Towards Multiple Soft Emissions in LHC Jet Production:

When Super-Leading Logarithms Go Non-Global

Josua Scholze | SCET Workshop
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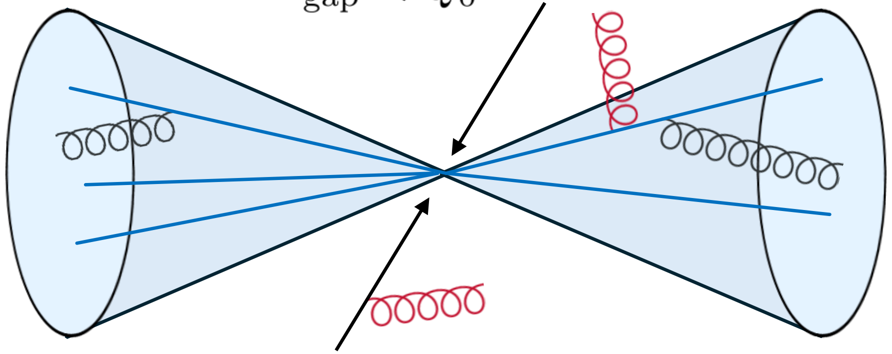
Super-Leading Logarithms



- Large logarithms in jet processes at hadron colliders (pp → jets):

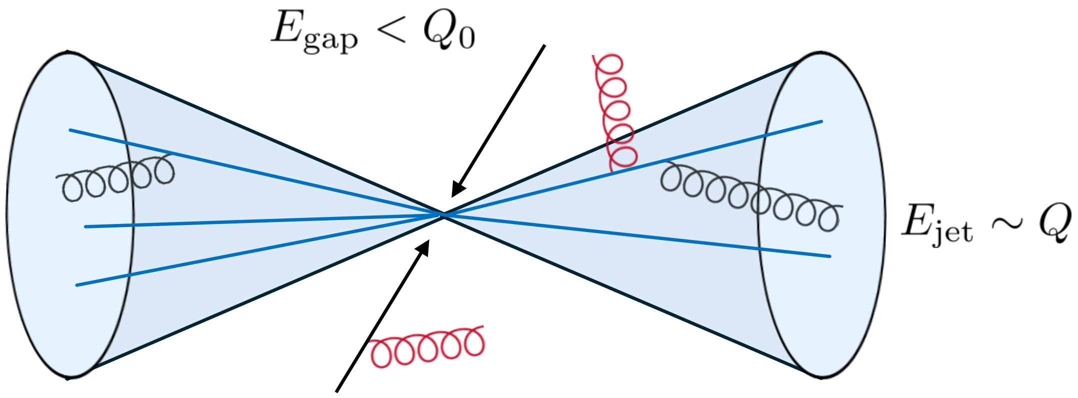
$$\sigma \sim \sigma_{\text{Born}} \times \{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 L^5 + \alpha_s^5 L^7}_{\text{"super-leading logarithms"}} + \ldots\}$$

$$L = \ln(Q/Q_0) \gg 1$$

[J. R. Forshaw, A. Kyrieleis, M. H. Seymour (2006)]

- Non-global observable: gap between jets

Super-Leading Logarithms



- Large logarithms in jet processes at hadron colliders (pp → jets):

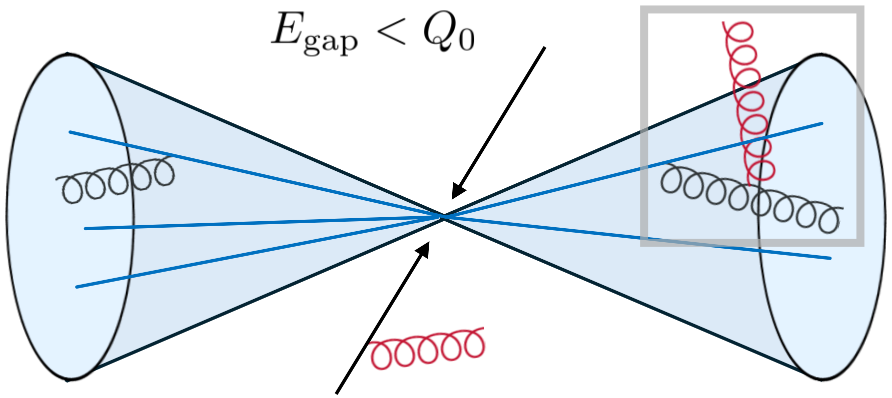
$$\sigma \sim \sigma_{\text{Born}} \times \{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 L^5 + \alpha_s^5 L^7}_{\text{“super-leading logarithms”}} + \dots\} \qquad L = \ln(Q/Q_0) \gg 1$$

[J. R. Forshaw, A. Kyrieleis, M. H. Seymour (2006)]

- Non-global observable: gap between jets

Non-Global Logarithms

- From secondary emissions
- Were studied for processes without Glaubers



Super-Leading Logarithms (SLLs)

- SCET factorization theorem: [T. Becher, M. Neubert, D. Shao (2021); + M. Stillger (2023)]

$$\sigma_{2\to M}(Q_0) = \int d\xi_1 \int d\xi_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, s, \xi_1, \xi_2, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, \xi_1, \xi_2, \mu) \rangle$$

Hard functions

Low-energy matrix elements

with $n_i = p_i / E_i \,, \quad n_i^2 = 0$

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Hard functions

Low-energy matrix elements

with $n_i = p_i/E_i\,, \quad n_i^2 = 0$

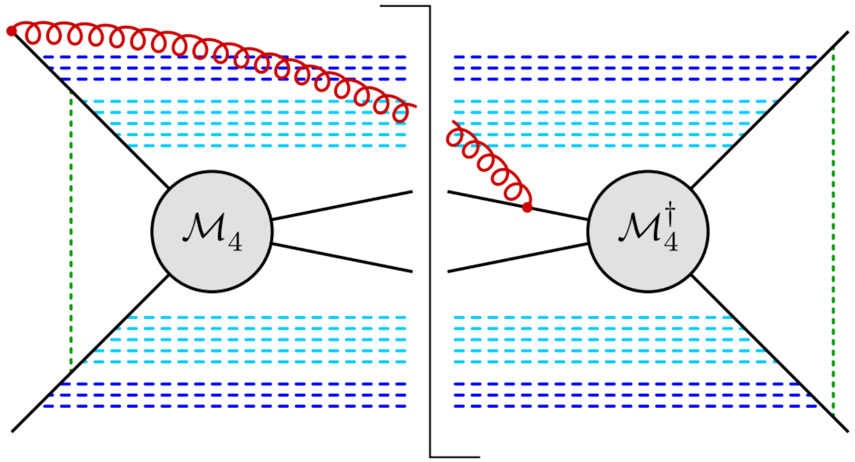
- For resummation, set: $\mu = \mu_s \sim Q_0$

PDFs

$$\boldsymbol{\mathcal{W}}_m(\xi_1, \xi_2, \mu_s) = f_1(\xi_1, \mu_s) f_2(\xi_2, \mu_s) \mathbf{1} + \mathcal{O}(\alpha_s)$$

$$\boldsymbol{\mathcal{H}}_m(\mu_s) = \sum_{l \le m} \boldsymbol{\mathcal{H}}_l(\mu_h) \times \mathbf{P} \exp \left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}^{\mathcal{H}}(\mu) \right]_{lm}$$

Anomalous dimension matrix



Anomalous dimension

- Full anomalous dimension: $\mathbf{\Gamma}^{\mathcal{H}}(\xi_1,\xi_2) = \delta(1-\xi_1)\delta(1-\xi_2)\mathbf{\Gamma}^S + \underbrace{\delta(1-\xi_2)\mathbf{\Gamma}_1^C + \delta(1-\xi_1)\mathbf{\Gamma}_2^C}$

  Purely collinear part: subleading

- Ingredients of soft anomalous dimension:

$$\mathbf{\Gamma}^S = \frac{\alpha_s}{4\pi}\overline{\mathbf{\Gamma}} + \gamma_{\text{cusp}}(\alpha_s)\left[\mathbf{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right) + \boldsymbol{V}^G\right] + \mathcal{O}(\alpha_s^2)$$

Purely soft part — Soft+collinear part — Glauber phase

$$\overline{\mathbf{\Gamma}} = 2\sum_{(i,j)}\left(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R}\right)\int\frac{d^2\Omega_k}{4\pi}\overline{W}_{ij}^k - 4\sum_{(i,j)}\overline{W}_{ij}^k\theta_{\text{hard}}(n_k)\,\boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{j,R}$$

$$\boldsymbol{V}^G = -2\pi i\left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right)$$

$$\mathbf{\Gamma}^c = \sum_{i=1,2}\left[C_i\,\mathbf{1} - \delta(n_i - n_k)\,\boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{i,R}\right]$$

Roadmap for Resummation of SLLs
(selected)

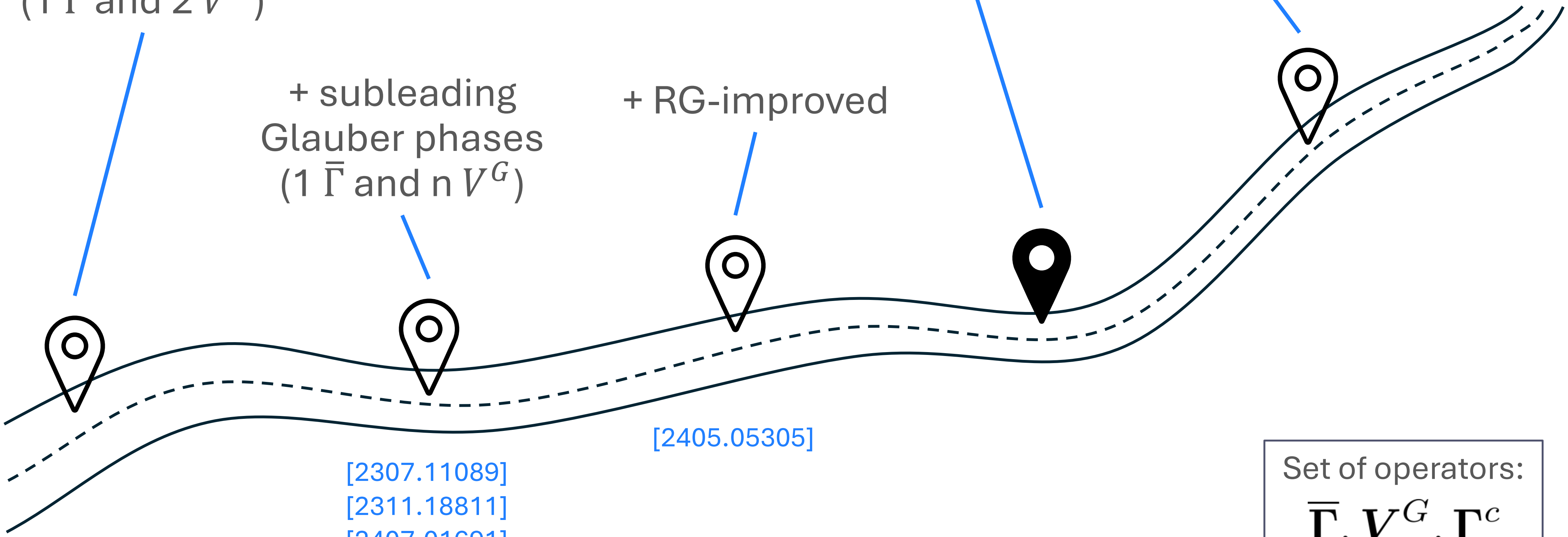
Leading-logarithmic contributions
($1\ \overline{\Gamma}$ and $2\ V^G$)

+ subleading Glauber phases
($1\ \overline{\Gamma}$ and n V^G)

+ RG-improved

Multiple soft emissions

($2\ \overline{\Gamma}$ and $1\ V^G$) ($2\ \overline{\Gamma}$ and $2\ V^G$)



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Set of operators:

$$\overline{\Gamma}, V^G, \Gamma^c$$

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Color Structures (2 $\bar{\Gamma}$ and 1 V^G)

- Cyclicity of the trace: $\langle \boldsymbol{\mathcal{H}} \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0$ and $\langle \boldsymbol{\mathcal{H}} \boldsymbol{V}^G \otimes \mathbf{1} \rangle = 0$

$$\langle \boldsymbol{\mathcal{H}} \dots \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

- Color coherence: $\left[\boldsymbol{\Gamma}^c, \overline{\boldsymbol{\Gamma}}\right] = 0$

$$\langle \boldsymbol{\mathcal{H}} \dots \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle \quad \text{and} \quad \langle \boldsymbol{\mathcal{H}} \dots \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}}\,\overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

- Filling up with $\boldsymbol{\Gamma}^c$ insertions:

$$\langle \boldsymbol{\mathcal{H}} (\boldsymbol{\Gamma}^c)^r \overline{\boldsymbol{\Gamma}} (\boldsymbol{\Gamma}^c)^{n-r} \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle \quad \text{and} \quad \langle \boldsymbol{\mathcal{H}} (\boldsymbol{\Gamma}^c)^n \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}}\,\overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

$$= \langle \boldsymbol{\mathcal{H}} \overline{\boldsymbol{\Gamma}} (\boldsymbol{\Gamma}^c)^n \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

Known from SLLs with one soft emission

New type of structures

Color Structures (2 $\overline{\Gamma}$ and 1 V^G)

- Insertions of $\mathbf{\Gamma}^c$ in $\langle \boldsymbol{\mathcal{H}} \ldots \boldsymbol{V}^G \overline{\mathbf{\Gamma}}\overline{\mathbf{\Gamma}} \otimes \mathbf{1} \rangle$ a priori very technical:

  Interplay of the real emissions and other color structures

$$\mathbf{\Gamma}^c = \sum_{i=1,2} \left[C_i \, \mathbf{1} - \delta(n_i - n_k) \, \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R} \right]$$

Containing e.g.: $\{\boldsymbol{T}_1^a, \boldsymbol{T}_1^b\}$

Color Structures (2 $\overline{\Gamma}$ and 1 V^G)

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- Trick: observe the relation $\langle \boldsymbol{\mathcal{H}} \boldsymbol{V}^G \overline{\mathbf{\Gamma}}\,\overline{\mathbf{\Gamma}} \otimes \mathbf{1} \rangle = \langle \boldsymbol{\mathcal{H}} \big(\alpha \overline{\mathbf{\Gamma}} \boldsymbol{V}^G \overline{\mathbf{\Gamma}} + \sum_j \beta(j) i f^{abc} \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \big) \otimes \mathbf{1} \rangle$

- To perform insertion of $\mathbf{\Gamma}^c$:

  Use: $[\mathbf{\Gamma}^c, \overline{\mathbf{\Gamma}}] = 0$

  Color structure in $\boldsymbol{V}^G \overline{\mathbf{\Gamma}}$

- $\dots \boldsymbol{V}^G \overline{\mathbf{\Gamma}} \otimes \mathbf{1} \rangle$ is an eigenvector of $\mathbf{\Gamma}^c$ with eigenvalue N_c

Obtaining a Symmetrized Color Basis (2 $\bar{\Gamma}$ and 1 V^G)

- With 3 color operators (familiar):

$$if^{abc}\boldsymbol{T}_1^a\boldsymbol{T}_2^b\boldsymbol{T}_i^c$$

- With 5 color operators (new):

$$if^{abc}\{\boldsymbol{T}_1^a,\boldsymbol{T}_1^d\}\{\boldsymbol{T}_2^b,\boldsymbol{T}_2^d\}\boldsymbol{T}_i^c$$

$$if^{abc}\boldsymbol{T}_1^a\{\boldsymbol{T}_2^b,\boldsymbol{T}_2^d\}\boldsymbol{T}_i^c\boldsymbol{T}_j^d\,, \qquad if^{abc}\{\boldsymbol{T}_1^b,\boldsymbol{T}_1^d\}\boldsymbol{T}_2^a\boldsymbol{T}_i^c\boldsymbol{T}_j^d$$

$$if^{abc}\boldsymbol{T}_1^a\{\boldsymbol{T}_2^b,\boldsymbol{T}_2^d\}\{\boldsymbol{T}_i^c,\boldsymbol{T}_i^d\}\,, \quad if^{abc}\{\boldsymbol{T}_1^b,\boldsymbol{T}_1^d\}\boldsymbol{T}_2^a\{\boldsymbol{T}_i^c,\boldsymbol{T}_i^d\}$$

$$if^{abc}\boldsymbol{T}_1^a\boldsymbol{T}_2^b\{\boldsymbol{T}_i^c,\boldsymbol{T}_i^d\}\boldsymbol{T}_j^d$$

$$if^{abc}\boldsymbol{T}_1^a\boldsymbol{T}_2^b\boldsymbol{T}_i^c\boldsymbol{T}_j\cdot\boldsymbol{T}_m$$

<u>Initial states</u>

$1, 2$

<u>Final states</u>

$i, j, m > 2$

and distinct

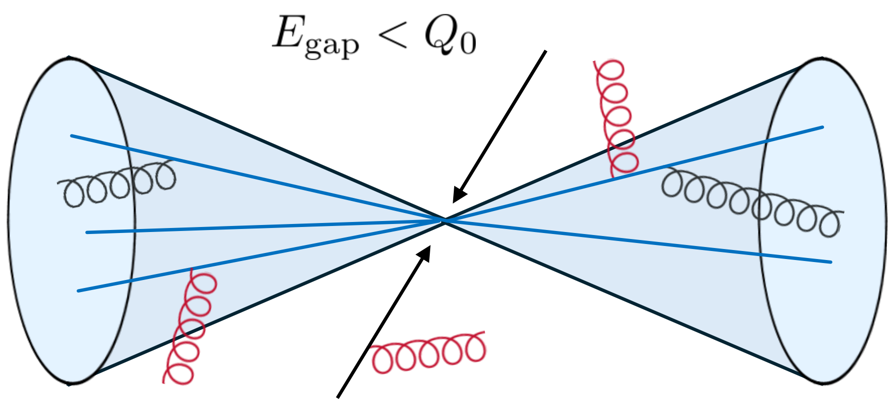
- Contains now symmetrized products of final states and up to 3 distinct final states

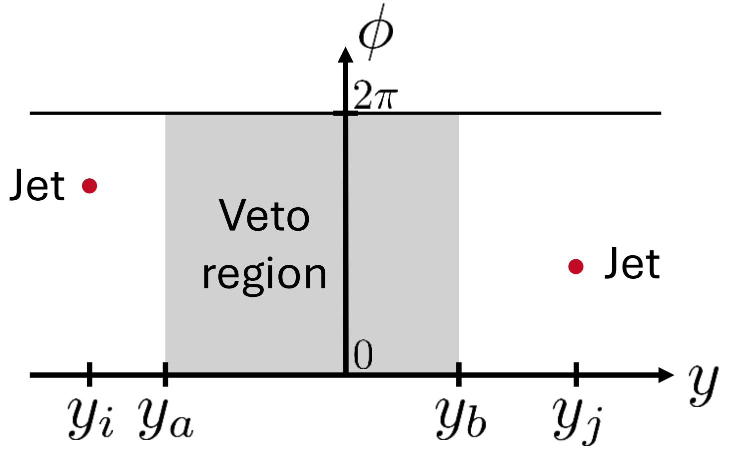
Angular integrals

- For one soft emission: J_{12} and $J_i = J_{1i} - J_{2i}$

$$J_{ij} = \int \frac{d^2\Omega_k}{4\pi} W_{ij}^k \underbrace{(1 - \theta_{\text{hard}}(n_k))}_{= \theta_{\text{veto}}(n_k)}$$

Soft dipole operator: $W_{ij}^k = \dfrac{n_i \cdot n_j}{n_i \cdot n_k \, n_j \cdot n_k}$





Rapidity: $y = \dfrac{1}{2} \ln\left(\dfrac{1+\cos\theta}{1-\cos\theta}\right)$

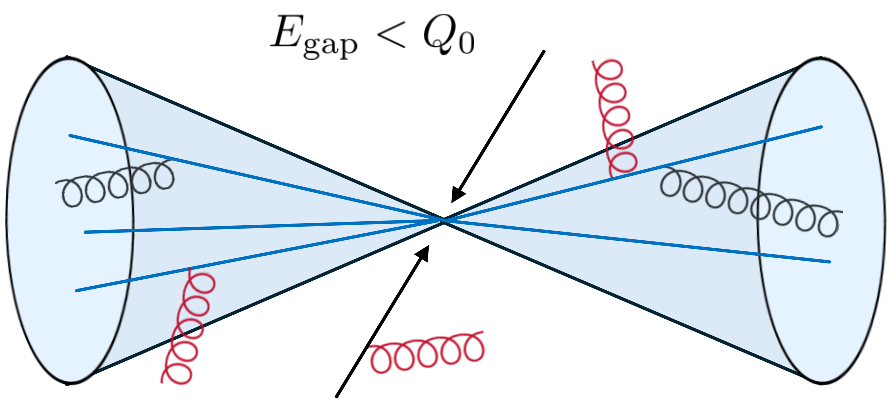
Angular integrals

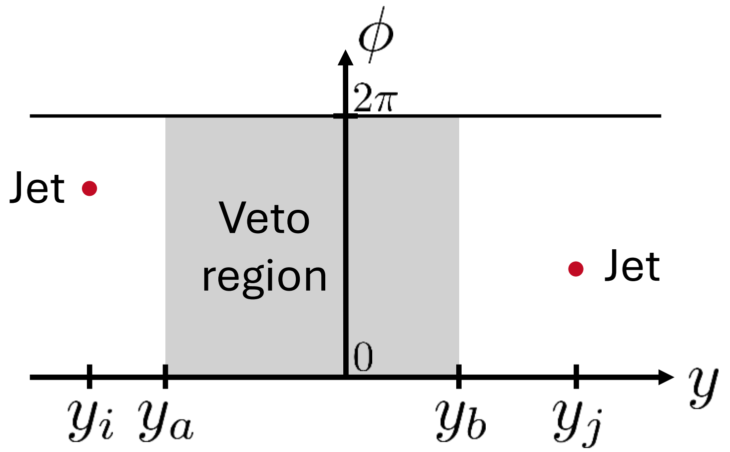
- For one soft emission: J_{12} and $J_i = J_{1i} - J_{2i}$
- Now products, e.g.: $J_i\, J_{1i}$ or $J_i\, J_{jm}$

$$J_{ij} = \int \frac{d^2\Omega_k}{4\pi} W_{ij}^k \underbrace{(1 - \theta_{\text{hard}}(n_k))}_{= \theta_{\text{veto}}(n_k)}$$

Soft dipole operator: $W_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k \, n_j \cdot n_k}$

$$J_{ij} = \frac{1}{2} \log \left[\frac{\sinh(y_a - y_j)\sinh(y_b - y_i)}{\sinh(y_a - y_i)\sinh(y_b - y_j)} \right] , \quad yi < yk < yj$$





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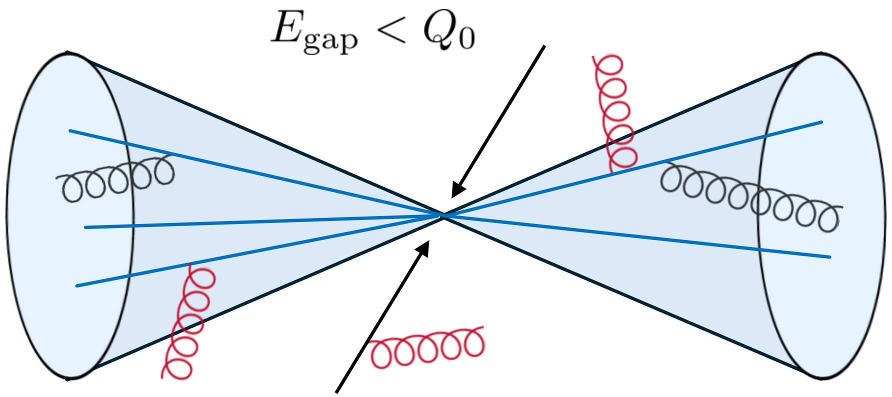
Angular integrals

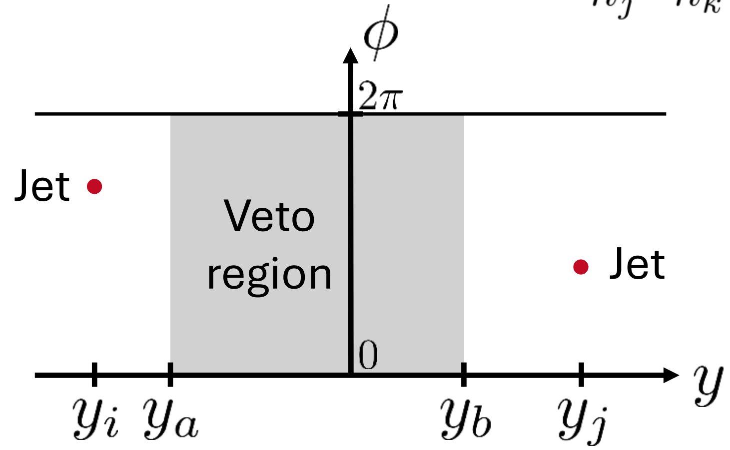
- For one soft emission: J_{12} and $J_i = J_{1i} - J_{2i}$
- Now products, e.g.: $J_i\, J_{1i}$ or $J_i\, J_{jm}$
- Additionally products like $J_i\, J_{12}^+$, because one $\overline{\Gamma}$ emission can stay in the jet

$$J_{ij} = \int \frac{d^2\Omega_k}{4\pi} W_{ij}^k (1 - \theta_{\text{hard}}(n_k)) \, ,$$

$$J_{ij}^+ = \int \frac{d^2\Omega_k}{4\pi} \overline{W}_{ij}^k (1 + \theta_{\text{hard}}(n_k))$$

Subtracted dipole: $\overline{W}_{ij}^k = W_{ij}^k - \frac{1}{n_i \cdot n_k} \delta(n_i - n_k) - \frac{1}{n_j \cdot n_k} \delta(n_j - n_k)$





Rapidity: $y = \frac{1}{2} \ln \left(\frac{1 + \cos\theta}{1 - \cos\theta} \right)$

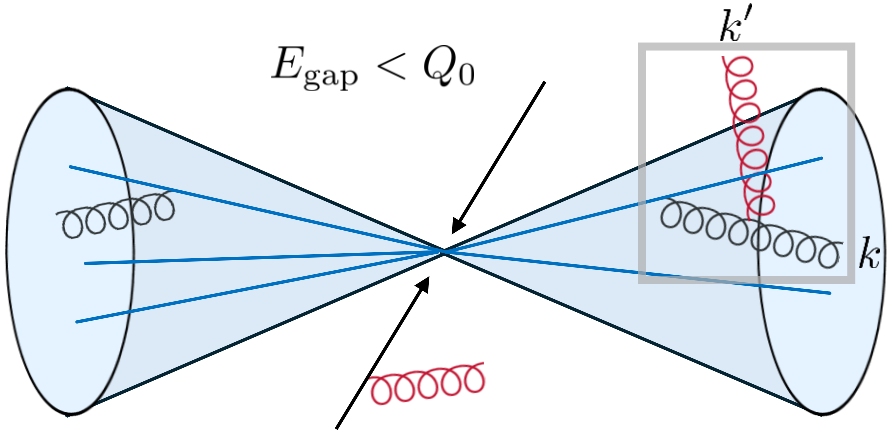
Angular integrals

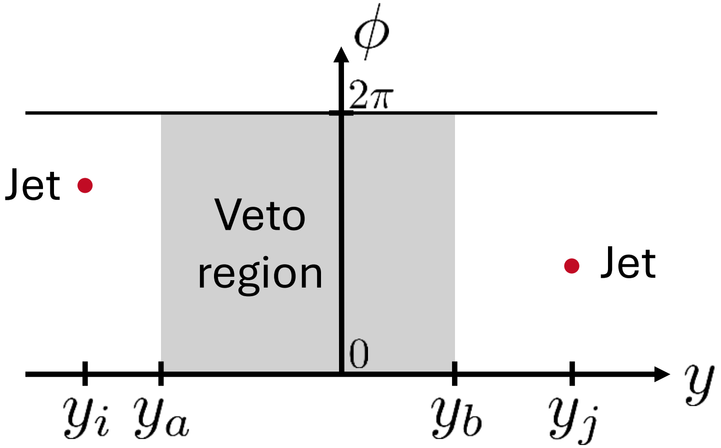
- For one soft emission: J_{12} and $J_i = J_{1i} - J_{2i}$
- Now products, e.g.: $J_i\, J_{1i}$ or $J_i\, J_{jm}$
- Additionally products like $J_i\, J_{12}^{+}$
- And non-global structures:

$$J_{ij} = \int \frac{d^2\Omega_k}{4\pi} W_{ij}^{k} (1 - \theta_{\text{hard}}(n_k)) \,,$$

$$J_{ij}^{+} = \int \frac{d^2\Omega_k}{4\pi} \overline{W}_{ij}^{k} (1 + \theta_{\text{hard}}(n_k))$$

$$\left[J_{ik} \overline{J}_{mn}\right]_k = \int \frac{d^2\Omega_k}{4\pi} \int \frac{d^2\Omega_{k'}}{4\pi} W_{ik}^{k'} \theta_{\text{veto}}(n_{k'}) \overline{W}_{mn}^{k} \theta_{\text{hard}}(n_k)$$

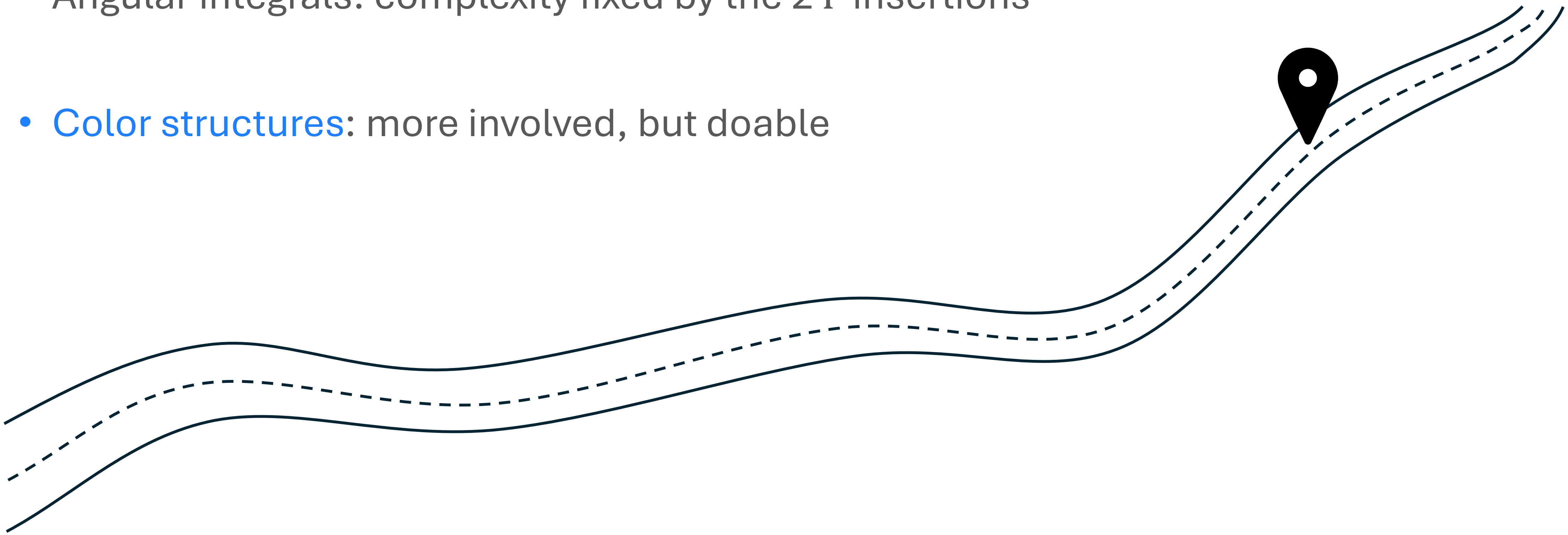




Rapidity: $y = \frac{1}{2} \ln\left(\frac{1 + \cos\theta}{1 - \cos\theta}\right)$

Outlook: case with 2 $\overline{\Gamma}$ and 2 V^G

- For a real contribution: 2 Glauber phases V^G required
- Angular integrals: complexity fixed by the 2 $\overline{\Gamma}$ insertions
- Color structures: more involved, but doable



Color Structures (2 $\bar{\Gamma}$ and 2 V^G)

- Starting from $\langle \boldsymbol{\mathcal{H}} \dots \boldsymbol{V}^G \bar{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$ and $\langle \boldsymbol{\mathcal{H}} \dots \boldsymbol{V}^G \bar{\boldsymbol{\Gamma}} \bar{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$

- Adding $\boldsymbol{\Gamma}^c$ and the second $\boldsymbol{V}^G$ (using $\left[\boldsymbol{\Gamma}^c, \bar{\boldsymbol{\Gamma}}\right] = 0$):

Remember: $[\boldsymbol{V}^G, \boldsymbol{\Gamma}^c] \neq 0$

$$\langle \boldsymbol{\mathcal{H}} (\boldsymbol{\Gamma}^c)^r \boldsymbol{V}^G (\boldsymbol{\Gamma}^c)^{n-r} \boldsymbol{V}^G \bar{\boldsymbol{\Gamma}} \bar{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

$$\langle \boldsymbol{\mathcal{H}} (\boldsymbol{\Gamma}^c)^r \boldsymbol{V}^G \bar{\boldsymbol{\Gamma}} (\boldsymbol{\Gamma}^c)^{n-r} \boldsymbol{V}^G \bar{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

} $(\Gamma^c)^r V^G$ applied at color structure discussed before

$$\langle \boldsymbol{\mathcal{H}} \bar{\boldsymbol{\Gamma}} (\boldsymbol{\Gamma}^c)^r \boldsymbol{V}^G (\boldsymbol{\Gamma}^c)^{n-r} \boldsymbol{V}^G \bar{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

} $\bar{\Gamma}$ applied at color structures with one soft emission

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- Starting from $\langle \boldsymbol{\mathcal{H}} \ldots \boldsymbol{V}^G \bar{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$ and $\langle \boldsymbol{\mathcal{H}} \ldots \boldsymbol{V}^G \bar{\boldsymbol{\Gamma}}\bar{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$

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$(\Gamma^c)^r V^G$ applied at color structure discussed before

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$\bar{\Gamma}$ applied at color structures with one soft emission

Remember:
$\left[\boldsymbol{V}^G, \boldsymbol{\Gamma}^c\right] \neq 0$

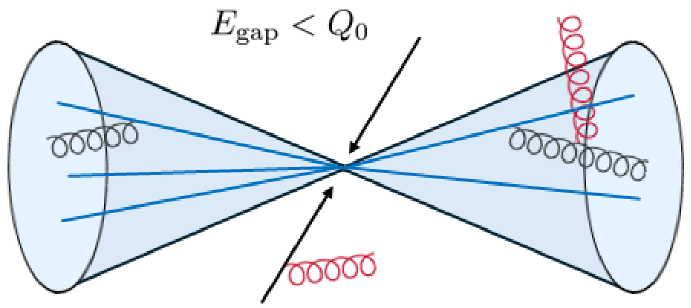
- Much more color structures (order of 100), e.g.:

$$\{\boldsymbol{T}_1^a, \boldsymbol{T}_1^b\}\{\boldsymbol{T}_2^a, \boldsymbol{T}_2^b\}\boldsymbol{T}_i \cdot \boldsymbol{T}_j$$

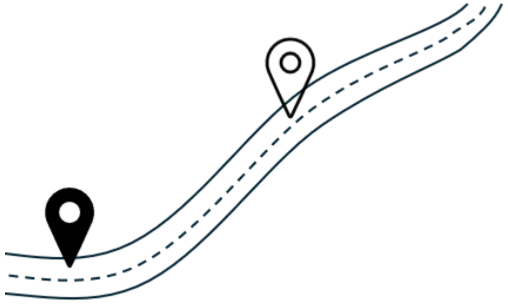
$$d^{ade} d^{bce} (\boldsymbol{T}_1^b \boldsymbol{T}_1^c \boldsymbol{T}_1^d \boldsymbol{T}_1^f)_+ \boldsymbol{T}_2^a \boldsymbol{T}_i^f$$

Towards Multiple Soft Emissions in LHC Jet Production: When Super-Leading Logarithms Go Non-Global

- Resummed subleading SLLs (2 $\bar{\Gamma}$ and 1 V^G)
  - Color structures
  - Angular integrals
- Observed non-global logarithms in combination with SLLs
- Contributions to real cross sections (2 $\bar{\Gamma}$ and 2 V^G)



Multiple soft emissions



Open questions:

- Phenomenology for LHC
- More than 2 insertions of $\bar{\Gamma}$