Super-Leading Logarithms and Sommerfeld Effect in $t\bar{t}$-Production

2510.24848 in collaboration with:

Upalaparna Banerjee
Matthias König
Yibei Li
Matthias Neubert
Josua Scholze

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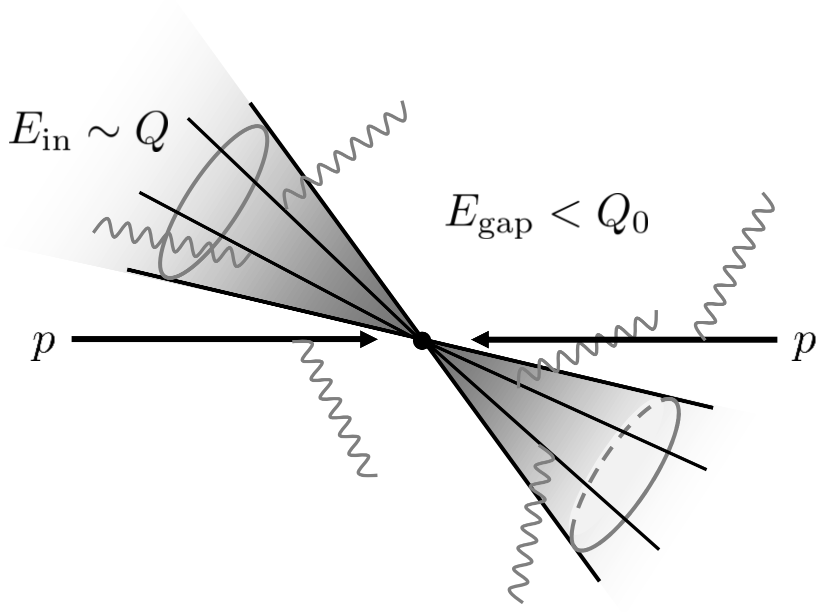
JG|U

Super-Leading Logarithms for Massive Final States

Origin of SLLs

Large Logarithms in $pp \to \mathrm{jets}$ processes: $\ln\left(\frac{Q}{Q_0}\right) \gg 1$

(gap-between-jets cross sections)



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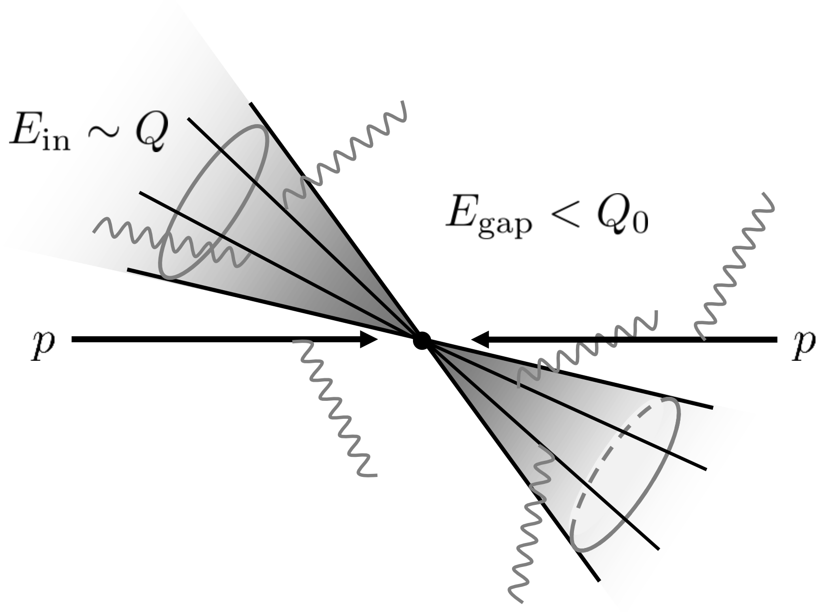

Partonic cross section evaluated at $\mu = \mu_s \sim Q_0$:

$$\hat{\sigma}^{\text{SLL}}_{2\to M} = \langle \boldsymbol{\mathcal{H}}(Q, \mu = \mu_s) \otimes \mathbf{1} \rangle$$

Hard function

Trivial low energy matrix element

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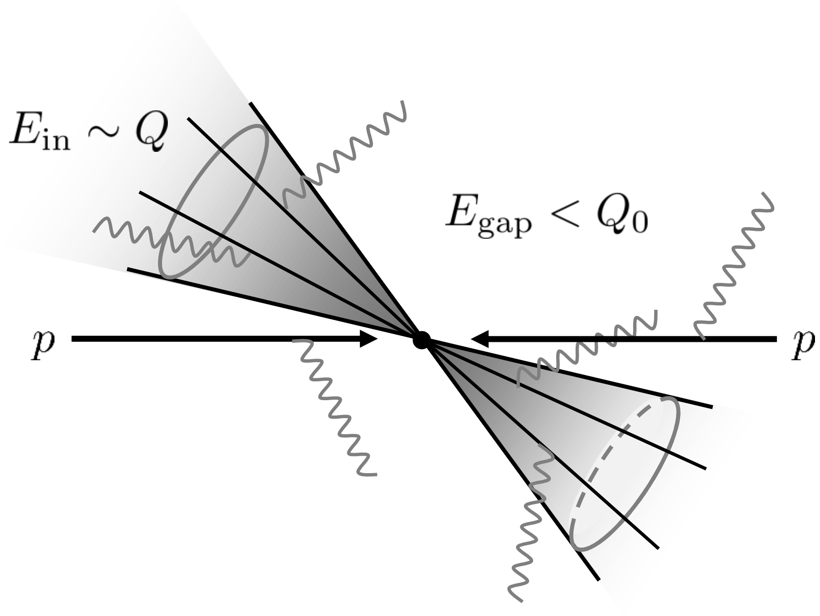

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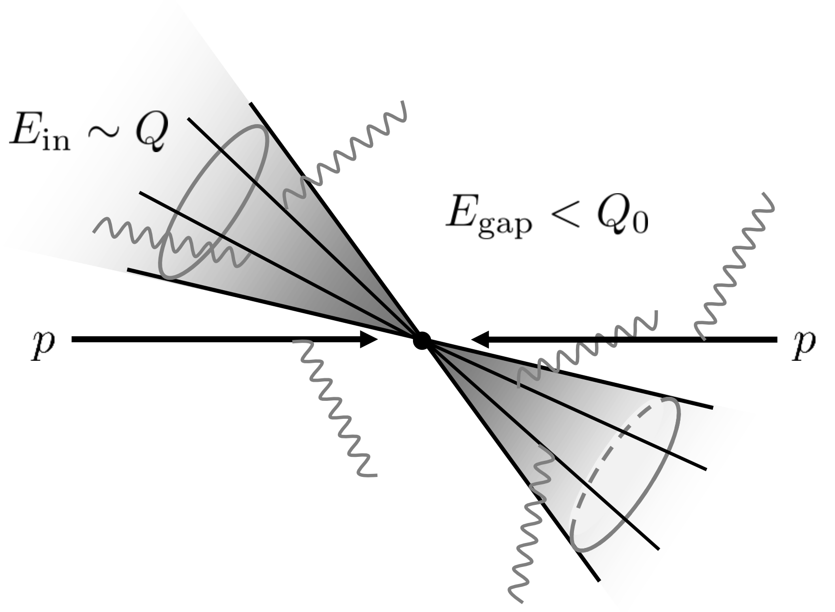
Evaluated at: $\mu_s \sim Q_0$

Natural scale: $\mu_h \sim Q$



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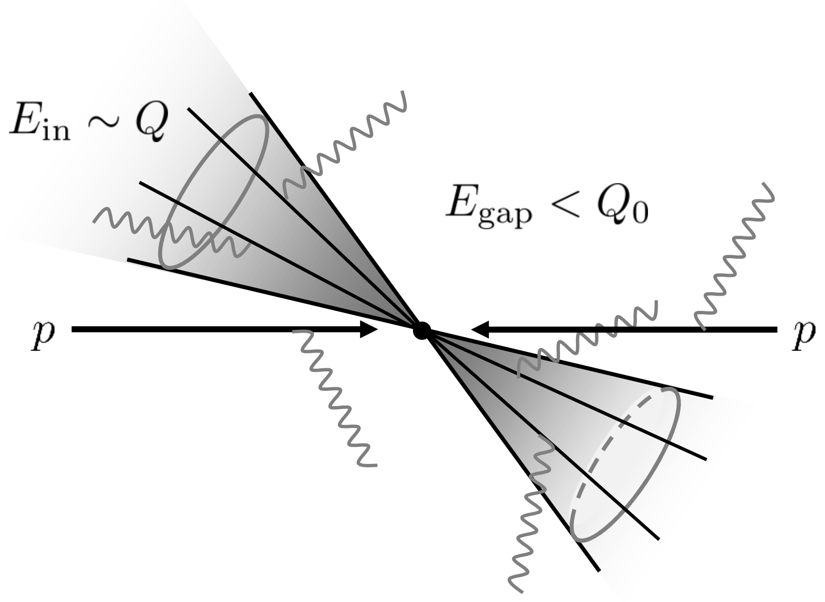

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$$\hat{\sigma}^{\text{SLL}}_{2\to M} = \langle \boldsymbol{\mathcal{H}}(Q, \mu = \mu_s) \otimes \mathbf{1} \rangle$$

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Evaluated at: $\mu_s \sim Q_0$

Natural scale: $\mu_h \sim Q$

Large logs!

$$\Longrightarrow \boldsymbol{\mathcal{H}}(Q, \mu_s) = \boldsymbol{\mathcal{H}}(Q, \mu_h) \times \mathbf{P} \exp\left[\int_{\mu_s}^{\mu_h} \frac{\mathrm{d}\mu}{\mu} \boldsymbol{\Gamma}^{\mathcal{H}}(\mu)\right]$$

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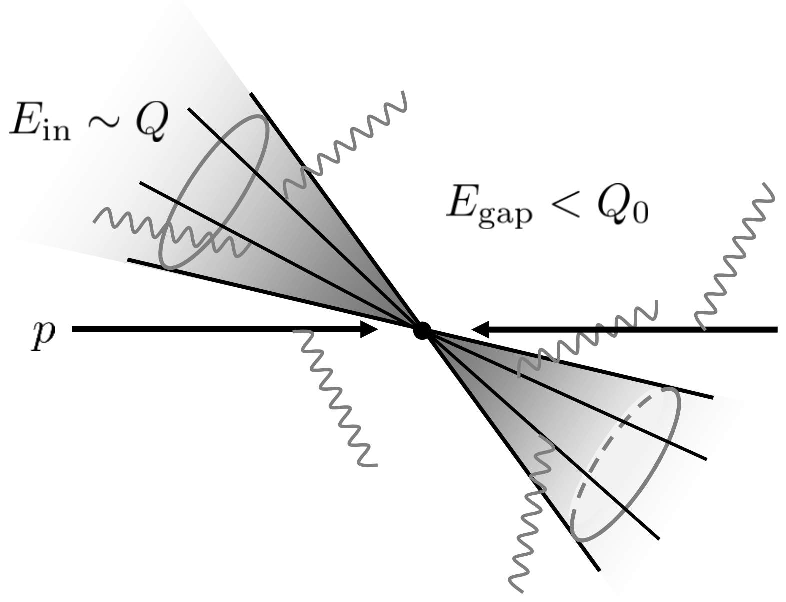

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Anomalous dimension matrix

Super-Leading Logarithms for Massive Final States

Anomalous Dimension

$$\boldsymbol{\mathcal{H}}(Q,\mu_s) = \boldsymbol{\mathcal{H}}(Q,\mu_h) \times \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{\mathrm{d}\mu}{\mu} \boldsymbol{\Gamma}^{\mathcal{H}}(\mu)\right]$$

$$\boldsymbol{\Gamma}^{\mathcal{H}}(\mu) = \left[\text{Soft and soft-collinear part}\right] + \left[\text{Collinear parts}\right]$$

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$\Longrightarrow$ subleading

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Soft and soft-collinear part $\Longrightarrow$ SLLs!

Collinear parts $\Longrightarrow$ subleading

$$\boldsymbol{\Gamma}^{S}(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left[\gamma_0 \boldsymbol{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right) + \overline{\boldsymbol{\Gamma}} + \gamma_0 \boldsymbol{V}^G + \gamma_0 \boldsymbol{V}^{\text{Coul}}\right] + \mathcal{O}(\alpha_s^2)$$

Super-Leading Logarithms for Massive Final States

Anomalous Dimension

$$\mathcal{H}(Q,\mu_s) = \mathcal{H}(Q,\mu_h) \times \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \mathbf{\Gamma}^{\mathcal{H}}(\mu)\right]$$

$$\mathbf{\Gamma}^{\mathcal{H}}(\mu) = \left[\text{Soft and soft-collinear part}\right] + \left[\text{Collinear parts}\right]$$

Soft and soft-collinear part ⟹ SLLs!

Collinear parts ⟹ subleading

$$\mathbf{\Gamma}^{S}(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left[\gamma_0 \mathbf{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right) + \overline{\mathbf{\Gamma}} + \gamma_0 \boldsymbol{V}^G + \gamma_0 \boldsymbol{V}^{\text{Coul}}\right] + \mathcal{O}(\alpha_s^2)$$

Cusp contribution

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Soft and soft-collinear part ⟹ SLLs!

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Cusp contribution

Purely soft emissions

⟹ Josua‘s talk

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$\boldsymbol{\Gamma}^{\mathcal{H}}(\mu) =$ [Soft and soft-collinear part ⟹ SLLs!] + [Collinear parts] ⟹ subleading

$$\boldsymbol{\Gamma}^{S}(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left[\gamma_0 \boldsymbol{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right) + \overline{\boldsymbol{\Gamma}} + \gamma_0 \boldsymbol{V}^G + \gamma_0 \boldsymbol{V}^{\text{Coul}}\right] + \mathcal{O}(\alpha_s^2)$$

- Cusp contribution
- Purely soft emissions ⟹ Josua‘s talk
- Glauber phase
- Coulomb phase ⟹ only for massive partons!

Super-Leading Logarithms for Massive Final States

Anomalous Dimension

Resummed cross section:

$$\hat{\sigma}_{2\to M}^{\text{SLL}} = \langle \boldsymbol{\mathcal{H}}(Q,\mu_h) \times \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{\mathrm{d}\mu}{\mu} \mathbf{\Gamma}^{\mathcal{H}}(\mu)\right] \otimes \mathbf{1} \rangle$$

Super-Leading Logarithms for Massive Final States

Anomalous Dimension

Resummed cross section:

$$\hat{\sigma}^{\text{SLL}}_{2\to M} = \left\langle \boldsymbol{\mathcal{H}}(Q,\mu_h) \times \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{\mathrm{d}\mu}{\mu} \mathbf{\Gamma}^{\mathcal{H}}(\mu)\right] \otimes \mathbf{1} \right\rangle$$

$$\begin{aligned} &= \boldsymbol{\mathcal{H}}(Q,\mu_h) + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \boldsymbol{\mathcal{H}}(Q,\mu_h) * \mathbf{\Gamma}^{\mathcal{H}}(\mu_1) \\ &\quad + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_s}^{\mu_1} \frac{d\mu_2}{\mu_2} \boldsymbol{\mathcal{H}}(Q,\mu_h) * \mathbf{\Gamma}^{\mathcal{H}}(\mu_1) * \mathbf{\Gamma}^{\mathcal{H}}(\mu_2) \\ &\quad + \dots \end{aligned}$$

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Anomalous Dimension

Resummed cross section:

$$\hat{\sigma}_{2\to M}^{\mathrm{SLL}} = \left\langle \boldsymbol{\mathcal{H}}(Q,\mu_h) \times \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{\mathrm{d}\mu}{\mu} \boldsymbol{\Gamma}^{\mathcal{H}}(\mu)\right] \otimes \mathbf{1} \right\rangle$$

$$\begin{aligned} = \boldsymbol{\mathcal{H}}(Q,\mu_h) &+ \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \boldsymbol{\mathcal{H}}(Q,\mu_h) * \boldsymbol{\Gamma}^{\mathcal{H}}(\mu_1) \\ &+ \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_s}^{\mu_1} \frac{d\mu_2}{\mu_2} \boldsymbol{\mathcal{H}}(Q,\mu_h) * \boldsymbol{\Gamma}^{\mathcal{H}}(\mu_1) * \boldsymbol{\Gamma}^{\mathcal{H}}(\mu_2) \\ &+ \dots \end{aligned}$$

$\overline{\boldsymbol{\Gamma}},\ \mathbf{V}^{G},\ \mathbf{V}^{\mathrm{Coul}} \longleftrightarrow \ln(\dots)$

Scale integrals

$\boldsymbol{\Gamma}^{c} \ln\left(\frac{\mu^2}{\mu_h^2}\right) \longleftrightarrow \ln^2(\dots)$

Super-Leading Logarithms for Massive Final States

Colour Traces

$$\boldsymbol{\Gamma}^S(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left[\gamma_0 \boldsymbol{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right) + \overline{\boldsymbol{\Gamma}} + \gamma_0 \boldsymbol{V}^G + \gamma_0 \boldsymbol{V}^{\text{Coul}}\right] + \mathcal{O}(\alpha_s^2)$$

- Want as many $\boldsymbol{\Gamma}^c$ as possible $\overset{?}{\Longrightarrow}$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\,(\boldsymbol{\Gamma}^c)^n \otimes \mathbf{1}\rangle$

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- Want as many $\boldsymbol{\Gamma}^c$ as possible $\overset{?}{\Longrightarrow}$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\,(\boldsymbol{\Gamma}^c)^n \otimes \mathbf{1}\rangle$ vanishes since $\langle \boldsymbol{\mathcal{H}}\boldsymbol{\Gamma}^c \otimes \mathbf{1}\rangle = 0$

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- Use soft emission (introduces Q_0) $\stackrel{?}{\Longrightarrow}$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\,(\boldsymbol{\Gamma}^c)^n\,\overline{\boldsymbol{\Gamma}} \otimes \mathbf{1}\rangle$

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- Use Glauber phase $\overset{?}{\Longrightarrow}$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\, (\boldsymbol{\Gamma}^c)^n \mathbf{V}^G \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$
  $[\mathbf{V}^G, \boldsymbol{\Gamma}^c] \neq 0$
  $[\mathbf{V}^{\text{Coul}}, \boldsymbol{\Gamma}^c] = 0$

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Colour Traces

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- Use Glauber phase $\overset{?}{\Longrightarrow}$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\,(\boldsymbol{\Gamma}^c)^n\, \mathbf{V}^G \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1}\rangle$ cross section should be real
  $[\mathbf{V}^G, \boldsymbol{\Gamma}^c] \neq 0$
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Super-Leading Logarithms for Massive Final States

Colour Traces

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- Use Glauber phase $[\mathbf{V}^G, \mathbf{\Gamma}^c] \neq 0$, $[\mathbf{V}^{\text{Coul}}, \mathbf{\Gamma}^c] = 0$ $\overset{?}{\Longrightarrow}$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\,(\mathbf{\Gamma}^c)^n\, \mathbf{V}^G\overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle$ cross section should be real
- Use second phase

$$\Longrightarrow \langle \boldsymbol{\mathcal{H}}(\mu_h)\mathbf{V}^G (\mathbf{\Gamma}^c)^n\, \mathbf{V}^G\overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle + \text{ different orderings}$$

$$\Longrightarrow \langle \boldsymbol{\mathcal{H}}(\mu_h)\mathbf{V}^{\text{Coul}} (\mathbf{\Gamma}^c)^n\, \mathbf{V}^G\overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle + \text{ different orderings}$$

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Colour Traces

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SLLs contribute like $\alpha_s^{3+n} L^{3+2n}$ with $L = \ln\left(\frac{Q}{Q_0}\right) \gg 1$:

$$\sigma_{\mathrm{SLL}} \sim \sigma_{\mathrm{Born}} \times \left[1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \alpha_s^4 L^5 + \alpha_s^5 L^7 + \ldots\right]$$

- Use second phase $\Longrightarrow$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\mathbf{V}^G\,(\mathbf{\Gamma}^c)^n\,\mathbf{V}^G\overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle$ + different orderings
  $\Longrightarrow$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\mathbf{V}^{\mathrm{Coul}}\,(\mathbf{\Gamma}^c)^n\,\mathbf{V}^G\overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle$ + different orderings

Super-Leading Logarithms for Massive Final States

Colour Traces

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Colour Traces

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- Want as many $\mathbf{\Gamma}^c$ as possible $\stackrel{?}{\Longrightarrow}$ $\langle \mathcal{H}(\mu_h)\,(\mathbf{\Gamma}^c)^n \otimes \mathbf{1}\rangle$ vanishes since $\langle \mathcal{H}\mathbf{\Gamma}^c \otimes \mathbf{1}\rangle = 0$
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- Use second phase $\Longrightarrow$ $\langle \mathcal{H}(\mu_h)\mathbf{V}^G\,(\mathbf{\Gamma}^c)^n\,\mathbf{V}^G\overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle$ + different orderings

$\Longrightarrow$ $\langle \mathcal{H}(\mu_h)\mathbf{V}^{\text{Coul}}\,(\mathbf{\Gamma}^c)^n\,\mathbf{V}^G\overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle$ + different orderings

$\Longrightarrow$ $\langle \mathcal{H}(\mu_h)\mathbf{V}^{\text{Coul}}\mathbf{V}^{\text{Coul}}\overline{\mathbf{\Gamma}} \otimes \mathbf{1}\rangle$

Super-Leading Logarithms for Massive Final States

Colour Traces

$$\boldsymbol{\Gamma}^S(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left[\gamma_0 \boldsymbol{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right) + \overline{\boldsymbol{\Gamma}} + \gamma_0 \boldsymbol{V}^G + \gamma_0 \boldsymbol{V}^{\text{Coul}}\right] + \mathcal{O}(\alpha_s^2)$$

- Want as many $\boldsymbol{\Gamma}^c$ as possible $\overset{?}{\Longrightarrow}$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\,(\boldsymbol{\Gamma}^c)^n \otimes \mathbf{1}\rangle$ vanishes since $\langle \boldsymbol{\mathcal{H}}\boldsymbol{\Gamma}^c \otimes \mathbf{1}\rangle = 0$
- Use soft emission (introduces Q_0) $\overset{?}{\Longrightarrow}$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\,(\boldsymbol{\Gamma}^c)^n\,\overline{\boldsymbol{\Gamma}} \otimes \mathbf{1}\rangle$ vanishes since $[\boldsymbol{\Gamma}^c, \overline{\boldsymbol{\Gamma}}] = 0$
- Use Glauber phase $\overset{?}{\Longrightarrow}$ $\langle$
  $[\mathbf{V}^G, \boldsymbol{\Gamma}^c] \neq 0$
  $[\mathbf{V}^{\text{Coul}}, \boldsymbol{\Gamma}^c] = 0$

SLLs contribute like $\alpha_s^{3+n} L^{3+2n}$ with $L = \ln\left(\frac{Q}{Q_0}\right) \gg 1$:

$$\sigma_{\text{SLL}} \sim \sigma_{\text{Born}} \times \left[1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \alpha_s^4 L^5 + \alpha_s^5 L^7 + \ldots\right]$$

- Use second phase $\Longrightarrow$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\mathbf{V}^G\,(\boldsymbol{\Gamma}^c)^n\,\mathbf{V}^G\overline{\boldsymbol{\Gamma}} \otimes \mathbf{1}\rangle$ + different orderings [Becher, Neubert, Shao, Stillger (2023)]
  $\Longrightarrow$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\mathbf{V}^{\text{Coul}}\,(\boldsymbol{\Gamma}^c)^n\,\mathbf{V}^G\overline{\boldsymbol{\Gamma}} \otimes \mathbf{1}\rangle$ + different orderings
  $\Longrightarrow$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\mathbf{V}^{\text{Coul}}\mathbf{V}^{\text{Coul}}\overline{\boldsymbol{\Gamma}} \otimes \mathbf{1}\rangle$

$2 \to t\bar{t}$ Processes

Kinematics and Choice of Scales

For $t\bar{t}$ -production:

- $q\bar{q} \to t\bar{t}$
- $gg \to t\bar{t}$

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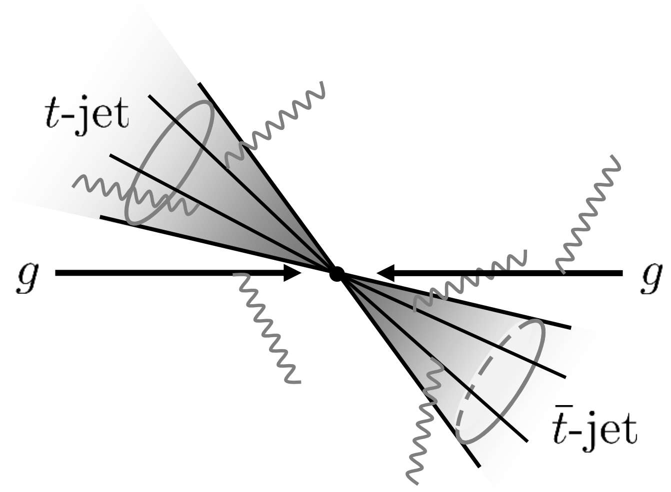

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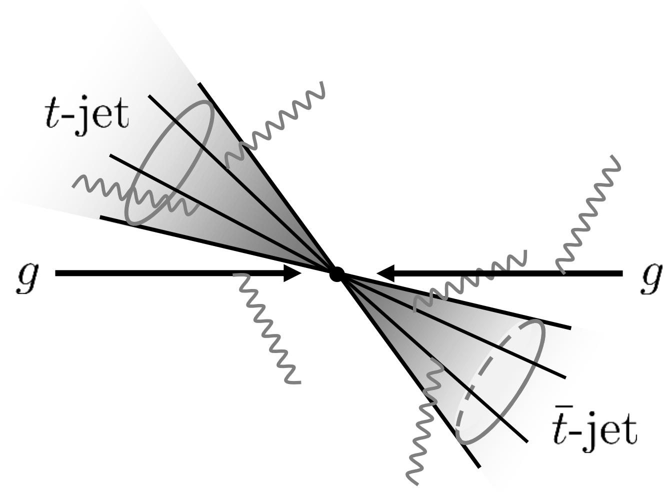

Center-of-mass frame: simplified kinematics

- $\beta \equiv \beta_t = \beta_{\bar{t}}$ where $\beta_I = \sqrt{1 - \frac{m_I^2}{E_I^2}}$
- $\eta \equiv \eta_t = -\eta_{\bar{t}}$ where $\eta_I = \mathrm{artanh}\left(\cos(\theta_I)\right)$

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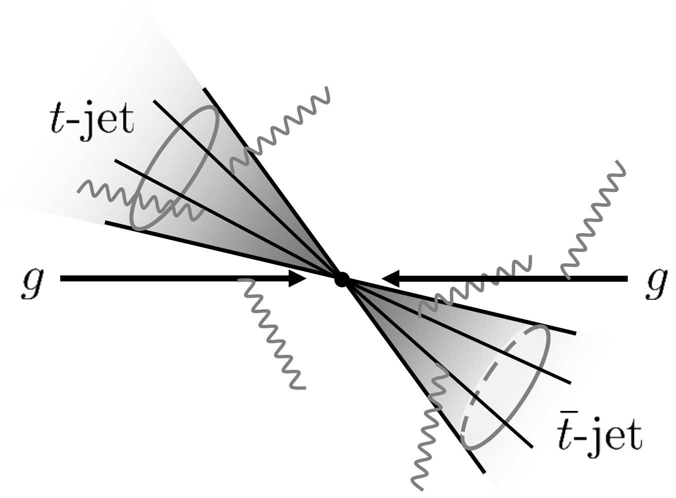

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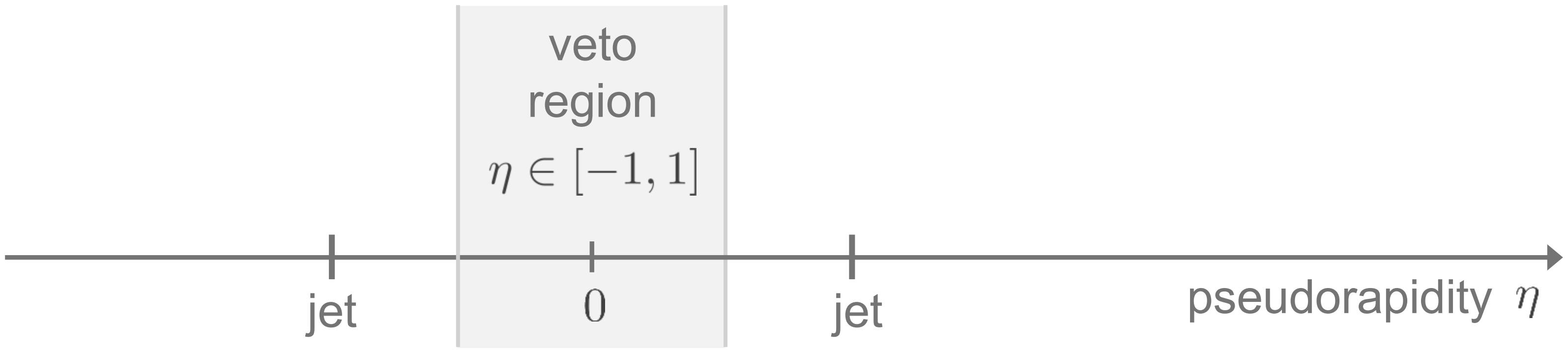

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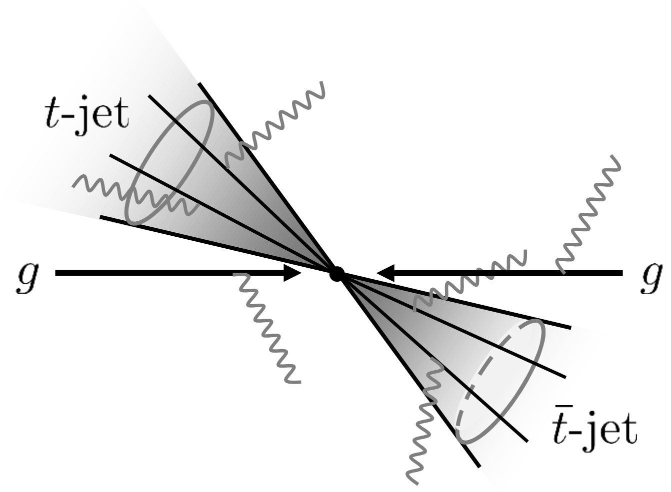

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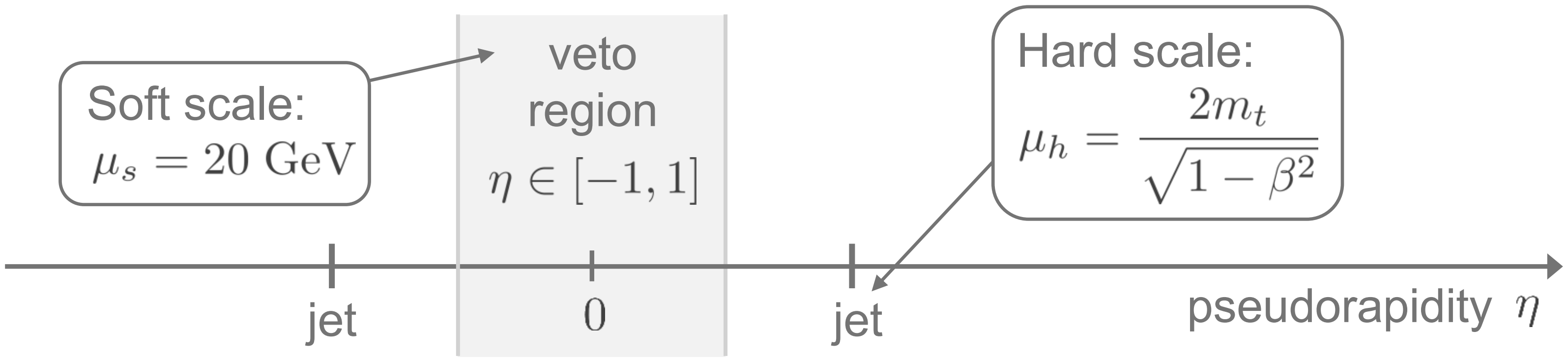
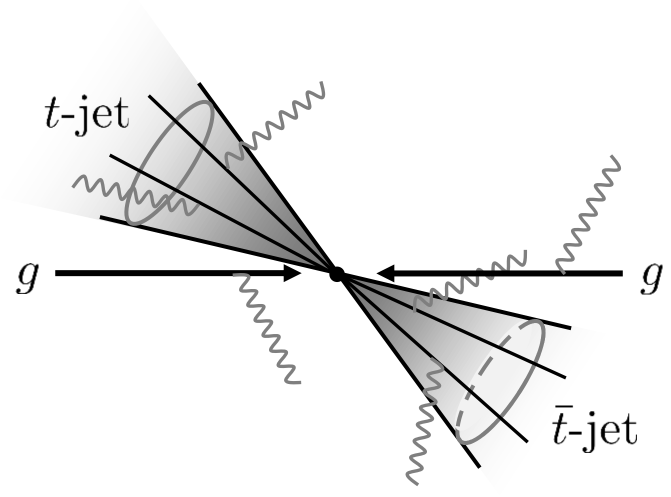


$2 \to t\bar{t}$ Processes

Kinematics and Choice of Scales

Coulomb phase:

$$\mathbf{V}^{\text{Coul}} = -\frac{1}{2}\pi i \sum_{(IJ)} \left(\boldsymbol{T}_{I,L} \cdot \boldsymbol{T}_{J,L} - \boldsymbol{T}_{I,R} \cdot \boldsymbol{T}_{J,R}\right) v_{IJ}$$



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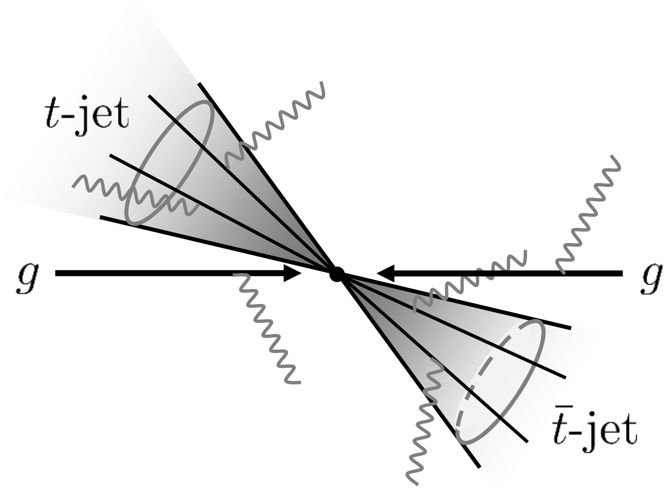
Sum over all pairs of massive final states



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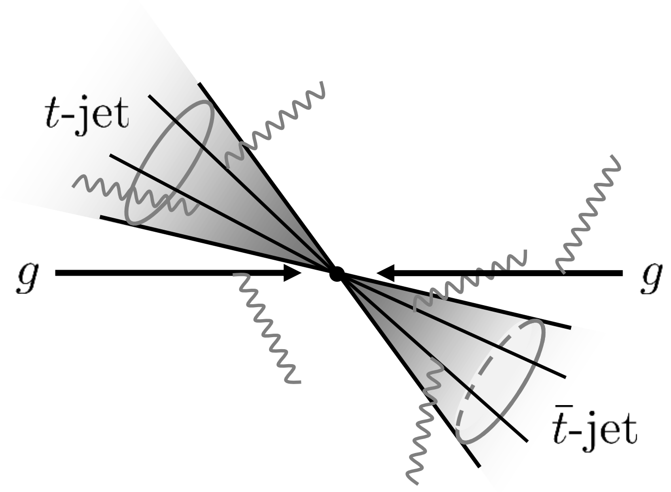
Colour generators (acting from the left and right on $\mathcal{H}$)



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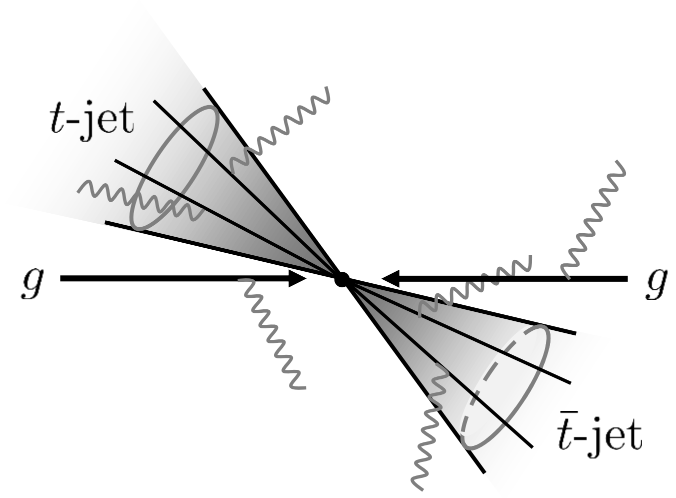

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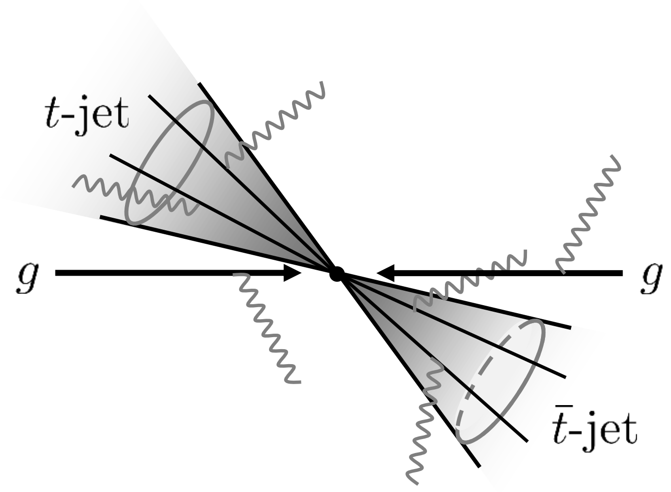

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Colour generators (acting from the left and right on $\mathcal{H}$)

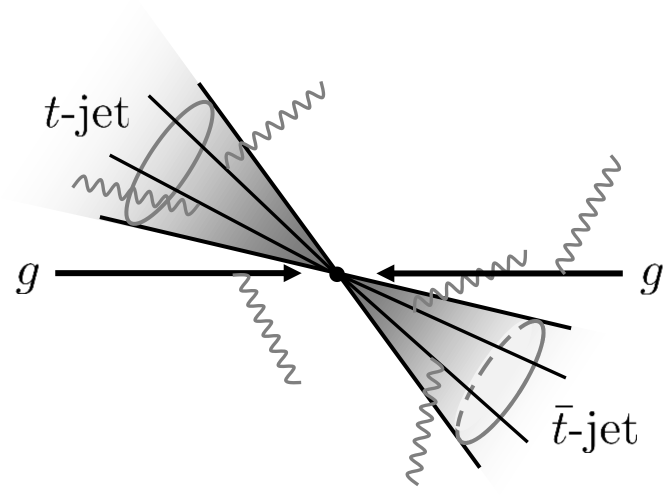
Kinematical factor

In the center-of-mass frame for $gg \to t\bar{t}$:

$$\mathbf{V}^{\text{Coul}} = -i\pi \left(\boldsymbol{T}_{t,L} \cdot \boldsymbol{T}_{\bar{t},L} - \boldsymbol{T}_{t,R} \cdot \boldsymbol{T}_{\bar{t},R}\right) v_{t\bar{t}}$$

$2 \to t\bar{t}$ Processes

Kinematics and Choice of Scales



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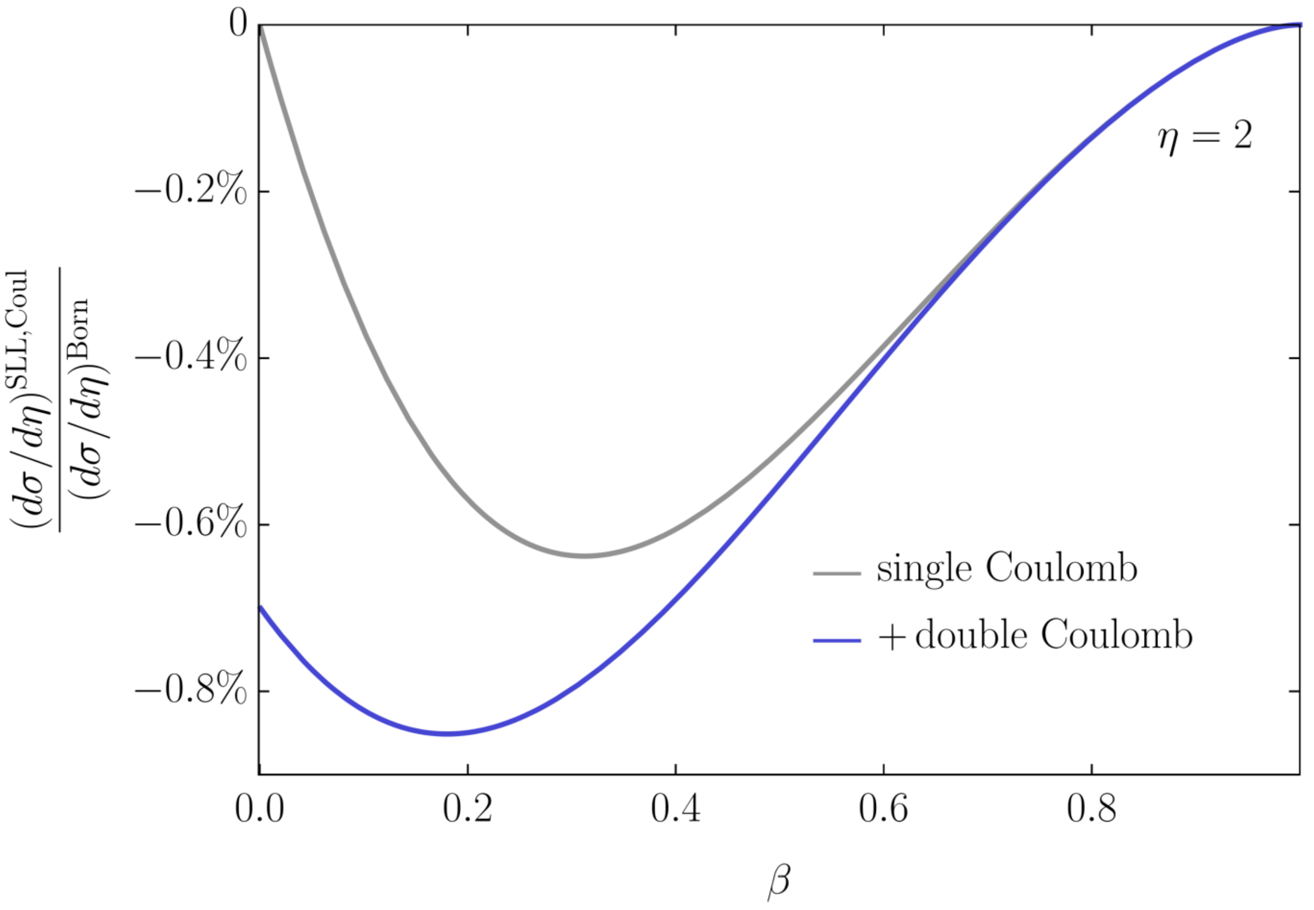
$$v_{t\bar{t}} = \left[\frac{(1-\beta)^2}{2\beta} \right]$$

- For $\beta \to 1$: $[\ldots] \to 0$ (massless limit)
- For $\beta \to 0$: $[\ldots] \to \infty$ (Sommerfeld enhancement)

Sommerfeld Enhancement

$gg \to t\bar{t}$



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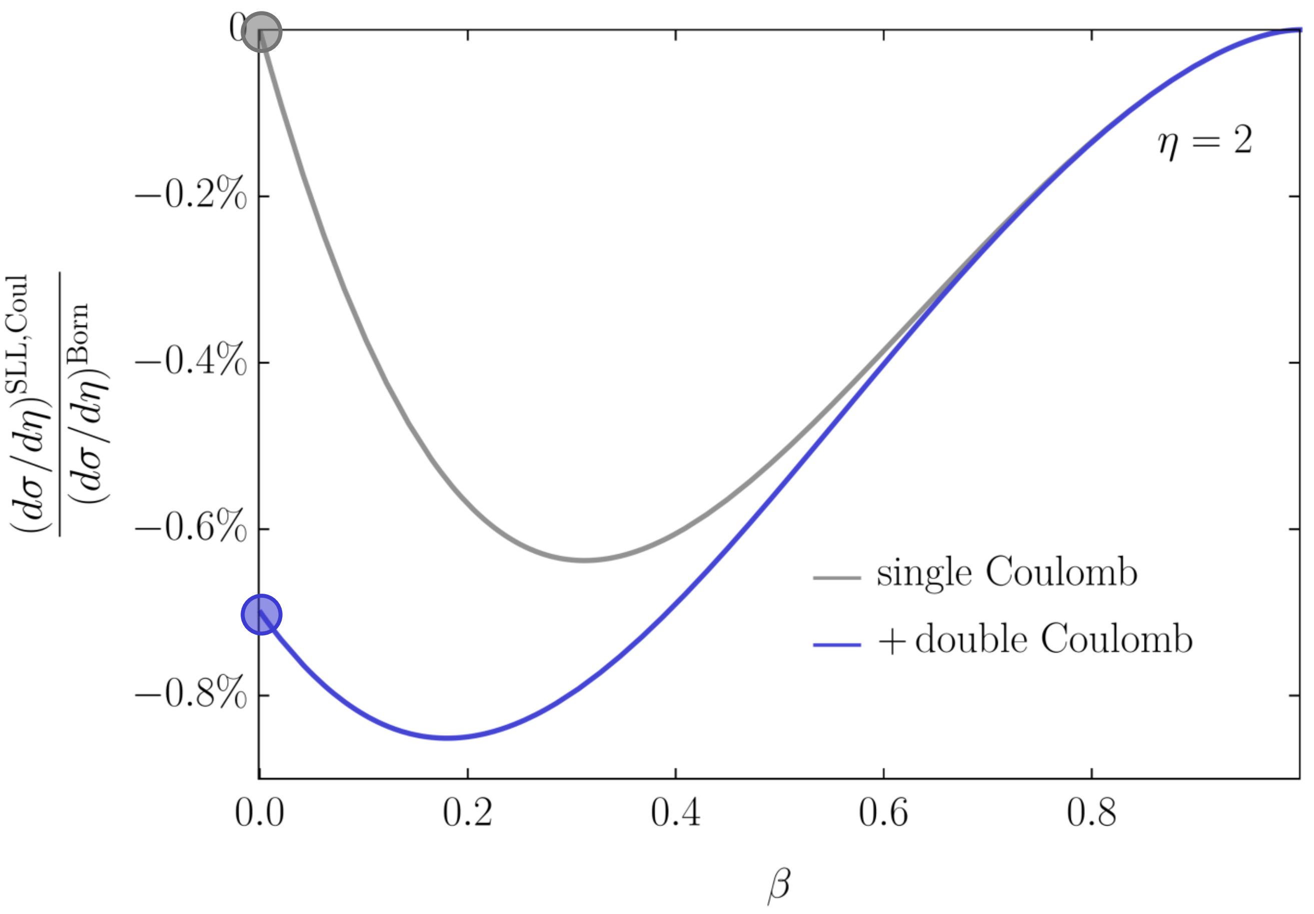

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for small β: $v_{t\bar{t}} \sim \frac{1}{\beta}$

- $\left(\mathbf{V}^{\mathrm{Coul}}\right)^1$: $\mathcal{O}(\beta^1)$ $\Longrightarrow$ 0 for $\beta \to 0$
- $\left(\mathbf{V}^{\mathrm{Coul}}\right)^2$: $\mathcal{O}(\beta^0)$ $\Longrightarrow$ constant for $\beta \to 0$

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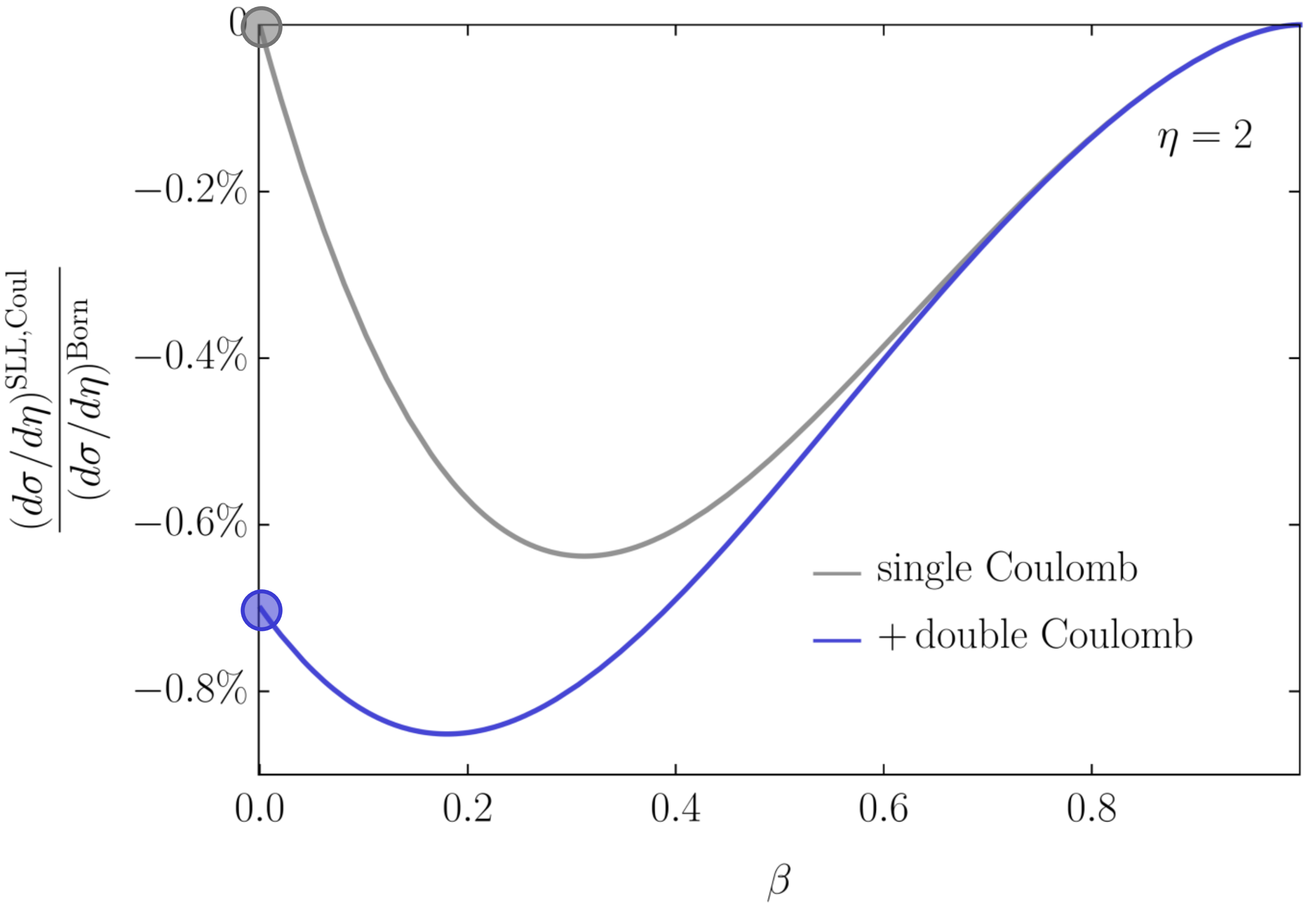

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- $\left(\mathbf{V}^{\text{Coul}}\right)^3$: $\mathcal{O}(\beta^{-1})$ $\Longrightarrow$ diverges for $\beta \to 0$
- $\ldots$

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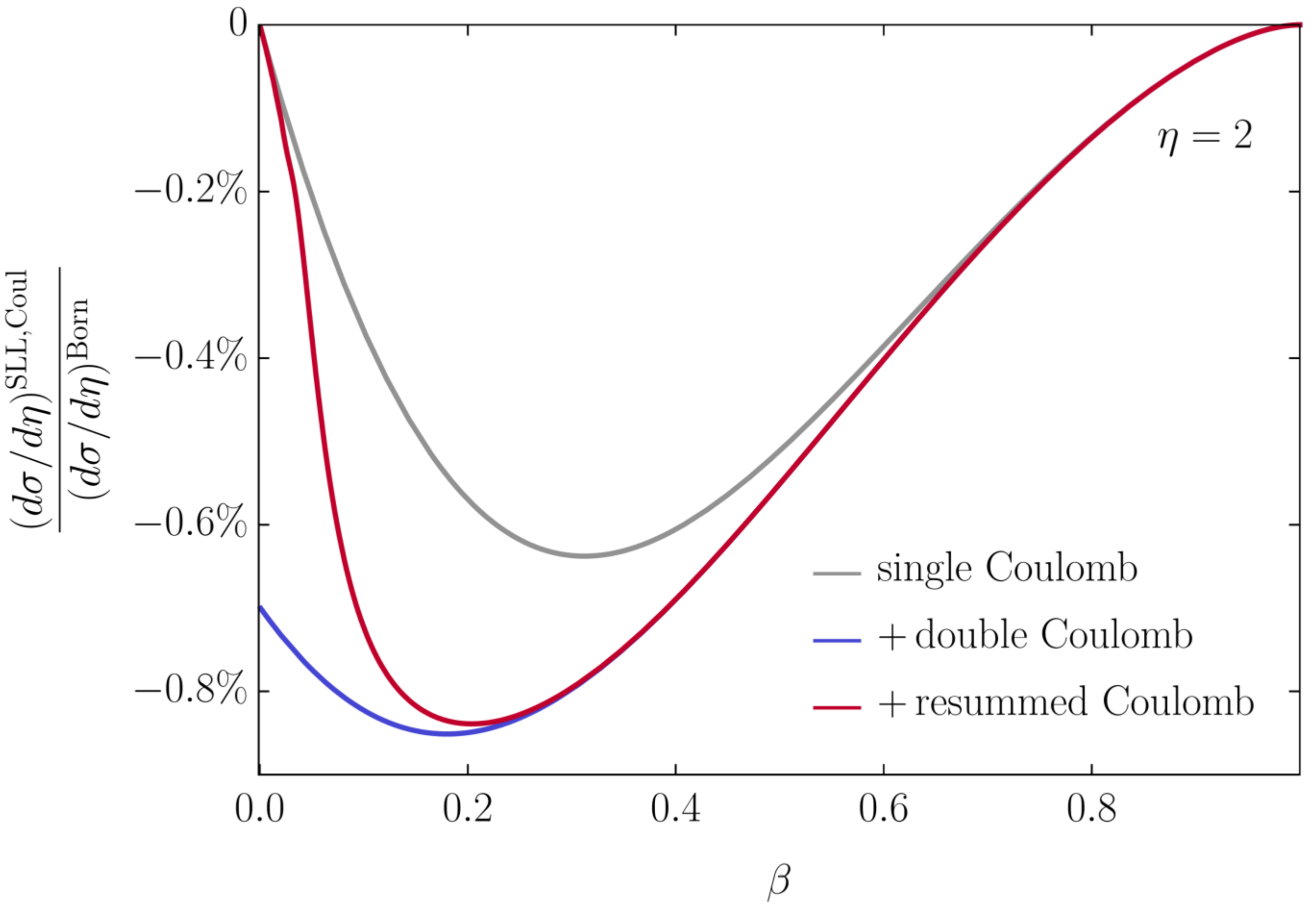

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⟹ Sommerfeld enhancement

⟹ Requires resummation of arbitrary (even) number of Coulomb insertions!

$$\langle \boldsymbol{\mathcal{H}}(\mu_h) \left(\mathbf{V}^{\text{Coul}} \right)^{2n} \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

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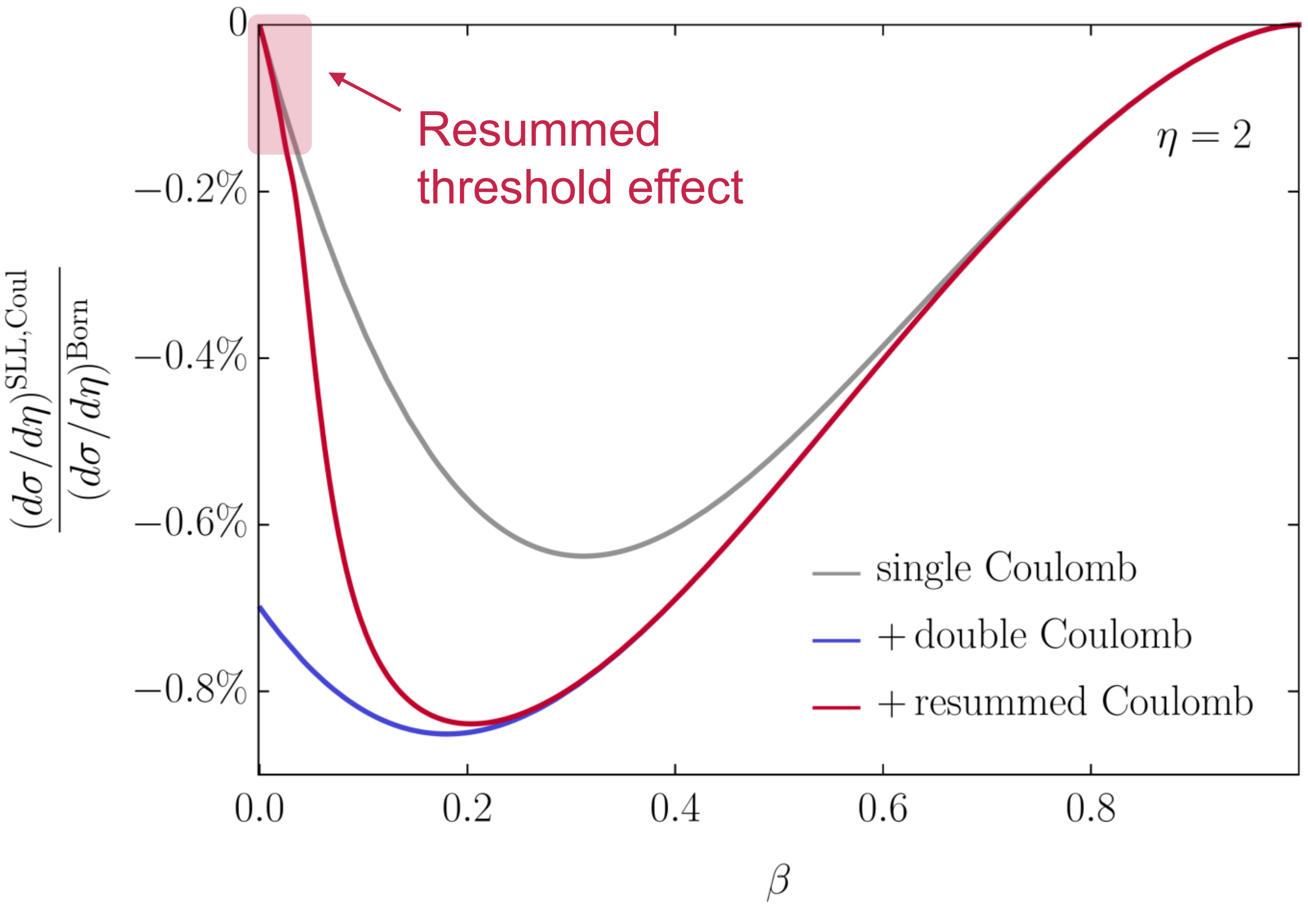

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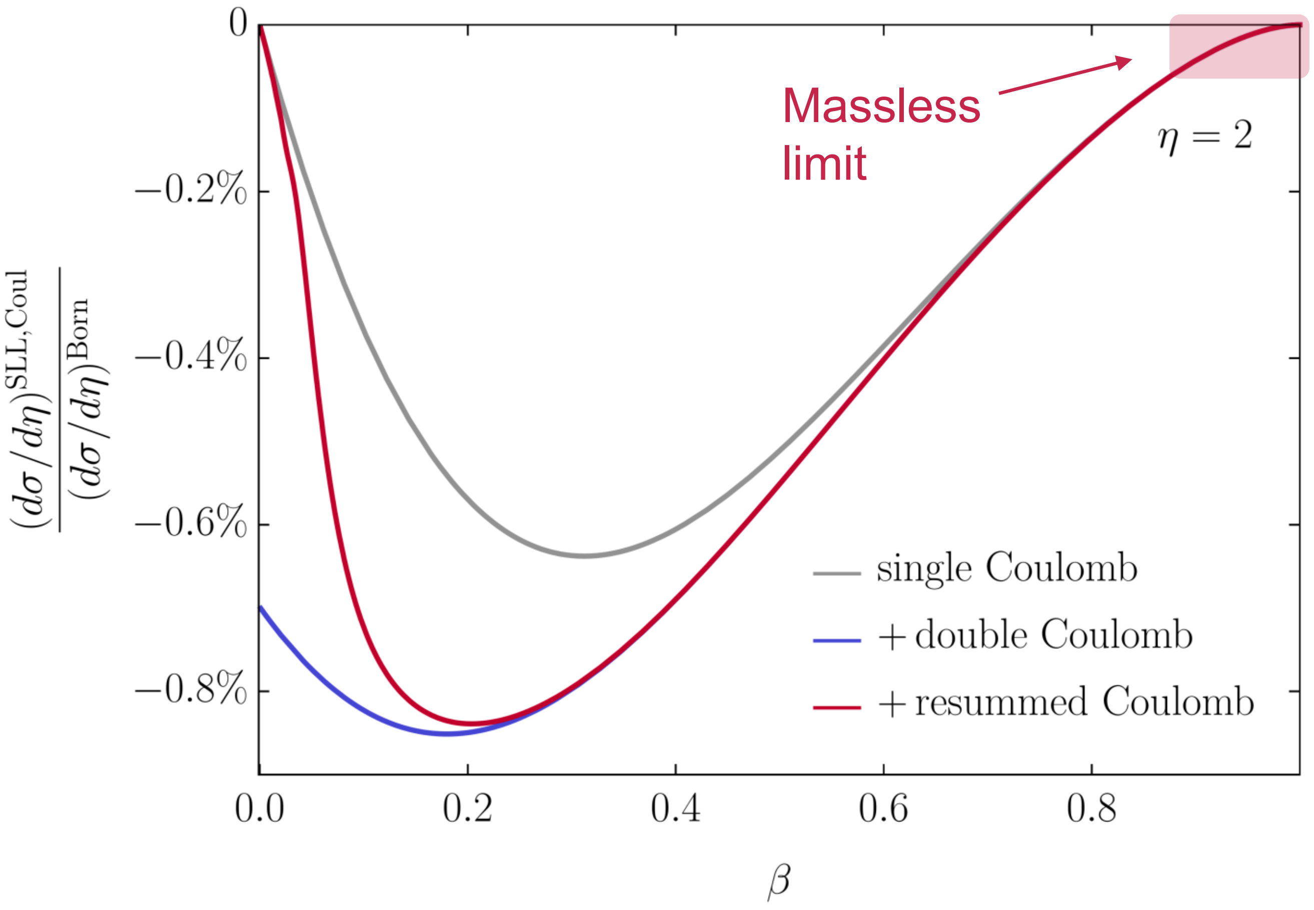

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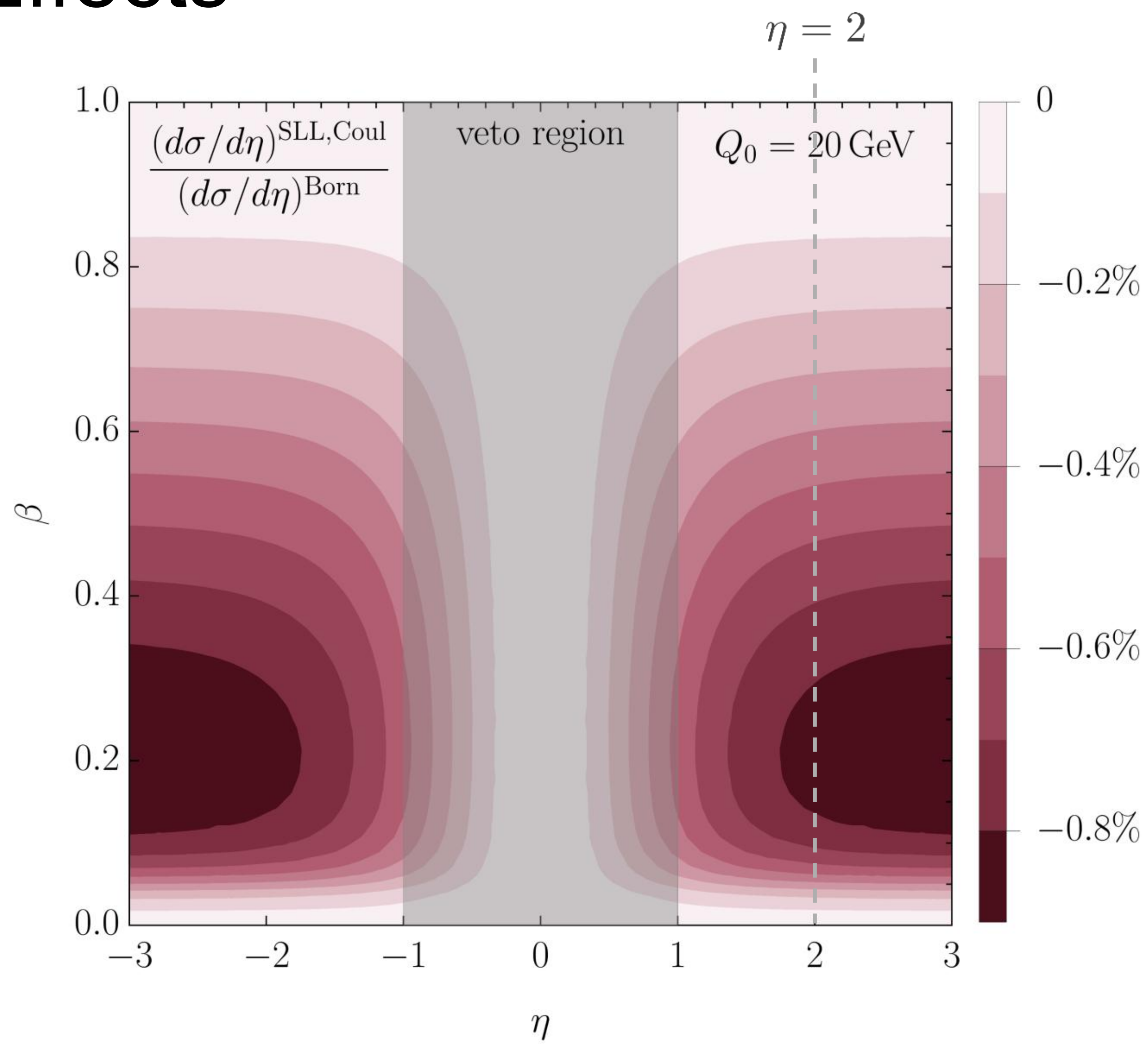
Numerical Effects

$gg \to t\bar{t}$

Coulomb SLLs



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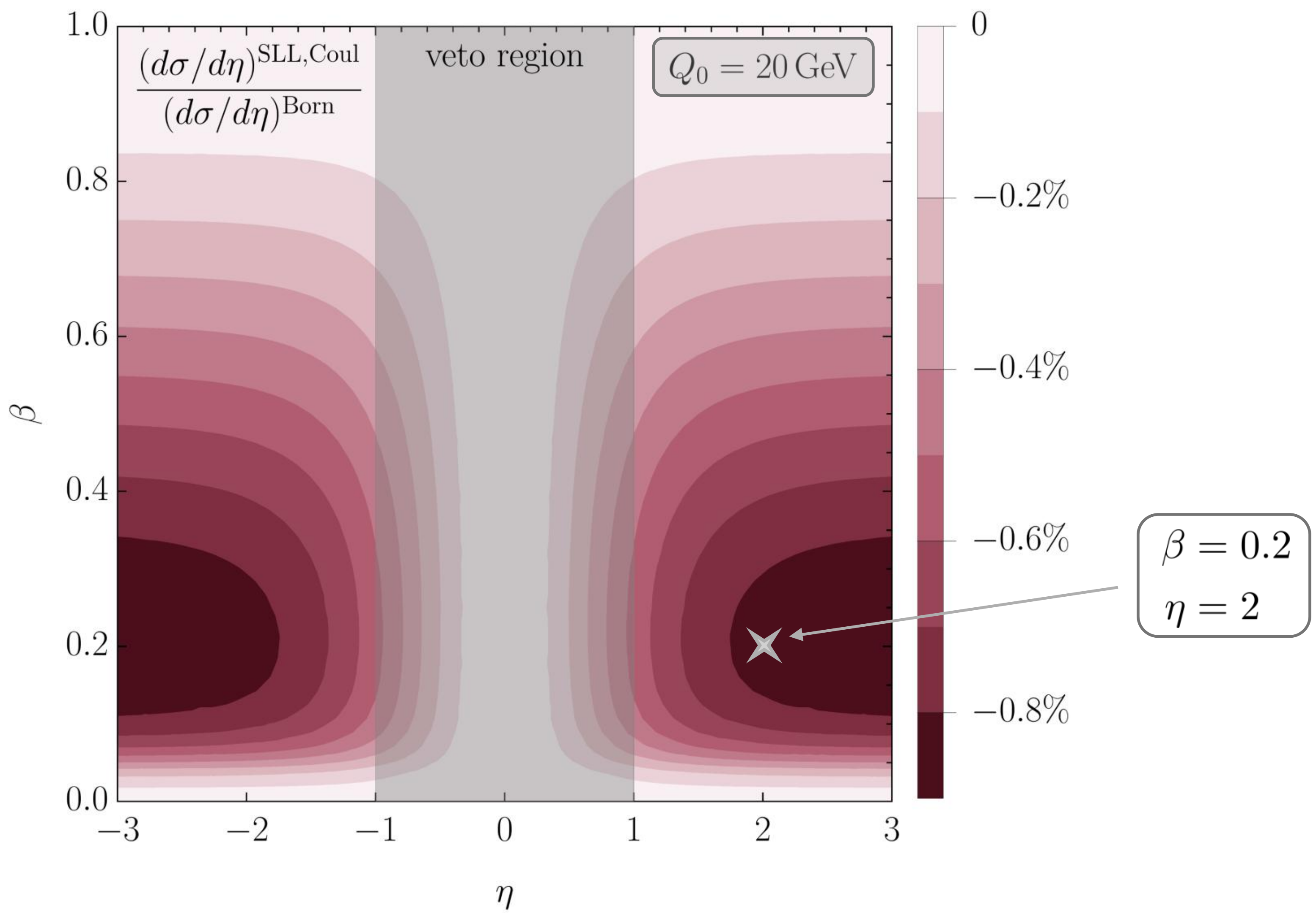

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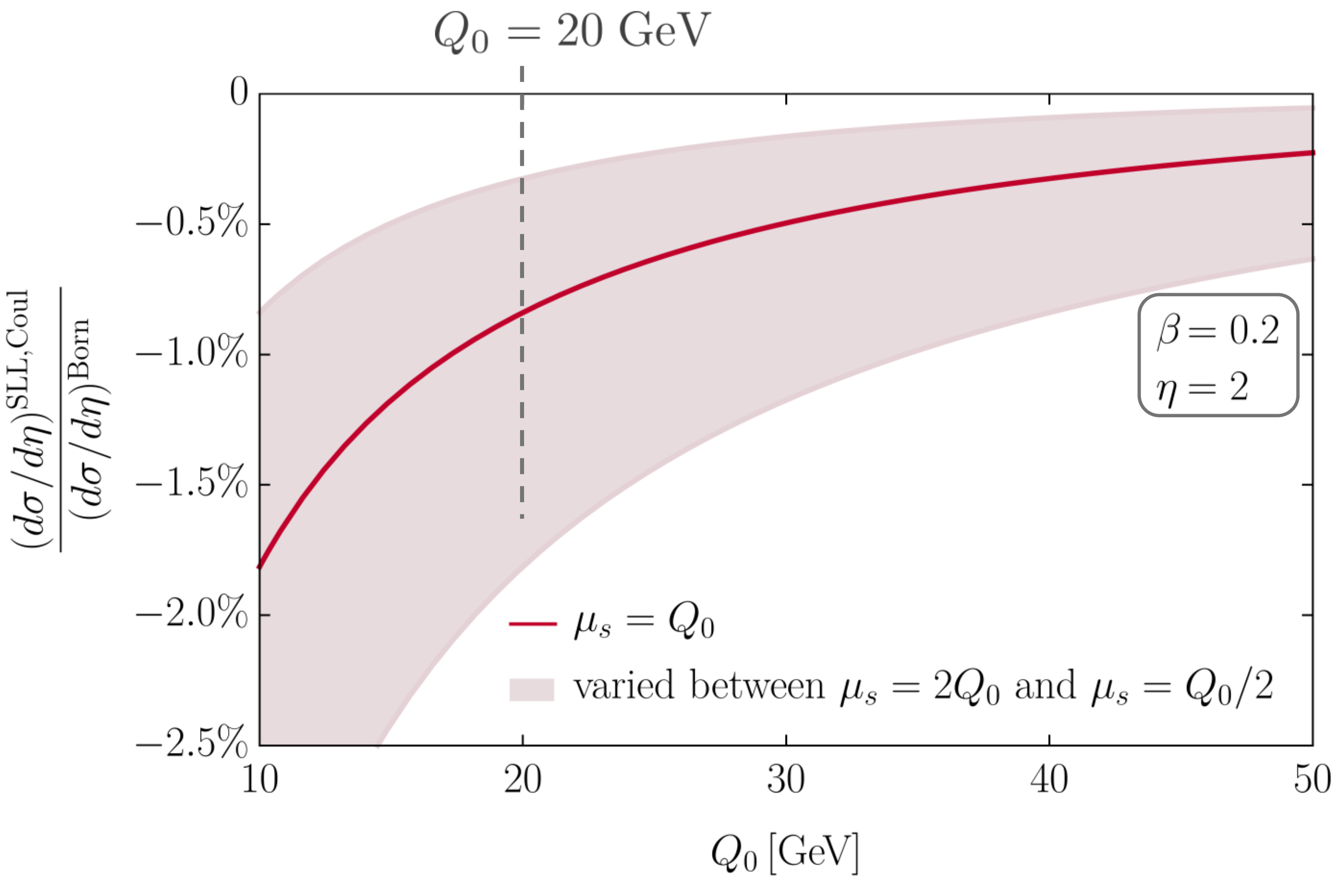

Numerical Effects

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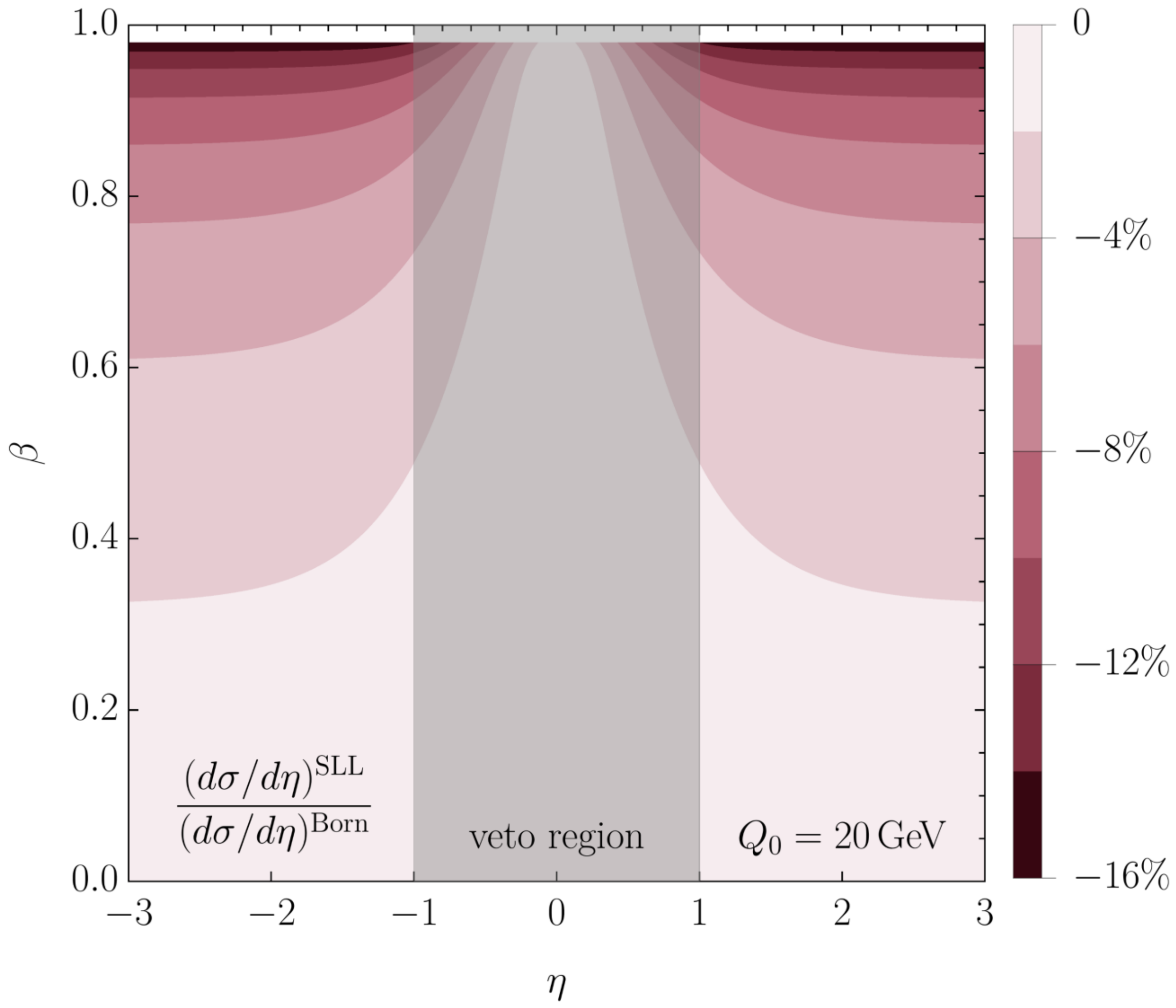
Coulomb and Glauber SLLs



Conclusion

New source of super-leading logarithms for massive final states:

$$\mathbf{V}^{\mathrm{Coul}} = -\frac{1}{2}\pi i \sum_{(IJ)} \left(\boldsymbol{T}_{I,L} \cdot \boldsymbol{T}_{J,L} - \boldsymbol{T}_{I,R} \cdot \boldsymbol{T}_{J,R}\right) v_{IJ}$$

- Vanishes for $\beta \to 1$ (masslesss limit)
- Enhanced for $\beta \to 0$ (threshold limit): Sommerfeld effect

$\Longrightarrow$ Requires additional resummation close to threshold

Numerical impact:

- $q\bar{q} \to t\bar{t}$: no contribution
- $gg \to t\bar{t}$: up to $\sim 1\%$ effects in the differential cross section

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Thank you!

Backup-Slides

Anomalous Dimension

$$\overline{\mathbf{\Gamma}} = \frac{1}{2}\gamma_0 \sum_{\alpha,\beta} (\boldsymbol{T}_{\alpha,L} \cdot \boldsymbol{T}_{\beta,L} + \boldsymbol{T}_{\alpha,R} \cdot \boldsymbol{T}_{\beta,R}) \int \frac{d^2\Omega_k}{4\pi} \overline{W}^k_{\alpha\beta} - 4 \sum_{\alpha,\beta} \theta_{\text{hard}}(n_k) \overline{W}^k_{\alpha\beta} \boldsymbol{T}_{\alpha,L} \circ \boldsymbol{T}_{\beta,R}$$

$$\mathbf{\Gamma}^c = \sum_{i=1,2} \left[C_i \, \mathbf{1} - \delta(n_i - n_k) \, \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R} \right]$$

$$\boldsymbol{V}^G = -2\pi i \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right)$$

$$\boldsymbol{V}^{\text{Coul}} = -\frac{1}{2}\pi i \sum_{(IJ)} \left(\boldsymbol{T}_{I,L} \cdot \boldsymbol{T}_{J,L} - \boldsymbol{T}_{I,R} \cdot \boldsymbol{T}_{J,R} \right) v_{IJ}$$

Cross section

$$U^c(1;\mu_i,\mu_j) = \exp\left[N_c \int_{\mu_j}^{\mu_i} \frac{d\mu}{\mu}\gamma_{\text{cusp}}(\alpha_s(\mu))\ln\left(\frac{\mu^2}{\mu_h^2}\right)\right]$$

$$\left(\frac{d\sigma}{d\eta}\right)^{\text{SLL,Coul}} = -\frac{1}{\cosh^2(\eta)}\frac{\beta}{32\pi M^2}\frac{1}{\mathcal{N}_1\mathcal{N}_2} \times \Big\{16\pi^2\,\text{Tr}\big(\boldsymbol{\mathcal{H}}_{2\to 2}(\mu_h)\boldsymbol{X}^{\text{Coul}}\big)\int_1^{x_s}\frac{dx}{x}\frac{1}{\beta_0^3}U^c(1;\mu_h,\mu)\left(\ln^2(x_s)-\ln^2(x)\right) + \frac{3}{2}\pi^2\,\text{Tr}\big(\boldsymbol{\mathcal{H}}_{2\to 2}(\mu_h)\boldsymbol{X}^{2\text{Coul}}\big)\frac{1}{\beta_0^3}\ln^3(x_s)\Big\}$$

where $\boldsymbol{X}_{2\to t\bar{t}}^{\text{Coul}} = J_{43}v_{t\bar{t}}f^{abe}f^{cde}\boldsymbol{T}_1^a\boldsymbol{T}_2^b\boldsymbol{T}_3^c\boldsymbol{T}_4^d$

$$\boldsymbol{X}_{2\to t\bar{t}}^{2\text{Coul}} = v_{t\bar{t}}^2 f^{abe}f^{cde}\left(\boldsymbol{T}_3^c\{\boldsymbol{T}_4^b,\boldsymbol{T}_4^d\} - \boldsymbol{T}_4^c\{\boldsymbol{T}_3^b,\boldsymbol{T}_3^d\}\right)\left(\tilde{J}_1^{34}\boldsymbol{T}_1^a + \tilde{J}_2^{34}\boldsymbol{T}_2^a\right)$$