Mixing mechanism for the $J^P = 0^+$ mesons in the light quark system

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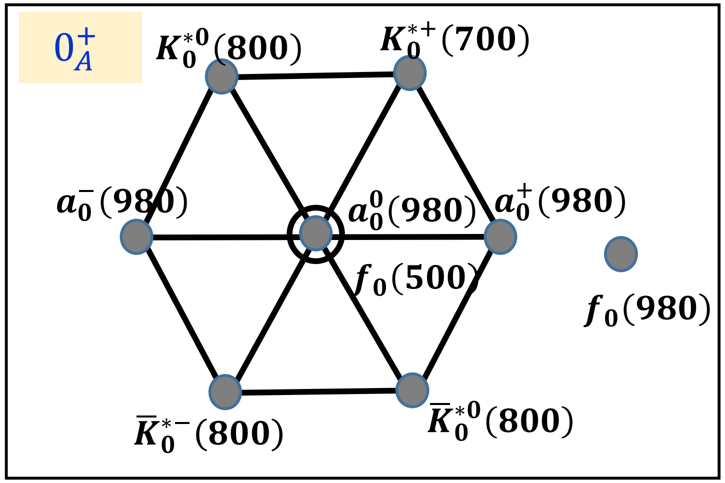
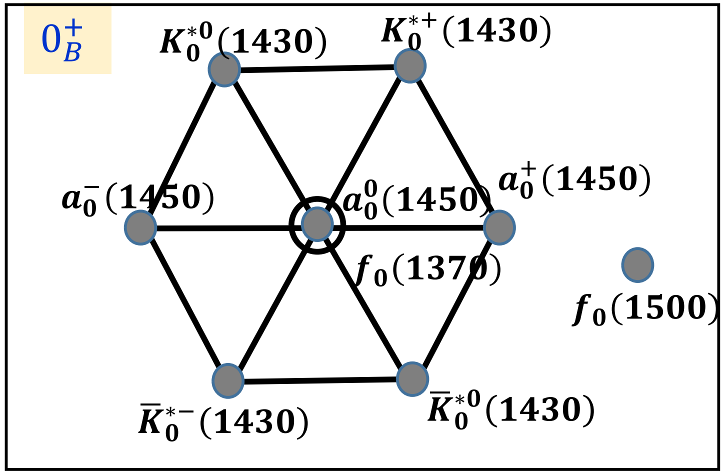
Overview

- Among high-mass resonances in $J^P = 0^+$, there appear to be three flavor nonets in PDG,

$$0_A^+: \ a_0(980), K_0^*(700), f_0(500), f_0(980)$$
$$0_B^+: \ a_0(1450), K_0^*(1430), f_0(1370), f_0(1500)$$
$$0_C^+: \ a_0(1710), K_0^*(1950), f_0(1710), f_0(1770)$$

- Currently, the marginal ordering in 0_B^+, $M[a_0(1450)] \gtrsim M[K_0^*(1430)]$, is difficult to understand.
- In this work, we propose a mixing mechanism involving two-quark states and tetraquarks to account for this mass ordering.



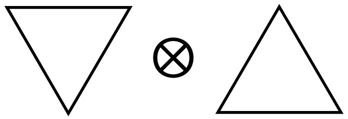


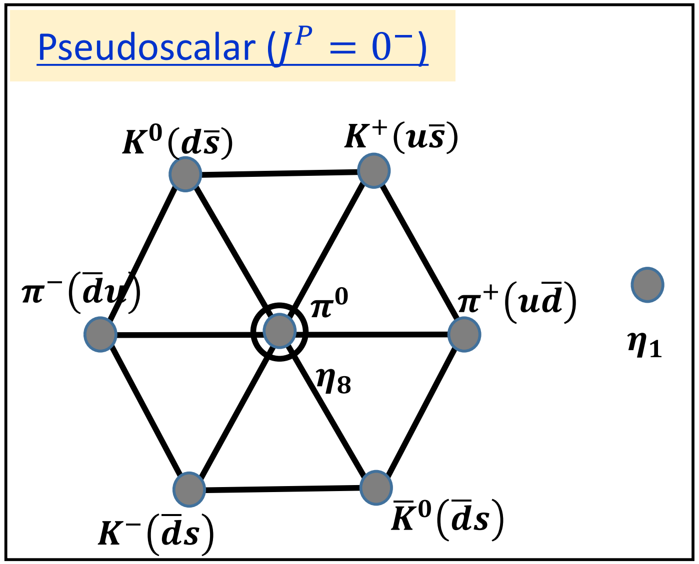
Outline

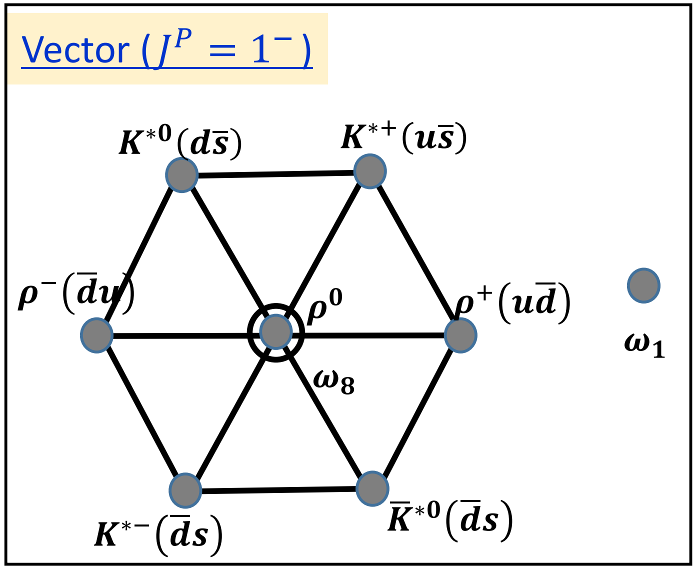
- Lowest-Lying mesons
- High-Mass resonances
- Two-Quark $[q\bar{q}(\ell = 1)]$ Description of High-Mass Resonances
- Tetraquark Description
  Tetraquark Structure for the 0^+_A and 0^+_B Nonets
- Mass Ordering of the 0^+_B Nonet
- Mixing Mechanism
  and its formulation
- Results

0^+_A: $a_0(980), K^*_0(700), f_0(500), f_0(980)$
0^+_B: $a_0(1450), K^*_0(1430), f_0(1370), f_0(1500)$
0^+_C: $a_0(1710), K^*_0(1950), f_0(1710), f_0(1770)$

Lowest-lying mesons

- As well known, the lowest-lying mesons are composed of two-quarks, $q\bar{q}$ $(q = u, d, s)$.
- Because of this, the lowest-lying mesons have
  - spin and parity as $J^P = 0^-, 1^-$.
  - They form a flavor nonet, $q\bar{q} \in 9_f$ $(3_f \otimes \bar{3}_f = 1_f \oplus 8_f)$ in each spin channel
  - Their isospins:  $\Rightarrow I = 0, \frac{1}{2}, 1$





- Another characteristics, particularly relevant in this talk, is the mass ordering :

$$M[(q\bar{q})_{I=1}] < M[(q\bar{q})_{I=1/2}]$$

$$m_\pi(u\bar{d}) \; < \; m_K(u\bar{s}), \quad m_\rho(u\bar{d}) \; < \; m_{K^*}(u\bar{s})$$

※ For isoscalar ($I = 0$) members, the mass ordering is obscured by ideal mixing and the anomaly.

High-mass resonances

up to spin-2

※Note that the PDG is continuously updated. $a_0(1710)$ and $f_0(1770)$ are recently included in the PDG.

$J^P = 0^+$

	I	Meson	Mass(MeV)	Γ(MeV)
	0	$f_0(500)$	400–800	100–800
	0	$f_0(980)$	990	10–100
	1	$a_0(980)$	980	50–100
	0	$f_0(1370)$	1200–1500	200–500
	1	$a_0(1450)$	1439	258
	0	$f_0(1500)$	1522	108
0^{++}	0	$f_0(1710)$	1733	150
	1	$a_0(1710)$	1713	107
	0	$f_0(1770)$	1784	161
	1	$a_0(1950)$	1931	271
	0	$f_0(2020)$	1982	436
	0	$f_0(2100)$	2095	287
	0	$f_0(2200)$	2187	210
	0	$f_0(2330)$	2314(?)	?
	1/2	$K_0^*(700)$	845	468
0^+	1/2	$K_0^*(1430)$	1425	270
	1/2	$K_0^*(1950)$	1957	170

$J^P = 2^+$

	I	Meson	Mass (MeV)	Γ (MeV)
	0	$f_2(1270)$	1275.4	187
	1	$a_2(1320)$	1318.2	107
	0	$f_2(1430)$	1430	?
	0	$f_2(1525)$	1517.3	72
	0	$f_2(1565)$	1571	132
	0	$f_2(1640)$	1639	100
2^{++}	1	$a_2(1700)$	1706	380
	0	$f_2(1810)$	1815	197
	0	$f_2(1910)$	1900	?
	0	$f_2(1950)$	1936	464
	0	$f_2(2010)$	2011	202
	0	$f_2(2150)$	2157	152
	0	$f_2(2300)$	2297	149
	0	$f_2(2340)$	2346	331
2^+	1/2	$K_2^*(1430)$	1427.3	100
	1/2	$K_2^*(1980)$	1990	348

$J^P = 1^+$

	I	Meson	Mass (MeV)	Γ (MeV)
	0	$h_1(1170)$	1166	375
1^{+-}	1	$b_1(1235)$	1229.5	142
	1	$h_1(1415)$	1409	78
	0	$h_1(1595)$	1594	384
	1	$a_1(1260)$	1230	250-600
	0	$f_1(1285)$	1281.8	23
1^{++}	0	$f_1(1420)$	1428.4	56.7
	0	$f_1(1510)$	1518	73
	1	$a_1(1640)$	1655	250
	1/2	$K_1(1270)$	1253	90
1^+	1/2	$K_1(1400)$	1403	174
	1/2	$K_1(1650)$	1650	150

- The high-mass states have spin, parity, charge conjugation, $J^{PC} = 0^{++}, 1^{++}, 1^{+-}, 2^{++}$.
- Isospins are restricted to $I = 0, \frac{1}{2}, 1$.

What could be the structure of the high-mass states?

$q\bar{q}(\ell = 1)$ description

One way to view is that the high-mass resonances are orbital excitations of the lowest-lying nonets, $[q\bar{q}(\ell = 1)]$.

- This description can reproduce the quantum numbers of the high-mass resonances.

$$[q\bar{q}\ (S = 0)] \otimes (\ell = 1) \Rightarrow J^{PC} = 1^{+-}$$
$$[q\bar{q}\ (S = 1)] \otimes (\ell = 1) \Rightarrow J^{PC} = 0^{++}, 1^{++}, 2^{++}$$

$C = (-)^{\ell+S}$
for charge neutral members

- Moreover, the $q\bar{q}(\ell = 1)$ description forms a flavor nonet, which explains the isospin states ($I = 0, \frac{1}{2}, 1$) of the high-mass resonances.

- There seems more than one nonet in each quantum number (radial excitations $nJ^{PC}, n = 1,2,\cdots$.?)

- The mass ordering, $M[(q\bar{q})_{I=1}] < M[(q\bar{q})_{I=1/2}]$, is still expected to be valid for the $q\bar{q}(\ell = 1)$ structure; however, it does not appear to hold universally.

- For $J^P = 1^+, 2^+$, the mass ordering, $M[(q\bar{q})_{I=1}] < M[(q\bar{q})_{I=1/2}]$, is satisfied to some extent.

	$I=1$		$I=1/2$
$J^P = 1^{+-}$: $b_1(1235), K_1(1270), h_1(1170), h_1(1415)$	$M[b_1(1235)]$	$\lesssim$	$M[K_1(1270)]$
1^{++}: $a_1(1260), K_1(1400), f_1(1285), f_1(1420)$.	$M[a_1(1260)]$	$<$	$M[K_1(1400)]$
2_A^{++} : $a_2(1320), K_2^*(1430), f_2(1270), f_2(1430)$	$M[a_2(1320)]$	$<$	$M[K_2^*(1430)]$
2_B^{++} : $a_2(1700), K_2^*(1980), f_2(1640), f_2(1810)$	$M[a_2(1700)]$	$<$	$M[K_2^*(1980)]$

J^{PC}	I	Meson	Mass (MeV)	Γ (MeV)
1^{+-}	0	$h_1(1170)$	1166	375
	1	$b_1(1235)$	1229.5	142
	1	$h_1(1415)$	1409	78
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J^{PC}	I	Meson	Mass (MeV)	Γ (MeV)
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	1	$a_2(1320)$	1318.2	107
	0	$f_2(1430)$	1430	?
	0	$f_2(1525)$	1517.3	72
	0	$f_2(1565)$	1571	132
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	0	$f_2(2300)$	2297	149
	0	$f_2(2340)$	2346	331
2^+	1/2	$K_2^*(1430)$	1427.3	100
	1/2	$K_2^*(1980)$	1990	348

- For $J^P = 0^+$, the following nonets can be identified, but the expected mass ordering, $M[(q\bar{q})_{I=1}] < M[(q\bar{q})_{I=1/2}]$ is not satisfied for the first two nonets.

$J^P = 0^+$

J^{PC}	I	Meson	Mass(MeV)	Γ(MeV)
0^{++}	0	$f_0(500)$	400–800	100–800
	0	$f_0(980)$	990	10–100
	1	$a_0(980)$	980	50–100
	0	$f_0(1370)$	1200–1500	200–500
	1	$a_0(1450)$	1439	258
	0	$f_0(1500)$	1522	108
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	0	$f_0(2330)$	2314(?)	?
0^+	1/2	$K_0^*(700)$	845	468
	1/2	$K_0^*(1430)$	1425	270
	1/2	$K_0^*(1950)$	1957	170

0_A^+: $a_0(980), K_0^*(700), f_0(500), f_0(980)$
0_B^+: $a_0(1450), K_0^*(1430), f_0(1370), f_0(1500)$
0_C^+: $a_0(1710), K_0^*(1950), f_0(1710), f_0(1770)$

	$I = 1$	$I = 1/2$	
0_A^+:	$M[a_0(980)]$	$> K_0^*[(700)]$	(inverted!)
0_B^+:	$M[a_0(1450)]$	$\gtrsim K_0^*[(1430)]$	(inverted!)
0_C^+:	$M[a_0(1710)]$	$< M[K_0^*(1950)]$	

The **inverted mass ordering** can be understood if they are tetraquarks. [Jaffe(1977)]

Tetraquark description

- Tetraquark nonet ($qq\bar{q}\bar{q}$) can be constructed by combining diquark ($\bar{3}_f$)-antidiquark (3_f).

$$\bar{3}_f \otimes 3_f = 1_f \oplus 8_f$$

qq $\bar{q}\bar{q}$

$$1_f = \frac{1}{\sqrt{3}} T^m \bar{T}_m \qquad T^i = \frac{1}{\sqrt{2}} \epsilon^{ijk} q^j q^k \equiv [q^j q^k]$$

$$[8_f]^i_j = T^i \bar{T}_j - \frac{1}{3}\delta^i_j\, T^m \bar{T}_m \qquad \bar{T}_i = \frac{1}{\sqrt{2}} \epsilon_{ijk} \bar{q}_j \bar{q}_k \equiv [\bar{q}_j \bar{q}_k]$$

$$1_f = \frac{1}{\sqrt{3}} \{[ds][\bar{d}\bar{s}] + [su][\bar{s}\bar{u}] + [ud][\bar{u}\bar{d}]\}$$

$(I = 1/2)$ $\boxed{[8_f]^3_1 = [ud][\bar{d}\bar{s}]}$

$$[8_f]^3_2 = [ud][\bar{s}\bar{u}]$$

$(I = 1)$ $\boxed{[8_f]^2_1 = [su][\bar{d}\bar{s}]}$

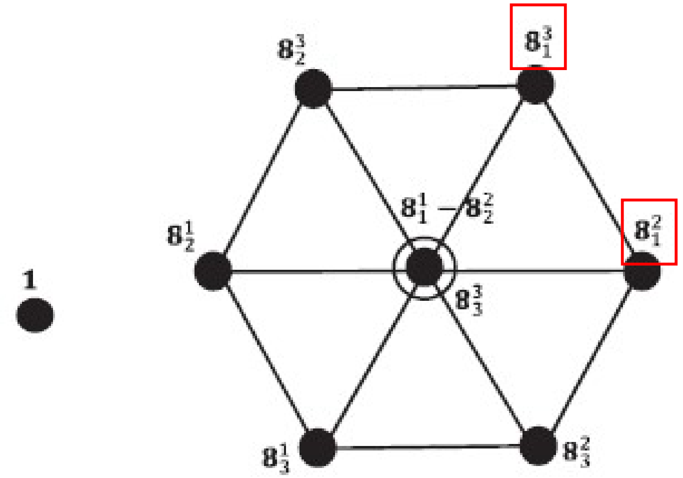
$$\frac{1}{\sqrt{2}}\{[8_f]^1_1 - [8_f]^2_2\} = \frac{1}{\sqrt{2}} \{[ds][\bar{d}\bar{s}] - [su][\bar{s}\bar{u}]\}$$

$$[8_f]^1_2 = [ds][\bar{s}\bar{u}]$$

$$\sqrt{\frac{3}{2}}[8_f]^3_3 = \frac{1}{\sqrt{6}} \{2[ud][\bar{u}\bar{d}] - [ds][\bar{d}\bar{s}] - [su][\bar{s}\bar{u}]\}$$

$$[8_f]^2_3 = [su][\bar{u}\bar{d}]$$

$$[8_f]^1_3 = [ds][\bar{u}\bar{d}]$$



Tetraquarks exhibit the "inverted mass ordering", $[8_f]^2_1(su\bar{d}\bar{s}) > [8_f]^3_1(ud\bar{d}\bar{s})$.
$a_0(980) > K_0^*(700)$, $a_0(1450) \gtrsim K_0^*(1430)$ (marginal)

Tetraquark structure for the 0_A^+ and 0_B^+ nonets

- $\bar{3}_f$ diquark has two types that differ by spin-color.
- Spin-0 $qq \Rightarrow |\mathrm{Type1}\rangle$, Spin-1 $qq \Rightarrow |\mathrm{Type2}\rangle$

⟨qq structure⟩

Flavor	Spin	Color
$\bar{3}_f$	0	$\bar{3}_c$
$\bar{3}_f$	1	6_c

$|\mathrm{Type1}\rangle$: $\boxed{[qq \in (J=0, \bar{3}_c, \bar{3}_f)]} \otimes [\bar{q}\bar{q} \in (J=0, 3_c, 3_f)] \Longrightarrow qq\bar{q}\bar{q} \in (J=0, 1_c, 9_f)$

Well-known tetraquark type, originally proposed for the 0_A^+ nonet (Jaffe).

$|\mathrm{Type2}\rangle$: $\boxed{[qq \in (J=1, 6_c, \bar{3}_f)]} \otimes [\bar{q}\bar{q} \in (J=1, \bar{6}_c, 3_f)] \Longrightarrow qq\bar{q}\bar{q} \in (J=0, 1_c, 9_f)$

Another type proposed in 2017 [EPJC 77, 3 (2017)].

$$\begin{aligned} & \qquad\quad\ \underline{\text{spin}} \quad\ \ \underline{\text{color}} \qquad\quad \underline{\text{flavor}} \\ |\mathrm{Type1}\rangle &= |000\rangle \otimes |1_c \bar{3}_c 3_c\rangle \otimes |9_f \bar{3}_f 3_f\rangle \\ |\mathrm{Type2}\rangle &= |011\rangle \otimes |1_c 6_c \bar{6}_c\rangle \otimes |9_f \bar{3}_f 3_f\rangle \end{aligned}$$

$|\mathrm{Type1}\rangle, |\mathrm{Type2}\rangle$ mix through V_{CS} !,
$\langle \mathrm{Type2} | V_{CS} | \mathrm{Type1} \rangle \neq 0$
$\Rightarrow$None of them alone can describe physical states.

$$V_{CS} \propto -\sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i\, m_j}$$

- Instead, the mixture of $|\text{Type1}\rangle$ and $|\text{Type2}\rangle$, which diagonalizes V_{CS}, can represent the tetraquark structure of the two nonets, 0_A^+ and 0_B^+.

$$\boxed{\begin{aligned} |0_B^+\rangle_4 &= -\alpha|\text{Type1}\rangle + \beta|\text{Type2}\rangle \\ |0_A^+\rangle_4 &= \quad \beta|\text{Type1}\rangle + \alpha|\text{Type2}\rangle \end{aligned}}$$

$\Rightarrow 0_B^+$: $a_0(1450), K_0^*(1430), f_0(1370), f_0(1500)$

$\Rightarrow 0_A^+$: $a_0(980), K_0^*(800), f_0(500), f_0(980)$

The mixing parameters, α, β, are determined also by the diagonalization,
($\alpha \approx 0.815, \beta \approx 0.58$).

We call this the **tetraquark mixing model** for the two nonets.

EPJC(2017) 77, 173, PRD(2018) 97, 094005,
PRD(2019) 99, 014005, PRD(2019) 100, 034021,
EPJC(2022) 82, 1113, PRD(2023) 108, 074016.

Lessons

- $|0_A^+\rangle_4$ and $|0_B^+\rangle_4$ describe the tetraquark part of the 0_A^+ and 0_B^+ nonets.
- These tetraquarks are orthogonal to each other.

$$\boxed{{}_4\langle 0_A^+ | 0_B^+ \rangle_4 = 0}$$

Mass ordering of the 0_B^+ nonet

- The tetraquark mixing model resolves the mass ordering in the 0_A^+ nonet: $M[a_0(980)] > M[K_0^*(700)]$ $\Rightarrow$ the 0_A^+ nonet could be tetraquarks.

- However, the marginal mass ordering within the 0_B^+ nonet remains unexplaind !

If the tetraquark picture is valid, we expect $M[a_0(1450)] > M[K_0^*(1430)]$ by approximately 150 MeV ($\approx m_s - m_u$)!
But experimentally, $\boxed{M[a_0(1450)] \gtrsim M[K_0^*(1430)]}$ by only 20 MeV !

- This close mass ordering can be explained if the 0_B^+ nonet includes a $q\bar{q}(\ell = 1)$ component ($|0^+\rangle_2$) in addition to the tetraquark component !

$$\boxed{|0_B^+\rangle = a|0_B^+\rangle_4 + b|0^+\rangle_2}$$

- This is because the $q\bar{q}(\ell = 1)$ component reduces the mass gap from the tetraquark ordering.

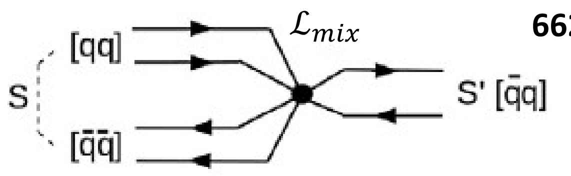
- Is there a relevant framework that generates the $|0^+\rangle_2$ component?

Mixing Mechanism

- Black et al. [PRD(2000) 61, 074001] proposed a mixing mechanism to explain the marginal mass ordering in the 0_B^+ nonet.
- They introduced a mixing Lagrangian, $\mathcal{L}_{mix}$, that connects two nonets, $|0^+\rangle_2$ and $|0^+\rangle_4$,

$|0^+\rangle_2$

$|0^+\rangle_4$



G. 't Hooft et al., PLB **662** (2008), 424-430.

$$\Rightarrow \boxed{\begin{aligned} |\text{nonet1}\rangle &= a\,|0^+\rangle_4 - b\,|0^+\rangle_2 \\ |\text{nonet2}\rangle &= b\,|0^+\rangle_4 + a\,|0^+\rangle_2 \end{aligned}}$$

- This mixing mechanism generates two distinct nonets, nonet1 and nonet2, whose wave functions are linear combinations of the tetraquark ($|0^+\rangle_4$) and two-quark ($|0^+\rangle_2$) nonets.

- There are three nonets in the PDG.

0_A^+: $a_0(980), K_0^*(700), f_0(500), f_0(980)$
0_B^+: $a_0(1450), K_0^*(1430), f_0(1370), f_0(1500)$
0_C^+: $a_0(1710), K_0^*(1950), f_0(1710), f_0(1770)$

Different tetraquark structure

$$\boxed{\begin{aligned} |0_B^+\rangle_4 &= -\alpha|\text{Type1}\rangle + \beta|\text{Type2}\rangle \\ |0_A^+\rangle_4 &= \beta|\text{Type1}\rangle + \alpha|\text{Type2}\rangle \end{aligned}}$$

$$\boxed{\begin{aligned} |\text{nonet1}\rangle &= a\,|0^+\rangle_4 - b\,|0^+\rangle_2 \\ |\text{nonet2}\rangle &= b\,|0^+\rangle_4 + a\,|0^+\rangle_2 \end{aligned}} \Rightarrow \boxed{\begin{aligned} |0_A^+\rangle &= a\,|\text{Type1}\rangle_4 - b\,|0^+\rangle_2 \\ |0_B^+\rangle &= b\,|\text{Type1}\rangle_4 + a\,|0^+\rangle_2 \end{aligned}}$$

- Black et al. mixed $|\text{Type1}\rangle$ and $|0^+\rangle_2$ to generate nonet1 $\Rightarrow 0_A^+$, nonet2 $\Rightarrow 0_B^+$ and explained the mass ordering of the 0_B^+ nonet.
- However, it also changes the 0_A^+ structure, diluting its tetraquark nature.
- Furthermore, this mixing provides the same tetraquark structure ($|\text{Type1}\rangle$) for the 0_A^+ and 0_B^+ nonets, making it inconsistent with the tetraquark mixing model.

0_A^+: $a_0(980), K_0^*(700), f_0(500), f_0(980)$

$\left\{ \begin{array}{l} 0_B^+: a_0(1450), K_0^*(1430), f_0(1370), f_0(1500) \\ 0_C^+: a_0(1710), K_0^*(1950), f_0(1710), f_0(1770) \end{array} \right.$

$$\boxed{\begin{aligned} |\text{nonet1}\rangle &= a\,|0^+\rangle_4 - b\,|0^+\rangle_2 \\ |\text{nonet2}\rangle &= b\,|0^+\rangle_4 + a\,|0^+\rangle_2 \end{aligned}} \Rightarrow \boxed{\begin{aligned} |0_B^+\rangle &= a\,|0_B^+\rangle_4 - b\,|0^+\rangle_2 \\ |0_C^+\rangle &= b\,|0_B^+\rangle_4 + a\,|0^+\rangle_2 \end{aligned}}$$

- Instead, we propose to mix $|0_B^+\rangle_4$ and $|0^+\rangle_2$ to produce nonet1 $\Rightarrow$ 0_B^+, nonet2 $\Rightarrow$ 0_C^+.
- The 0_B^+ and 0_C^+ nonets share the same tetraquark structure, $|0_B^+\rangle_4$. (no problem here, as we have no constraint on the 0_C^+ nonet.)
- The 0_A^+ nonet is not affected by this mixing and therefore remains a pure tetraquark state of $|0_A^+\rangle_4$.
- If this mixing occurs maximally, the other mixing between $|0_A^+\rangle_4$ and $|0^+\rangle_2$ is unlikely to occur. The $|0^+\rangle_2$ component, which couples maximally with $|0_B^+\rangle_4$, is unlikely to couple with the orthogonal state of $|0_A^+\rangle_4$.

Tetraquark mixing model

$$\boxed{\begin{aligned} |0_B^+\rangle_4 &= -\alpha|\text{Type1}\rangle + \beta|\text{Type2}\rangle \\ |0_A^+\rangle_4 &= \quad \beta|\text{Type1}\rangle + \alpha|\text{Type2}\rangle \end{aligned} \quad \Longrightarrow \quad {}_4\langle 0_A^+ | 0_B^+ \rangle_4 = 0}$$

Black et al. PRD(2000) 61, 074001

Formulation of the Mixing Mechanism

- Following Black et al., we represent the $|0^+\rangle_2$ and $|0^+\rangle_4$ nonets as tensors,

$$\boxed{\begin{array}{ll} |0^+\rangle_2: \; N'^b_a = q_a \bar{q}^b \quad (q_a = u, d, s) & \\ |0^+\rangle_4: \; N^b_a = T_a \bar{T}^b & T_a = \frac{1}{\sqrt{2}}\epsilon_{abc} q^b q^c, \\ & \bar{T}^a = \frac{1}{\sqrt{2}}\epsilon_{abc} \bar{q}_b \bar{q}_c \end{array}}$$

and propose a mixing Lagrangian $\boxed{\mathcal{L}_{mix} = -\gamma \mathrm{Tr}[NN'].}$

- This is a simplest invariant term that can be constructed from N'^b_a, N^b_a.

- For the $I = 1$ and $I = 1/2$ members,

$$\boxed{\mathcal{L}_{mix} = -\gamma\phi_{I=1}\phi'_{I=1} - \gamma\phi_{I=1/2}\phi'_{I=1/2}}$$

$\Rightarrow$ The same mathematical procedure can be applied for the $I = 1$ and $I = 1/2$ members.

- Note, for the $I = 1$ and $I = 1/2$ members, the mixing Lagrangian takes the same form whether it is derived from $\mathcal{L}_{mix}$ or its SU(3) symmetric form of $-\gamma\mathrm{Tr}[\mathbf{8}_f \mathbf{8}'_f]$.
- Here we consider the mix $N'^b_a(= |0^+\rangle_2)$ and $N^b_a(=|0^+_B\rangle_4)$ that generates the 0^+_B and 0^+_C nonets, $|0^+_B\rangle_4 = -\alpha|\mathrm{Type1}\rangle + \beta|\mathrm{Type2}\rangle$

Diagonalizing the mass matrix

- To explain this in general, we denote the states as follows:

the pre-mixing states
$|b\rangle$: tetraquark, $|b'\rangle$: two-quark

$\Rightarrow$ After mixing : $|B\rangle, |B'\rangle$

- Mass terms in Lagrangian:

$$-\frac{1}{2}m_b^2\phi_b^2 - \frac{1}{2}m_{b'}^2\phi_{b'}^2 - \gamma\phi_b\phi_{b'} = -\frac{1}{2}(\phi_b, \phi_{b'})M^2\begin{pmatrix}\phi_b \\ \phi_{b'}\end{pmatrix}$$

$$M^2 = \begin{array}{c|cc} & |b\rangle & |b'\rangle \\ \hline |b\rangle & m_b^2 & \gamma \\ |b'\rangle & \gamma & m_{b'}^2 \end{array} \quad \Rightarrow \quad \begin{array}{c|cc} & |B\rangle & |B'\rangle \\ \hline |B\rangle & M_B^2 & 0 \\ |B'\rangle & 0 & M_{B'}^2 \end{array}$$

- The diagonalization process expresses the physical masses, M_B^2, and $M_{B'}^2$ as:

$$M_{B'}^2 = \frac{1}{2}\left[m_{b'}^2 + m_b^2 + \sqrt{(m_{b'}^2 - m_b^2)^2 + 4\gamma^2}\right]$$

$$M_B^2 = \frac{1}{2}\left[m_{b'}^2 + m_b^2 - \sqrt{(m_{b'}^2 - m_b^2)^2 + 4\gamma^2}\right]$$

What we need is m_b^2 and $m_{b'}^2$ for given M_B^2, and $M_{B'}^2$.
By analyzing them, we can access the validity of the mixing mechanism.

- These can be inverted to :

$$m_b^2 = \frac{1}{2}\left[M_{B'}^2 + M_B^2 - \sqrt{(M_{B'}^2 - M_B^2)^2 - 4\gamma^2}\right]$$

$$m_{b'}^2 = \frac{1}{2}\left[M_{B'}^2 + M_B^2 + \sqrt{(M_{B'}^2 - M_B^2)^2 - 4\gamma^2}\right]$$

- For M_B^2, and $M_{B'}^2$, these yield m_b^2 and $m_{b'}^2$ as functions of the mixing parameter, γ^2.
- m_b^2 and $m_{b'}^2$ take real values if $\gamma^2 \le \gamma_{max}^2$.

$$\gamma_{\max}^2 = \frac{(M_{A'}^2 - M_A^2)^2}{4}$$

- The diagonalization also expresses the physical states in terms of the pre-mixing states,

$$\boxed{\begin{aligned}|B\rangle &= C|b\rangle - D|b'\rangle \\ |B'\rangle &= D|b\rangle + C|b'\rangle\end{aligned}}$$

$$\boxed{\begin{aligned}C &= \frac{m_{b'}^2 - M_B^2}{\sqrt{(m_{b'}^2 - M_B^2)^2 + \gamma^2}} \\ D &= \frac{M_{B'}^2 - m_{b'}^2}{\sqrt{(m_{b'}^2 - M_{B'}^2)^2 + \gamma^2}}\end{aligned}}$$

- C, D represent the overlaps of the physical states with the tetraquark and two-quark components.

- When $\gamma^2 = 0$, $C = 1$ and $D = 0$. i.e., $|B\rangle = |b\rangle$ and $|B'\rangle = |b'\rangle$
- When $\gamma^2 = \gamma_{max}^2$, $\Rightarrow$ $m_b^2 = m_{b'}^2 = \frac{1}{2}[M_B^2 + M_{B'}^2]$ $\quad C = D = \frac{1}{\sqrt{2}}$
- For $0 \le \gamma^2 \le \gamma_{max}^2$, we have $0 \le D \le \frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}} \le C \le 1$.

$$\Longrightarrow D \le C$$

- As we mentioned, this mixing mechanism can be applied to the $I = 1$ and $I = 1/2$ members separately.

For $I = 1$: $|B\rangle \Rightarrow a_0(1450)$ in the 0_B^+ nonet
$|B'\rangle \Rightarrow a_0(1710)$ in the 0_C^+ nonet

For $I = 1/2$: $|B\rangle \Rightarrow K_0^*(1430)$ in the 0_B^+ nonet
$|B'\rangle \Rightarrow K_0^*(1950)$ in the 0_C^+ nonet

- This mixing mechanism yields pre-mixing masses, m_b^2 and $m_{b'}^2$, in the $I = 1$ and $I = 1/2$ channels.
- Since these are the masses of pure tetraquarks or pure two-quark states, their mass ordering must be satisfied:

Tetraquarks: $m_b(I = 1) > m_b(I = 1/2)$
Two-quarks: $m_{b'}(I = 1) < m_{b'}(I = 1/2)$

- If these orderings are satisfied with a reasonable gap, it supports the validity of our mixing mechanism.

Results

$$m_b^2 = \frac{1}{2}\left[M_{B'}^2 + M_B^2 - \sqrt{(M_{B'}^2 - M_B^2)^2 - 4\gamma^2}\right]$$
$$m_{b'}^2 = \frac{1}{2}\left[M_{B'}^2 + M_B^2 + \sqrt{(M_{B'}^2 - M_B^2)^2 - 4\gamma^2}\right]$$

Pre-mixing masses

We calculate for $I = 1,\ M_{B'} = 1.713$ GeV, $M_B = 1.439$ GeV and for $I = 1/2,\ M_{B'} = 1.957$ GeV, $M_B = 1.425$ GeV.

$0 \leq \gamma^2 \leq 0.186$ GeV²

γ^2	$I=1$			$I=1/2$		
		m_a	$m_{a'}$		m_K	$m_{K'}$
0		1.439	1.713		1.425	1.957
0.05		1.461	1.695		1.435	1.950
0.10		1.486	1.672		1.445	1.942
0.15		1.520	1.641		1.455	1.935
0.17		1.541	1.622		1.460	1.931
$\gamma^2_{max} \Rightarrow$ 0.186		$\boxed{1.582}$	1.582		$\boxed{1.463}$	1.929

As γ increases
$m_a, m_K \uparrow$,
$m_{a'}, m_{K'} \downarrow$.

When $\gamma^2 = \gamma^2_{max}$, we find a reasonable tetraquark mass ordering.

- At $\gamma^2 = 0.186$ GeV², the pre-mixing tetraquark masses are $m_a = 1.582 >$ $m_K = 1.463$ GeV, with $\Delta m = 119$ MeV.
  This gap is consistent with the tetraquark mass ordering.
- Also, $m_{a'} = 1.582 < m_{K'} = 1.929$ GeV, roughly consistent with the two-quark structure.

This results also provide a generating mechanism for the marginal mass ordering in the 0_B^+ nonet.

- For $I = 1$, $m[a_0(1450)] = 1.582$ GeV $\Rightarrow$ $M[a_0(1450)] = 1.439$ GeV, the mass change, 140 MeV, is substantial !

- For $I = 1/2$, $m[K_0^*(1430)] = 1.463$ GeV $\Rightarrow$ $M[K_0^*(1430] = 1.425$ GeV, the mass change, 38 MeV, is very small !

$\Rightarrow$ Such a discriminating mass change leads to the marginal gap, $M[a_0(1450)] - M[K_0^*(1430)] = 20$ MeV.

In fact, this discriminating mass change originates from the two-quark component, which strongly affects the isovector channel but has a weaker influence on the isodoublet channel.
This aspect can be observed from the physical wave functions!

Physical wave functions

$|a_0(1450)\rangle = C_a|0_B^+\rangle_4 - D_a|0^+\rangle_2 \ (I=1)$ $|K_0^*(1430)\rangle = C_K|0_B^+\rangle_4 - D_K|0^+\rangle_2 \ (I=1/2)$

γ^2	$I=1$ C_a	D_a	$I=1/2$ C_K	D_K
0	1	0	1	0
0.05	0.963	0.269	0.992	0.125
0.10	0.917	0.399	0.984	0.179
0.15	0.849	0.528	0.975	0.221
0.17	0.805	0.593	0.972	0.236
$\gamma^2_{max} \Rightarrow$ 0.186	$\boxed{0.707}$	$\boxed{0.707}$	$\boxed{0.969}$	$\boxed{0.248}$

$$C = \frac{m_{b'}^2 - M_B^2}{\sqrt{(m_{b'}^2 - M_B^2)^2 + \gamma^2}}$$

$$D = \frac{M_{B'}^2 - m_{b'}^2}{\sqrt{(m_{b'}^2 - M_{B'}^2)^2 + \gamma^2}}$$

When $\gamma^2 = \gamma^2_{max}$, we find the following:

- For $I = 1$, $C_a = D_a = 0.707$
  $|0_B^+\rangle_4 \implies |a_0(1450)\rangle = 0.71|0_B^+\rangle_4 - 0.71|0^+\rangle_2$
- For $I = 1/2$, $C_K \gg D_K$
  $|0_B^+\rangle_4 \implies |K_0^*(1430)\rangle = 0.97|0_B^+\rangle_4 - 0.25|0^+\rangle_2$

This gives the discriminating mass change in the previous slide !

Summary

- In the PDG, there are three flavor nonets in $J^P = 0^+$.

  0_A^+: $a_0(980), K_0^*(700), f_0(500), f_0(980)$
  0_B^+: $a_0(1450), K_0^*(1430), f_0(1370), f_0(1500)$
  0_C^+: $a_0(1710), K_0^*(1950), f_0(1710), f_0(1770)$

- Currently, the mass ordering between $a_0(1450), K_0^*(1430)$ in the 0_B^+ nonet is difficult to understand.
- This marginal mass ordering might result from the 0_B^+ nonet being a state that includes both two-quark and tetraquark components.
- To investigate this, we developed a mixing mechanism to generate the 0_B^+ and 0_C^+ nonets and calculated the pre-mixing tetraquark states.
- We report that the pre-mixing tetraquark masses for the isovector and isodoublet members are separated by a reasonable mass gap of $\Delta m \approx 119$ MeV.
- The marginal mass ordering in the 0_B^+ nonet appears to arise from the two-quark component, which develops strongly in the isovector channel but weakly in the isodoublet channel.

Thank you for your attention !