# Compton Scattering Total Cross Section at NNLO and Resummation of LL

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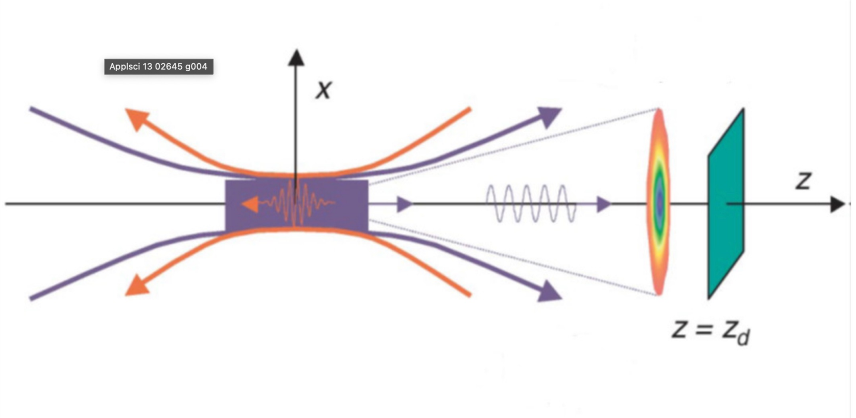
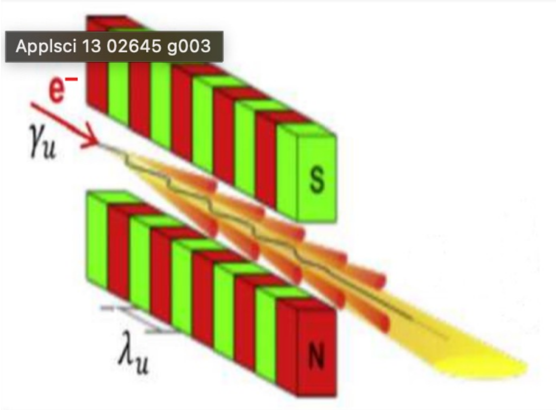
# Background




- Compton scattering $e^- + \gamma \to e^- + \gamma + X$
- Long history: Dates back to more than a century ago.
- Broad applications: medical physics, astronomy,
- Particle colliders: FCC, ILC, CEPC and muon collider.
- Clean benchmark, Beam diagnostics, luminosity measurements and polarization monitoring
- Double Logarithms at NLO: (R.N.Lee, M.D.Schwartz, X.Y.Zhang 2021)




$$\sigma_{\rm tot} \sim \frac{2\pi\alpha^2}{s} \ln\left(\frac{s}{m^2}\right)\left[1 + \frac{\alpha}{6\pi}\ln^2\left(\frac{s}{m^2}\right) + \cdots\right]$$

- 8.2% at $\sqrt{s} = 1GeV$ and 30% at $\sqrt{s} = 1TeV$.
- Does the NNLO contribution still offer a significant improvement?
- What is the origin of these double logarithms and how can one resum them to all orders?

# Double Logs and Regge Limit

- The logarithm at Leading Order originates from the limit of backward scattering, corresponding to $s \gg |t|$ (Regge limit).
- Origin of double logs?
  - Soft-collinear overlapping singularities: cancelled due to BN theorem.
  - Connection to DGLAP or BFKL: The LL in Regge Limit of QCD gluon is well-studied, which takes the form $[\alpha_s \ln(s/(-t))]^n$ , is not applicable to the double log structure in QED Compton scattering.
- Resummation of double logs at amplitude level(V.G.Gorshkov, V.N.Gribov, L.N. Lipatov, G.V.Frolov 1966,1967)
- Resummation related to endpoint divergence
  - Bottom-quark induced $h \to \gamma\gamma$ decay (J.Y.Hou, J.Wang, D.J.Zhang 2022, Z.L.Liu, B.Mecaj, M.Neubert, X.Wang 2019,2020 )
  - Muon-electron backward scattering (G.Bell P. Böer, T. Feldmann 2022)

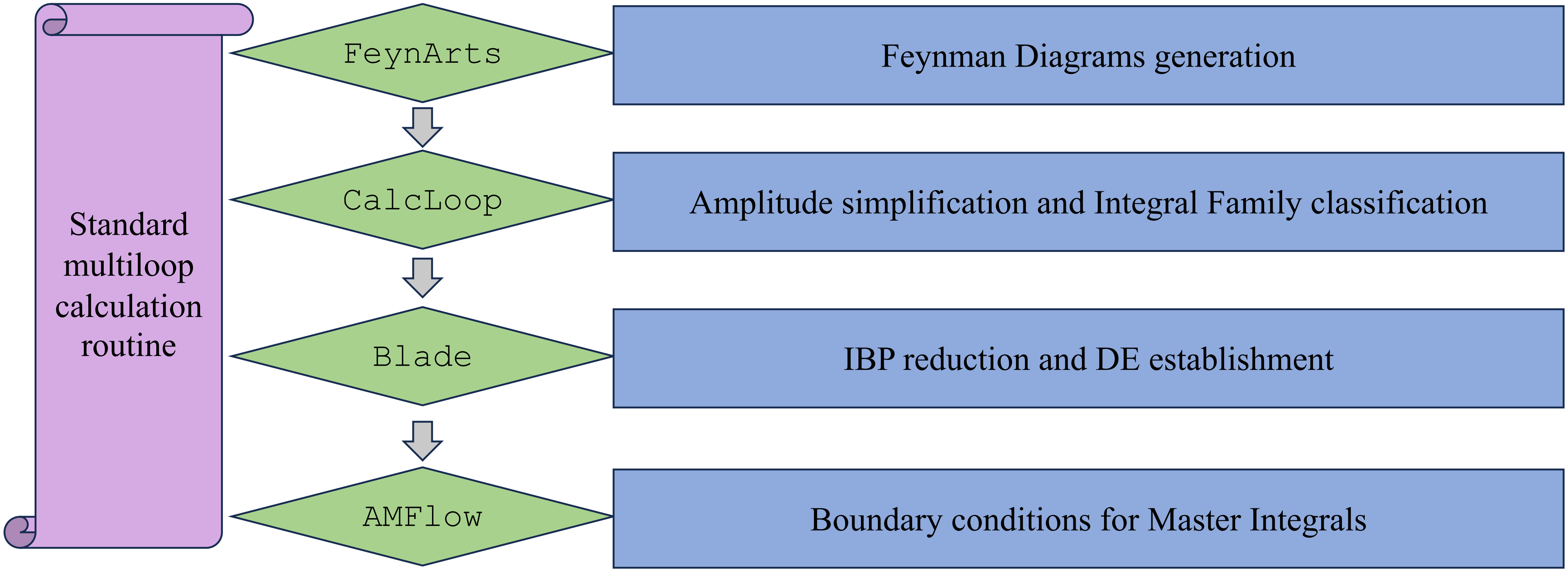
$$F_1(\lambda) = \sum_{n=0}^{\infty} \left(\frac{\alpha_{\text{em}}}{2\pi}\right)^n F_1^{(n)}(\lambda) \simeq \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha_{\text{em}}}{2\pi} \ln^2 \lambda^2\right)^n}{n!(n+1)!} = \frac{I_1\left(2\sqrt{\frac{\alpha_{\text{em}}}{2\pi} \ln^2 \lambda^2}\right)}{\sqrt{\frac{\alpha_{\text{em}}}{2\pi} \ln^2 \lambda^2}},$$

# NNLO Computation $e^-(p_1)\gamma(p_2) \to e^-(p_3)\gamma(p_4)\{\gamma(p_5)\gamma(p_6)\}$

QED NNLO corrections with full electron mass dependence.

No double logs with three charged particle production (R.Lee, A.Lyubyakin, V.A.Stotsky 2020)

$$\sigma_{e^-\gamma\to e^-e^-e^+} = \alpha^3\left[\frac{28\ln s}{9} - \frac{218}{27} - \frac{1}{s}\left(\frac{8}{3}\ln^3 s - \frac{49}{6}\ln^2 s - \left(2\pi^2 - \frac{409}{9}\right)\ln s + 12\zeta_3 + \frac{31\pi^2}{18} - \frac{1081}{12}\right) + O\left(s^{-2}\right)\right]$$



# Frobenius series solutions

From the differential equations of the Master Integrals(MIs)

$$\frac{d\vec{I}}{d\tau} = M(\epsilon,\tau).\vec{I}, \quad \tau = \frac{m^2}{s}$$

The solution of the MIs can be represented by piecewise Frobenius series solutions

$$I(\epsilon,\tau) = \sum_{a,b} \tau^a \ln^b(\tau) \left( \sum_{n=0}^{\infty} C_{abn}(\epsilon)\tau^n \right)$$

where $a = a_0 + a_1\epsilon, \quad 2a_0, a_1 \in Z$ and $b$ is a non-negative integer.

The coefficients are fixed by the boundary condition at $\tau = 1/530$ , and checked at another regular point $\tau = 49/53$ .

# Renormalization

- IR divergences cancel after real and virtual contributions are combined together.
- UV divergence is renormalized in the on-shell scheme.
- Two methods to double check:
  - Calculate counterterm Feynman diagrams.
  - Replace bare paramters with renormalized ones.
- Instead of reconstructing $\epsilon$ dependence, we only take two values of $\epsilon$, $\epsilon = \pm 1/1000$ , resulting in an error of $\mathcal{O}(10^{-6})$ at the cross section level.

(H.-Y. Bi et al. 2024)

# Fixed Order Results

- The cross section in the full physical region $\tau \in (0,1)$ is obtained in terms of a piecewise power series with up to 16 segments.
- Threshold Limit ( $x = (1-\tau)/\tau$ )

$$\sigma(x) = \frac{\pi\alpha^2}{m^2}\left(\frac{8}{3} - \frac{8}{3}x + \frac{52}{15}x^2 - \frac{133}{30}x^3 + \frac{572}{105}x^4\right) + \frac{\alpha^3}{m^2}x^2\left(-\frac{16}{9}\ln x + \frac{4}{3}x\ln x - \frac{25}{9}x^2\ln x\right.$$
$$\left. + \frac{7}{15} - \frac{113}{90}x + \frac{203}{125}x^2\right) + \frac{\alpha^4}{\pi m^2}x^2(0.814x^2\ln^2 x + 0.797x^2\ln x - 2.05 + 7.16x - 18.6x^2) + \mathcal{O}(x^5),$$

- High-energy limit

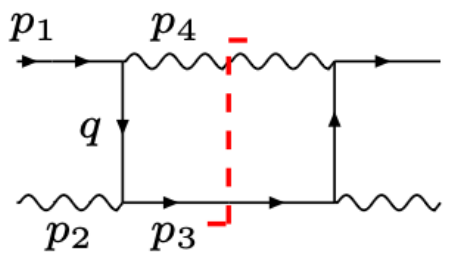
$$\sigma(\tau) = \frac{\pi\alpha^2}{s}[(2L+1) + \tau(-6L+17) + \tau^2(-30L+32) + \tau^3(-70L+48) + \tau^4(-126L+64) + \mathcal{O}(\tau^5)]$$
$$+ \frac{\alpha^3}{s}\left[\left(\frac{1}{3}L^3 - \frac{1}{2}L^2 + \frac{17}{4}L - 9.5016\right) + \tau(2L^3 + 13L^2 - 36.5340L + 0.55139) +\right.$$
$$\left. + \tau^2\left(\frac{38}{3}L^3 + \frac{151}{2}L^2 - 89.091L + 47.062\right) + \tau^3\left(\frac{157}{3}L^3 + \frac{344}{3}L^2 - 220.07L + 149.73\right) + \mathcal{O}(\tau^4)\right]$$
$$+ \frac{\alpha^4}{\pi s}\left[\left(\frac{1}{48}L^5 - 0.149L^4 + 1.34L^3 - 1.17L^2 - 16.6L + 23.8\right)\right.$$
$$\left. + \tau(1.13L^5 - 6.54L^4 - 44.6L^3 + 442L^2 - 36.0L - 2.64\times 10^3) + \mathcal{O}(\tau^2)\right],$$

$1/(96\pi^2)$ suppression

- 0.2% at $\sqrt{s} = 1GeV$ and 2.5% at $\sqrt{s} = 1TeV$

# Region Analysis of the Leading Logarithms(Leading Order)

- At LO, LL comes from t-channel diagram in the backward scattering limit.



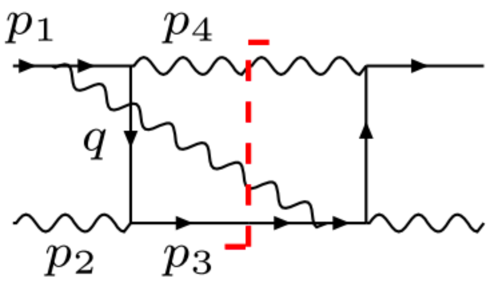

- Four regions: hard ($h$): $\sqrt{s}(1,1,1)$ , collinear ($c$): $\sqrt{s}(1,\lambda^2,\lambda)$, anticollinear ($\bar{c}$): $\sqrt{s}(\lambda^2,1,\lambda)$, soft (s): $\sqrt{s}(\lambda,\lambda,\lambda)$ where $\lambda=\sqrt{\tau}$.
- Endpoint divergences in the (anti)collinear and soft regions:
  - Imposing analytic regulator $\nu^\eta/(n_+q+n_-q+i0)$^$\eta$
  - The imaginary part of the forward scattering loop amplitude comes from the combination of hard and anticollinear region:

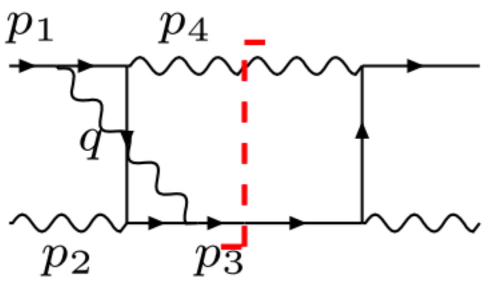
$$\mathrm{Im}\left[\left(\frac{\mu^2}{-s-i0}\right)^\epsilon\frac{-1}{\epsilon^2}+\left(\frac{\mu^2}{m^2}\right)^\epsilon\left(\frac{\nu}{-\sqrt{s}+i0}\right)^\eta\frac{1}{\epsilon\eta}\right]$$
$$=-\pi\left[\ln\frac{\mu^2}{s}-\ln\frac{\mu^2}{m^2}\right]=-\pi\ln\tau.$$

  - First applying the Cutkosky rule: anticollinear mode turns to Glauber mode ($G$): $\sqrt{s}(\lambda^2,\lambda^2,\lambda)$

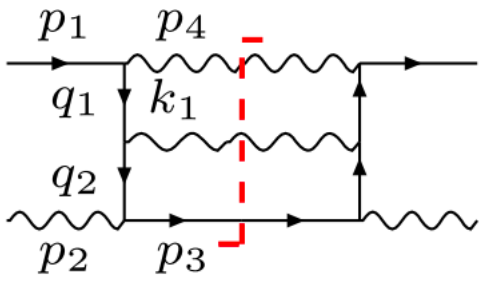
# NLO Analysis

- Double logs produced by diagrams with additional photon radiated from external electrons.



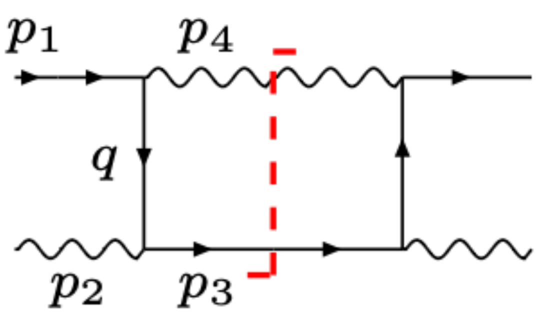

- Only uncancelled diagram contributing to LL is




$$= \frac{\alpha^3}{s}\left(\frac{L}{\epsilon} + \frac{1}{3}L^3 + \frac{1}{2}L^2 + L + \frac{5}{2} - \frac{2\pi^2}{3} - 2\zeta_3 + \mathcal{O}(\tau)\right)$$

- Relevant regions: $(q_1, q_2) \sim (h, h), (c, h), (h, \bar{c}), (c, G), (G, \bar{c})$

# Calculation of the ladder diagrams



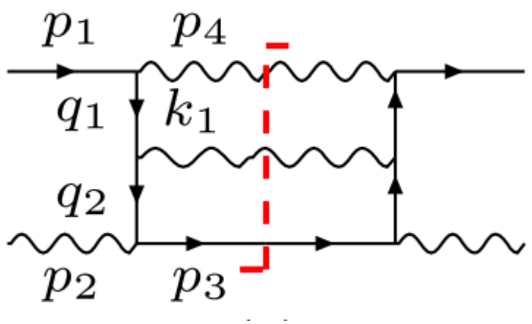

$$\sigma_{\rm LO,LL} = \int d^d q \delta_+(p_3^2)\delta_+(p_4^2)\frac{e^4}{4q^4}\mathrm{Tr}\left[\not{p}_1\gamma^{\mu_1}\ \not{q}\gamma^{\mu_2}\ \not{p}_3\gamma_{\mu_2}\ \not{q}\gamma_{\mu_1}\right]$$

$$d^d q\delta_+(p_3^2)\delta_+(p_4^2) \sim \delta_+(q^2 - 2q^-)\delta_+(q^2+2q^+)$$

$$q^- = \frac{1}{2}q_T^2, \quad q^+ = -\frac{1}{2}q_T^2$$ → Glauber mode

$$\sigma_{\rm LO,LL} = \frac{2\alpha^2}{s}(2\pi)\int_{m^2}^{1}\frac{d^2 q_T}{q_T^2} = -\frac{2\pi\alpha^2}{s}\ln\tau$$

Lower bound is from the propagator $1/(q^2 - m^2)$




Similarly, the on-shell conditions at NLO leads to

$$q_1^- = \frac{1}{2}q_{1T}^2, \quad q_2^- = q_1^- + \frac{(q_{1T}-q_{2T})^2}{2(q_1^+ - q_2^+)}, \quad q_2^+ = -\frac{1}{2}q_{2T}^2$$

$$\sigma_{\rm NLO,LL} = \frac{\alpha^3}{s}\int_{m^2}^{1} dq_1^+ \int_{m^2}^{q_1^+} dq_{1T}^2 \int_{m^2}^{q_1^+} dq_{2T}^2 \frac{1}{q_{1T}^2 q_{2T}^2 (q_1^+ + \frac{1}{2}q_{2T}^2)} = -\frac{\alpha^3}{3s}\ln^3\tau$$

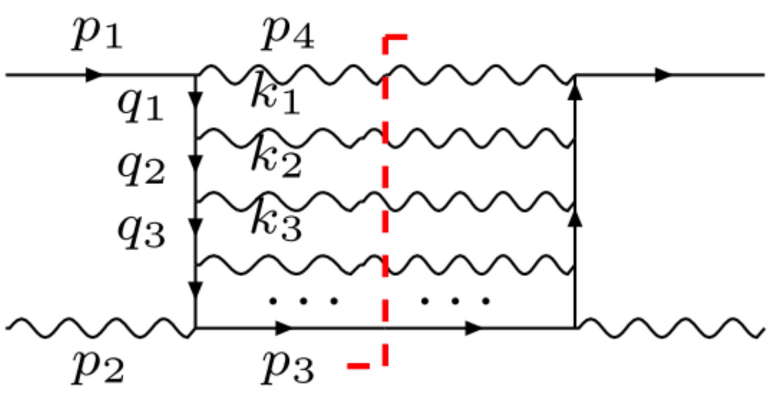
# All Orders calculation

- The LL structure extends to all orders.
- Relevant regions are $r_i = (c, \dots, c, G, \bar{c}, \dots, \bar{c})$

(J.F.Donoghue, B.K.El-Menoufi, G. Ovanesyan 2014)

- Ladder diagram with momenta satisfying

strong ordering (overlap of all $r_i$ regions)




$$p_{1+} > q_{1+} > \cdots > q_{n+} > 0,$$
$$0 < -q_{1-} < \cdots < -q_{n-} < p_{2-}.$$

contributes to LLs.

The calculation simplifies in this momentum scaling

$$\delta(p_4^2) \approx \delta((n_+p_1)(n_-p_1 - n_-q_1) - q_{1T}^2)$$

$$\delta(k_i^2) \approx \delta(-(n_+q_i)(n_-q_{i+1}) - (q_i - q_{i+1})_T^2$$

$$\delta(p_3^2 - m^2) \approx \delta(n_+q_n n_-p_2 - q_{nT}^2 - m^2)$$

$$q_i^2 - m^2 \approx -q_{iT}^2 - m^2 - \left(\frac{n_+q_i}{n_+q_{i-1}}\right) q_{(i-1)T}^2$$

# All Orders Calculation

- The phase space integration is simplified to

$$\sigma_{\mathrm{LL}}^{\mathrm{N}^{n-1}\mathrm{LO}} = \frac{4\pi^2}{s}\frac{\alpha^{n+1}}{(2\pi)^n}\int_1^{1/\tau}\frac{\mathrm{d}x_1}{x_1}\prod_{j=2}^{n-1}\int_1^{x_{j-1}}\frac{\mathrm{d}x_j}{x_j}$$
$$\times\int_1^{x_1}\frac{\mathrm{d}y_1}{y_1}\prod_{i=2}^{n-1}\int_{\max\{1,\frac{x_i}{x_{i-1}}y_{i-1}\}}^{x_i}\frac{\mathrm{d}y_i}{y_i}\int_1^{x_{n-1}}\frac{\mathrm{d}y_n}{y_n},$$

where $x_i = n_+ q_i\sqrt{s}/m^2$, $\mathrm{y_i} = (\mathrm{q_{iT}^2} + \mathrm{m^2})/\mathrm{m^2}$ .

The integration bound is determined by the momentum scaling and thus the dimensional regularization is not needed.

- The integration can be carried out by constructing a recursive relation

$$\sigma_{\mathrm{LL}}^{\mathrm{N}^{n-1}\mathrm{LO}} = -\frac{2\alpha^{n+1}}{(2\pi)^{n-2}s}\frac{1}{(n-1)!(n+1)!}\ln^{2n-1}\tau.$$

Validated at N3LO

# Resummation

- The LLs can be directly resummed as

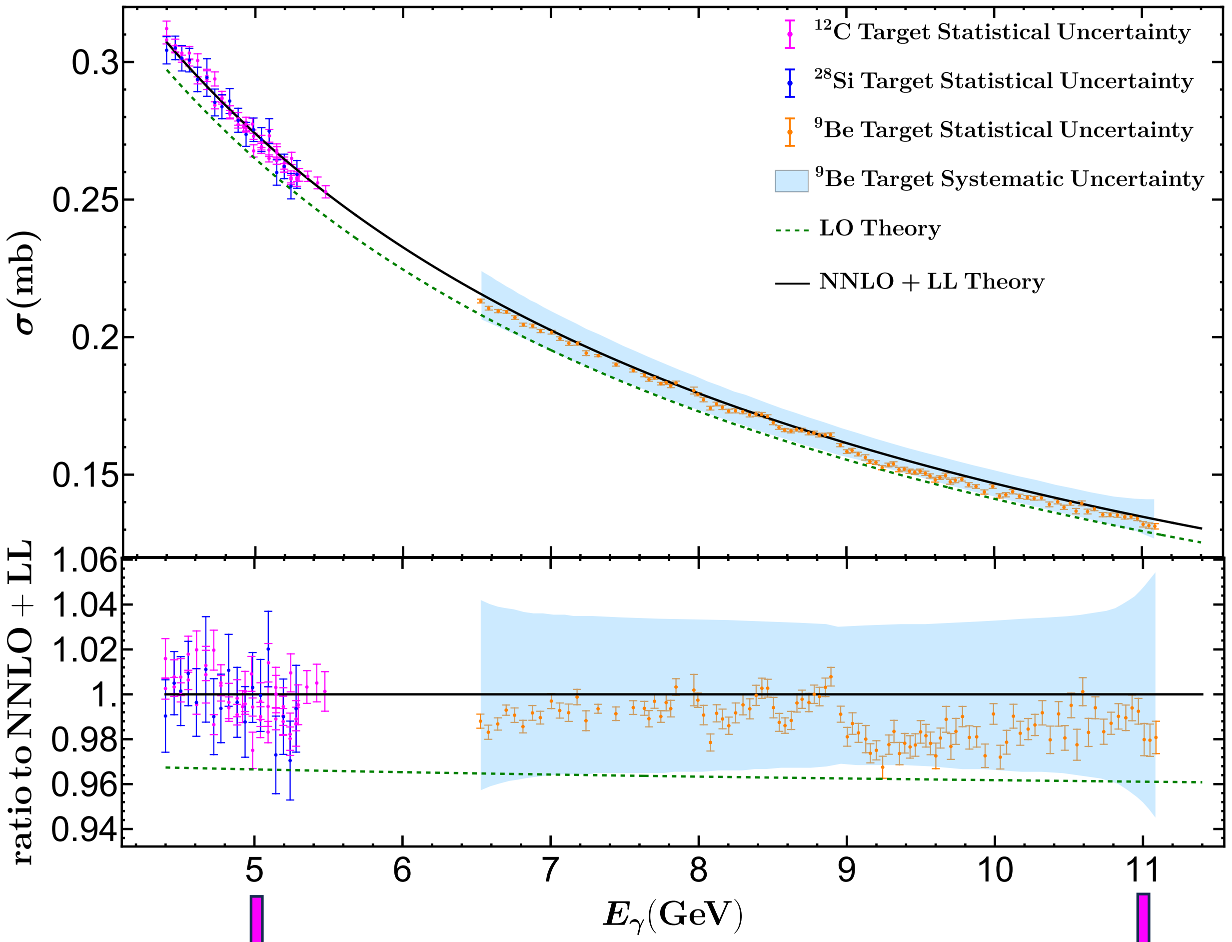
$$\sigma_{\mathrm{LL}} = -\frac{8\pi^2\alpha}{s\ln\tau} I_2\left(\sqrt{\frac{2\alpha}{\pi}}\ln\tau\right)$$
$$= \frac{2^{9/4}\pi^{7/4}\alpha^{3/4}}{s(-\ln\tau)^{3/2}\tau^{\sqrt{2\alpha/\pi}}} \quad \text{as } \tau\to 0,$$

For fixed $m$, the cross section scales as $s^{-1+\sqrt{2\alpha}}/\log^{3/2}(\frac{s}{m^2})$ as $s\to\infty$.

For fixed $s$, the cross section becomes divergent as $m\to 0$ since new IR divergences emerge in this limit.

# Numerical Comparison of theory and experiments

- the combined NNLO + LL resummation effects are minuscule (at the 0.05% level) and thus provides an exceptionally precise prediction.
- Compared with PrimEx(2019) and GlueX(2025) (at JLab) experimental data.




$\sqrt{s} = 0.07GeV$

$\sqrt{s} = 0.1GeV$

# Conclusion

- We performed complete NNLO calculation of the Compton-scattering total cross section with full electron mass dependence, revealing that the anticipated double-logarithmic enhancement is indeed present but numerically suppressed.
- We traced the origin of these logarithms to a specific kinematic configuration induced by a Glauber electron exchange, allowing us to resum the leading logarithms to all orders, yielding a compact result expressed in terms of a modified Bessel function.
- We validated our combined NNLO+LL results with high-precision data from PrimEx and GlueX collaborations.
- Resummation using SCET(Ongoing): the factorization formula has more complicate structure.
- The methodology developed here can be directly applied to other fundamental processes with similar logarithmic structure arising from massive fermion exchanges, e.g. $e^+e^- \to \gamma\gamma$ and $gg \to t\bar{t}$.