leptons in theory and experiment

**Theory: Born-projected leptons**

- Defined directly from $\ell^+\ell^- + X$
- Projection onto two-body kinematics
- Uses thrust axis

**Experiment: MC-Born leptons**

- Start from Born-level $\ell^+\ell^-$
- `PHOTOS` adds QED FSR and builds the response matrix.
- Born leptons reconstructed by unfolding the response matrix.

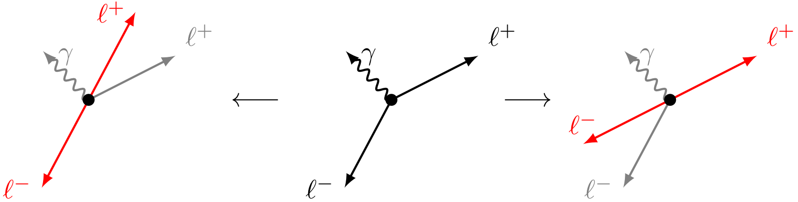



Goal: Is there a Born projection that reproduces **exactly** the MC-Born leptons?

# Why MC-Born are not calculable

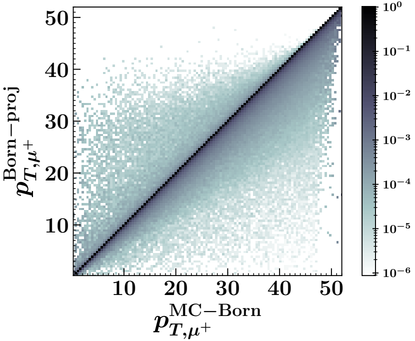
**The PHOTOS map is not injective**

- Distinct MC-Born configuration can radiate into the **same** final state.
- Photon angles and energies are not uniquely reversible.
- PHOTOS does not recoil non-emitting particles, so at NLO there is a twofold degeneracy.



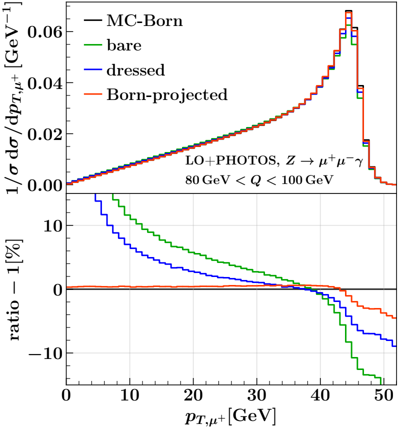

**Therefore:** a true inverse PHOTOS map **does not exist**.

# Born-Projections: Response Matrix and $p_T$ Distributions

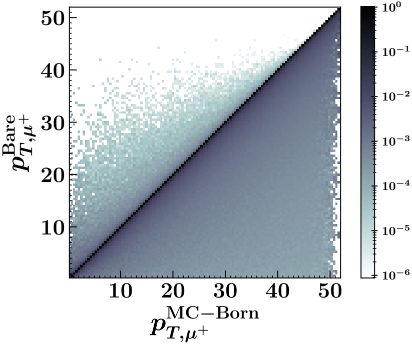



- Born-projected leptons closely reproduce PHOTOS Born kinematics.
- Small devistions arise from occasional "misreconstruction" of the MC-Born configuration.

- $p_T$ agreement with PHOTOS reflects clossenes between the two definitions. It is **not** a measure of the accuracy of Born-projected leptons.



# Born-Projections: Response Matrix and $p_T$ Distributions

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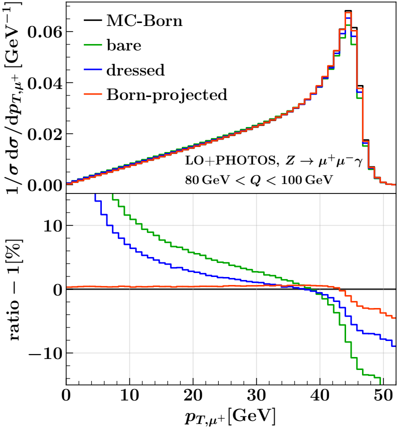


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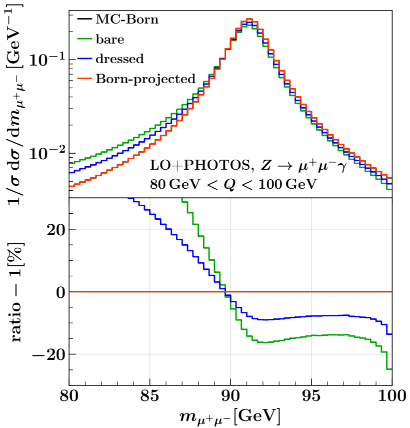
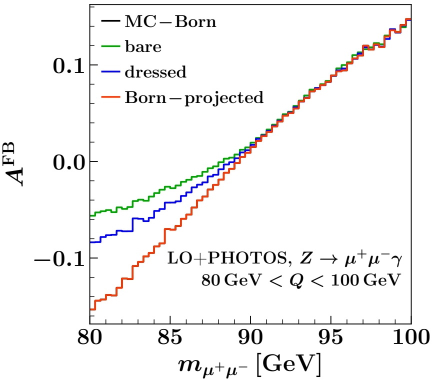
# Differential Distributions

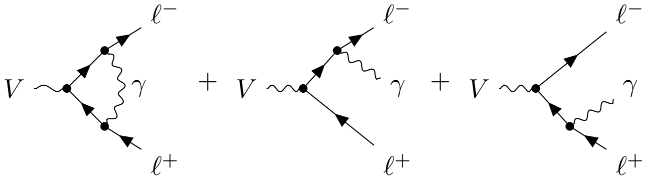



- Differences only due to $m_{\ell\ell}$.
- $m_{\ell\ell}$ is equal for all Born leptons, by definition.

- $A_{\mathrm{FB}}$ probes electroweak coupling: $\sin^2(\theta_W)$
- Both $A_{\mathrm{FB}}$ and $m_{\ell\ell}$ depend on lepton defintion.
- We see no difference between MC-Born and Born-projected.



# Results for $Z/\gamma^* \to \ell^-\ell^+$

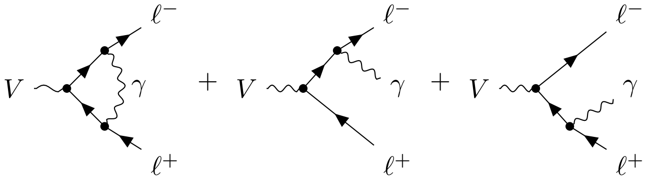



$$\frac{1}{12\pi} F^{\mu\nu}_{VV'}(P_1, P_2) = \frac{\pi}{3} \int L^{\mu\nu}_{VV'}(\Phi_L)\, \delta^{(4)}\big(P_1 - \hat{P}_1(\Phi_L)\big)\; \mathrm{d}\Phi_L$$

$$= (t^\mu t^\nu - g^{\mu\nu} - n_L^\mu n_L^\nu) F_+(q^2)$$

$$+ (t^\mu t^\nu - g^{\mu\nu}) F_0(q^2)$$

$$+ i\, \epsilon^{\mu\nu}{}_{\alpha\beta}\, n_L^\alpha t^\beta F_-(q^2)$$




$$F_{+VV'} = \frac{P_V^* P_{V'}}{Q^2}\alpha_{\rm em}(v_V v_{V'} + a_V a_{V'})\left(\frac{2}{3} + \alpha_{\rm em} Q_\ell^2 \frac{16\log(2) - 11}{2\pi}\right)$$

$$F_{0VV'} = \frac{P_V^* P_{V'}}{Q^2}\alpha_{\rm em}(v_V v_{V'} + a_V a_{V'})\left(\alpha_{\rm em} Q_\ell^2 \frac{12 - 16\log(2)}{3\pi}\right)$$

$$F_{-VV'} = \frac{P_V^* P_{V'}}{Q^2}\alpha_{\rm em}(v_V a_{V'} + a_V v_{V'})\left(\frac{2}{3} + \alpha_{\rm em} Q_\ell^2 \frac{5 + 4\log(2)}{6\pi}\right)$$

$$P_V = \frac{Q^2}{Q^2 - M_V^2 + iM_V\Gamma_V}$$

# Numerical Results from FSR

$$\sigma_{\text{incl}} = \frac{L_{\text{incl}} W_{\text{incl}}}{2Q^2} \propto \frac{4\pi\alpha_{\text{em}}^2}{3Q^2 N_c} \left( \frac{Q^4 (a_q^2 + v_q^2)(a_\ell^2 + v_\ell^2)}{(Q^2 - M_V^2)^2 + M_V^2 \Gamma_V^2} \right) \left( 1 + \frac{3\alpha_{\text{em}} Q_\ell^2}{4\pi} \right)$$

- $L_{\text{incl}} = F_+ + \frac{3}{2} F_0$: known $\rightarrow$ Cross-check.
- Global FSR normalisation $\Longrightarrow$ **No shape effects.**
- $\sigma_{\text{NLO}} = \sigma_{\text{LO}}(1 + \delta_{\text{FSR}}), \qquad \delta_{\text{FSR}} \sim 0.17\%$.

$$\tilde{A}_{\text{FB}} = \frac{3}{8} \frac{F_- W_4}{L_{\text{incl}} W_{\text{incl}}} = -\frac{6 a_q a_\ell v_q v_\ell}{(a_q^2 + v_q^2)(a_\ell^2 + v_\ell^2)} \frac{4\pi + \alpha_{\text{em}} Q_\ell^2 (5 + 4\log(2))}{4\pi + 3\alpha_{\text{em}} Q_\ell^2}$$

- New result
- Normalisation effects cancel in $\tilde{A}_{FB}$.
- FSR is also a global correction, $\sim 0.27\%$

# Conclusion & Outlook

- Born-projected leptons provide a **well-defined** framework for **including FSR** in Drell-Yan.

- Preserves the **angular decomposition** of the leptonic tensor at **all orders**.

$\Longrightarrow$ Allows a systematic way to combine **precision QCD** with potentially large **EW corrections** in the final state.

- Thrust Born-projected leptons behave **numerically very similar** to MC-Born leptons, which are extensively used in experimental analyses, but **theoretically ill-defined**.

**Future Work:**

- Include virtual EW corrections.

- Explore different projections $\hat{P}_1(\Phi_L)$.

# Backup Slides

Supplementary material

# Tensor Structure in Drell–Yan Factorisation

Thanks to initial-final factorisation, the Drell–Yan cross section can be written as a contraction of hadronic and leptonic tensors.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} \propto \sum_{V,V'} W^{\mu\nu}_{VV'}(q)\, L_{VV'\mu\nu}(q)$$

**Tensor properties**

- Rank-2 Lorentz tensors due to spin-1 nature of the intermediate boson.
- Hermitian $L^{\nu\mu} = (L^{\mu\nu})^*$, $W^{\nu\mu} = (W^{\mu\nu})^*$
- Transverse[1]: $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$

[1] *The chiral anomaly breaks the transverse property. All arguments are still valid in this case, but formulas are more complex, so we will ignore this.*

# Helicity Decomposition

**Helicity decomposition:**

- Any spin-1 tensor $W_{\mu\nu}$ can be expanded in **helicity components** $W_i$:

$$W_i = P_i^{\mu\nu} W_{\mu\nu},$$

- Projectors $P_i^{\mu\nu}$ are defined with the helicity states of the intermediate boson.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} \propto \sum_{i=-1}^{7} W_i \, L^i$$

**Why this matters:**

- **Independent** of specific leptonic final states.
- At $\mathrm{LO}$ there is a one-to-one connection between $W_i$ and $A_i$.

$$A_i = \frac{2L_{\pm(i)} W_i}{L_+(W_0 + 2W_{-1})}$$

- Electroweak effects can be included directly at the level of $W_i$ and $L_i$.

Helicity decomposition provides a generalisation of the $\mathrm{LO}$ angular decomposition to arbitrary $V \to L$ final states.

# Helicity Decomposition

**Helicity basis**

- Tensors can be expanded in a finite set of **projectors** $P_i^{\mu\nu}$
- Defines **helicity components**:

$$W_i = P_i^{\mu\nu} W_{\mu\nu}, \qquad L_i = P_i^{\mu\nu} L_{\mu\nu}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} \propto \sum_{i=-1}^{7} W_i \, L_i$$

$$P_{-1}^{\mu\nu} = \frac{1}{2}(\epsilon_+^{*\mu}\epsilon_+^{\nu} + \epsilon_-^{*\mu}\epsilon_-^{\nu})$$

$$P_0^{\mu\nu} = \frac{1}{2}(\epsilon_0^{*\mu}\epsilon_0^{\nu})$$

$$P_2^{\mu\nu} = \frac{-1}{4}(\epsilon_+^{*\mu}\epsilon_-^{\nu} + \epsilon_-^{*\mu}\epsilon_+^{\nu})$$

$$P_4^{\mu\nu} = \frac{1}{4}(\epsilon_+^{*\mu}\epsilon_+^{\nu} - \epsilon_-^{*\mu}\epsilon_-^{\nu})$$

$$P_5^{\mu\nu} = \frac{i}{2}(\epsilon_+^{*\mu}\epsilon_-^{\nu} - \epsilon_-^{*\mu}\epsilon_+^{\nu})$$

$$P_1^{\mu\nu} = \frac{-1}{2}(\epsilon_+^{*\mu}\epsilon_0^{\nu} + \epsilon_0^{*\mu}\epsilon_+^{\nu} + \epsilon_-^{*\mu}\epsilon_0^{\nu} + \epsilon_0^{*\mu}\epsilon_-^{\nu})$$

$$P_3^{\mu\nu} = \frac{-1}{4\sqrt{2}}(\epsilon_+^{*\mu}\epsilon_0^{\nu} + \epsilon_0^{*\mu}\epsilon_+^{\nu} - \epsilon_-^{*\mu}\epsilon_0^{\nu} - \epsilon_0^{*\mu}\epsilon_-^{\nu})$$

$$P_6^{\mu\nu} = \frac{i}{2\sqrt{2}}(\epsilon_+^{*\mu}\epsilon_0^{\nu} - \epsilon_0^{*\mu}\epsilon_+^{\nu} - \epsilon_-^{*\mu}\epsilon_0^{\nu} + \epsilon_0^{*\mu}\epsilon_-^{\nu})$$

$$P_7^{\mu\nu} = \frac{i}{4\sqrt{2}}(\epsilon_+^{*\mu}\epsilon_0^{\nu} - \epsilon_0^{*\mu}\epsilon_+^{\nu} + \epsilon_-^{*\mu}\epsilon_0^{\nu} - \epsilon_0^{*\mu}\epsilon_-^{\nu})$$

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$$W_i = P_i^{\mu\nu} W_{\mu\nu}, \qquad L_i = P_i^{\mu\nu} L_{\mu\nu}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} \propto \sum_{i=-1}^{7} W_i \, L_i$$

$$P_{-1}^{\mu\nu} = \frac{1}{2}(x^\mu x^\nu + y^\mu y^\nu)$$

$$P_0^{\mu\nu} = \frac{1}{2} z^\mu z^\nu$$

$$P_2^{\mu\nu} = \frac{-1}{4}(x^\mu x^\nu - y^\mu y^\nu)$$

$$P_4^{\mu\nu} = \frac{1}{4i}(x^\mu y^\nu - y^\mu x^\nu)$$

$$P_5^{\mu\nu} = \frac{-1}{2}(x^\mu y^\nu + y^\mu x^\nu)$$

$$P_1^{\mu\nu} = \frac{-1}{2}\left(x^\mu z^\nu + z^\mu x^\nu\right)$$

$$P_3^{\mu\nu} = \frac{1}{4i}\left(y^\mu z^\nu - z^\mu y^\nu\right)$$

$$P_6^{\mu\nu} = \frac{-1}{2}\left(y^\mu z^\nu + z^\mu y^\nu\right)$$

$$P_7^{\mu\nu} = \frac{1}{4i}\left(x^\mu z^\nu - z^\mu x^\nu\right)$$

# Projected Leptonic Tensor in $D$ Dimensions

## Definition in $D = 4 - 2\epsilon$

$$F^{\mu\nu}_{VV'}(P_1, P_2) = (2\pi)^{D-2} \int L^{\mu\nu}_{VV'}(\Phi_L)\, \delta^{(D)}\big(P_1 - \hat{P}_1(\Phi_L)\big)\, \mathrm{d}\Phi_L,$$

with the Born momentum

$$\hat{P}^\mu_1(\Phi_L) = \frac{Q}{2}\,(t^\mu + n^\mu_L)\,.$$

## Tensor decomposition

The tensor can be decomposed into scalar coefficients $F_i(q^2)$ as:

$$\frac{1}{12\pi}\, F^{\mu\nu}_{VV'}(P_1, P_2) = (t^\mu t^\nu - g^{\mu\nu} - n^\mu_L n^\nu_L) F_+(q^2) + (t^\mu t^\nu - g^{\mu\nu}) F_0(q^2) + i\,\epsilon^{\mu\nu}{}_{\alpha\beta}\, n^\alpha_L t^\beta F_-(q^2).$$

Transversality $q_\mu F^{\mu\nu} = 0$ is manifest.

# Virtual and Real Divergent Contributions

## Virtual contribution ($F^{\mathrm{virt}}$)

$$F^{\mathrm{virt}}_{+VV'} = -\frac{Q^2\alpha_{\mathrm{em}}^2 Q_\ell^2(v^2+a^2)}{3\pi}\left(\frac{Q^2 e^{\gamma_E}}{4\pi\mu^2}\right)^{-\varepsilon}\left[\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}-\frac{7\pi^2}{6}+8\right]$$

$$F^{\mathrm{virt}}_{-VV'} = -\frac{2Q^2\alpha_{\mathrm{em}}^2 Q_\ell^2 va}{3\pi}\left(\frac{Q^2 e^{\gamma_E}}{4\pi\mu^2}\right)^{-\varepsilon}\left[\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}-\frac{7\pi^2}{6}+5\right]$$

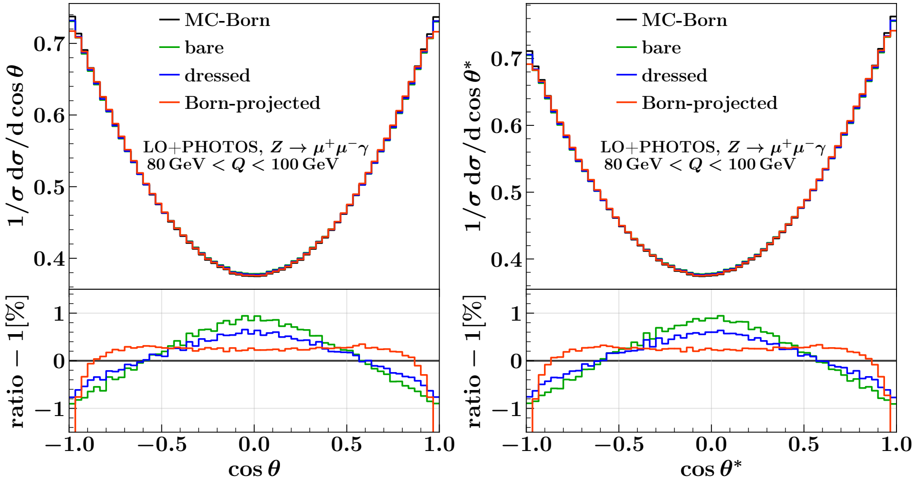
## Real contribution ($F^{\mathrm{real}}$)

$$F^{\mathrm{real}}_{+VV'} = +\frac{Q^2\alpha_{\mathrm{em}}^2 Q_\ell^2(v^2+a^2)}{3\pi}\left(\frac{Q^2 e^{\gamma_E}}{4\pi\mu^2}\right)^{-\varepsilon}\left[\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}-\frac{7\pi^2}{6}-\frac{17}{2}+24\log(2)\right]$$

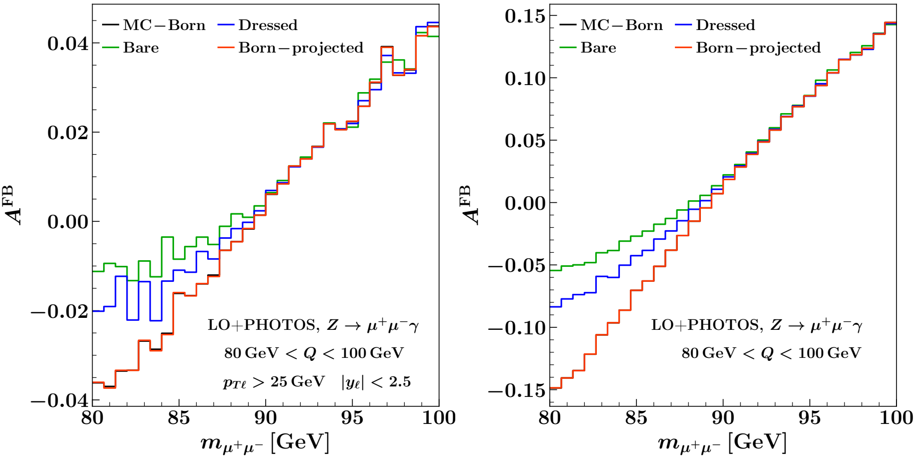
$$F^{\mathrm{real}}_{-VV'} = +\frac{2Q^2\alpha_{\mathrm{em}}^2 Q_\ell^2 va}{3\pi}\left(\frac{Q^2 e^{\gamma_E}}{4\pi\mu^2}\right)^{-\varepsilon}\left[\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}-\frac{7\pi^2}{6}+\frac{15}{2}+2\log(2)\right]$$

Note the cancellation between virtual and real divergences in the sum.

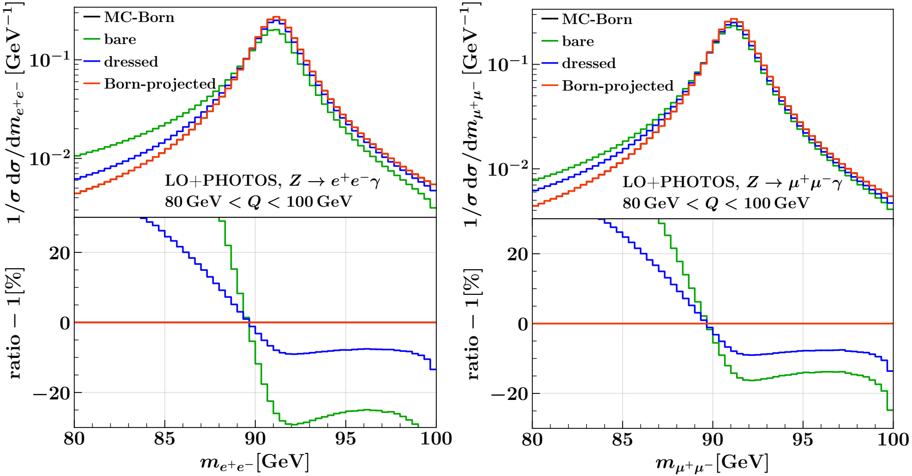
# Differential Angular Distributions



# Forward-Backward Asymmetry



# Born Leptons for $e$ and $\mu$



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