Event Shapes with ARES and CAESAR

SCET 2026

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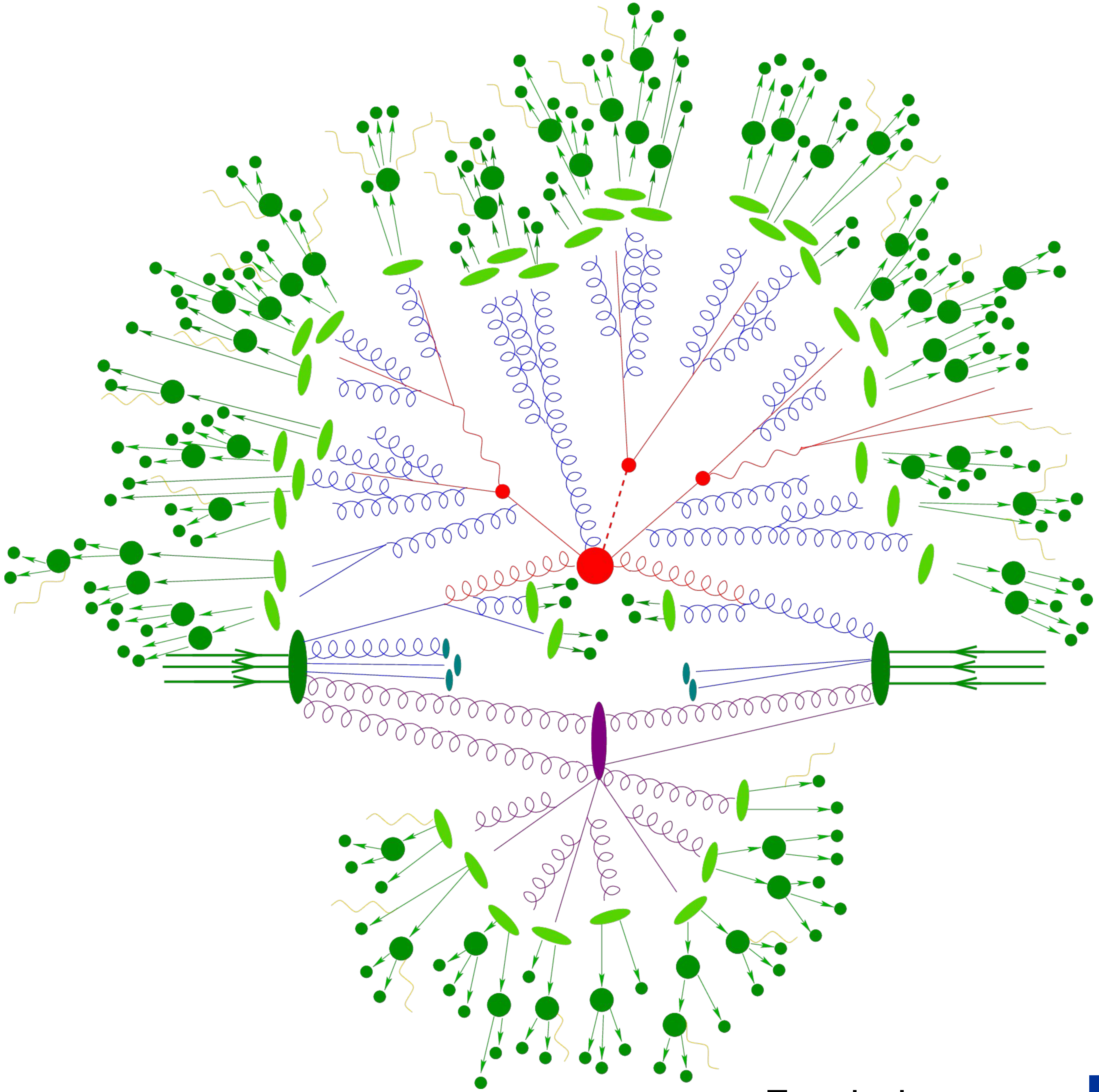


Funded by the European Union

Motivation

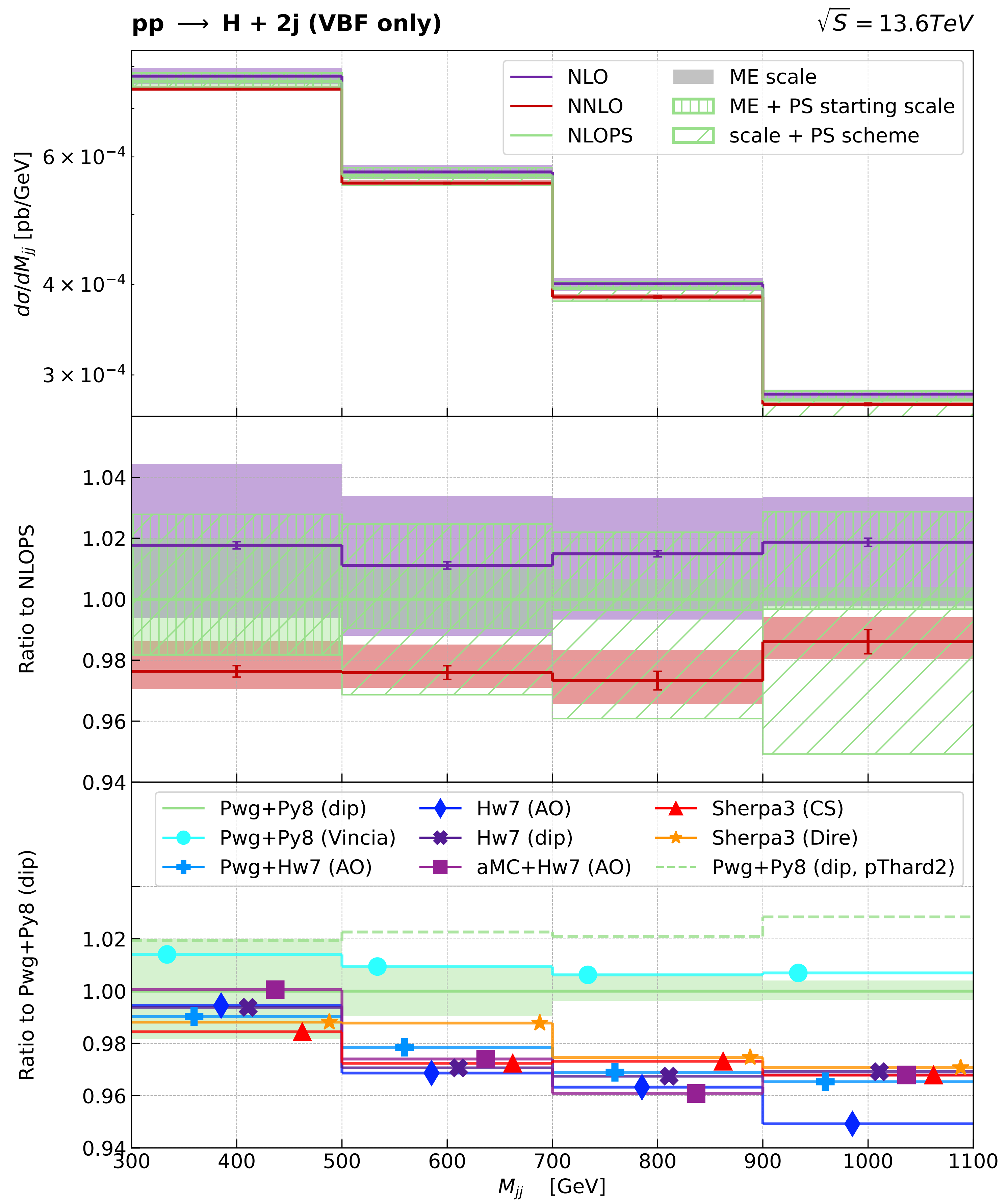
- For collider phenomenology, fully differential radiation pattern essential

- Typical implemented in parton showers

- Especially in recent years, developments driven by asymptotic limit and exact matching resummed calculations
  [Dasgupta, Dreyer, Hamilton,Monni, Salam, Soyez '20], [vanBeekveld, Ferrario Ravasio, Hamilton, Salam, Soto-Ontoso,Soyez '22]

- A lot of insights possible by comparison to resummation formalism that directly works with amplitudes and a notion of "multiple emissions"

Example: Shower systematics in VBF Higgs production
[Barone et al. '25]



Motivation — Parton Showers at NLL

- Several solutions/re-evaluations of parton shower concepts:
- [Dasgupta, Dreyer, Hamilton,Monni, Salam, Soyez ’20], [vanBeekveld, Ferrario Ravasio, Hamilton, Salam, Soto-Ontoso,Soyez ’22]
  - partitioning of splitting functions and appropriate choice of evolution variable can lead to NLL accurate shower for local and global recoil strategies
- [Forshaw, Holguin, Plätzer ’20]
  - Connections between angular ordered and dipole showers
- [Nagy, Soper ’11]
  - local transverse, global longitudinal recoil
- [Herren, Krauss, DR, Schönherr, Höche ’22]
  - dipole shower Alaric, global recoil, enables analytic comparison to resummation and proof of NLL accuracy
- [Preuss ’24]
  - global recoil in antenna shower Apollo

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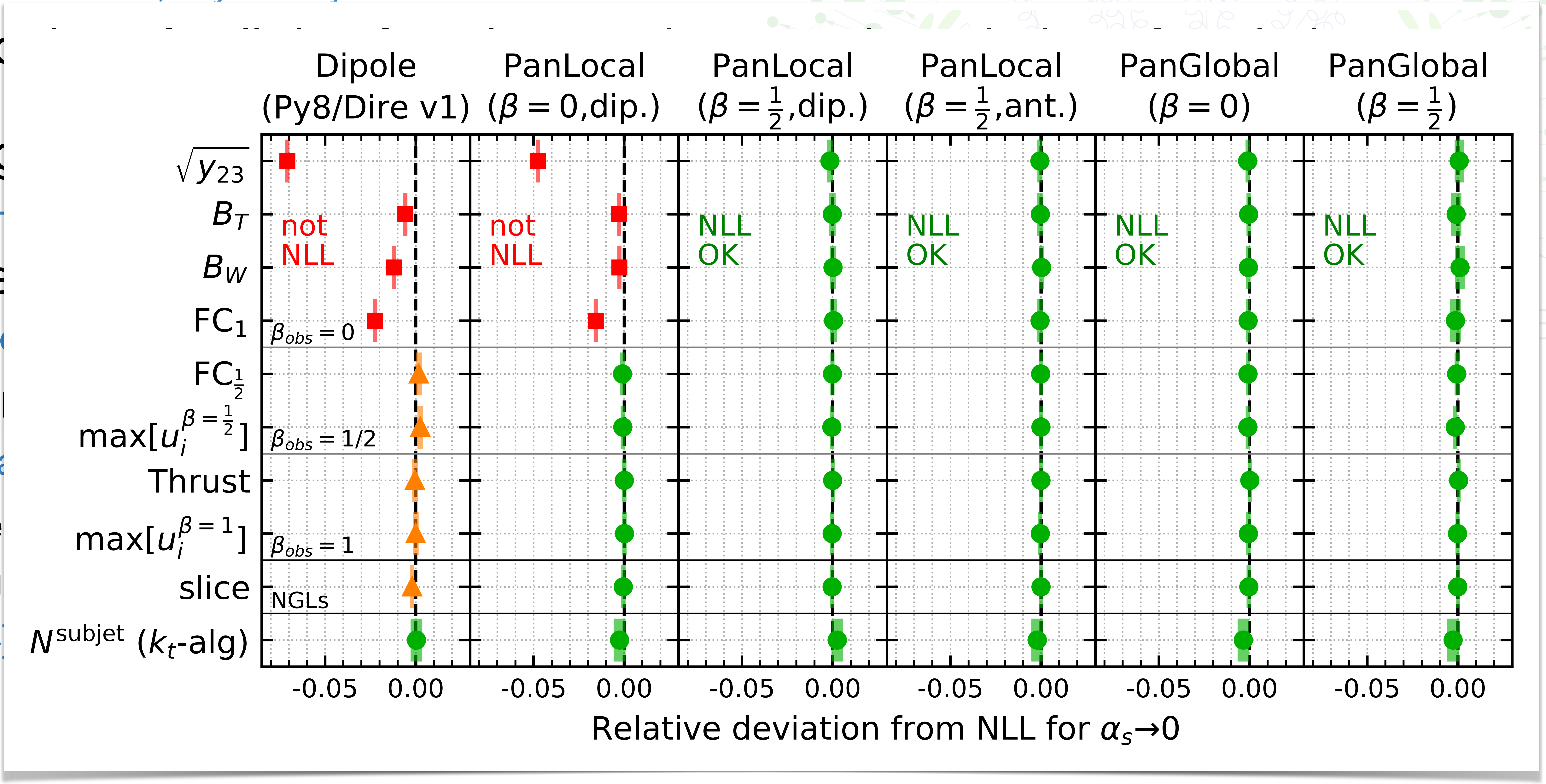
- [Preuss '24]
  - global



Outline

1. Basics of the CAESAR formalism
2. Putting things together: implementation in Sherpa framework
   - + Extension towards jet shapes with masses
3. Extension towards CAESAR @ NNLL
4. Outlook/Summary

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1. **Basics of the CAESAR formalism**

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4. Outlook

CAESAR formalism

- Consider observable $V(\{p_i\},\{k_i\})$ calculated with set of Born momenta $\{p_i\}$ and ensemble of radiated gluons $\{k_i\}$ [Banfi, Salam, Zanderighi '04]

- Applicability condition: recursive IRC safety ⇒ soft-collinear emissions with $V(\{p_i\},k) < \epsilon v$ contribute to only power corrections to $V(\{p_i\},\{k_i\})$

- Then, general cross section with cut on $V(\{p_i\},\{k_i\})$ reads

$$H(Q) e^{\int_0^{\epsilon v} dk_i |M(\{p_i\},\{k_i\})|^2} \sum_n \frac{1}{n!} \int_{\epsilon v} dk_i |M(\{p_i\},\{k_i\})|^2 \theta(v - V(\{p_i\},\{k_i\}))$$

Virtual Unresolved Resolved real radiation

Analytic cancellation of IR poles

Finite integral over multiple emissions, can be evaluated numerically

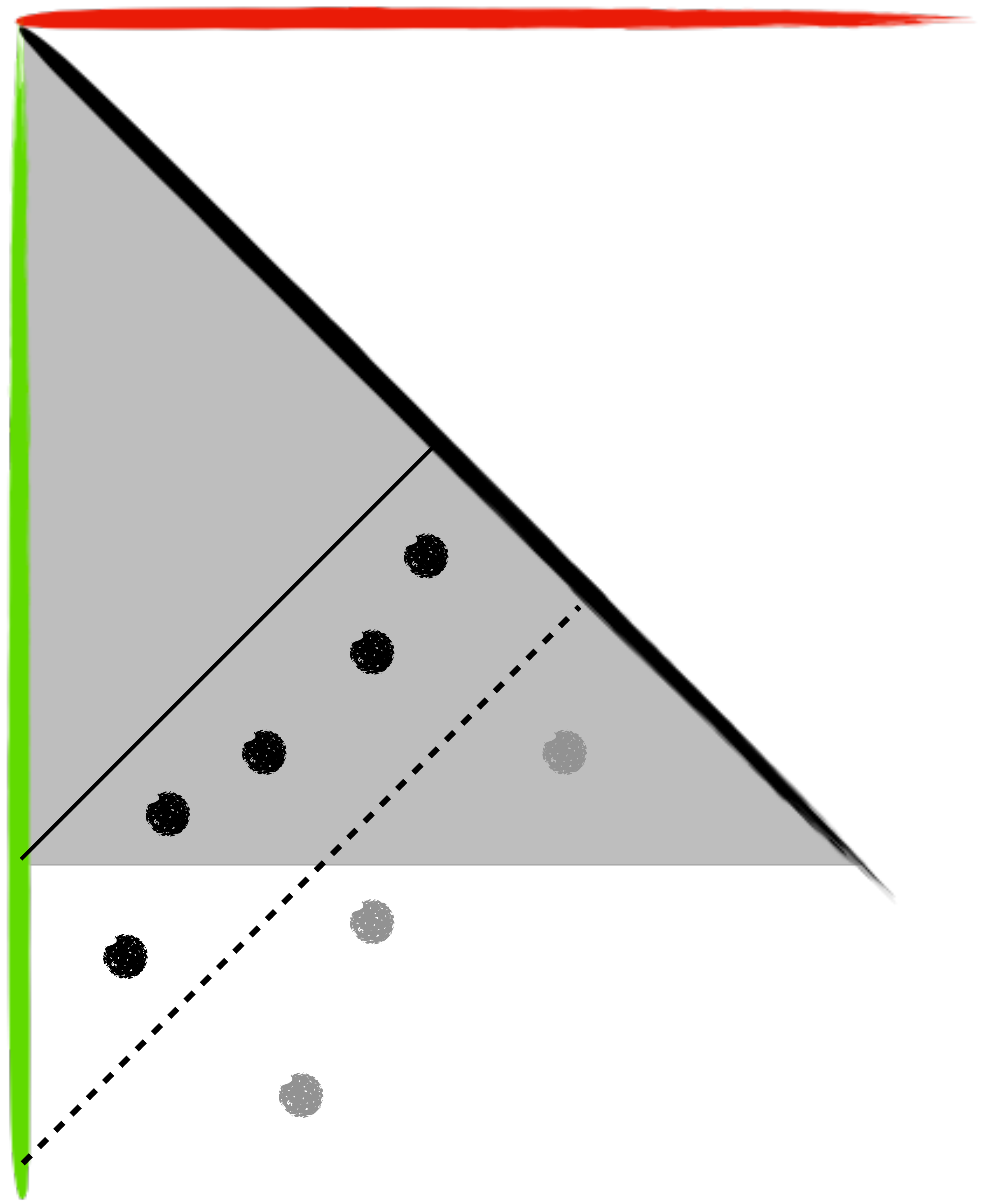
CAESAR formalism

- Overall structure

$$\Sigma(v) = e^{-R_{NLL}(v)} F(v) \qquad F(v) = e^{-\int_{\epsilon v}^{v} dk\, M^2(k)} \sum_n \frac{1}{n!} \int dk_i\, M^2(k_i)\, \theta(v - V(\{p_i\}, \{k_i\}))$$

- at NLL: approximate matrix element M^2 **and** observable dependence $V(k)$ in soft collinear limit, independent emissions strongly separated in rapidity

- limit needed to eliminate subleading contributions



Numerical evaluation of SC limit

- how to extract NLL observable independent (i.e. without additional information)?

- method from [Banfi, Salam, Zanderighi '05]: need explicit implementation of soft-collinear limit*:

$$k_t^\rho = k_t \rho$$

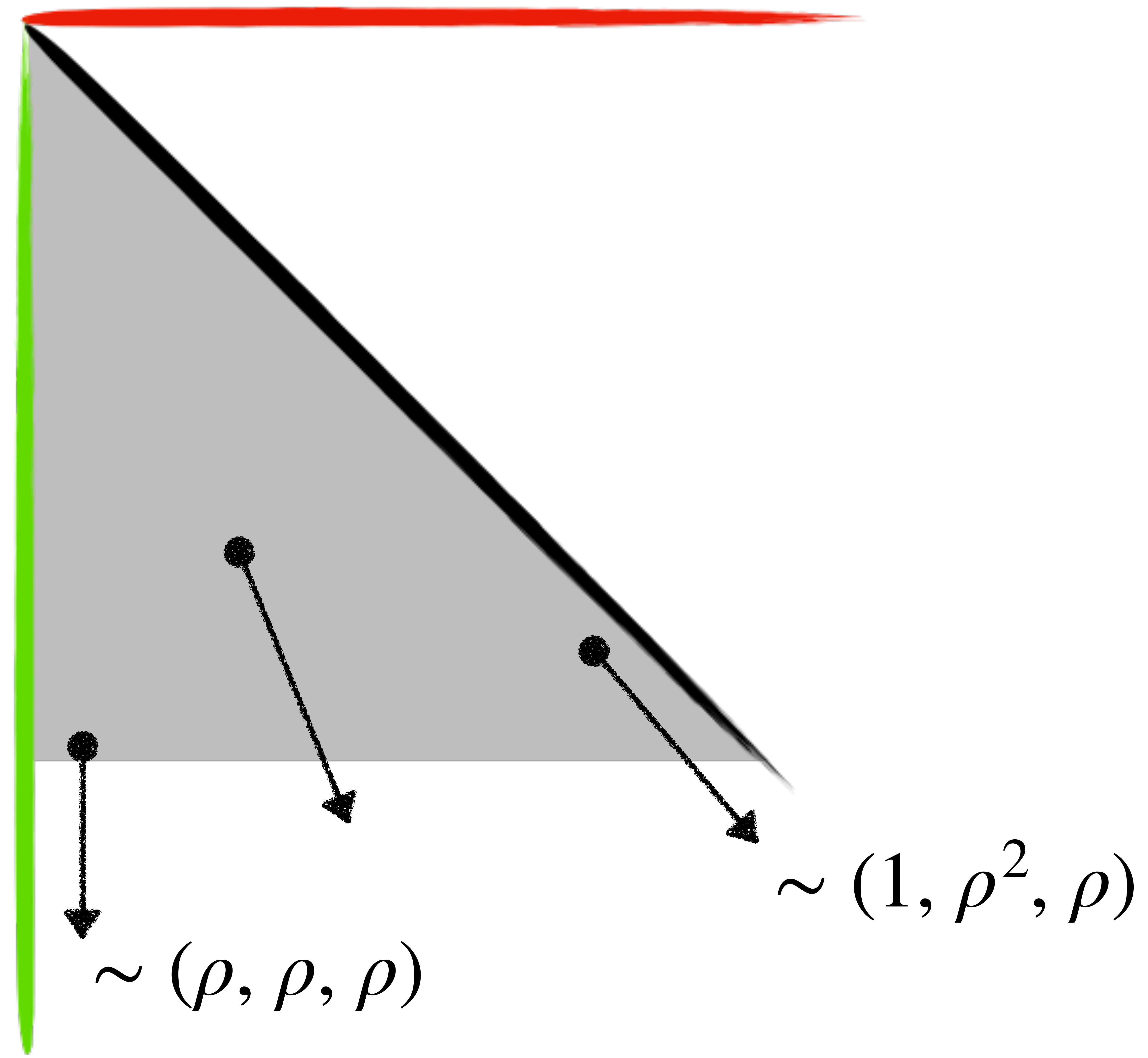
$$\eta^\rho = \eta - \xi \ln \rho$$

$$\xi = \frac{\eta}{\eta_{\max}}$$

and assume

$$V(k_i^\rho) = \rho V(k_i)$$

→ numerically evaluate integrals in this limit



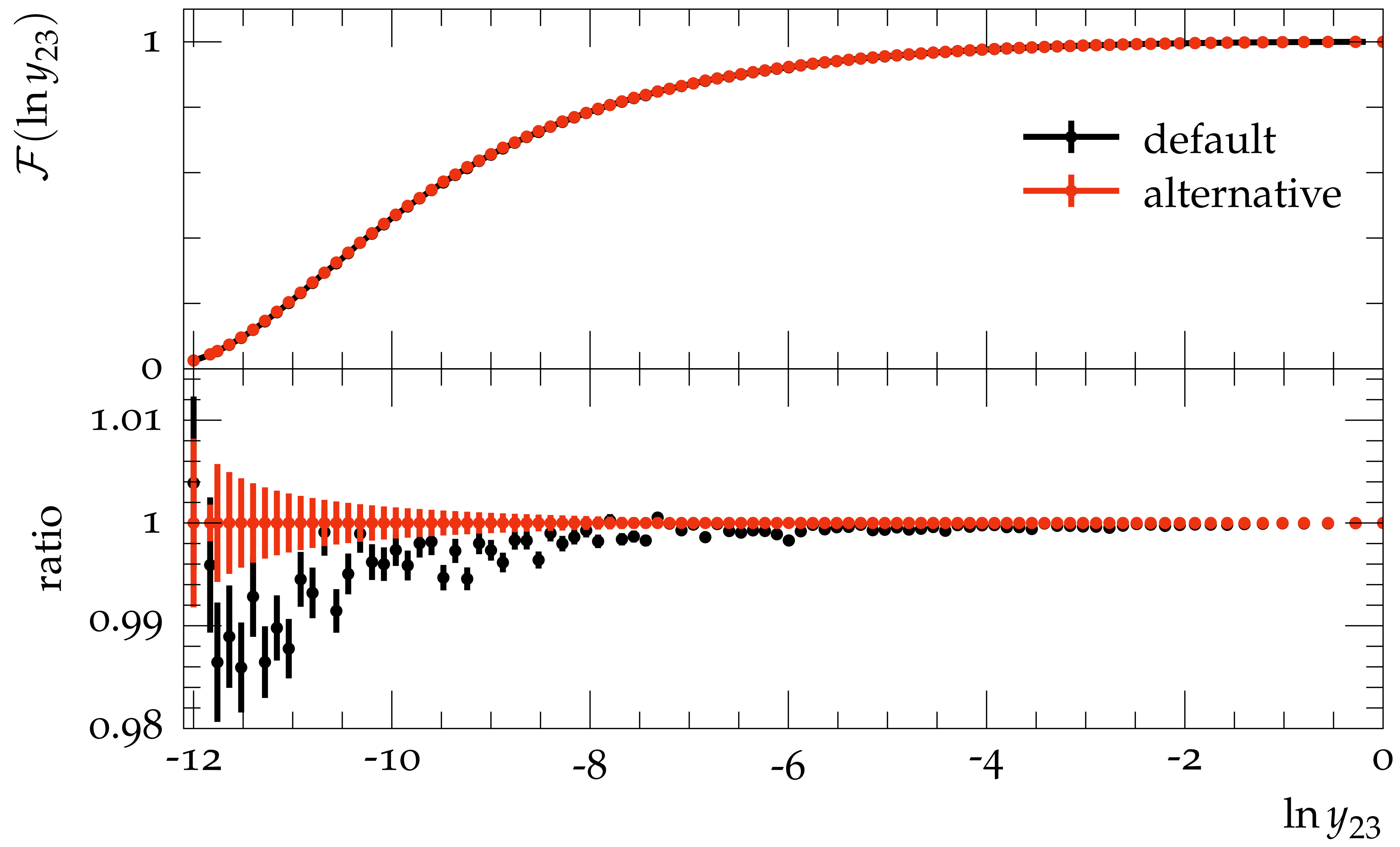
* for example $V(k_t, \eta) \sim k_t/Q$ for brevity

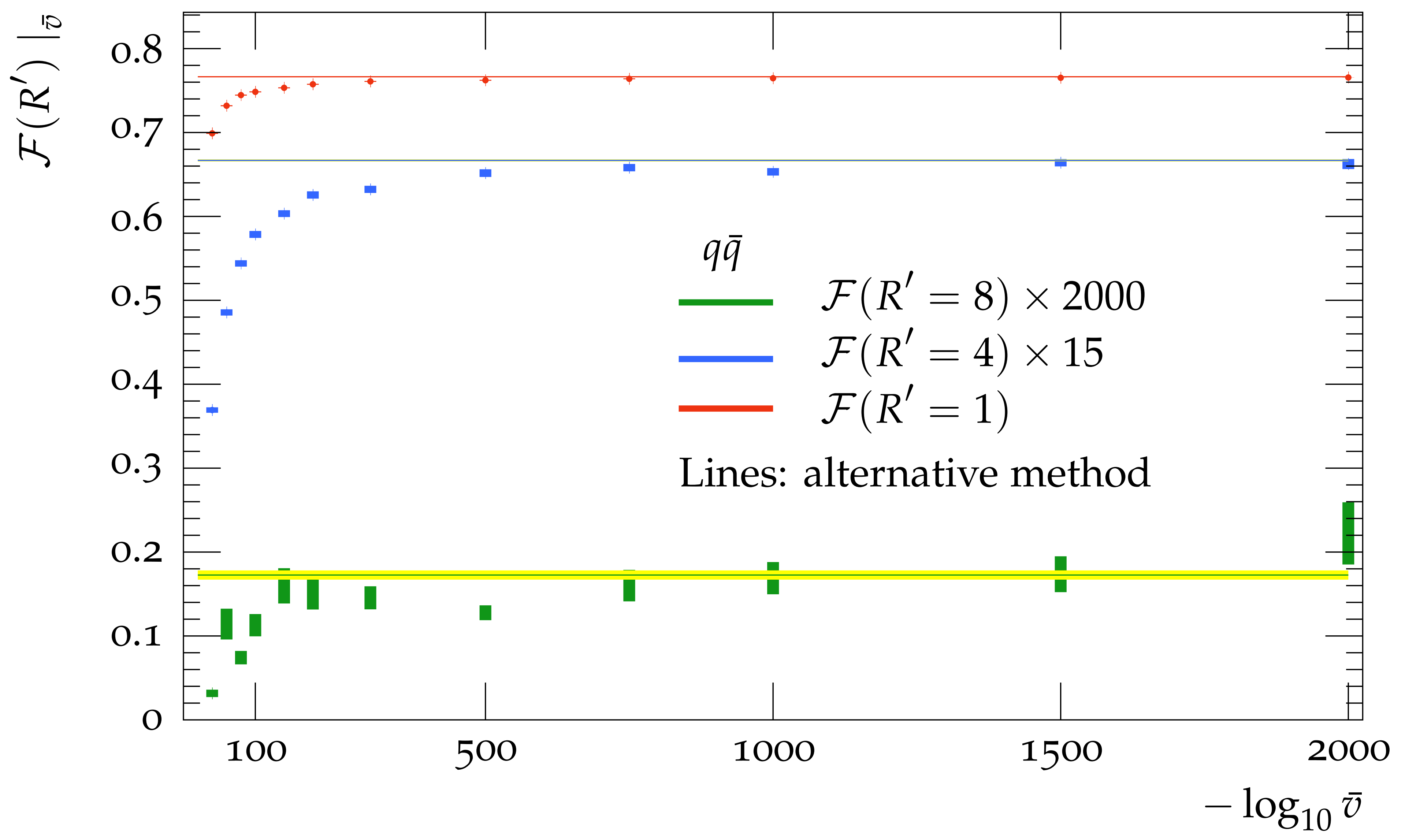
Example: jet resolution scales

- consider Durham jet cluster algorithm:

$$y_{i,j} = \frac{\min\left[E_i^2, E_j^2\right]}{Q^2}(1-\cos\theta_{ij}) \sim k_t^2/Q^2$$

- $y_{n\,n+1} \to$ min scale to resolve $(n+1)$ jets
- little analytic insight $\to$ numerical evaluation needed
- y_{23} first in [Banfi, Salam, Zanderighi '02]
- higher rates in [Baberuxki, Preuss, DR, Schumann '19]





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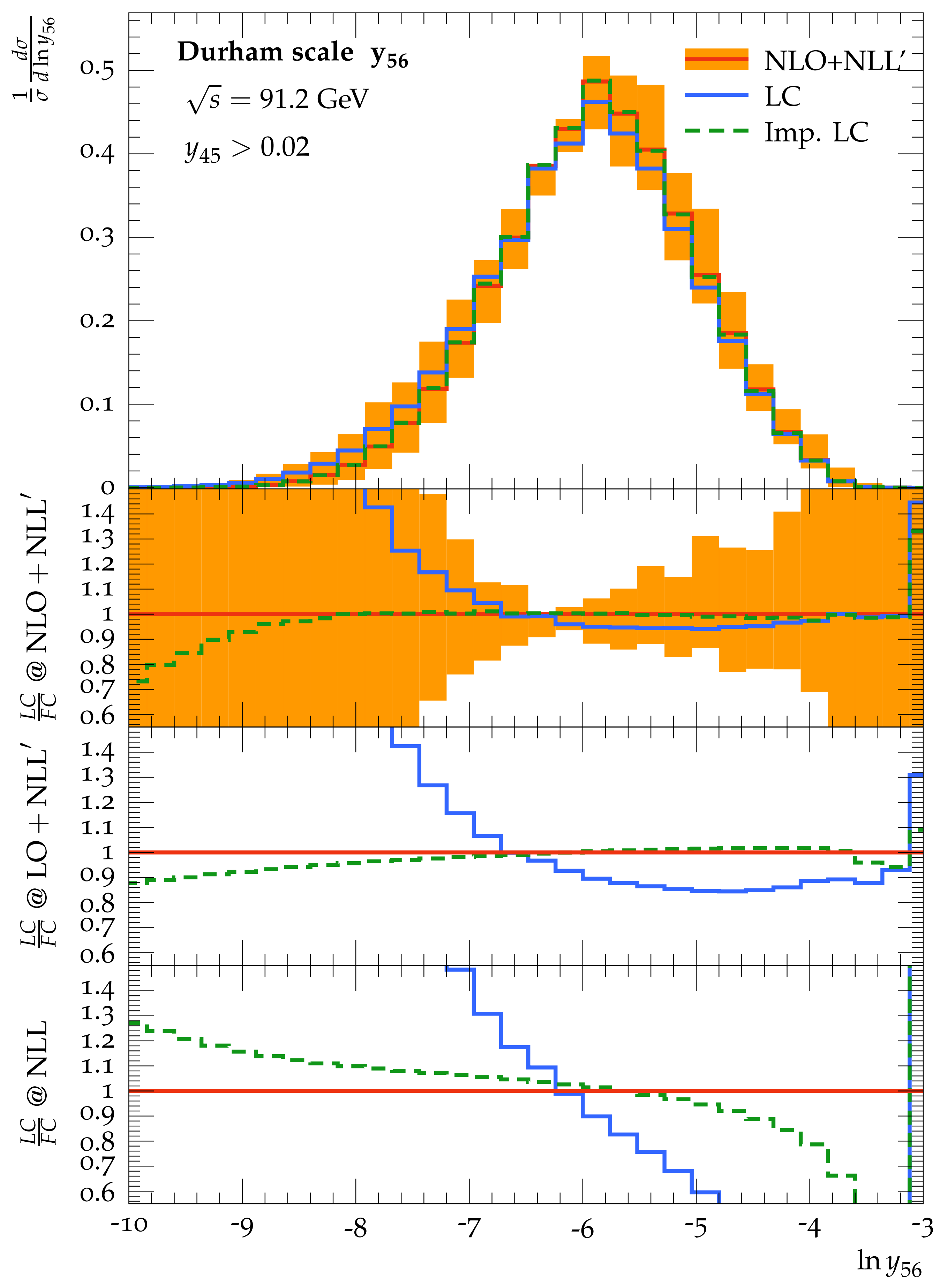
CAESAR formalism in Sherpa

- Putting everything together requires additional work [Gerwick, Höche, Marzani, Schumann '15]
- automation of several tasks conveniently done in traditional event generator framework, where we have easy access to
  - phase space integration
  - fixed order ingredients
  - color insertion operators
  - pdf handling
  - …



Example: Durham jet resolution up to y_{56}, color handling through interface to ME generator COMIX [Baberuxki, Preuss, DR, Schumann '19]

CAESAR+Sherpa framework — Overview

- CAESAR formalism for soft gluon resummation at NLL

[Banfi, Salam, Zanderighi ’04]

- available as implementation in Sherpa

[Gerwick, Höche, Marzani, Schumann ’15]
[Baberuxki, Preuss, DR, Schumann ’19]

- multiplicative matching (⇒ NLL’ accurate)

- necessary extensions for jet observables… :
  - modified wide angle behaviour
  - non-global logs

[Dasgupta, Khelifa-Kerfa, Marzani, Spannowski ‘12]
[Caletti, Fedkevych, Marzani, DR, Schumann, Soyez, Theeuwes ’21]
[DR, Caletti, Fedkevych, Marzani, Schumann, Soyez ‘22]
[Dasgupta, Salam ’01]

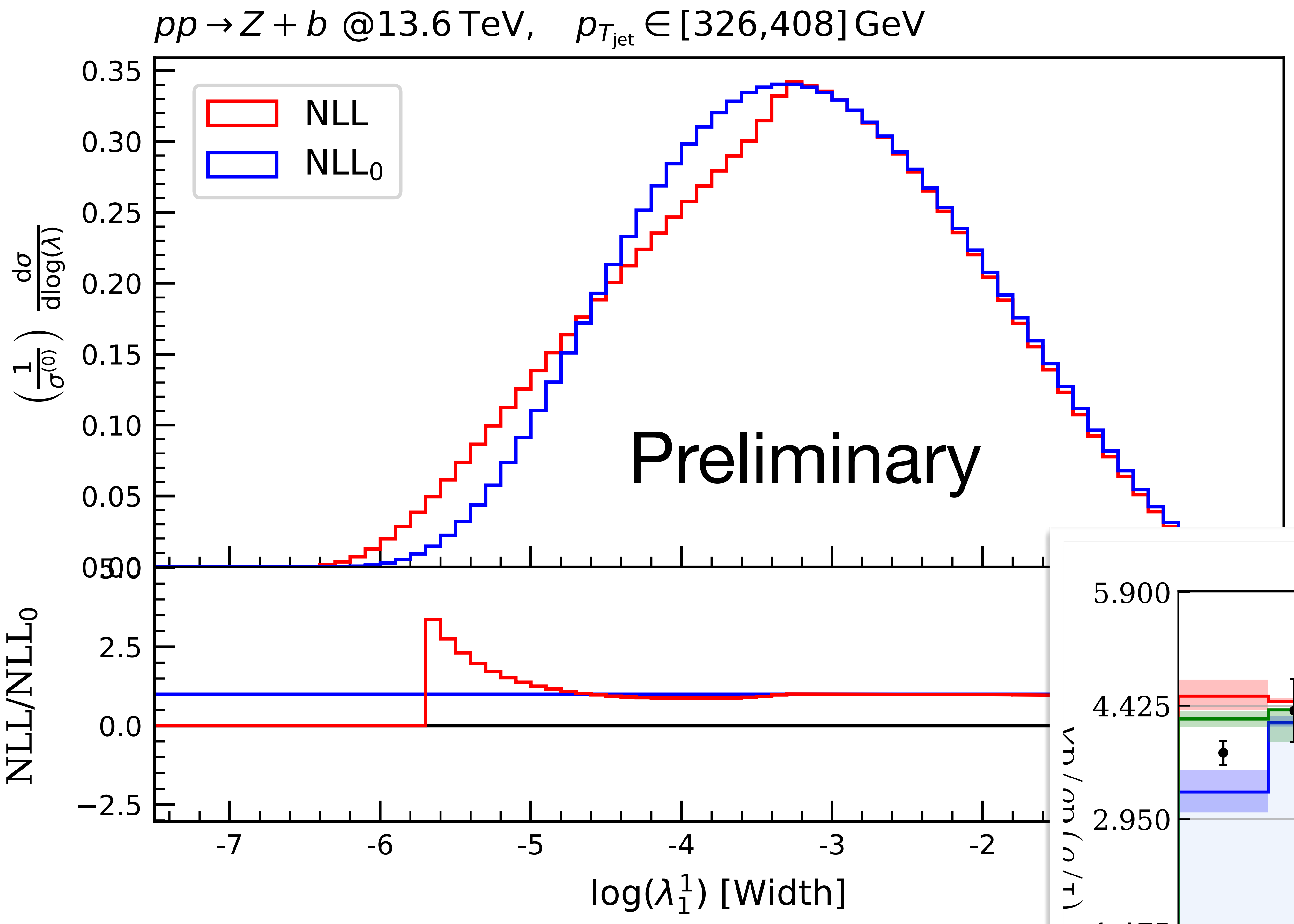
- … and for soft drop grooming
  - CEASAR style formulas available

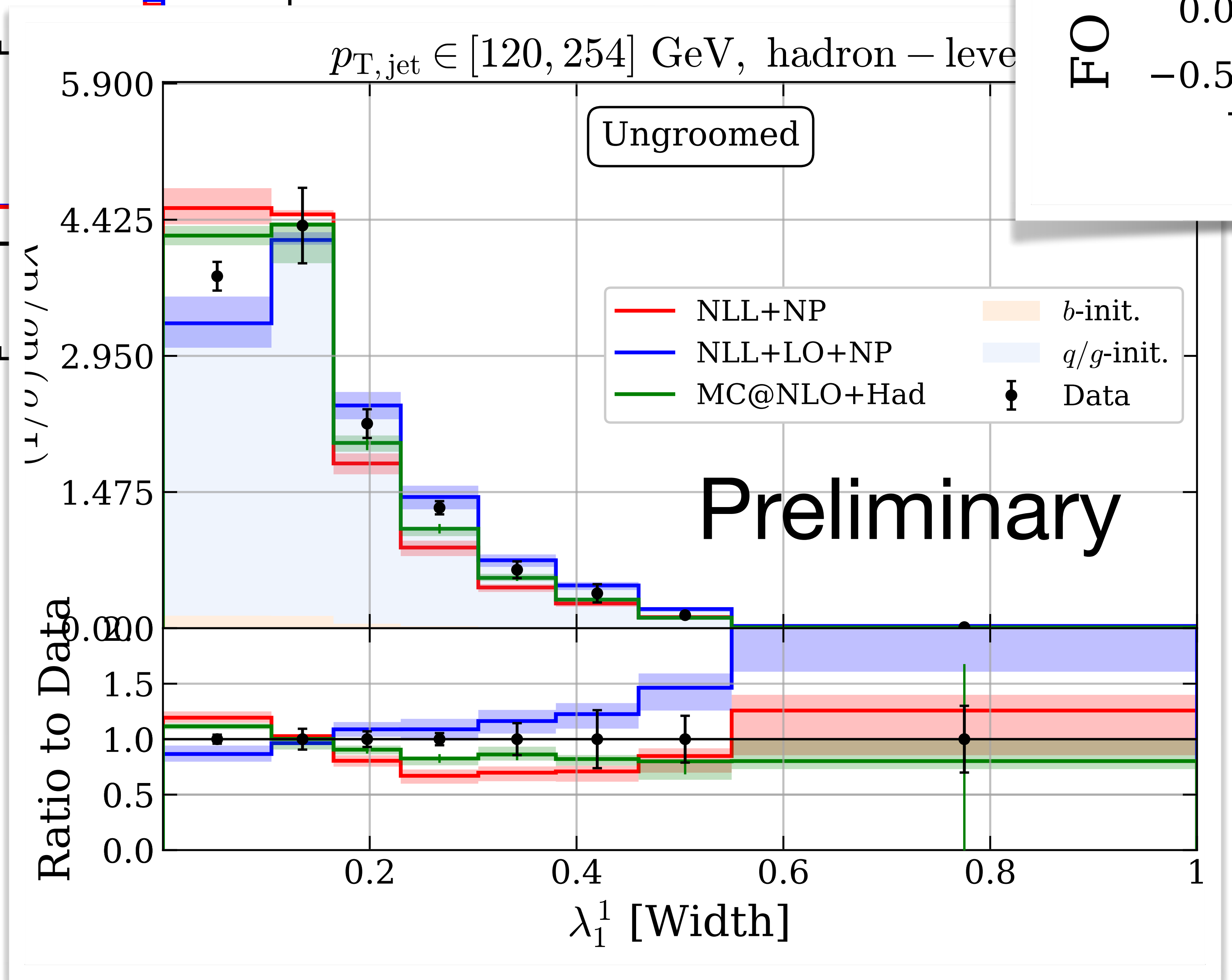
[Larkoski, Marzani, Soyez, Thaler ’14]
[Baron, DR, Schumann, Schwanemann, Theeuwes ‘20]

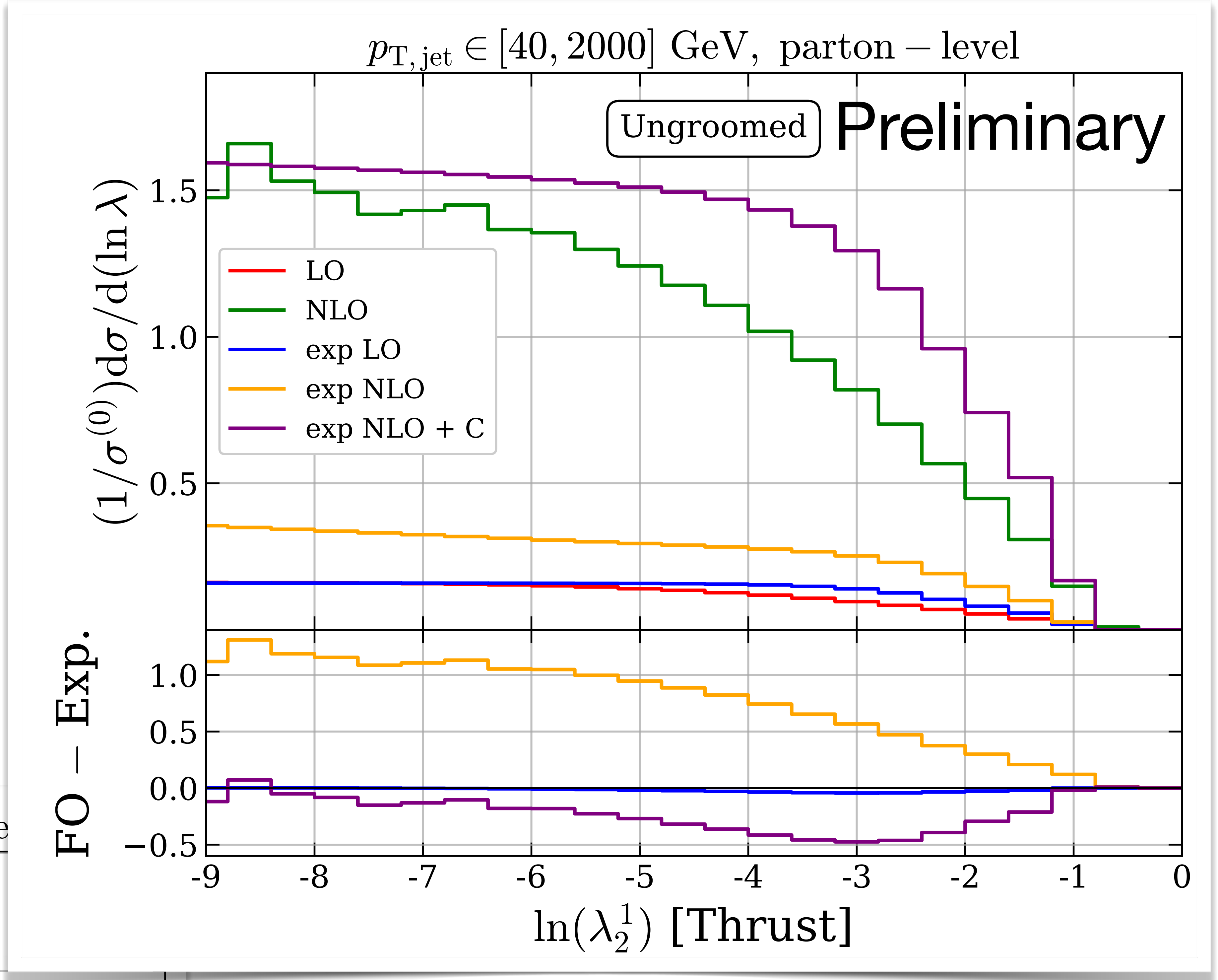
CAESAR+Sherpa: current developments

- quark mass effects on jet substructure

  [Ghira, Mai, Marzani, DR, Schumann, Stöcker WIP]







- Radiators with masses previously calculated [Ghira, Marzani, Soyez '24]

- First result: including mass effects in results from [DR, Caletti, Fedkevych, Marzani, Schumann '21]
- Compare to [CMS '21]

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Back to the CAESAR formalism…

- Overall structure

$$\Sigma(v) = e^{-R_{NLL}(v)} F(v) \qquad F(v) = e^{-\int_{\epsilon v}^{v} dk\, M^2(k)} \sum_n \frac{1}{n!} \int dk_i\, M^2(k_i)\, \theta(v - V(\{p_i\},\{k_i\}))$$

- at NLL: approximate matrix element M^2 **and** observable dependence $V(k)$ in soft collinear limit

From CAESAR to ARES formalism

- Overall structure

[Banfi, McAslan, Monni, Zanderighi '14]

$$\Sigma(v) = e^{-R_{NLL}(v)} F(v) \qquad F(v) = e^{-\int_{\epsilon v}^{v} dk\, M^2(k)} \sum_n \frac{1}{n!} \int dk_i\, M^2(k_i)\, \theta(v - V(\{p_i\},\{k_i\}))$$

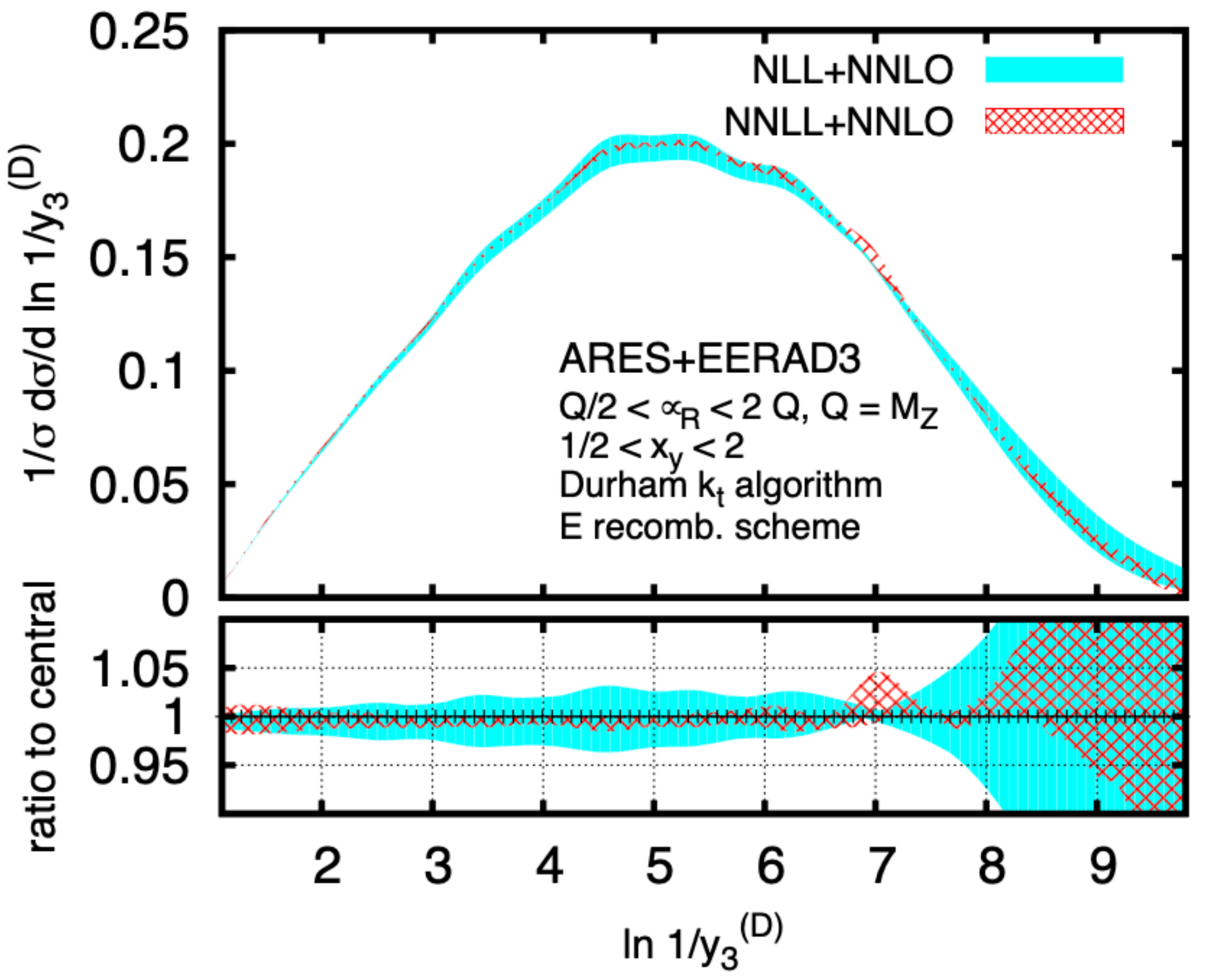
- at NLL: approximate matrix element M^2 **and** observable dependence $V(k)$ in soft collinear limit

- at NNLL: need to relax approximations one-by-one, gives rise to

$$\Sigma(v) = e^{-R_{NNLL}(v)} \{F(v) + \delta F_{NNLL}\}$$

$$\delta F_{NNLL} = \delta F_{sc} + \delta F_{hc} + \delta F_{rec} + \delta F_{wa} + \delta F_{correl} + \ldots$$

- Each δF corresponds to next order of expansions around soft collinear limit for one emission



[Banfi, McAslan, Monni, Zanderighi '16]

Example: δF_{wa} with ARES

- In sc limit, observables generally behave as $V(\{p_i\},k) \sim \frac{k_t}{Q} e^{-b\eta}$
- Not generally true for soft but wide angle: $V_{wa}(\{p_i\},k) \sim \frac{k_t}{Q} f(\eta,\phi)$
  - e.g. thrust has $f(\eta,\phi) = \frac{2}{\cosh\eta}$
- gives rise to correction [Banfi, McAslan, Monni, Zanderighi '14]

$$\mathcal{F}_{\rm wa}(v) = e^{-\int_{\epsilon v}^{v}[dk]M^2_{\rm sc}(k)} \sum_{n=0}^{\infty} \frac{1}{n!} \int_{\epsilon v} \prod_{i=1}^{n} [dk_i] M^2_{\rm sc}(k_i) 2C_F \int_0^\infty \frac{dk_t}{k_t} \frac{\alpha_s(k_t)}{\pi} \int_{-\infty}^{\infty} d\eta \int_0^{2\pi} \frac{d\phi}{2\pi} \times$$
$$\times \left[\Theta\left(1 - \lim_{v\to 0} \frac{V^{(k)}_{\rm wa}(\{\tilde{p}\},k,k_1,\ldots,k_n)}{v}\right) - \Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\},k,k_1,\ldots,k_n)}{v}\right)\right]$$

Towards CAESAR@NNLL

- ARES: no actual phase space generation, need to study a given observable in the limit corresponding to each δF and implement new routine calculating observable based on generated parameters

- CAESAR: generate actual phase space (i.e. four-momenta), scale all emissions towards the soft collinear limit, calculate observable based on actual definition (as it would be based on reconstructed four-momenta in experiments)

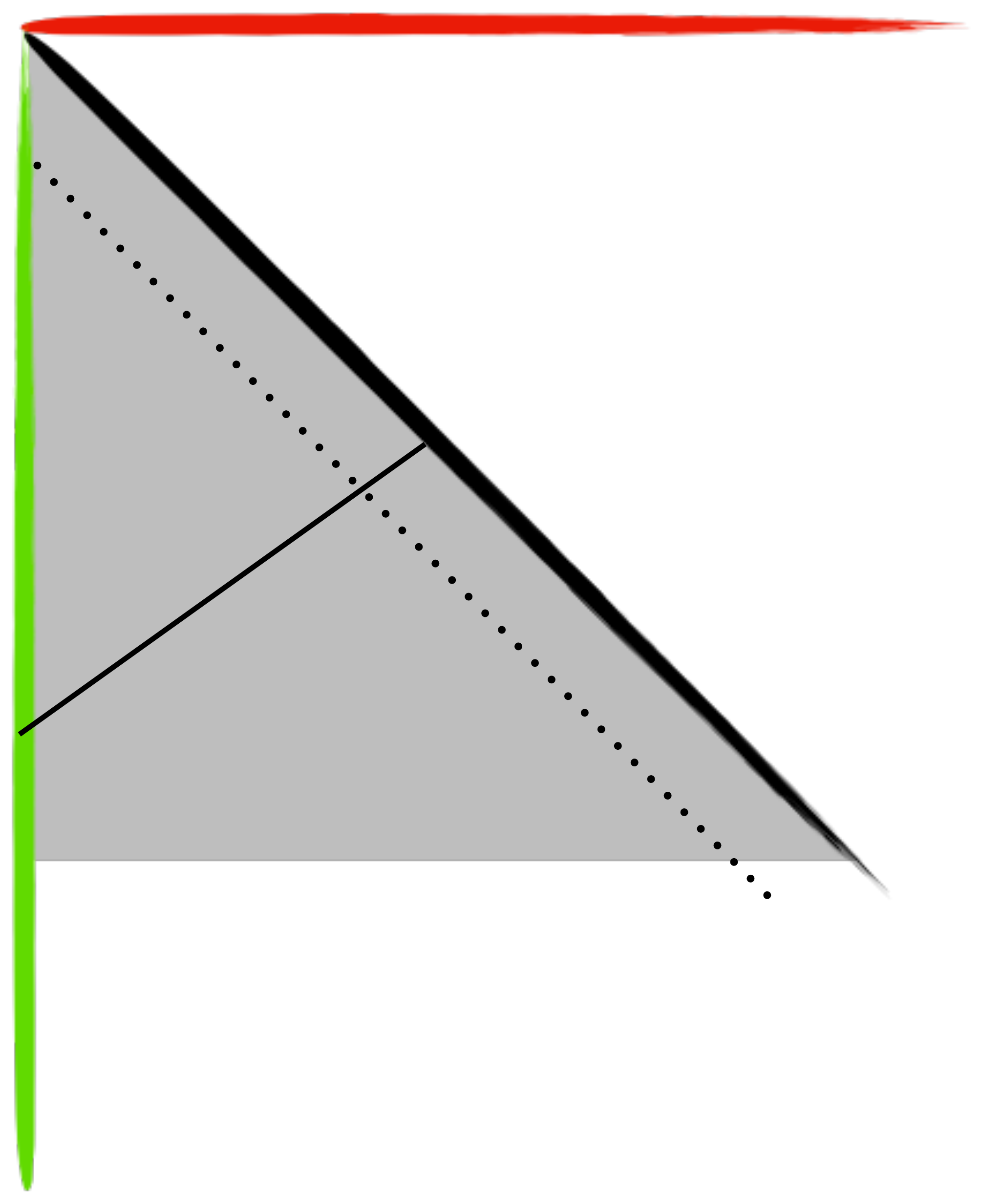
- $\Rightarrow$ numerically challenging, but no observable specific input needed (apart from definition of observable)

Example: recoil correction

- Recoil correction arises from finite recoil of hard-collinear emission [Banfi, McAslan, Monni, Zanderighi '14]

$$\delta\mathcal{F}_{\rm rec}(\lambda) = \sum_{\ell=1,2} \frac{\alpha_s(v^{1/(a+b_\ell)}Q)}{\alpha_s(Q)(a+b_\ell)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \int d\mathcal{Z}[\{R'_{\rm NLL,\ell_i}, k_i\}] \times$$
$$\times \int_0^1 dz\, p_\ell(z) \left[\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm hc}^{(k')}(\{\tilde{p}\}, k', \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, k, \{k_i\})}{v}\right)\right]$$

- For first term, need full recoil on hard legs, quickly becomes involved in particular with multiple legs that themselves have a non-trivial configuration



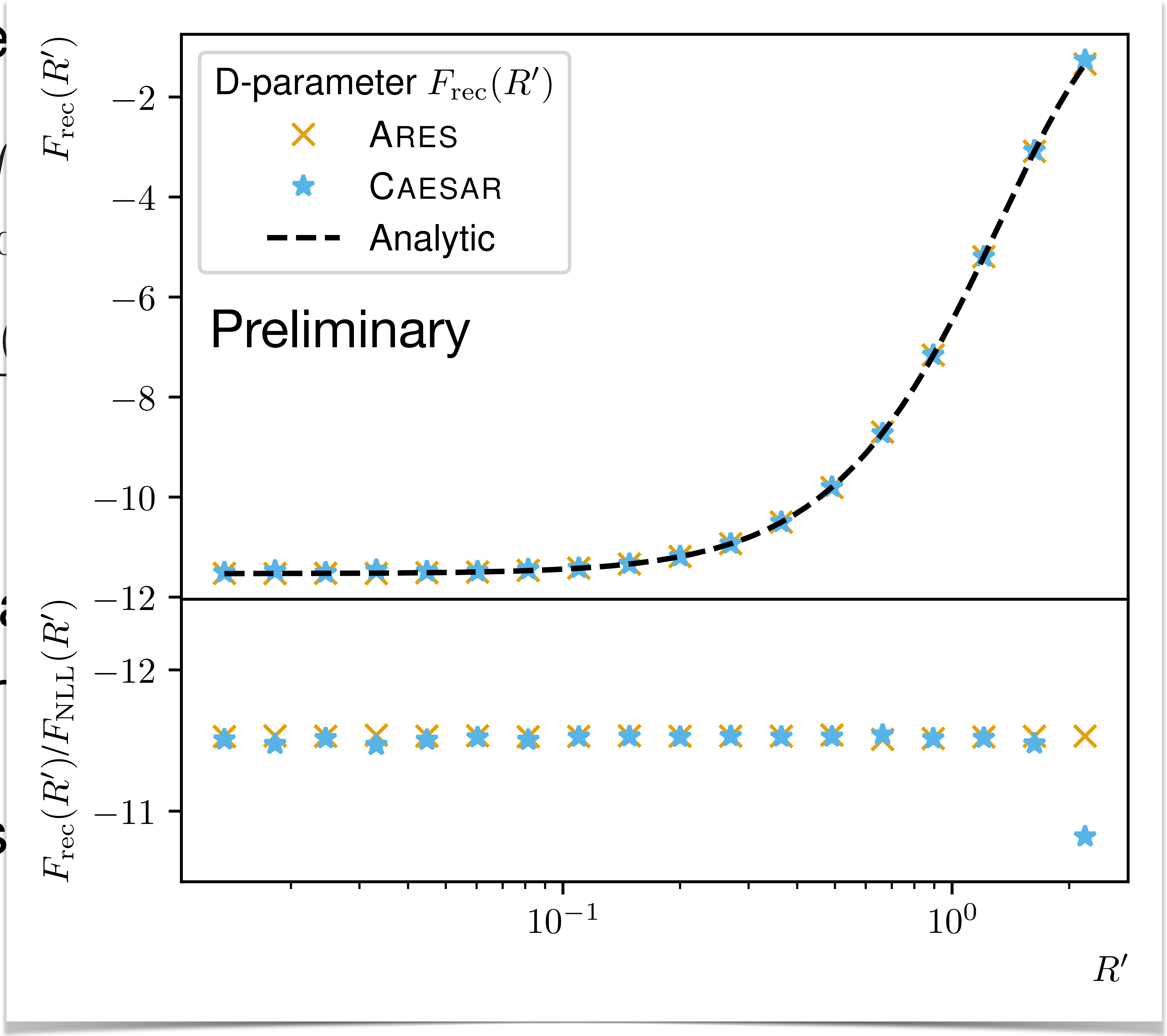
- In CAESAR, just generate phase space point, insert momenta with recoil, evaluate observable [Banfi, DR WIP] [Arpino, Banfi, El-Menoufi '13]
- Validate against known results for D-parameter

Example: recoil correction

- Recoil correction arises from finite

$$\delta\mathcal{F}_{\rm rec}(\lambda) = \sum_{\ell=1,2} \frac{\alpha_s(v^{1/(a+b_\ell)}Q)}{\alpha_s(Q)(a+b_\ell)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0$$

$$\times \int_0^1 dz\, p_\ell(z) \left[\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm hc}^{(k')}(}{}\right.\right.$$



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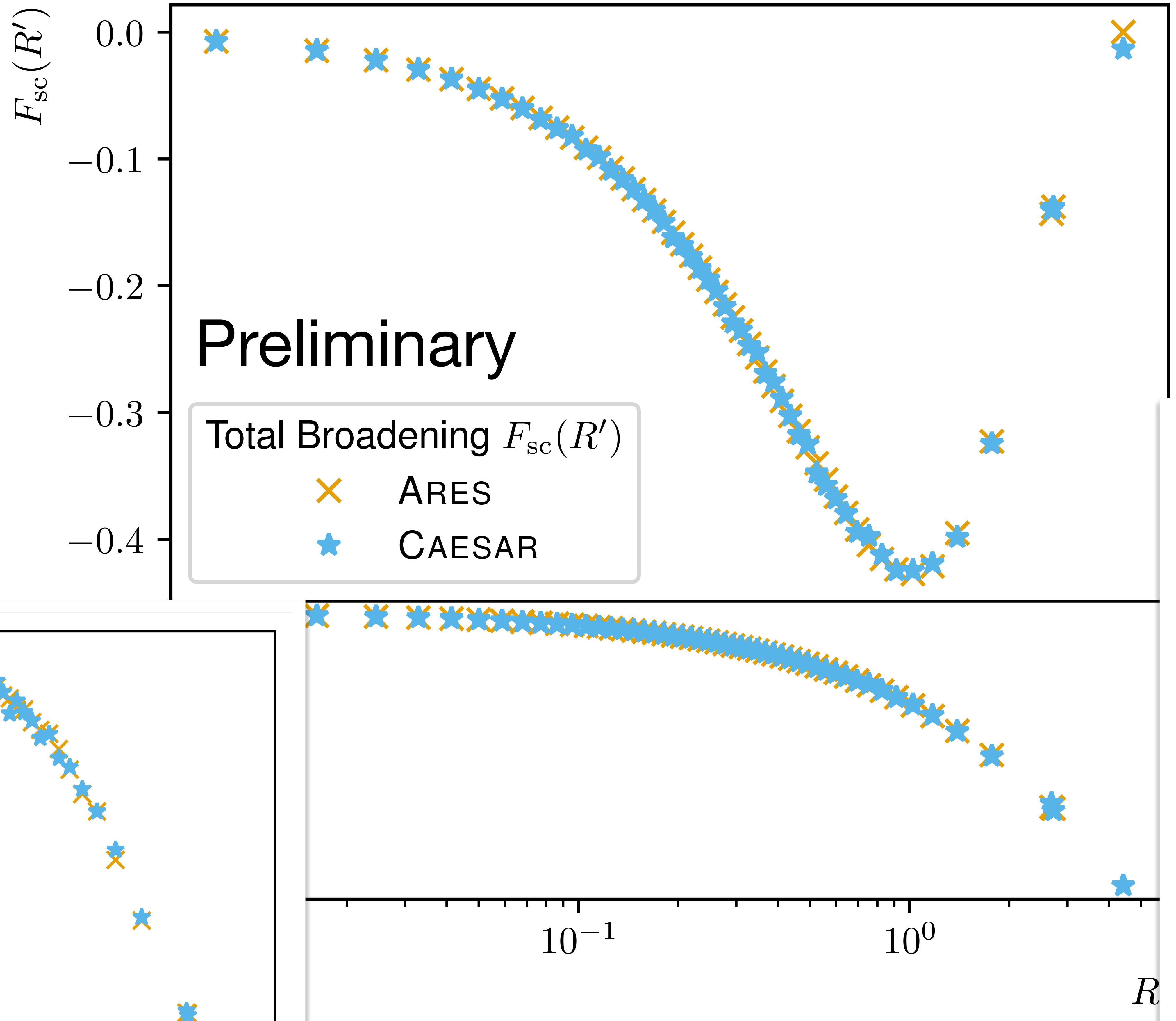
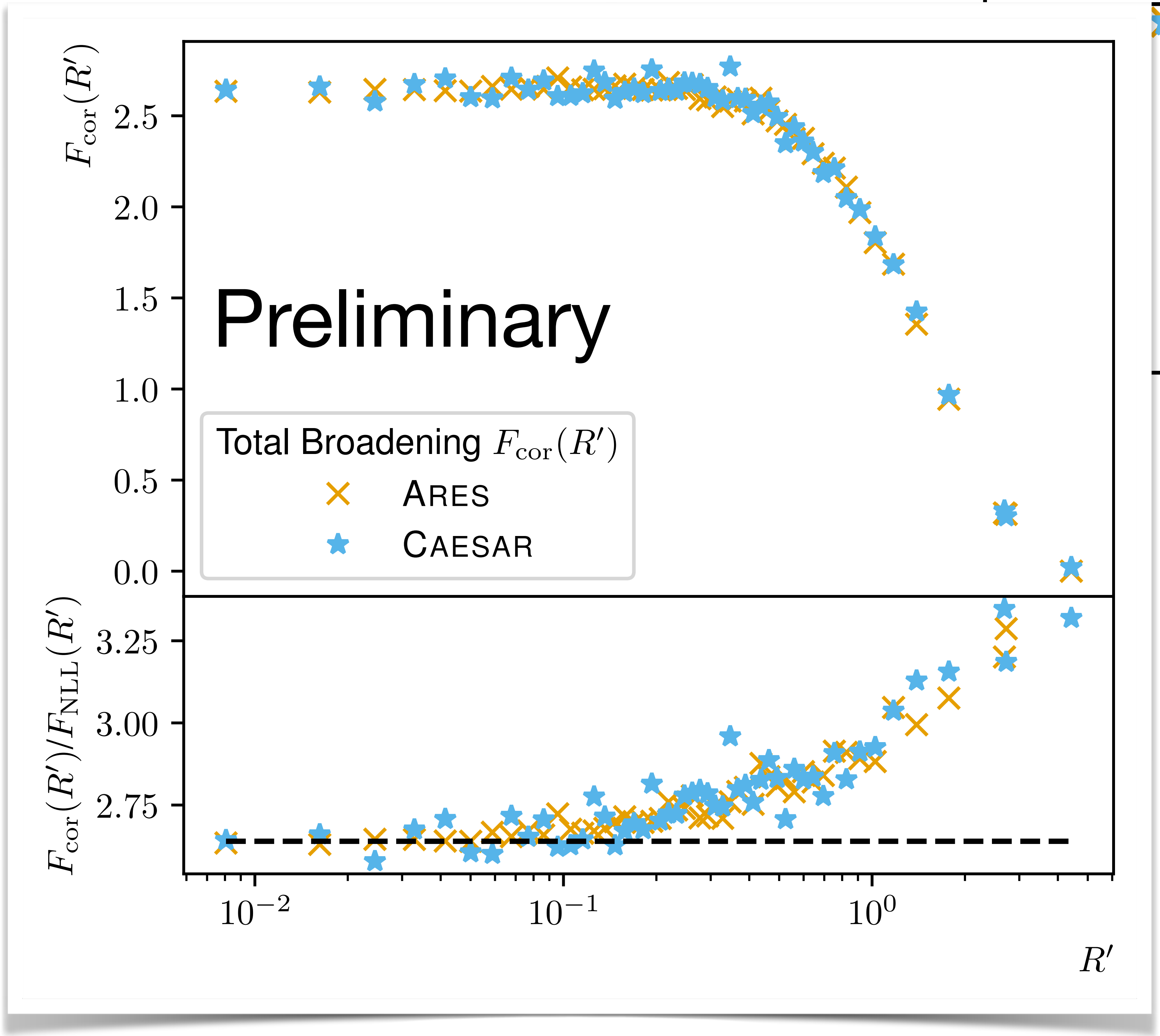
- In CAESAR, just generate phase s evaluate observable

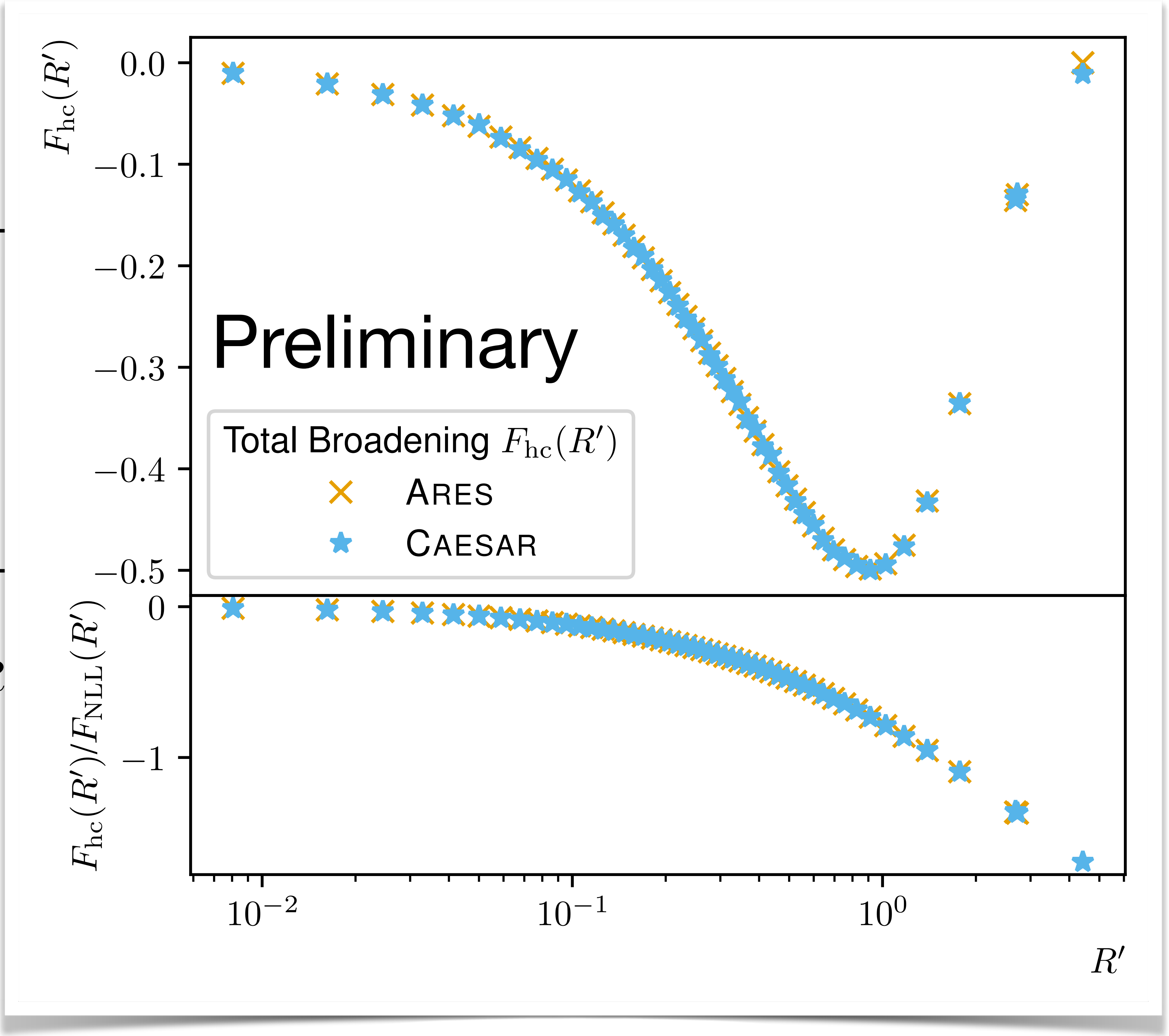
Other corrections

[Banfi, DR WIP]

- Validation of CAESAR@NNLL vs ARES
- Example: Total Broadening







Outlook: Towards LHC physics

- Goal: Extend to hadron collisions, starting with Z/H+jet process

  [Banfi, Lim, Pasha, Peake, DR, Wisnia WIP]

  - 1-jettiness, Transverse Thrust, Thrust Minor
  - checked analytically against known results
  - will need convolution of beam functions with pdfs etc

$$\delta F_X \sim \int dk \, |M|^2 \frac{f(x/z, Q^2)}{f(x, Q^2)} \left[\theta(1 - V_X(\{k_i\})) - \theta(1 - V_{sc}(\{k_i\}))\right]$$

Summary

- CAESAR/ARES formalism
  - access to observables like jet-resolution scales that do not easily factorize
  - direct interpretation in terms of multiple emissions from Born events eases comparison to parton shower methods
- Practical implementation in event generator framework Sherpa
  - new: inclusion of parton mass effects
- Extension of CAESAR to NNLL
  - avoid any observable specific investigation of limits
- Outlook: extension to hadron colliders