



# Dynamical generation of Exotic Heavy Mesons in the heavy meson scattering

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# Outline

- *Introduction*
- *Coupled-channel formalism*
- *Effective Lagrangian*
- *Heavy meson scattering in doubly charmed-channels*
- *Heavy meson scattering in hidden-charm channels*
- *Summary*

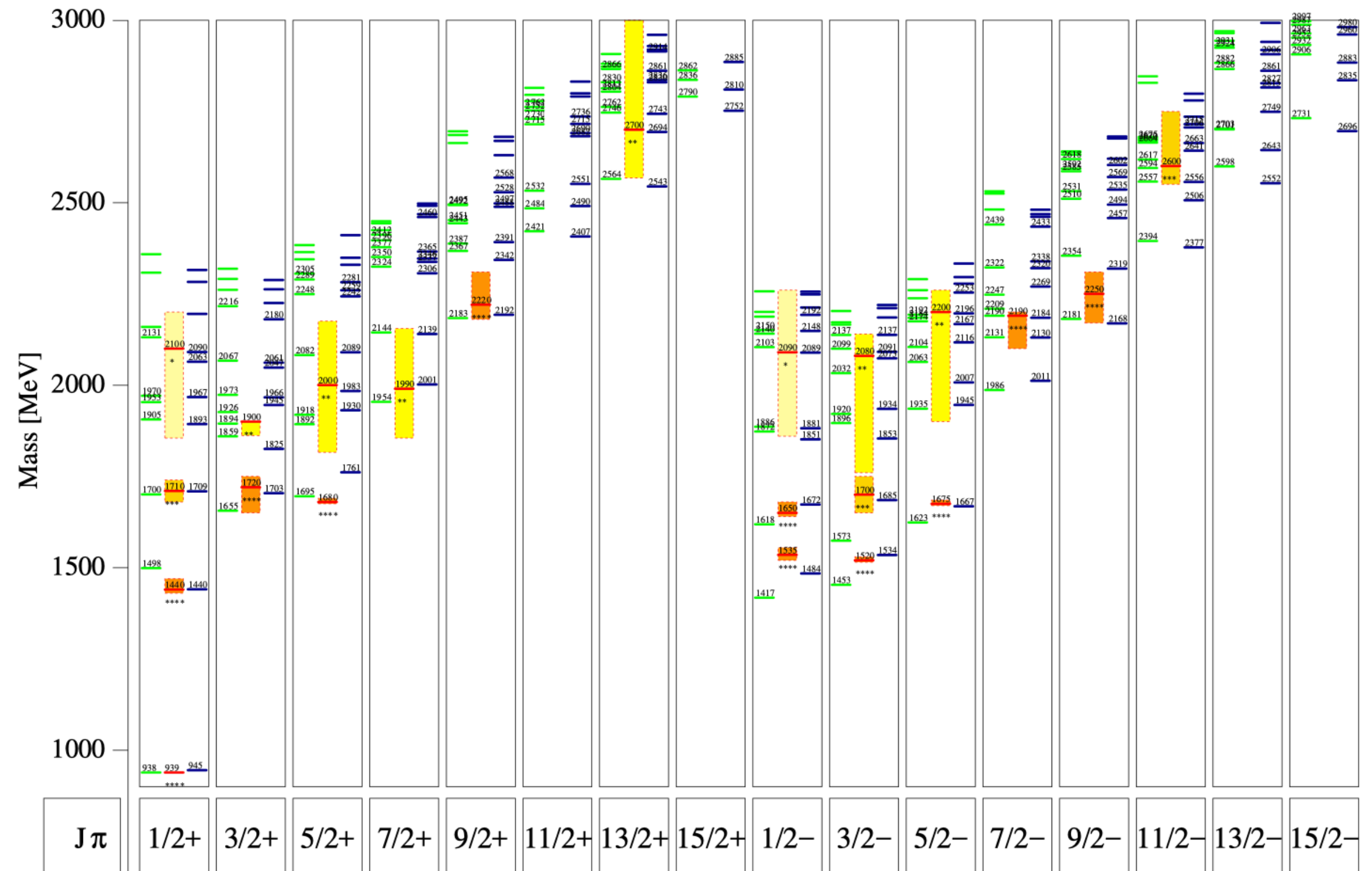
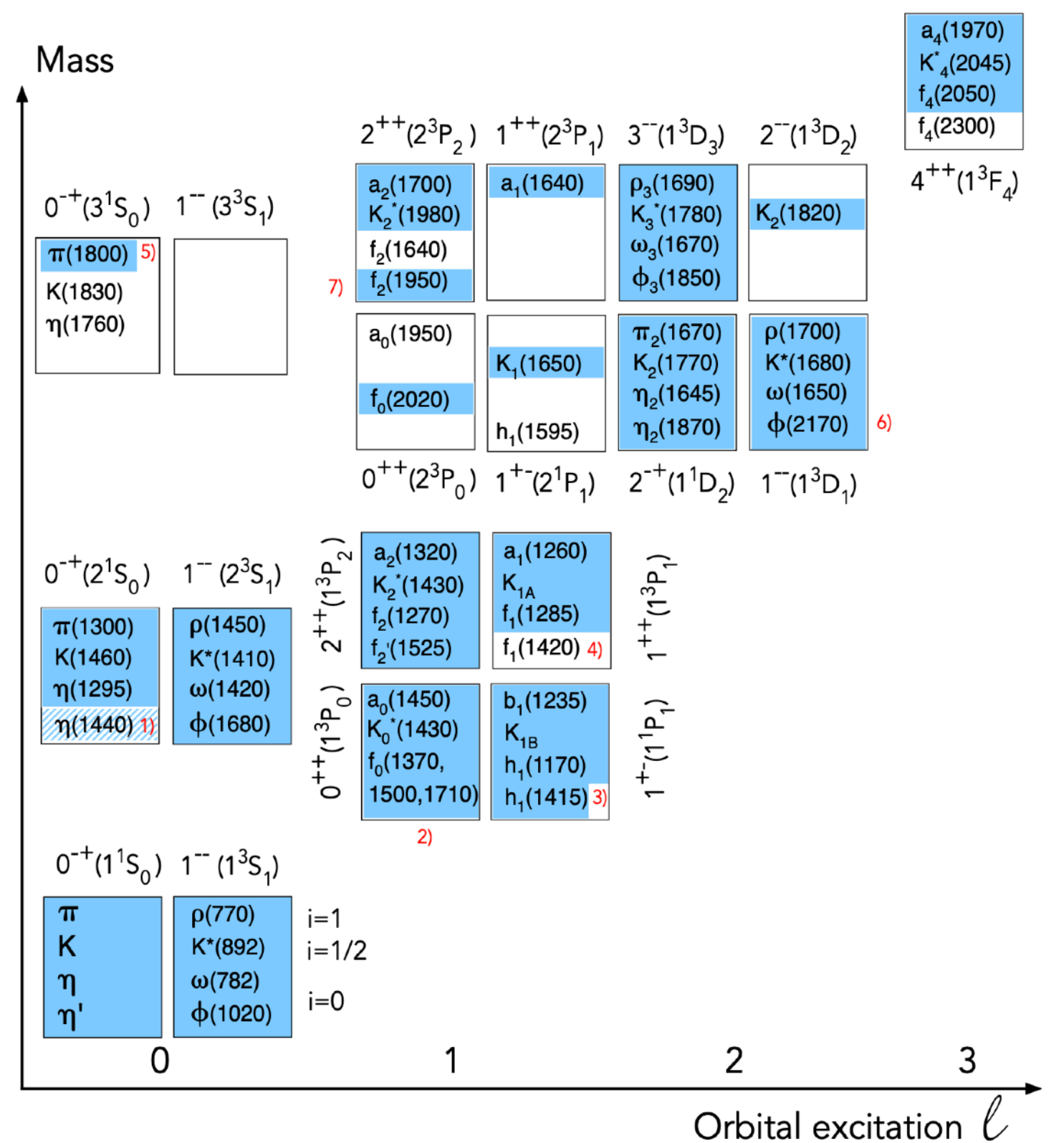
# Introduction

# Hadron spectroscopy

## Conventional Quark Model

- The spectrum of strongly interacting particles consists of a tower of many states.

Conventional hadrons : Mesons( $q\bar{q}$ ) and Baryons( $qqq$ )



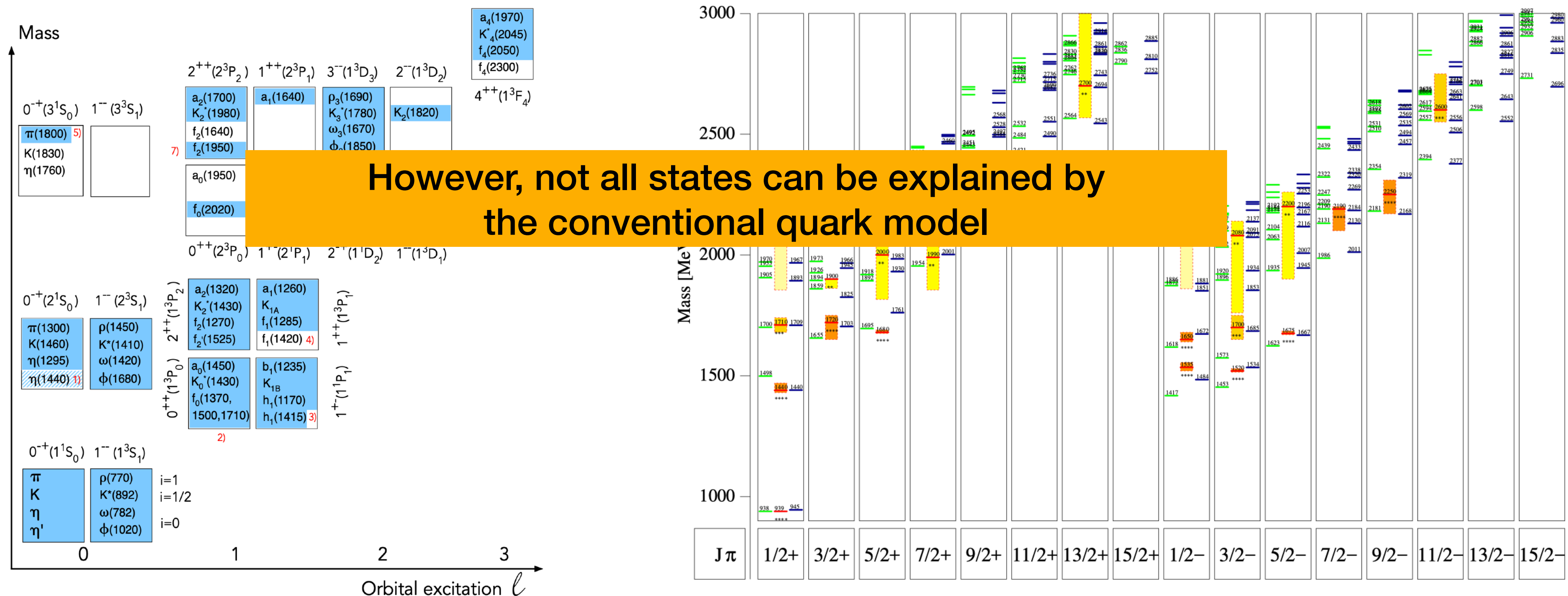


# Hadron spectroscopy

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# Beyond Quark Model

QCD allows many different types of color-neutral objects

Meson :  $q\bar{q}$       $qq\bar{q}\bar{q}$ (tetraquark),  $q\bar{q}g$ (hybrids), glueballs, ...

Baryon :  $qqq$       $qqqq\bar{q}$ (pentaquark),  $qqqqqq$ ...

Pentaquark



diquark-diquark-  
antiquark

*H*-dibaryon



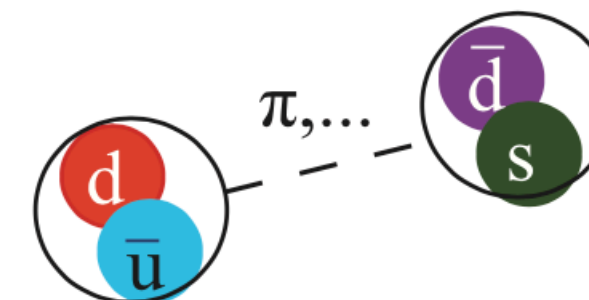
diquark-diquark-  
diquark

Tetraquark

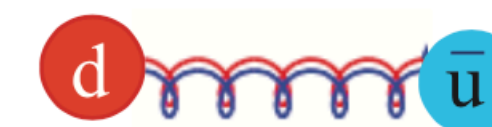


diquark-diantiquark

Molecule



Hybrid



Glueball



S.L.Olsen Front.Phys.(Beijing) 10 (2015) 2, 121-154

# Beyond Quark Model

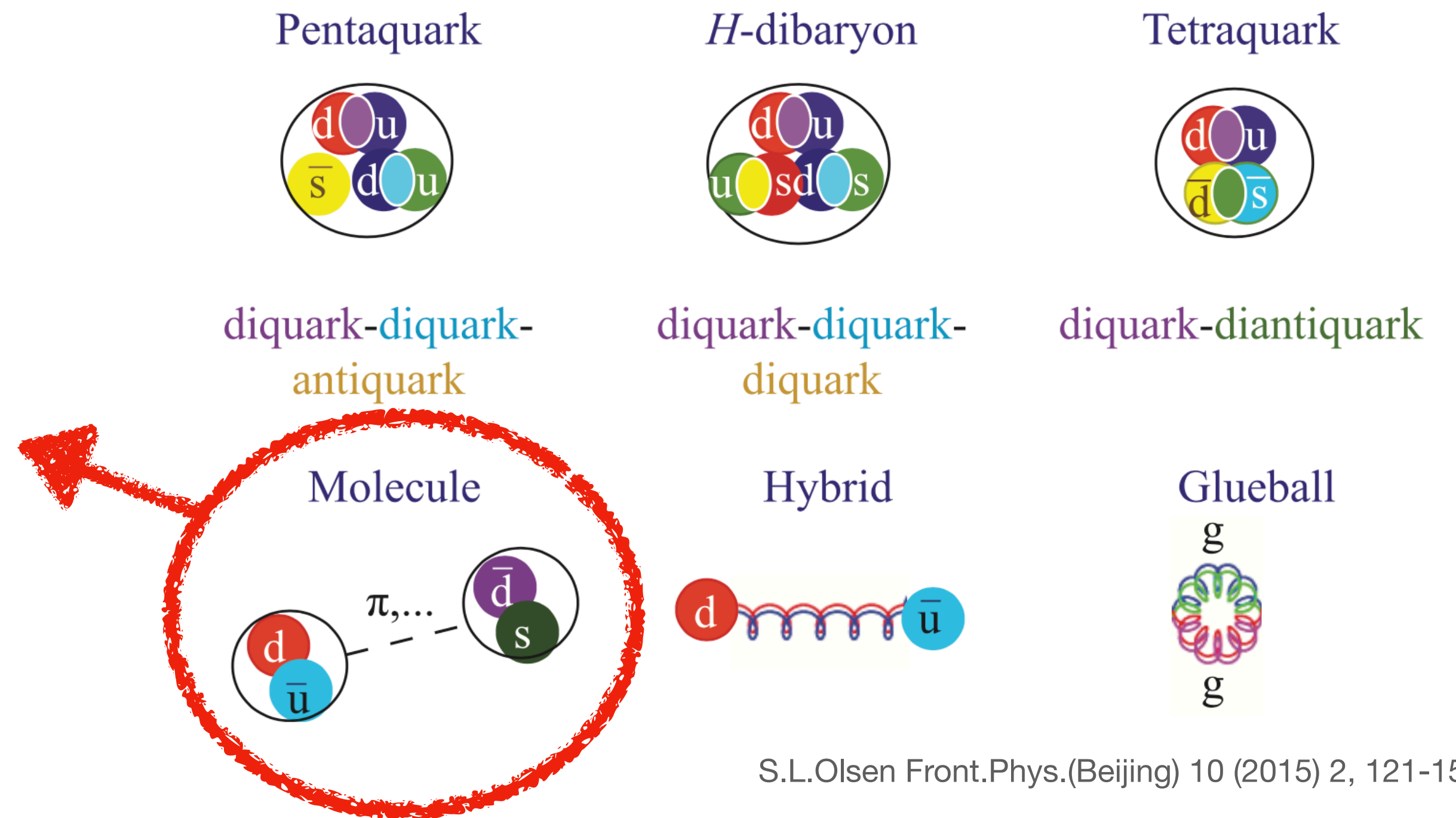
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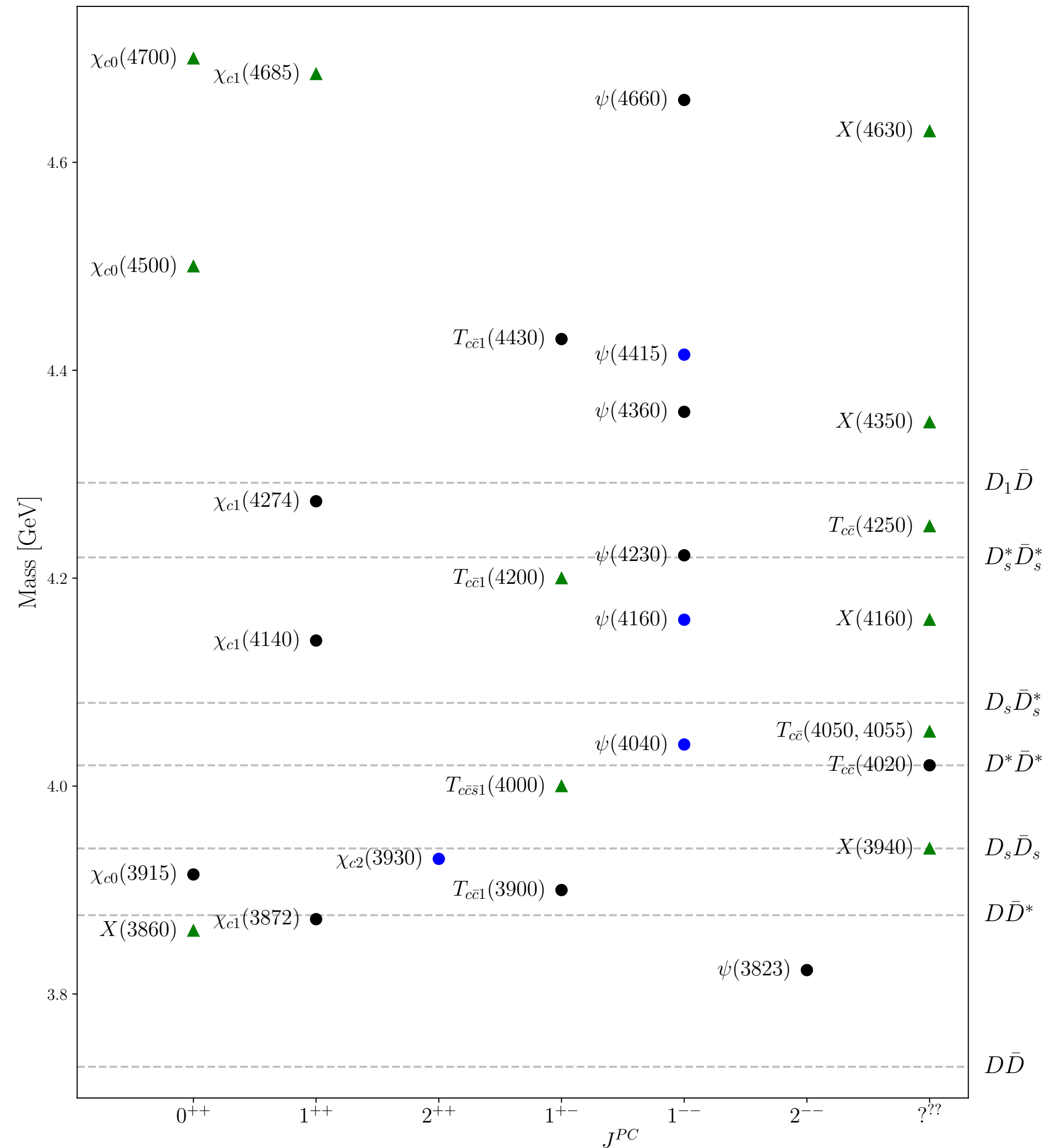
Baryon :  $qqq$       $qqqq\bar{q}$ (pentaquark),  $qqqqqq$ ...

## Hadronic molecule

- Bound states of color-neutral states via meson-exchanges
- Near certain two-particle thresholds
- Dominantly decay into the two-particle channels



# Exotic states

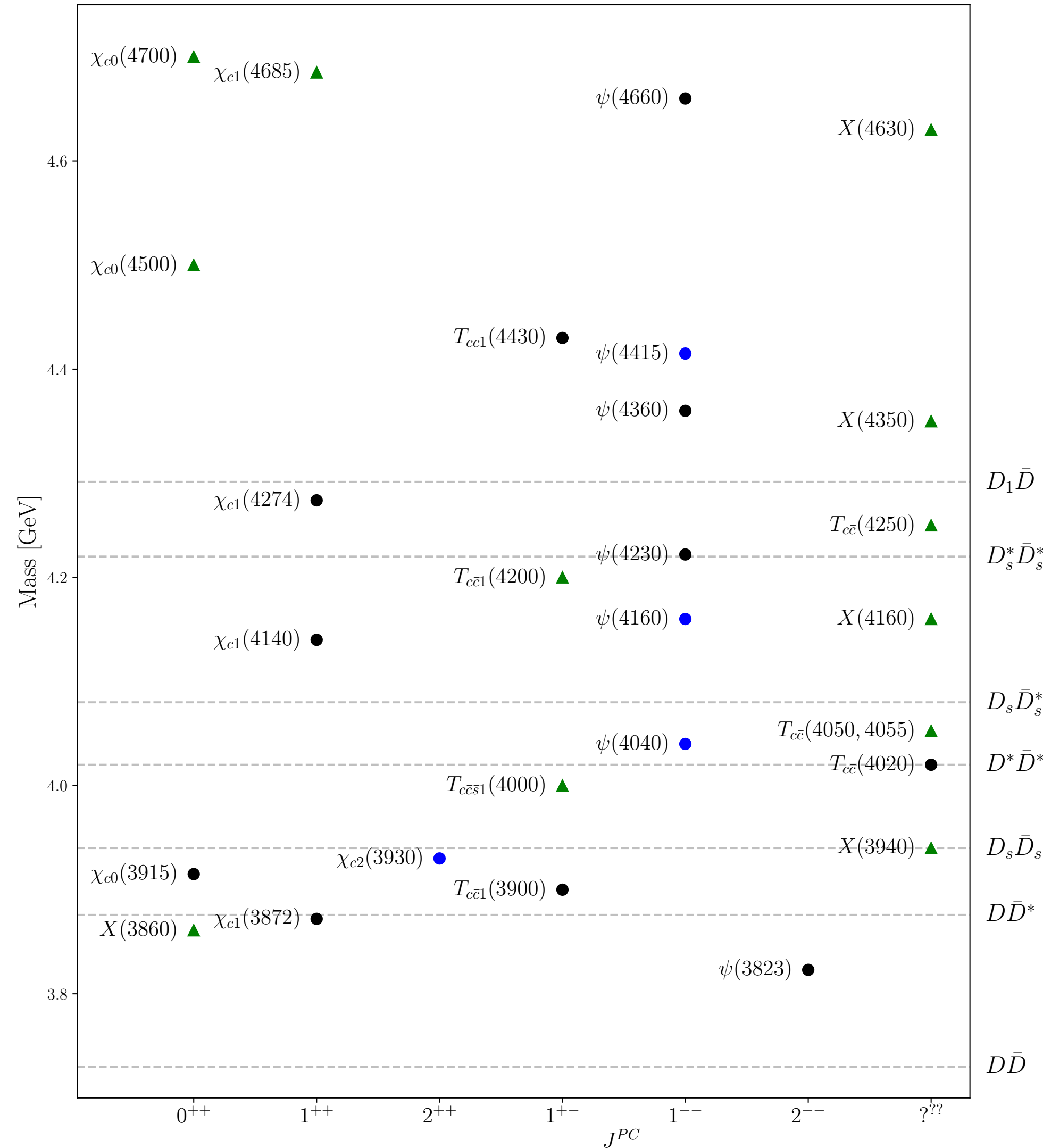


The representative hadronic-molecule candidate is  $\chi_{c1}(3872)$

$$I^G(J^{PC}) = 0^+(1^{++}) \quad \text{QM candidates: } M(2^3P_1 c\bar{c}) \approx 3950 \text{ MeV}$$

$$M_{\chi_{c1}} = 3871.84 \text{ MeV}$$

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Its mass is very close to the  $D^0\bar{D}^{*0}$  threshold:

$$M_{\chi_{c1}} - (m_{D^0} + m_{\bar{D}^{*0}}) = -0.09 \pm 0.28 \text{ MeV}$$

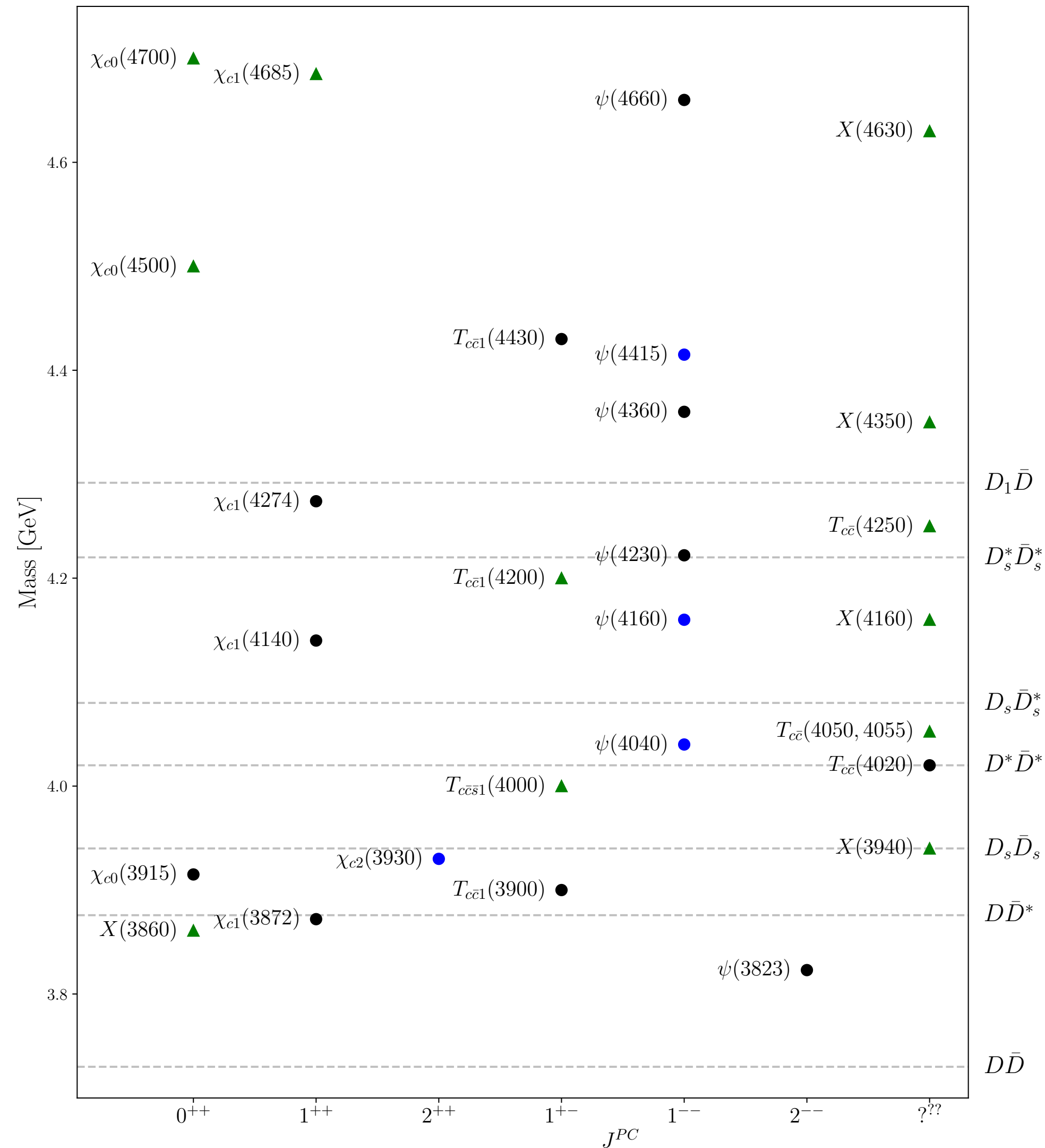
Dominantly decay into this channel:

$$\mathcal{B}(\chi_{c1}(3872) \rightarrow D^0\bar{D}^{*0}) > 34 \%$$

**Consistent with the characteristics of hadronic molecules**



# Exotic heavy mesons

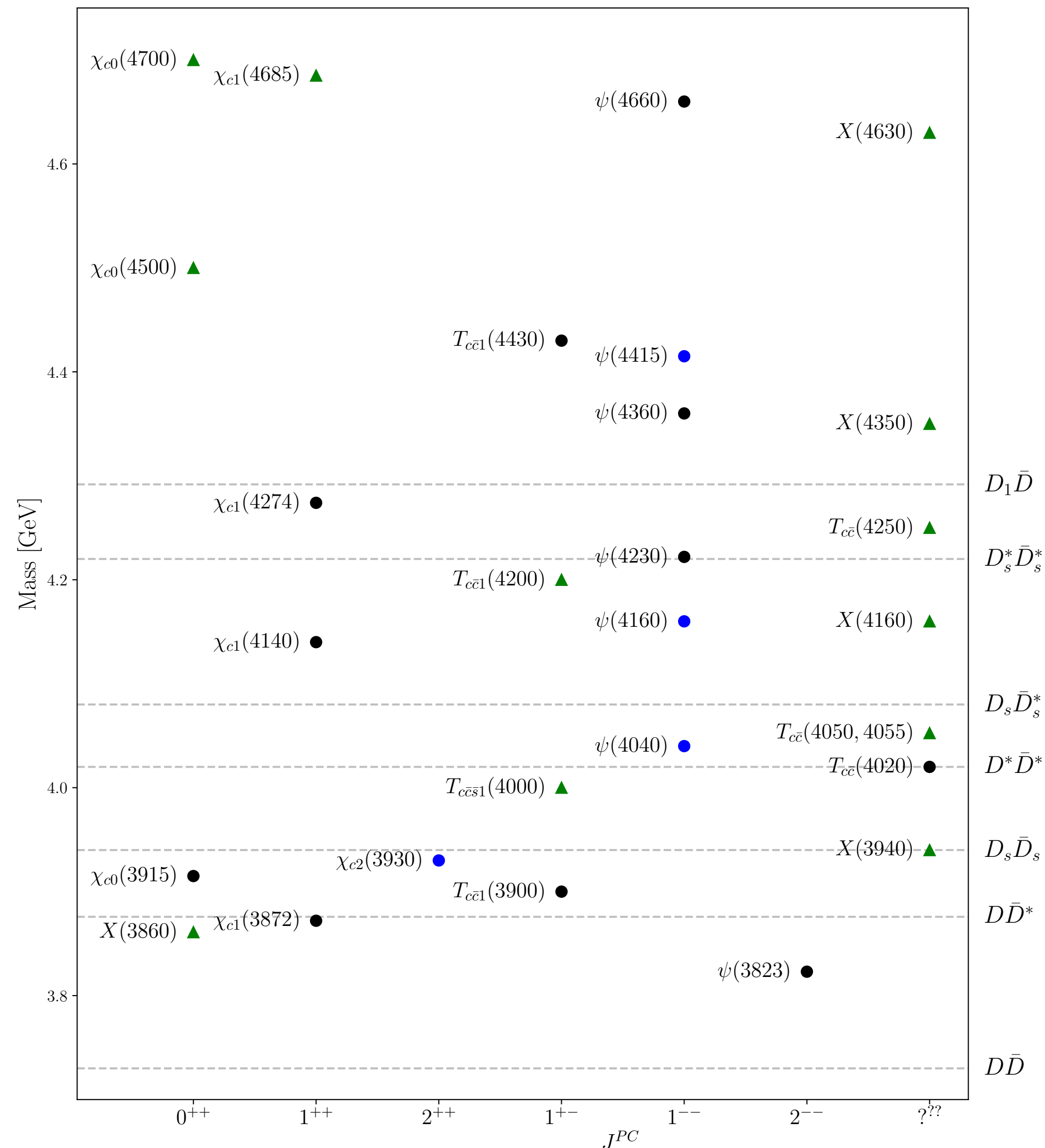


## Observed multiquark candidates listed in PDG:

- Low-lying scalar mesons :  $a_0/f_0(980), f_0(500), a_1(1260), b_1(1235)$ ...
- Exotic states with an heavy flavor :  $D_{s0}^*(2317), D^*(2400), \dots$
- Exotic  $c\bar{c}$  or  $b\bar{b}$  states :  $\chi_{c1}(3872), T_{c\bar{c}1}(3900), T_{b\bar{b}1}(10610), \dots$
- Open heavy-flavored state :  $T_{cc}^+$
- Fully heavy tetraquark :  $T_{cc\bar{c}\bar{c}}(6900)$



# Exotic heavy mesons



D. Lohse, Nucl. Phys. A516, 513

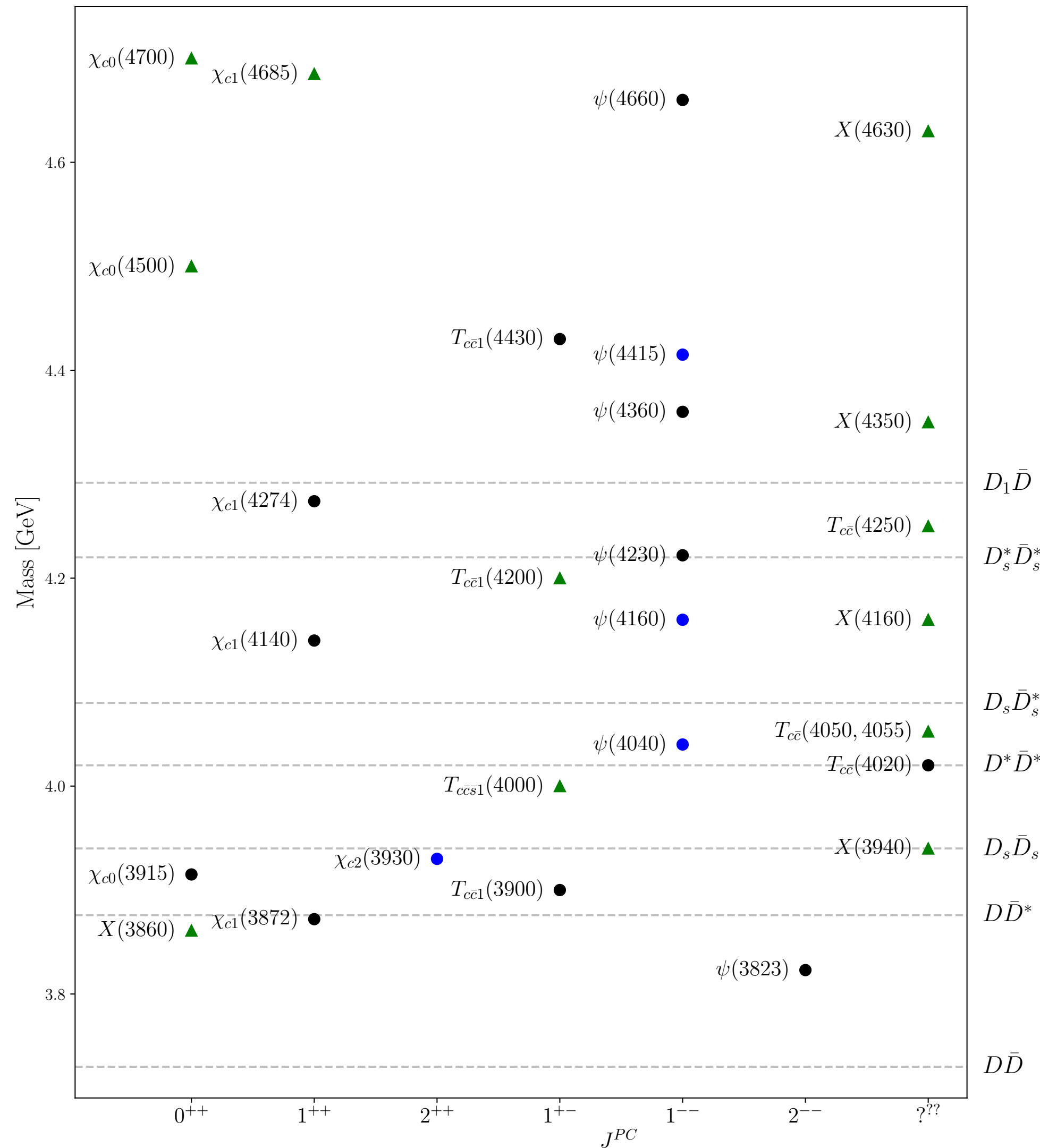
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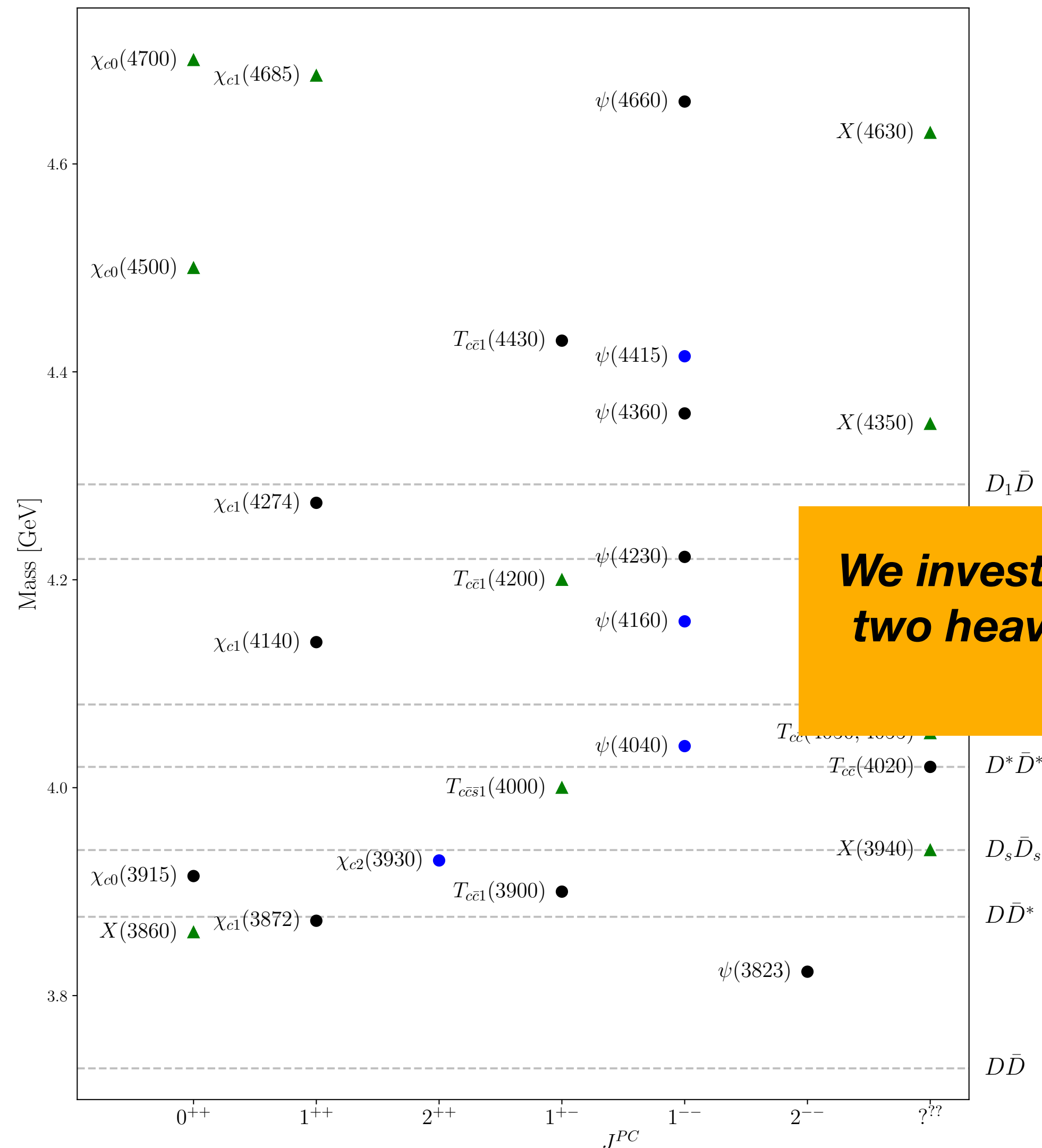
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**We investigate the hadron molecular features of the exotic states containing two heavy quarks using the *fully off-mass-shell* coupled-channel formalism within the meson-exchange framework.**

# Coupled-channel formalism

# Two-body scattering equation

- Blankenbecler-Sugar equation

$$T = V + VGT$$

- The two-body T-matrix are obtained by solving the Bethe-Salpeter equation:

$$T_{fi} = V_{fi} + \sum_n \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{V_{fn} T_{ni}}{(k_1^2 - m_1^2 + i\epsilon)(k_2^2 - m_2^2 + i\epsilon)}$$

- The three-dimensional reduction via the Blankenbecler-Sugar scheme preserves unitarity and off-shellness:

$$G_k(q) = \frac{\pi}{\omega_1^k \omega_2^k} \delta \left( q^0 - \frac{\omega_1^k - \omega_2^k}{2} \right) \frac{\omega_1^k + \omega_2^k}{s - (\omega_1^k + \omega_2^k)^2 + i\epsilon}$$



$$T_{fi}(\mathbf{p}, \mathbf{p}') = V_{fi}(\mathbf{p}, \mathbf{p}') + \sum_n \int \frac{d^3 q}{(2\pi)^3} \frac{\omega_1^n + \omega_2^n}{s - (\omega_1^n + \omega_2^n)^2 - i\epsilon} V_{ni}(\mathbf{p}, \mathbf{q}) T_{fn}(\mathbf{q}, \mathbf{p}')$$

# Two-body scattering equation

- **Blankenbecler-Sugar equation**
- Total angular momentum projection

$$T_{fi}^J(p, p') = V_{fi}^J(p, p') + \frac{1}{(2\pi)^3} \int_0^\infty dq \frac{\omega_1 + \omega_2}{2\omega_1\omega_2} \frac{q^2 V_{fk}^J(p, q) T_{ki}^J(q, p')}{s - (\omega_1 + \omega_2)^2 - i\varepsilon}$$

*Matrix inversion method:*  $T^J = V^J + V^J G T^J \implies T^J = (1 - V^J G)^{-1} V^J$



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*Matrix inversion method:*  $T^J = V^J + V^J G T^J \implies T^J = (1 - V^J G)^{-1} V^J$

We obtain the *off-mass-shell*  $T$  matrix in the **full-channel momentum space** :

$$\begin{array}{l}
 f = 1 \left\{ \begin{array}{l} p'_1 \\ \vdots \\ p'_n \end{array} \right. \\
 f = 2 \left\{ \begin{array}{l} p'_1 \\ \vdots \\ p'_n \end{array} \right. \\
 \vdots
 \end{array}
 \left(
 \begin{array}{c|c|c}
 \overbrace{p_1 \cdots p_n}^{i=1} & \overbrace{p_1 \cdots p_n}^{i=2} & \cdots \\
 \hline
 & & \\
 \hline
 & & \\
 \hline
 \vdots & & \ddots
 \end{array}
 \right)$$

# Two-body scattering equation

$$T_{fi}^J(p, p') = V_{fi}^J(p, p') + \frac{1}{(2\pi)^3} \int_0^\infty dq \frac{\omega_1 + \omega_2}{2\omega_1\omega_2} \frac{q^2 V_{fk}^J(p, q) T_{ki}^J(q, p')}{(\omega_1 + \omega_2)^2 - s}$$

*Regularization of the two-body propagator:*

- The two-body propagator is singular at the on-mass-shell momentum point,  $\omega^2(\tilde{q}) = s$ ,  $\tilde{q} = \frac{\lambda^{\frac{1}{2}}(s, m_1^2, m_2^2)}{2\sqrt{s}}$
- Change of the variable

$$\omega = \omega_1 + \omega_2, \quad d\omega = \frac{\omega_1 + \omega_2}{\omega_1\omega_2} q dq \quad \Longrightarrow \quad \int_{m_1+m_2}^\infty d\omega \frac{f(q)}{\omega^2 - s}, \quad \text{where } f(q) = \frac{1}{2} q V(q) T(q)$$

# Two-body scattering equation

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- Decompose into regular part and singular part.

$$\int_{m_1+m_2}^\infty d\omega \frac{f(q) - f(\tilde{q})}{\omega^2 - s} + \int_{m_1+m_2}^\infty d\omega \frac{f(\tilde{q})}{\omega^2 - s}$$

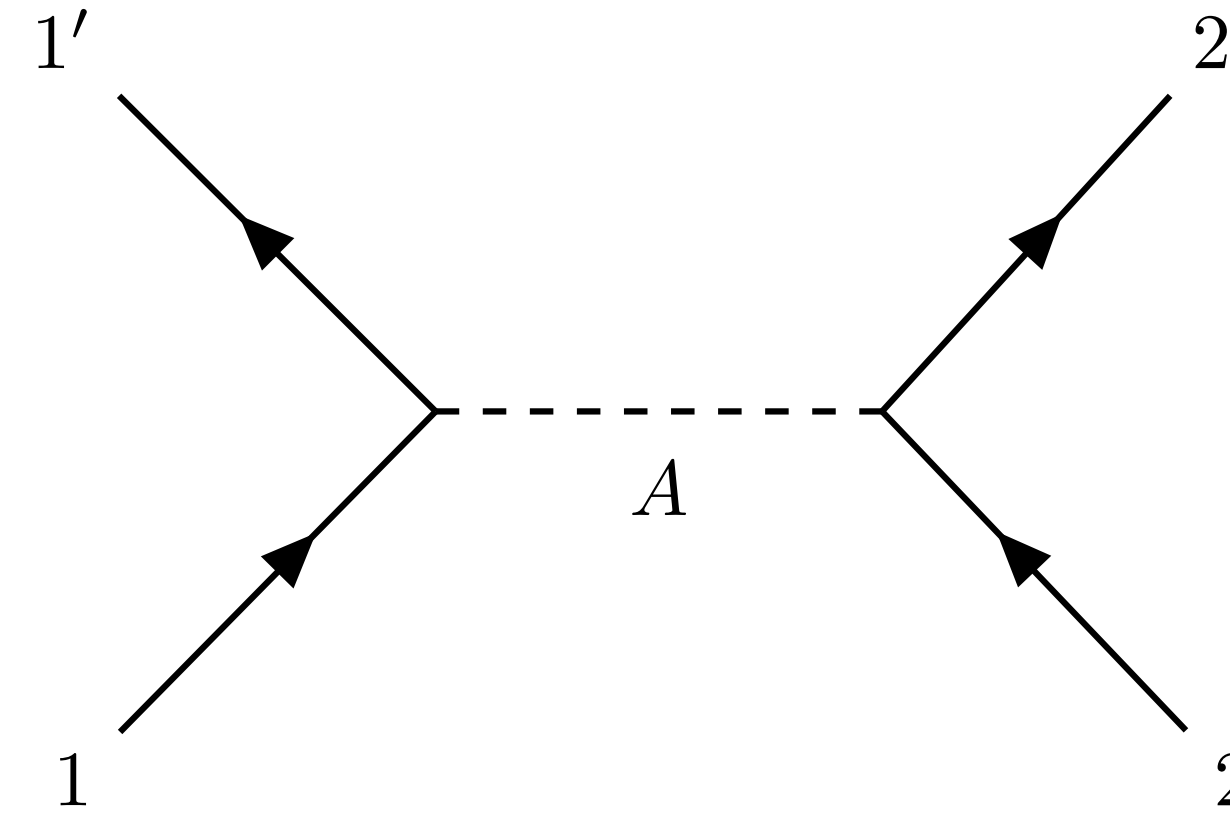
- One can regularize the singular part:

$$\int_{m_1+m_2}^\infty d\omega \frac{f(\tilde{q})}{\omega^2 - s} = P \int_{m_1+m_2}^\infty d\omega \frac{f(\tilde{q})}{\omega^2 - s} + \int_C d\omega \frac{f(\tilde{q})}{\omega^2 - s} = \frac{f(\tilde{q})}{2\sqrt{s}} \left( i\pi - \log \left| \frac{\sqrt{s} - m_1 - m_2}{\sqrt{s} + m_1 + m_2} \right| \right)$$

# Kernel amplitudes

- **Scattering amplitudes**

$$\mathcal{V}_{12 \rightarrow 1'2'} = \sum_A \mathcal{M}_{12 \rightarrow 1'2'}^A$$



$$\mathcal{M}_{12 \rightarrow 1'2'}^A = \text{IS} F_A^2 \Gamma_{12}^A P^A \Gamma_{1'2'}^A$$

Since the hadron has a finite size, form factor is need to be considered at each vertex:  $F(q^2) = \left( \frac{n\Lambda^2 - m_{\text{ex}}^2}{n\Lambda^2 - q^2} \right)^n$

For minimal uncertainty from the cutoff parameters, we strictly fixed the values about  $\Lambda = m_{\text{ex}} + 600 \text{ MeV}$ .

IS is the isospin symmetric factor from the isospin projection (for definite isospin channels)

# Effective Lagrangian



# Effective Lagrangian

- **Heavy chiral Lagrangian**

The coupling constants between heavy and light mesons are determined by the interaction Lagrangian based on the *Heavy Quark Effective Field Theory*(HQEFT).

$$\mathcal{L}_{\text{heavy}} = ig\text{Tr}[H_b\gamma_\mu\gamma_5\mathcal{A}_{ba}^\mu\bar{H}_a] + i\beta\text{Tr}[H_bv^\mu(\mathcal{V}_\mu - \rho_\mu)_{ba}\bar{H}_a] + i\lambda\text{Tr}[H_b\sigma_{\mu\nu}F_{ba}^{\mu\nu}(\rho)\bar{H}_a] + g_\sigma\bar{H}_aH_a\sigma$$

A heavy-light meson is made up by a **heavy** quark  $Q$  and a **light** antiquark  $\bar{q}$ .

→ **heavy quark spin symmetry**(HQSS), **heavy quark flavor symmetry**(HQFS) + **chiral symmetry**

- **Heavy superfield**: HQSS, HQFS, Lorentz invariance, Parity invariance

$$H^a = \frac{1 + \not{v}}{2}(P_\mu^{*a}\gamma^\mu - P^a\gamma_5), \quad \bar{H} = \gamma_0 H^\dagger \gamma_0 = (P_\mu^{*\dagger a}\gamma^\mu + P^{\dagger a}\gamma_5)\frac{1 + \not{v}}{2}$$

Pseudoscalar heavy field:  $P^a = \{D^+, D^0, D_s^+\}$  or  $\{B^-, \bar{B}^0, \bar{B}_s^0\}$

Vector heavy field:  $P_\mu^{*a} = \{D_\mu^{*+}, D_\mu^{*0}, D_{s\mu}^{*+}\}$  or  $\{B_\mu^{*-}, \bar{B}_\mu^{*0}, \bar{B}_{s\mu}^{*0}\}$



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- *Light pseudoscalar mesons*: chiral symmetry spontaneous break down

$$\mathcal{A}^\mu = \frac{i}{f_\pi}\partial^\mu\mathcal{M} + \dots \quad \mathcal{M} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

- *Light vector mesons*: dynamical gauge boson of the hidden local symmetry

$$\rho^\mu = i\frac{g_V}{\sqrt{2}}V^\mu, \quad V^\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega \end{pmatrix}^\mu$$

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- **Effective Lagrangian for charmonium interactions**

$$\mathcal{L}_J = g_\psi\text{Tr}[J\bar{H}_a^{\bar{Q}}\gamma_\mu\partial^\mu\bar{H}_a^Q]$$

Charminum superfield:  $J = \frac{1 + \psi}{2} [\psi^\mu\gamma_\mu - \eta_c\gamma_5] \frac{1 - \psi}{2}$

$$\eta_c(J^P = 0^-), \psi^\mu(J^P = 1^-)$$

SU(2) heavy quark spin symmetry :  $J \rightarrow S_c J S_c^\dagger$

$$\bar{H}^Q \rightarrow \bar{H}^Q S_c^\dagger$$

$$\bar{H}^{\bar{Q}} \rightarrow S_{\bar{c}} \bar{H}^{\bar{Q}}$$

# Heavy meson scattering in the doubly charm channel

# Kernel matrix in Doubly-charm sector

- **Hadron channels in doubly-charm sector**

Possible two-hadron states with  $cc\bar{q}\bar{q}'$  :

- $DD, DD^*, D^*D^*$  ( $I = 0, 1$ )

Four sets of coupled channels: mesons can be classified by quantum numbers,  $I^G(J^{PC})$

- $0^\pm(0^{+\pm}, 2^{+\pm})$ :  $DD, D^*D^*$
- $0^\pm(1^{+\pm})$ :  $DD^*, D^*D^*$
- $1^\pm(0^{+\mp})$ :  $DD, D^*D^*$
- $1^\pm(1^{+\mp})$ :  $DD^*, D^*D^*$

Kernel matrix element:

$$\mathcal{V} = \begin{pmatrix} \mathcal{V}_{DD \rightarrow DD} & \mathcal{V}_{DD \rightarrow DD^*} & \mathcal{V}_{DD \rightarrow D^*D^*} \\ \mathcal{V}_{DD^* \rightarrow DD} & \mathcal{V}_{DD^* \rightarrow DD^*} & \mathcal{V}_{D^*D^* \rightarrow D^*D^*} \\ \mathcal{V}_{D^*D^* \rightarrow DD} & \mathcal{V}_{D^*D^* \rightarrow DD^*} & \mathcal{V}_{D^*D^* \rightarrow D^*D^*} \end{pmatrix}$$

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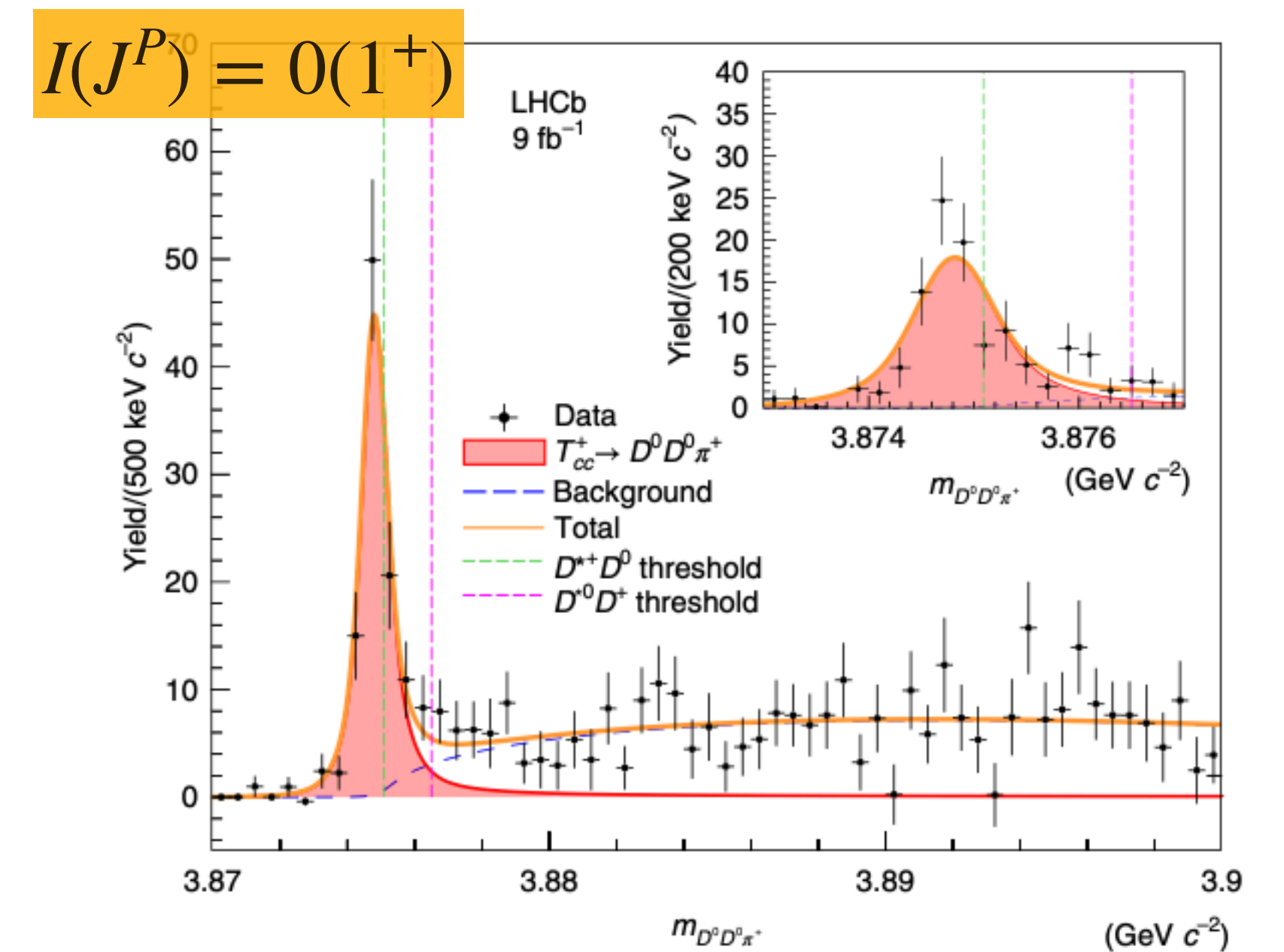
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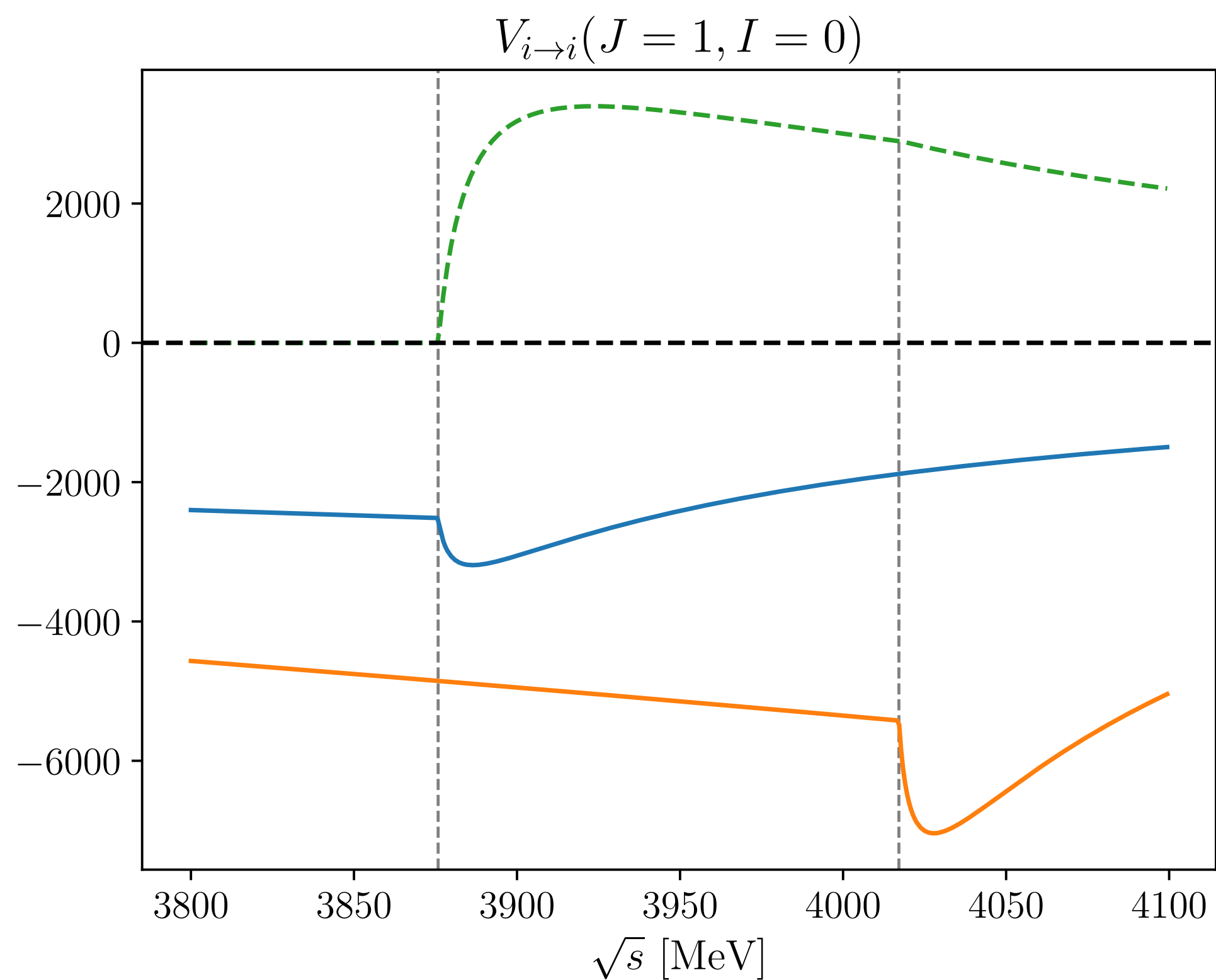


R. Aaij et.al.(LHCb Collabrator) Nature Physics. 18 (2022) 7, 751-754

# Dynamical generation of the poles

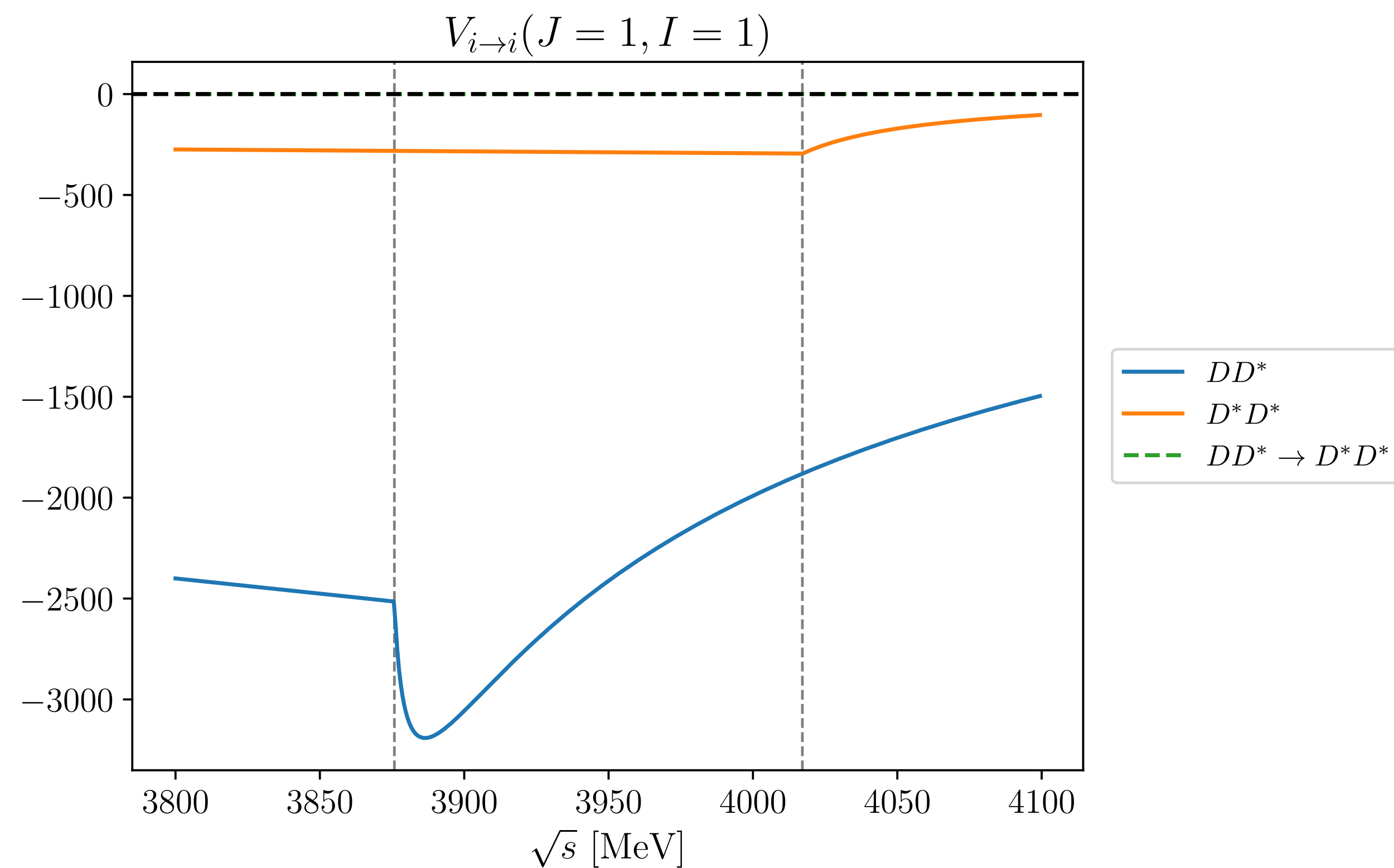
## Kernel amplitudes

### Vector channels ( $J=1$ )



**Strong attractions** for both diagonal elements

→ Pole is expected to generate in the coupled-channel solution



**Strong attraction** for  $DD^* \rightarrow DD^*$

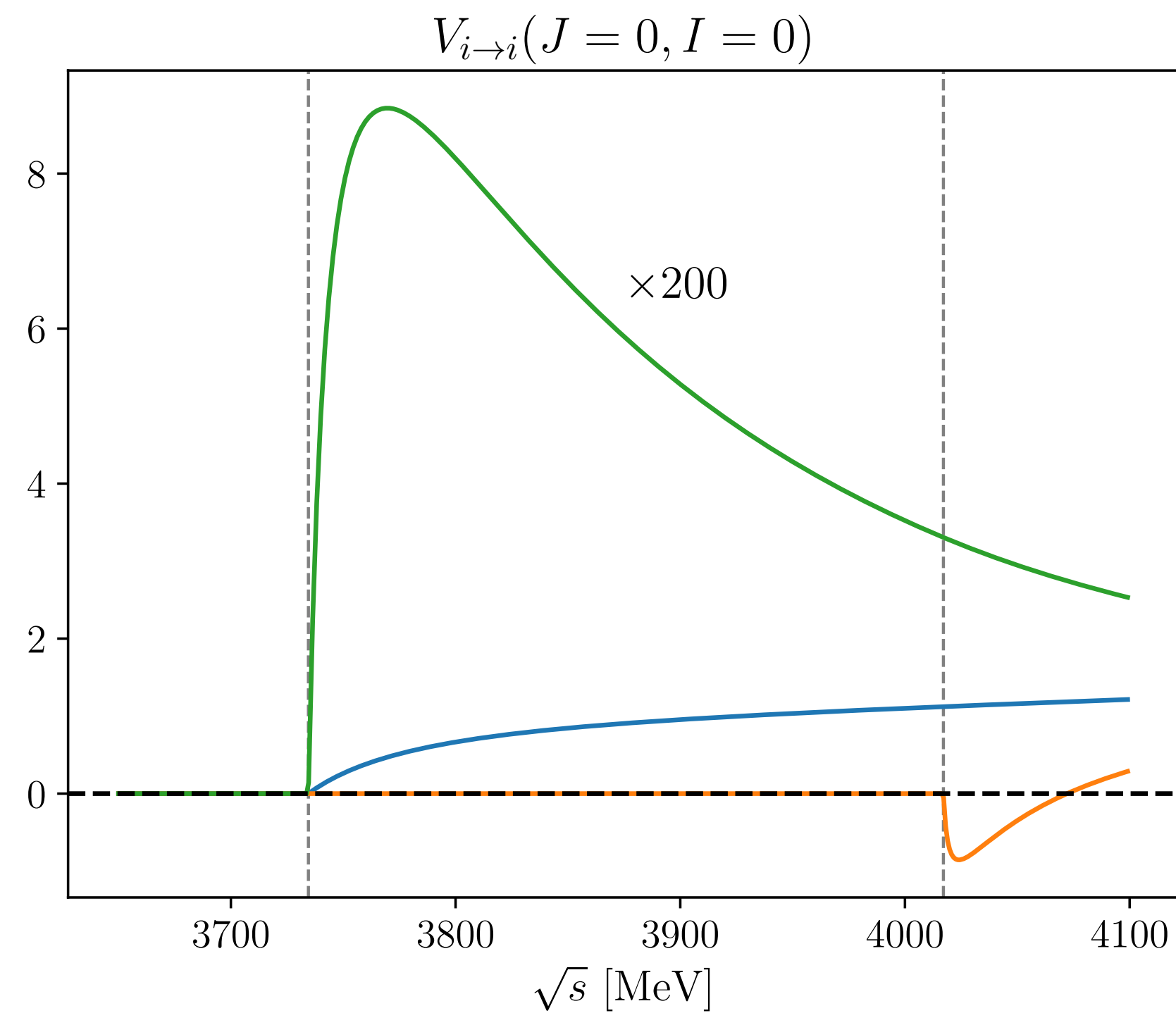
No transition appears in the  $DD^* \rightarrow D^*D^*$  process.



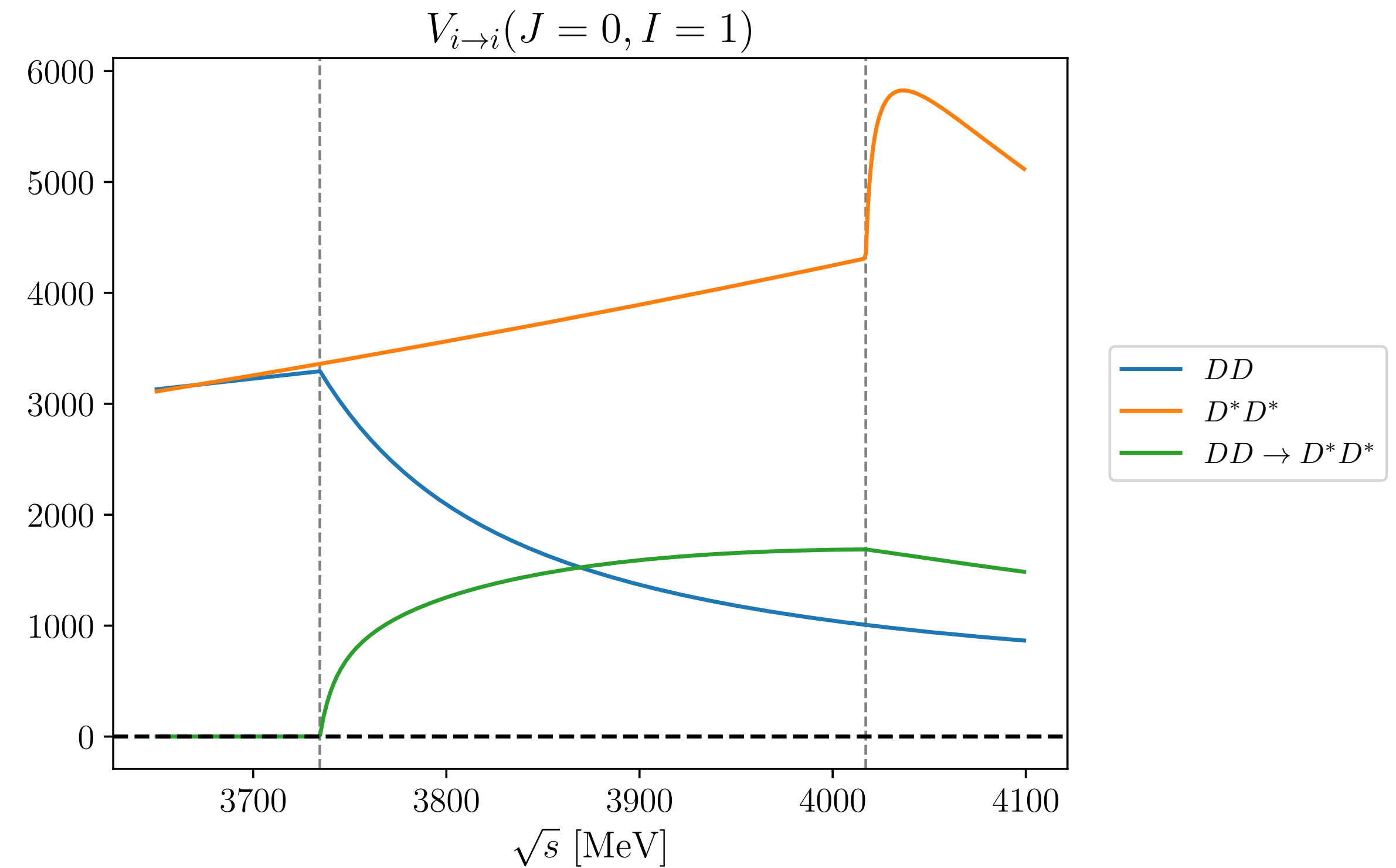
# Dynamical generation of the poles

## Kernel amplitudes

### Scalar channels ( $J=0$ )



**destructive** interference between kernel amplitudes in the diagonal elements due to the  $IS$  factors.

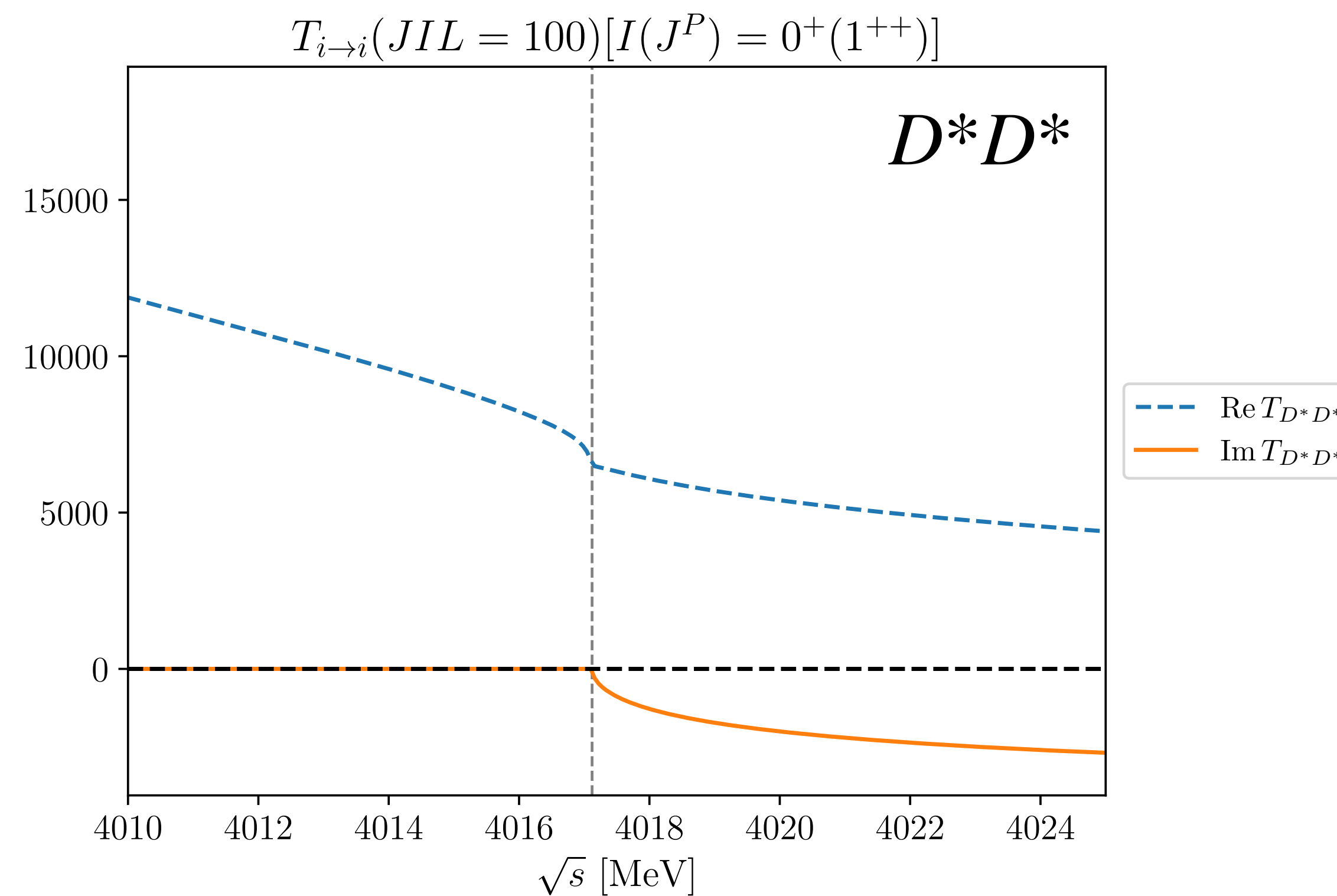
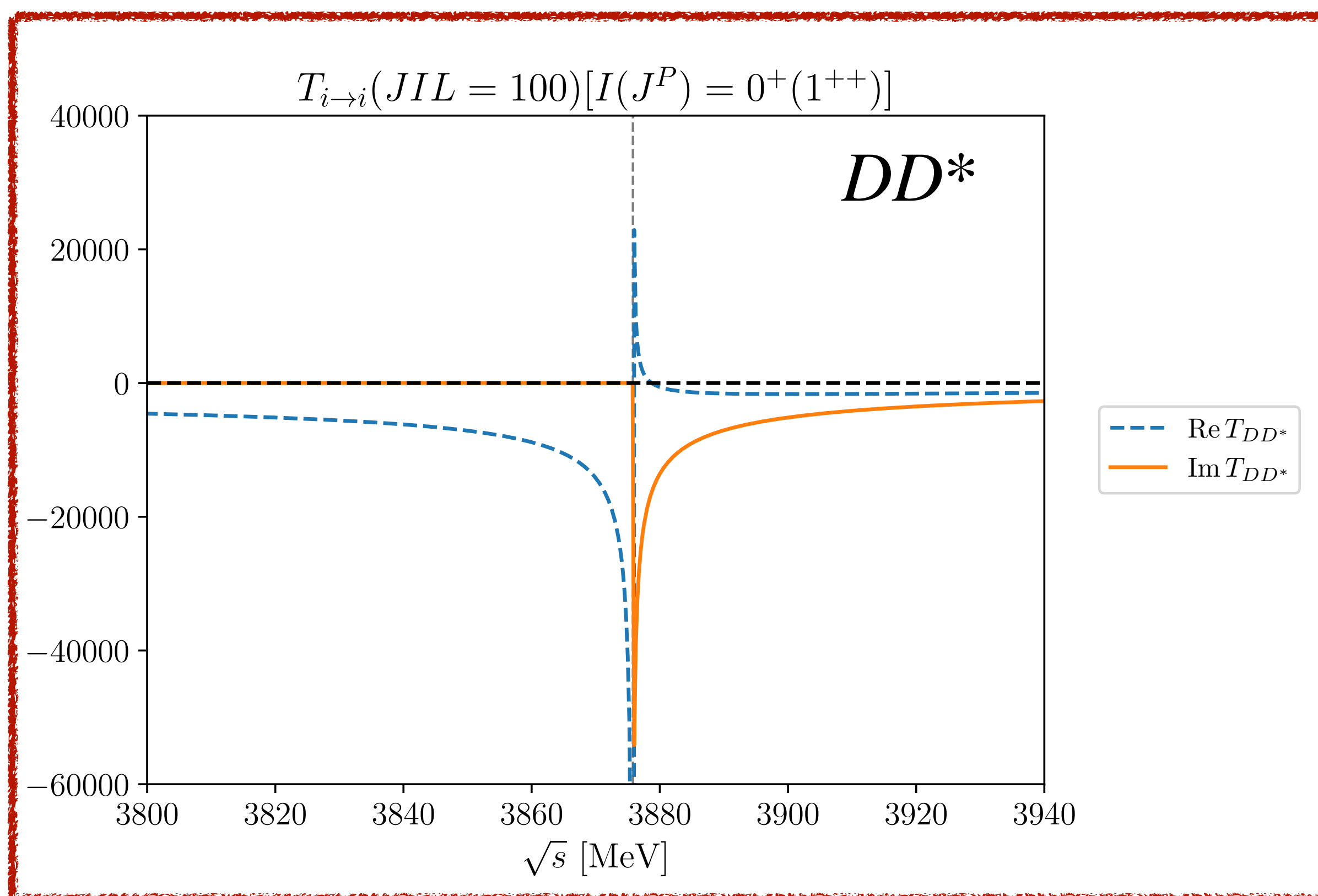


all interactions are strongly **repulsive**.

# Dynamical generation of the poles

## Single channel T matrix elements

### Vector-isoscalar channel ( $J=1, I=0$ )

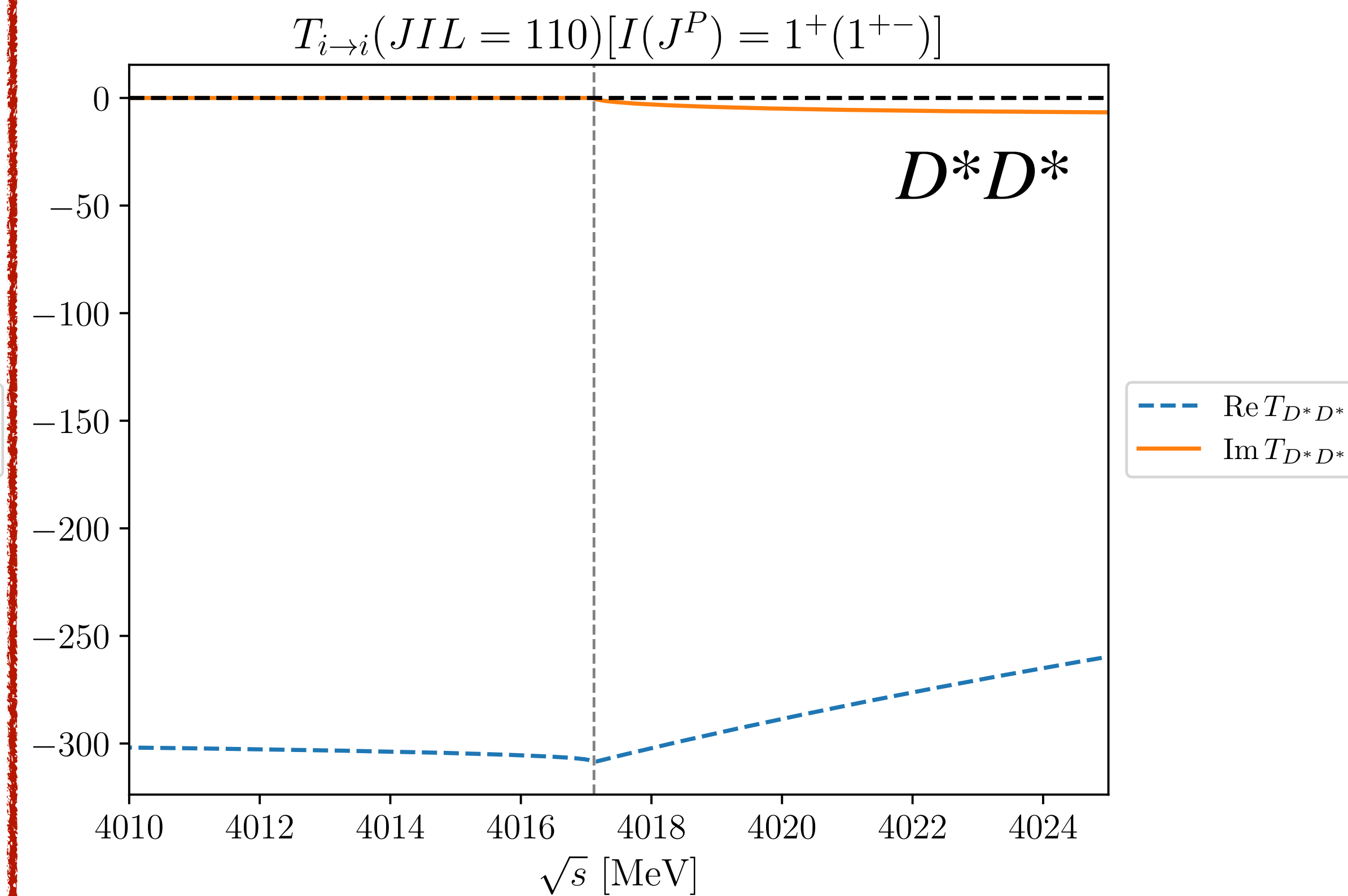
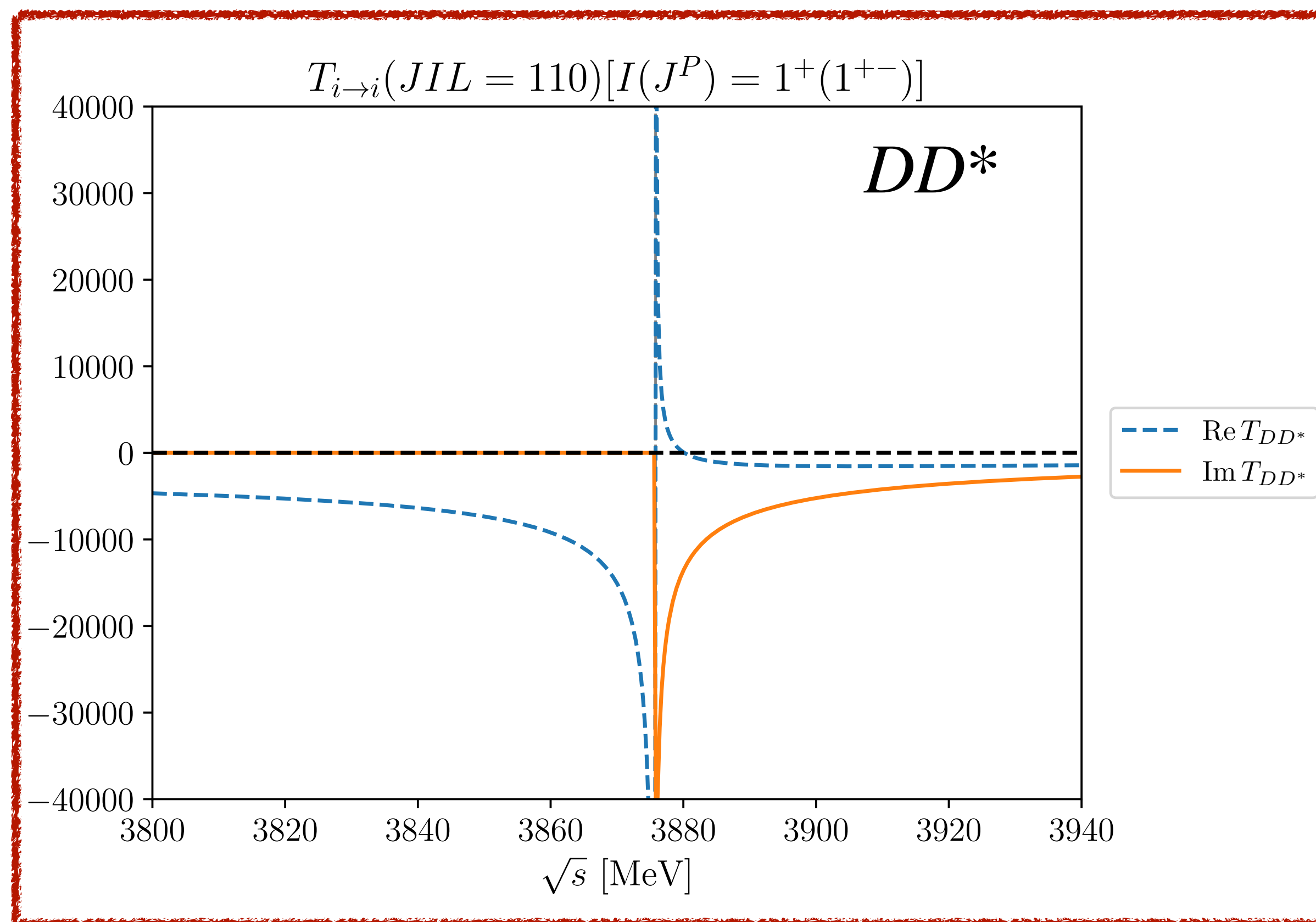


The attraction in  $DD^*$  kernel is nearly sufficient to generate a pole at the threshold.

# Dynamical generation of the poles

## Single channel T matrix elements

### Vector-isovector channel ( $J=1, I=1$ )

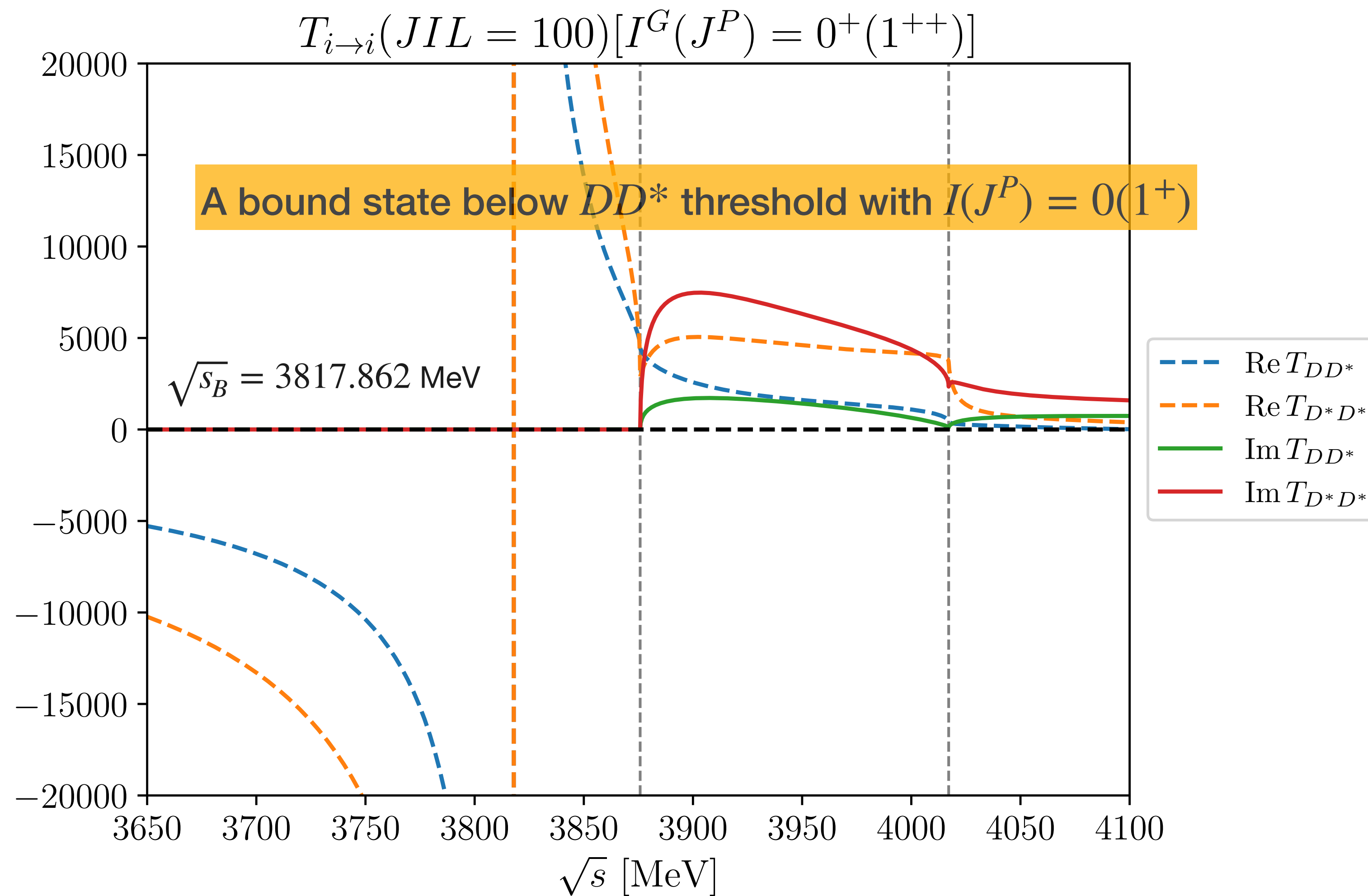


The pole is about to emerge at  $DD^*$  mass threshold.

# Dynamical generation of the poles

## Fully-coupled T-matrices

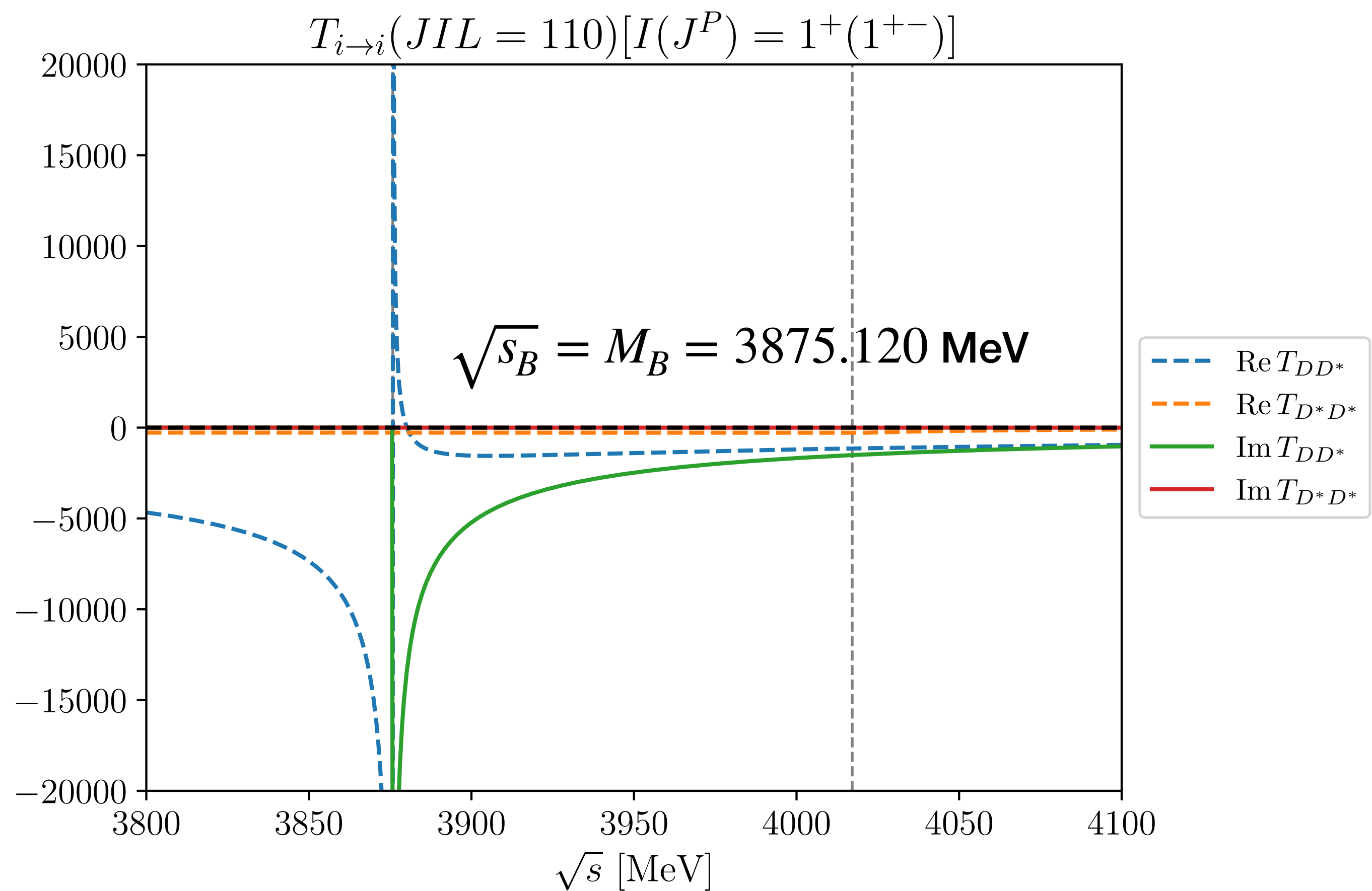
### Vector-isoscalar channel ( $I=0, J=1$ )



# Dynamical generation of the poles

## Fully-coupled T-matrices

### Vector-isovector channel ( $I=1, J=1$ )



# Heavy meson scattering in the hidden charm channel



# Feynman amplitudes

- **Hadron channels in hidden-charm sector**

Possible two-hadron states with  $c\bar{c}q\bar{q}'$  :

- $D\bar{D}, D\bar{D}^*, D^*\bar{D}^*$  ( $I = 0, 1$ )
- $D_s\bar{D}_s, D_s\bar{D}_s^*, D_s^*\bar{D}_s^*, \eta_c\omega, J/\psi\omega, \eta_c\phi, J/\psi\phi$  ( $I = 0$ )
- $\eta_c\pi, J/\psi\pi, \eta_c\rho, J/\psi\rho$  ( $I = 1$ ).

Four sets of coupled channels: mesons can be classified by quantum numbers,  $I^G(J^{PC})$

- $0^\pm(0^{+\pm}, 2^{+\pm})$ :  $D\bar{D}, J/\psi\omega, D_s\bar{D}_s, D^*\bar{D}^*, J/\psi\phi, D_s^*\bar{D}_s^*$
- $0^\pm(1^{+\pm})$ :  $\eta_c\omega, D\bar{D}^*, \eta_c\phi, D^*\bar{D}^*, D_s\bar{D}_s^*, D_s^*\bar{D}_s^*$
- $1^\pm(0^{+\mp})$ :  $D\bar{D}, J/\psi\rho, D^*\bar{D}^*$
- $1^\pm(1^{+\mp})$ :  $\eta_c\rho, D\bar{D}^*, D^*\bar{D}^*$

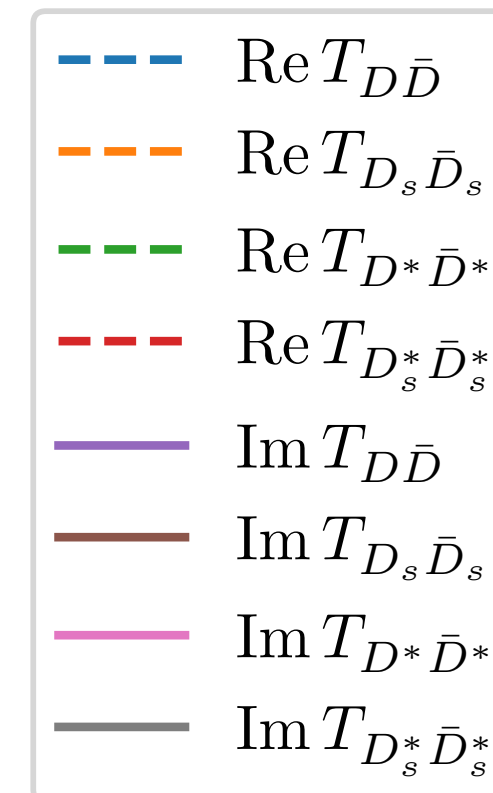
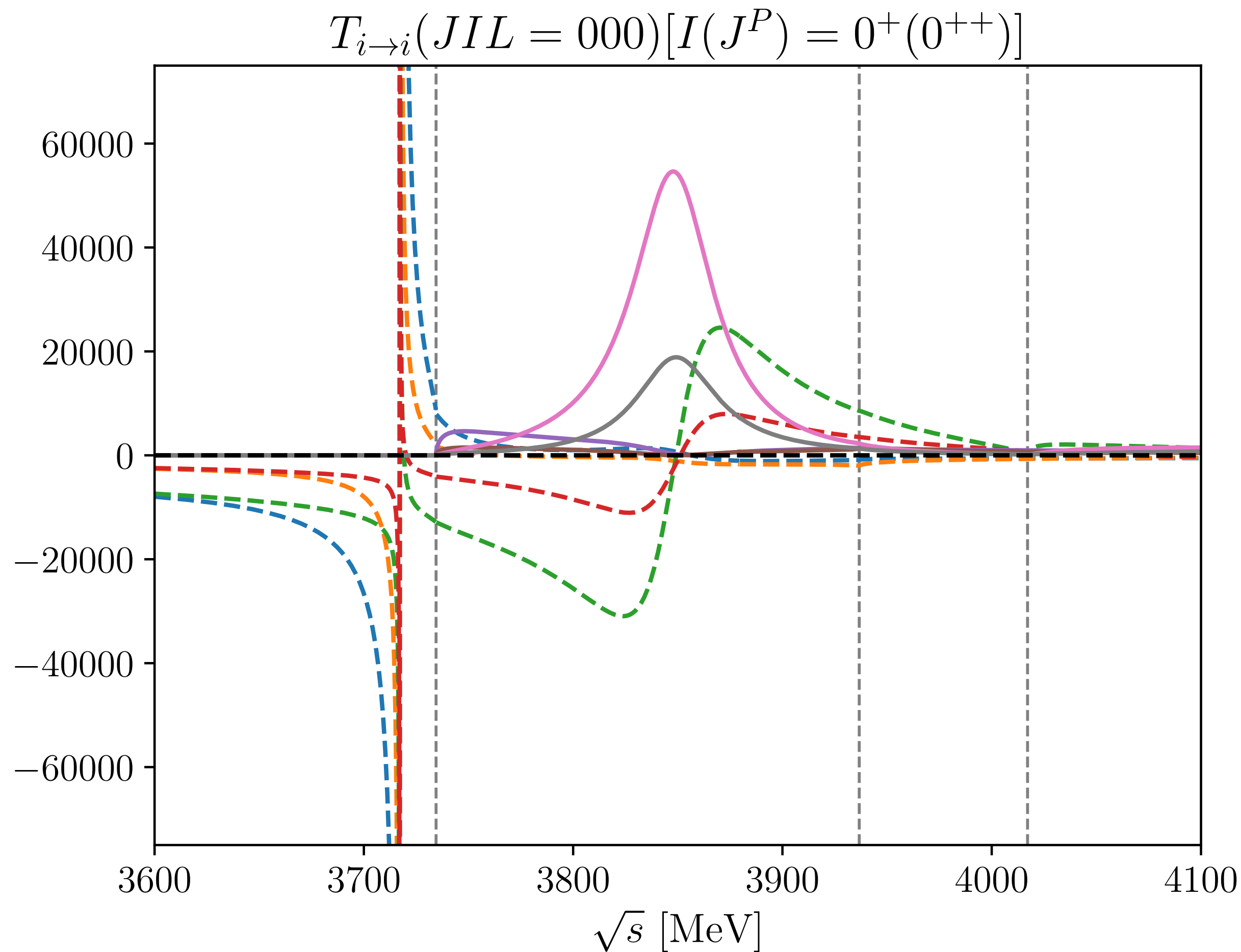
- $0^+(0^{++})$ :  $\chi_{c0}(3860), \chi_{c0}(3915)$
- $0^+(1^{++})$ :  $\chi_{c1}(3872), \chi_{c1}(4140), \chi_{c1}(4274)$
- $0^+(2^{++})$ :  $\chi_{c2}(3930)$
- $1^+(1^{+-})$ :  $T_{c\bar{c}1}(3900), T_{c\bar{c}1}(4200), T_{c\bar{c}1}(4430)$

**Isoscalar channels ( $I = 0$ )**

# Dynamical generation of the poles

## Fully-coupled T-matrices

### Scalar-isoscalar(J=0, I=0) channel



- A bound state below  $D\bar{D}$  threshold

$$\sqrt{s_B} = 3720 \text{ MeV}$$

lower charmonium channels( $\eta_c\eta, J/\psi\eta, \dots$ ) are additionally coupled, leading to resonance

- A resonance between  $D\bar{D}$  and  $D_s\bar{D}_s$

$$\sqrt{s_R} = 3861.34 - i22.76 \text{ MeV}$$

very close to the mass of **X(3860)**.

$$(M_{X(3860)} = 3862_{-35}^{+50} \text{ MeV})$$

but much narrower width than **X(3860)**.

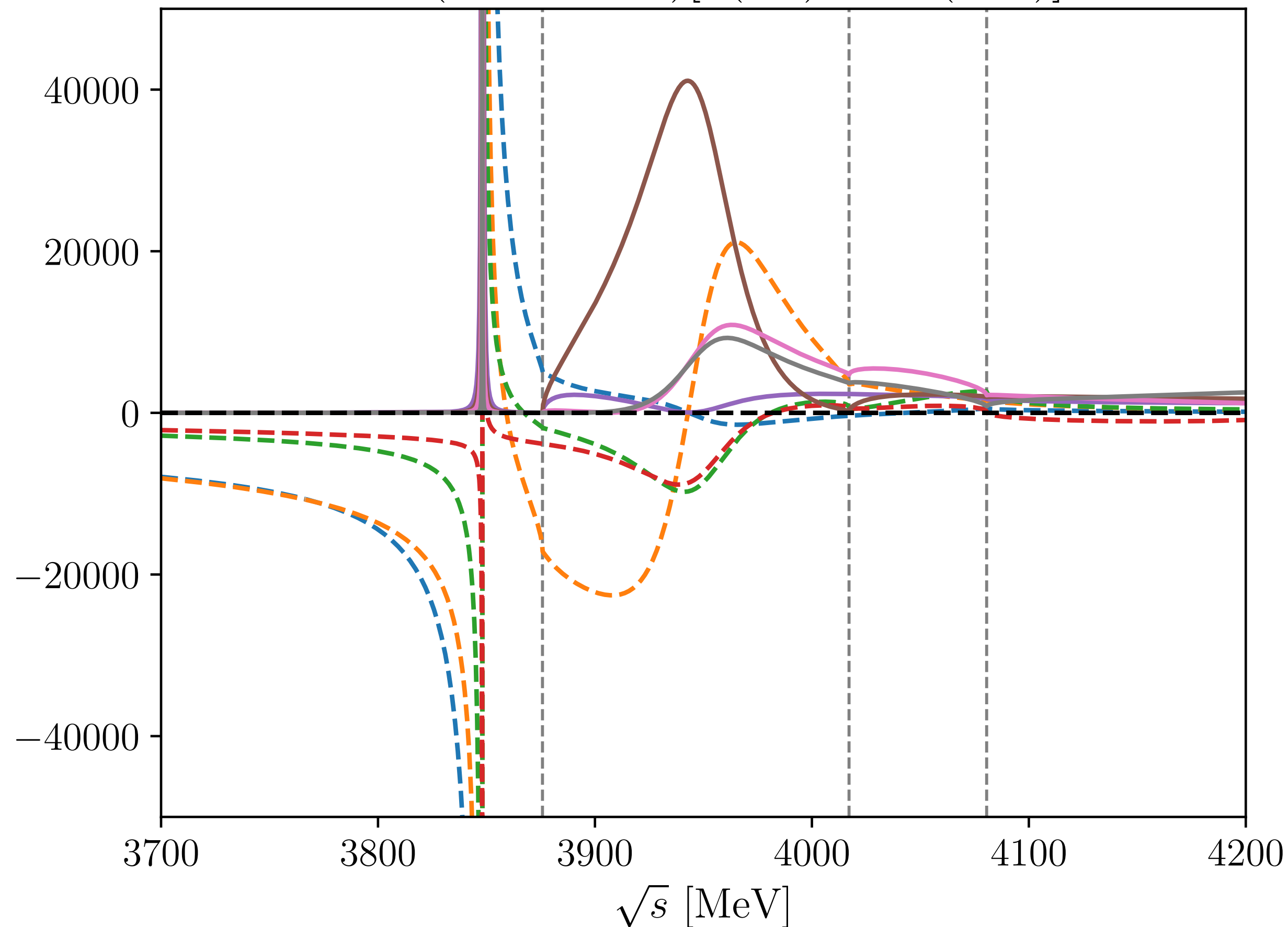
$$(\Gamma_{X(3860)} \simeq 200_{-110}^{+180} \text{ MeV})$$

# Dynamical generation of the poles

## Fully-coupled T-matrices

### Vector-isoscalar(J=1, I=0) channel

$$T_{i \rightarrow i}(JIL = 100)[I(J^P) = 0^+(1^{++})]$$



- Nearly bound state below  $D\bar{D}^*$  threshold

$$\sqrt{s_B} = 3848.11 \text{ MeV}$$

Within our cutoff scheme,  
larger binding energy than  $\chi_{c1}(3872)$

$$M_{\chi_{c1}} = 3871.84 \text{ MeV}$$

- A resonance between  $D\bar{D}^*$  and  $D^*\bar{D}^*$

$$\sqrt{s_R} = 3948.622 - i26.98 \text{ MeV}$$

nice candidate for  $X(3940)$ :

$$m_{X(3940)} = 3942 \pm 9 \text{ MeV}$$

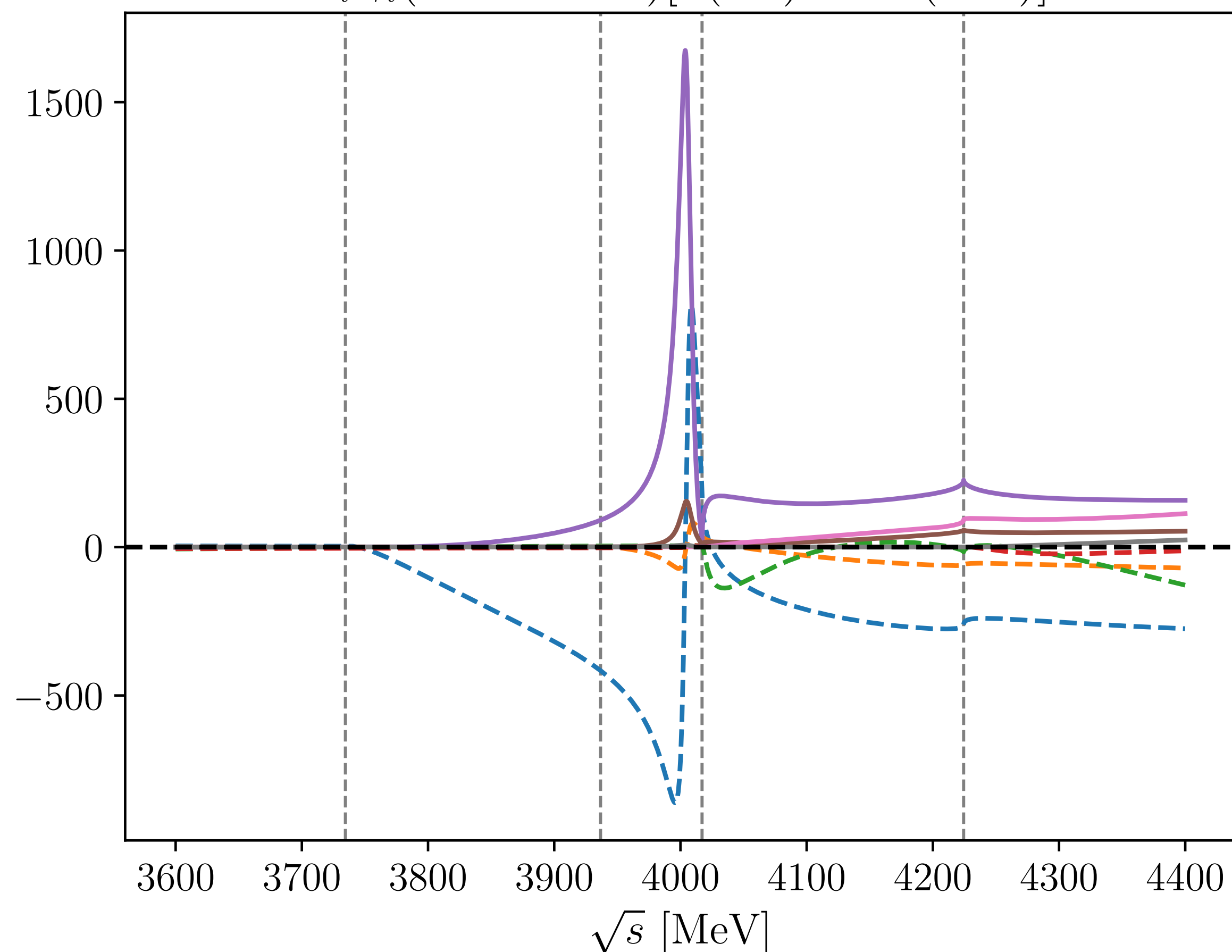
$$\Gamma_{X(3940)} = 37^{+27}_{-17} \text{ MeV}$$

# Dynamical generation of the poles

## Fully-coupled T-matrices

### Tensor-isoscalar(J=2, I=0) channel

$$T_{i \rightarrow i}(JIL = 202)[I(J^P) = 0^+(2^{++})]$$



- A very narrow resonance near  $D^* \bar{D}^*$  threshold

$$\sqrt{s_R} = 4005.26 - i5.95 \text{ MeV}$$

positioned near  $D^* \bar{D}^*$  threshold but dominantly coupled to the  $D \bar{D}$  like  $\chi_{c2}(3930)$ .

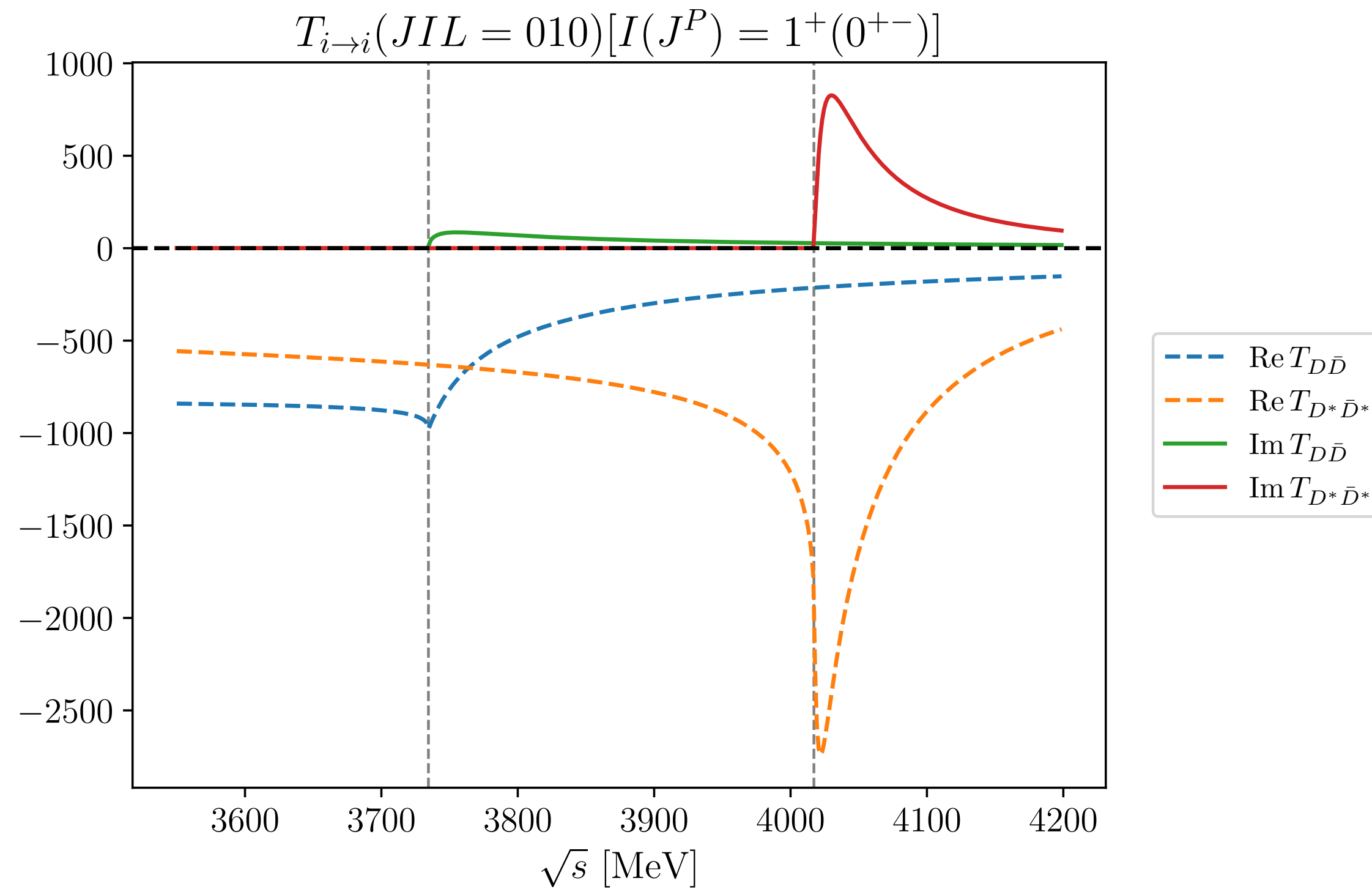
about 80 MeV heavier than  $\chi_{c2}(3930)$ ...

**Isovector channels ( $I = 1$ )**

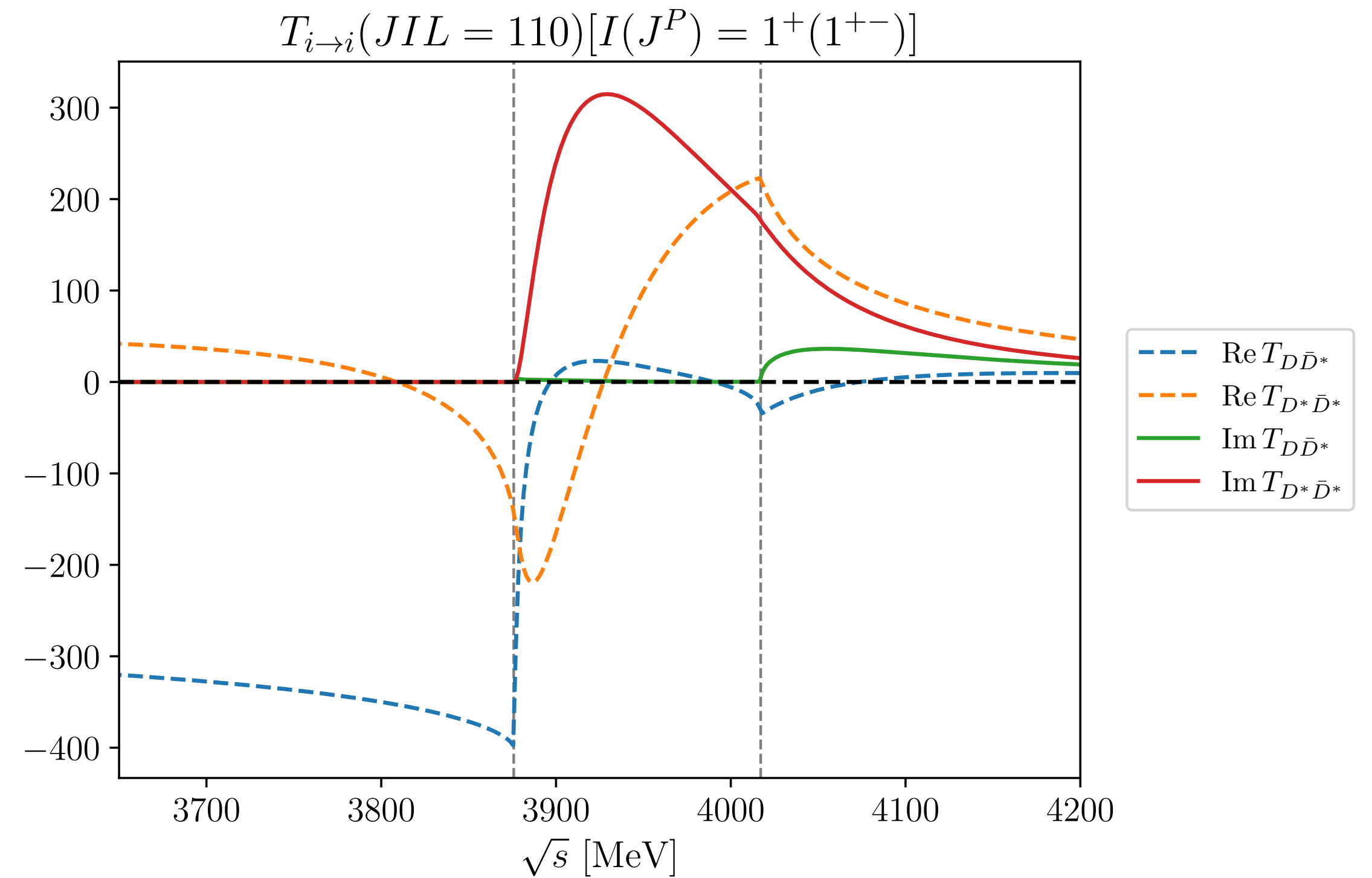


# Dynamical generation of the poles

## Fully-coupled T-matrices



- No resonant shape in scalar channels
- A pronounced cusp at the  $D\bar{D}^*$  mass threshold



- A peak appears between two mass thresholds
- This structure is found to be a virtual state

# Summary

# Summary

- We investigated the production of exotic mesons containing two heavy quarks via coupled-channel dynamics within meson-exchange framework.
- The tree-level interactions between heavy mesons and light-unflavored mesons are described through the effective Lagrangian approach based on HQEFT.
- In the doubly charm sector, we searched two vector bound states. The isoscalar state may be nice candidate for observed doubly charmed meson by LHCb:  $T_{cc}^+(3875)$ ,  $T_{cc}^{I=1}$ .
- Our investigation in the hidden-charm sector revealed three states: a new scalar bound state below the  $D\bar{D}$  threshold, along with scalar, vector and tensor resonances as  $D^*\bar{D}^*$ ,  $D\bar{D}^*$  and  $D\bar{D}$  molecular states, respectively:  $X(3860)$ ,  $\chi_{c1}(3872)$ ,  $X(3940)$ ,  $\chi_{c2}(3930)$ .
- A virtual state with quantum numbers  $I(J^P) = 1(1^+)$  between  $D\bar{D}^*$  and  $D^*\bar{D}^*$ :  $T_{c\bar{c}1}(3900)$
- This study enhances the understanding of production mechanism of exotic mesons as hadron molecule.

**Thank you**