

Axial- and vector-meson degeneracy at finite temperature via anisotropic lattice QCD

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Contents based on [arXiv:2412.20922](https://arxiv.org/abs/2412.20922) [hep-lat]



[Introduction]

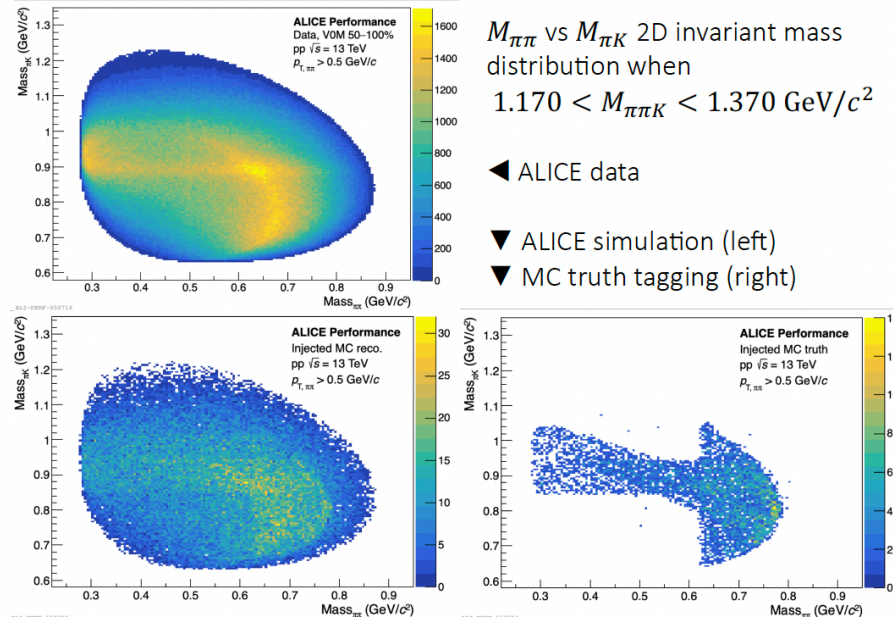
Produced hadrons experience heat bath before kinetic freeze-out from QGP

Hadron properties changed during transporting: Mass, width, couplings, etc.

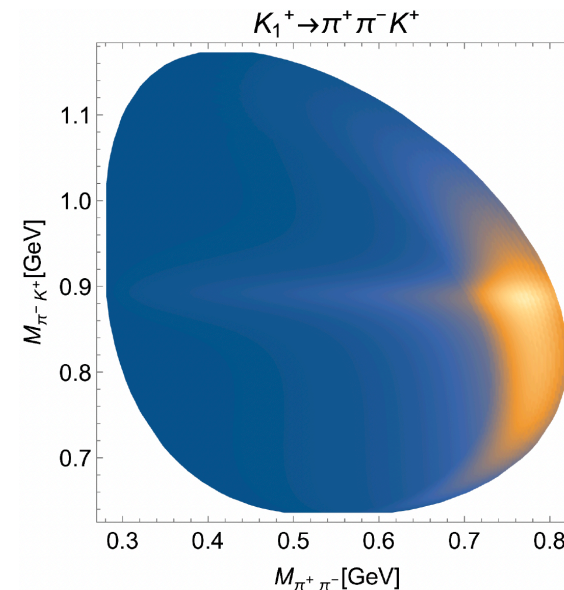
Understood by dynamics of chiral symmetry plus $U_A(1)$.

How can we OBSERVE these novel phenomena in relativistic heavy-ion collision?

2D INVARIANT MASS DISTRIBUTION

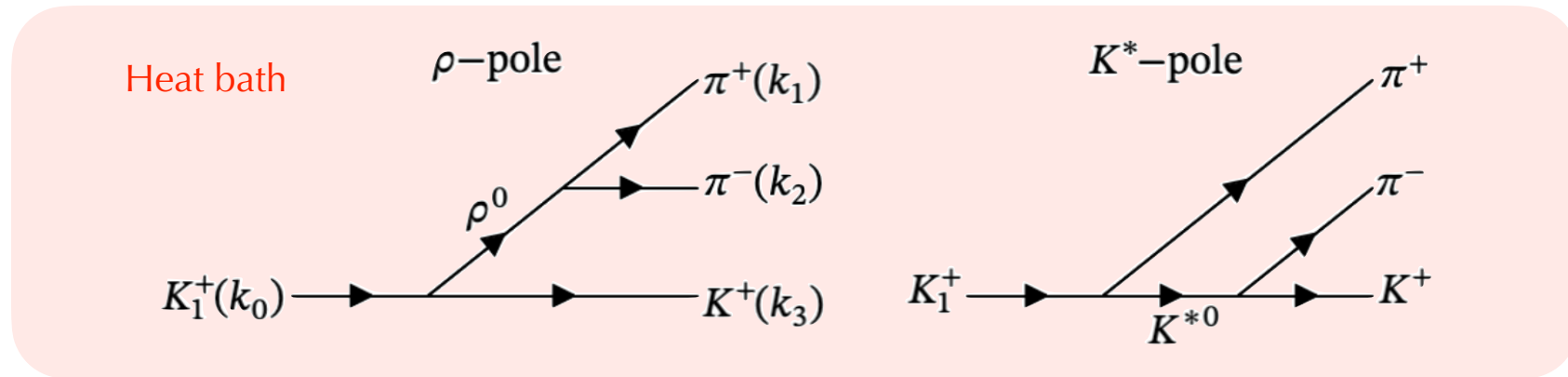


Momentum-dependent mass distribution (theory)



[Motivation]

A-V degeneracy from restoration of SBCS in medium: K_1 and K^*



Considering chemical freezeout $T \sim T_c \sim 160$ MeV, K_1 and K^* degenerated at initial stages.

This means there are NO DECAYS there, but as T gets cooler, decay increases.

Tracing-back this process by counting decays, one reaches upto chemical freezeout $T \sim T_c$.

For this study, one needs T and μ dependent hadron properties via

QCD-like effective models or Lattice QCD

Theoretical approaches

Lattice-QCD (LQCD) simulation is fundamental since it only depends on the 1st principle, i.e., QCD.

As the techniques developed, the computation power became increasingly powerful and reliable.

Basically, in LQCD, we are doing the following:

A. Preparation

Generate a gauge configuration, i.e., preparing a QCD vacuum and constructing a hadron correlator that we are interested in.

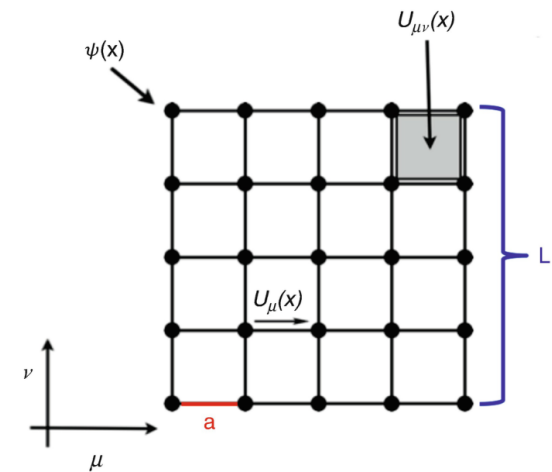
B. Measurement

Perform MCMC for the correlator with source: $\int \langle O \rangle \text{Exp}[-S(q,g)] dV$ and obtain numerics

C. Analyses

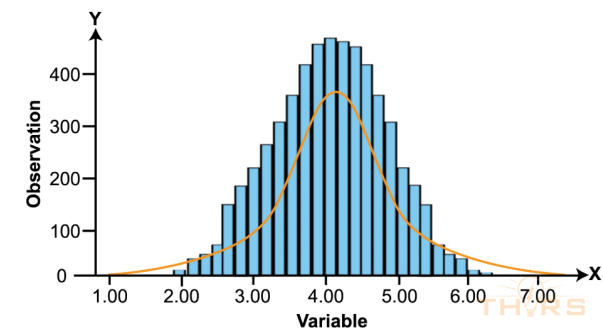
Using obtained data, extract what we want to research.

But, this is not that easy as is said: Step A is time consuming (also money).



$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N \mathcal{O}'(U_i)$$

STATISTICAL DISTRIBUTION



Theoretical approaches

Moreover, there are problems in the medium, such as *sign problems*.

$$\begin{aligned}
 D(\mu_q) &= \not{D} + m + \mu_q \gamma_0 \\
 D^\dagger(\mu_q) &= -\not{D} + m + \mu_q^* \gamma_0 = \gamma_5 D^\dagger(-\mu_q^*) \gamma_5 \\
 \{\det[D(\mu_q)]\}^* &= \det[D^\dagger(\mu_q)] = \det[\gamma_5 D(-\mu_q^*) \gamma_5] = \det[D(-\mu_q^*)]
 \end{aligned}$$

Probability for MCMC becomes complex for finite μ , and for high μ , it oscillates and kills physical observation.

$$\int dU O'(U) (R + iI) e^{-S_G} \sim \int dU O'(U) e^{-S_G + i\phi}$$

To overcome this problem, there proposed many methods:

- A. Canonical method: Performing simulation on imaginary μ then Wick-rotated to real μ : Problematic.
- B. Langevin method: Phase transition is considered as the noise of a thermodynamic system of Boltzman.
- C. Taylor expansion: Expansion of correlator around $\mu=0$ for small μ .

Each has pros and cons depending on resources, techniques, observables, etc.

In this presentation, I would like to show **my first LQCD analyses** for mesonic spectra in the medium.

How it started

As indicated, mesonic properties change in the medium at finite T and μ

To avoid the sign problem, we employ the Taylor expansion method at small μ .

A lattice working group in Korea:

Prof. Seyong Kim (Sejong, Korea), Prof. Yongsun Kim (Sejong, Korea),

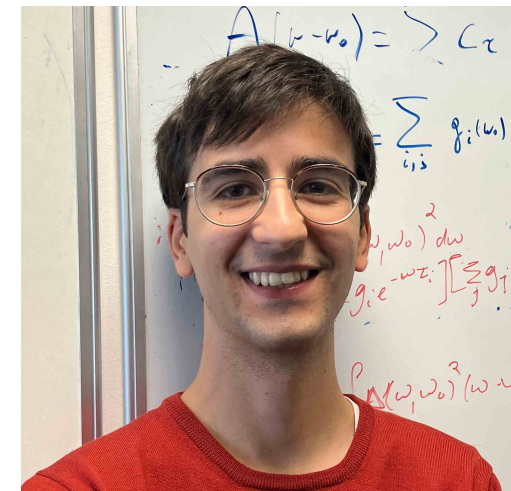
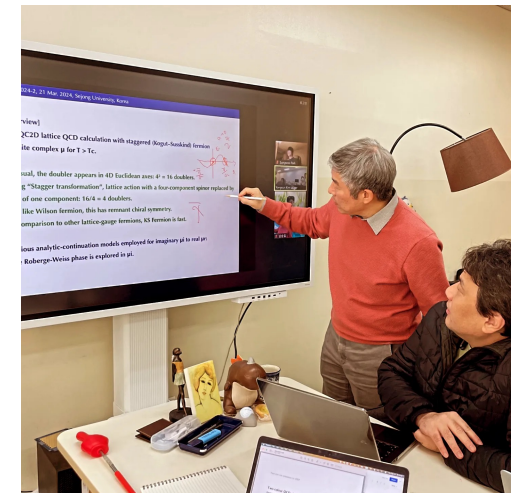
Dr. Jongwan Lee (CTPU/IBS, Korea), Dr. Sungwoo Park (LLNL, US),

and S.i.N. (PKNU).

In this work, we focus on vector- and axial-meson properties, considering the degeneracy of their masses in the medium.

With Prof. Seyong Kim's help, we collaborated with the LQCD group at Swansea Univ., UK, including Dr. Antonio Smecca.

Dr. Aleksandr Nikolaev started LQCD calculations for this subject but could not finish it for personal reasons.



Dr. Antonio Smecca

[Strategy]

a_1 and ρ meson masses studied via LQCD (will be extended to strangeness sector)

To overcome sign problem at finite μ , Taylor expansion method applied $\mu_B < 0.4$ GeV

Anisotropic lattice using FASTSUM ensemble with Wilson-clover fermion.

As a byproduct, QCD phase boundary obtained.

[Theory: Correlation]

Correlator for light mesons: $G_H(x) = \langle J_H(x) J_H^\dagger(0) \rangle$; $J_H \equiv \bar{\psi} \Gamma_H \psi$

Using quark determinant and gauge weight:

$$G(x) = \frac{\langle\langle g(x) \det M \rangle\rangle}{\langle\langle \det M \rangle\rangle} \equiv \langle g(x) \rangle \quad g(x) = \text{tr} [S(x) \Gamma S(-x) \Gamma^\dagger]$$

Taylor expansion w.r.t. small

$$G(x) = G(x) \Big|_{\mu=0} + \frac{\mu}{T} T G'(x) \Big|_{\mu=0} + \frac{1}{2} \left(\frac{\mu}{T} \right)^2 T^2 G''(x) \Big|_{\mu=0} + \mathcal{O} \left(\frac{\mu^4}{T^4} \right)$$

[Theory: Lattice setup]

Wilson-clover fermions (a-behavior improved)

Lattice size: $N_s=24$ and $N_t=16$

Anisotropy: $a_s/a_t=3.5$, $1/a_t=5.63$ GeV

Pion mass=384 MeV

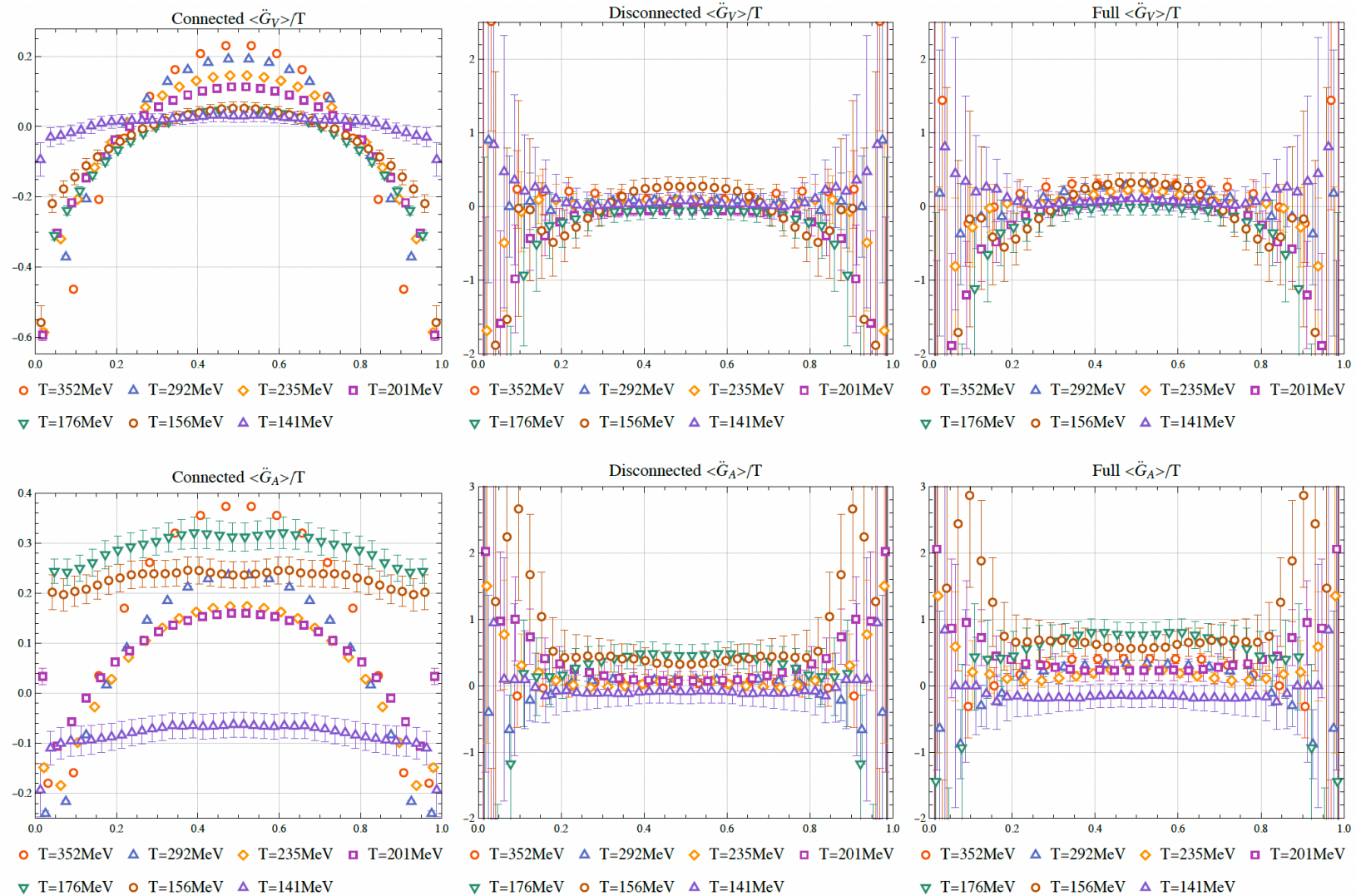
	Gen2	Gen2L
M_π [MeV]	384(4)	236(2)
T_{pc} [MeV]	181(1)	167(3)
a_τ [fm]	0.0350(2)	0.0330(2)
a_s [fm]	0.1205(8)	0.1136(6)

Gauge configuration: FASTSUM ensembles at Swansea Univ., UK.

N_τ	128*	40	36	32	28	24	20	16
T [MeV]	44	141	156	176	201	235	281	352
T/T_c	0.24	0.76	0.84	0.95	1.09	1.27	1.52	1.90
N_{cfg}	139	501	501	1000	1001	1001	1000	1001

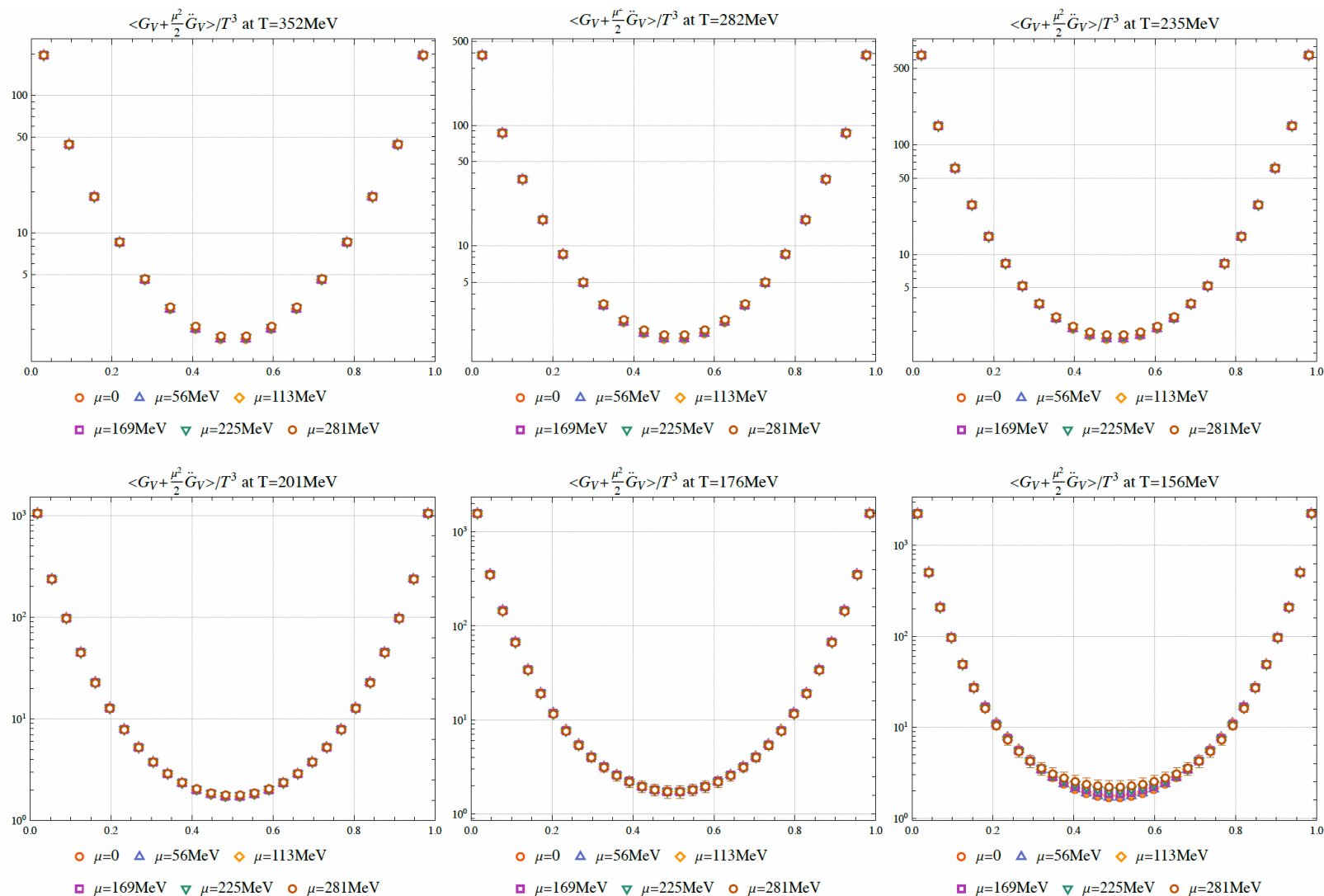
[Numerical results: LQCD data analyses]

2nd-derivatives of connected and disconnected correlators for axial and Vector mesons
as functions of Euclidean time



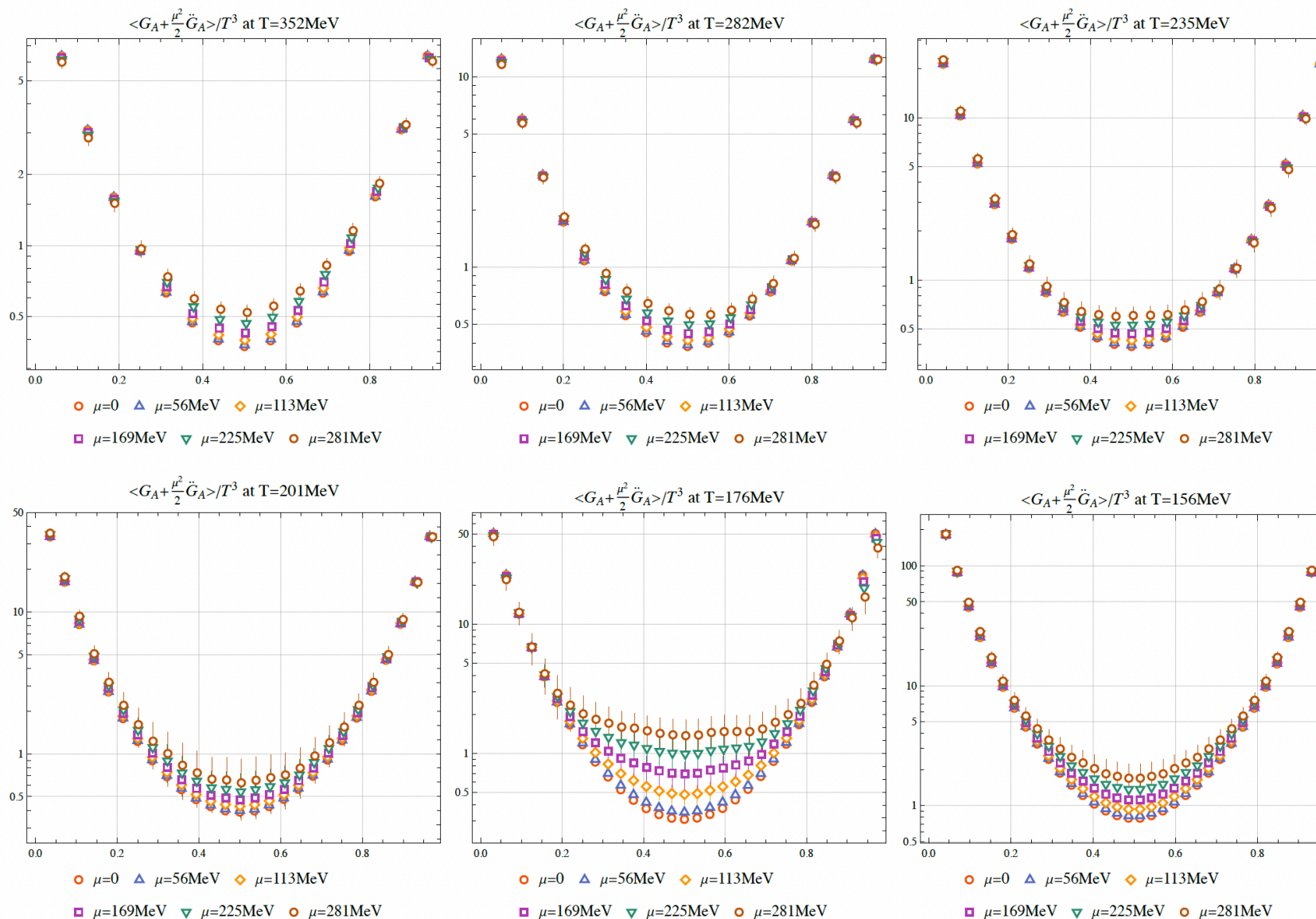
[Numerical results: LQCD data analyses]

Full correlators for **vector** mesons as functions of Euclidean time for T and μ

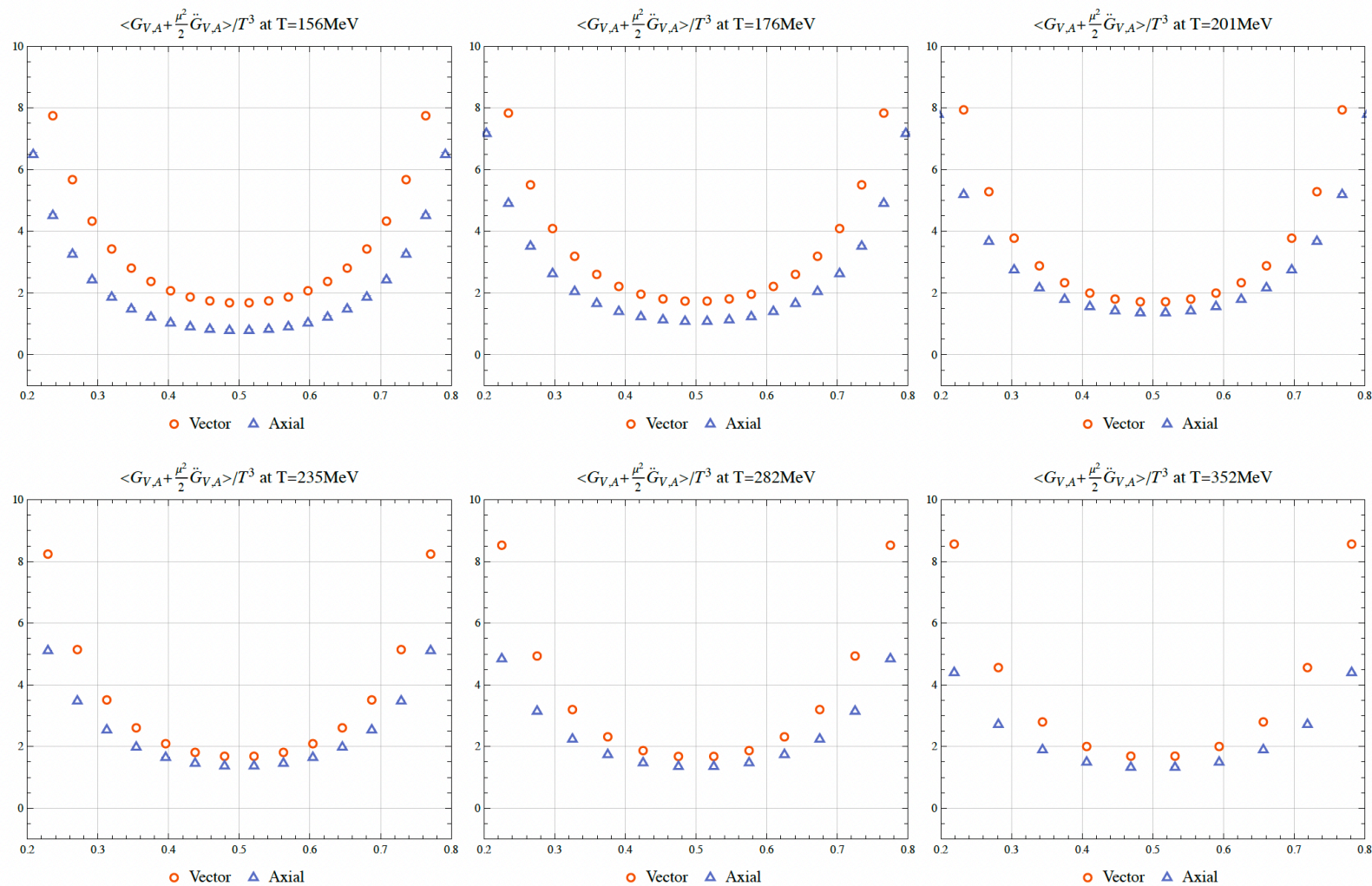


[Numerical results: LQCD data analyses]

Full correlators for **axial** mesons as functions of Euclidean time for T and μ



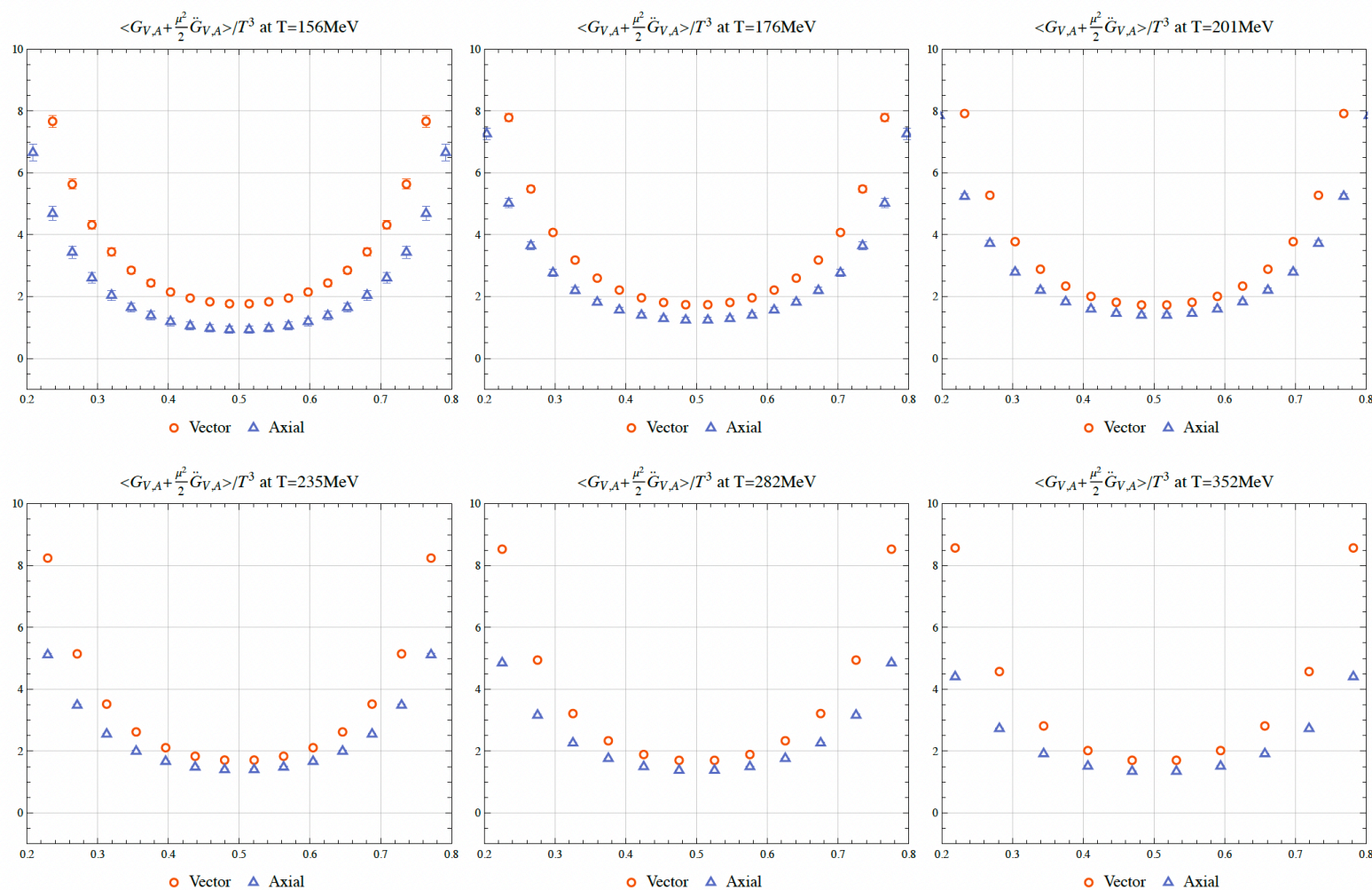
[Numerical results: LQCD data analyses]

Full correlators for **axial** and **vector** mesons as functions of Euclidean time for T and $\mu=0$ 

Indication of partial restoration of SQCS

[Numerical results: LQCD data analyses]

The same for **axial** and **vector** mesons as functions of Euclidean time for T and $\mu=130$ MeV

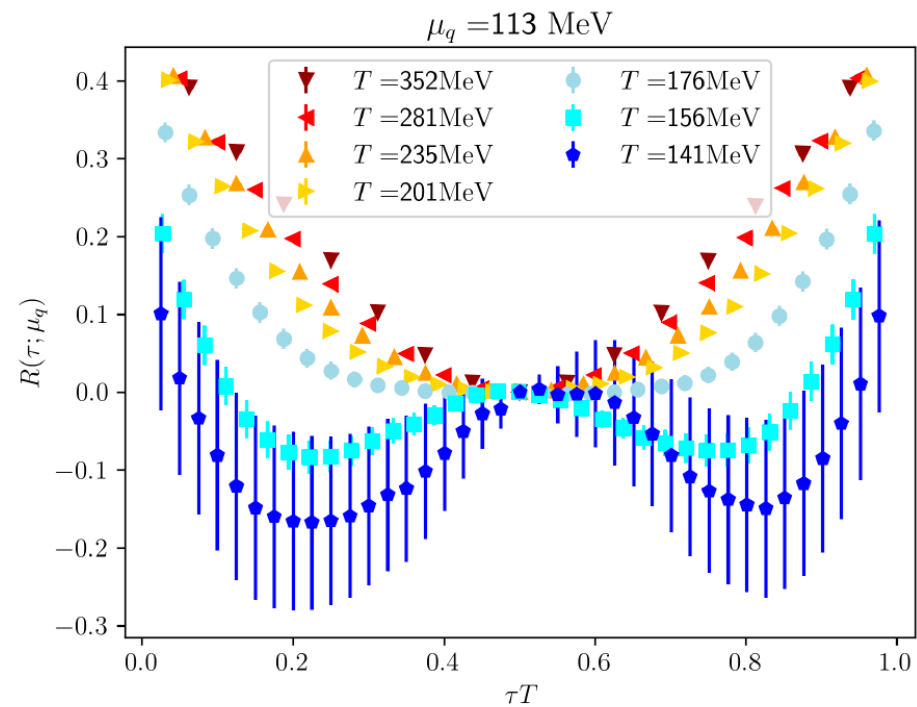
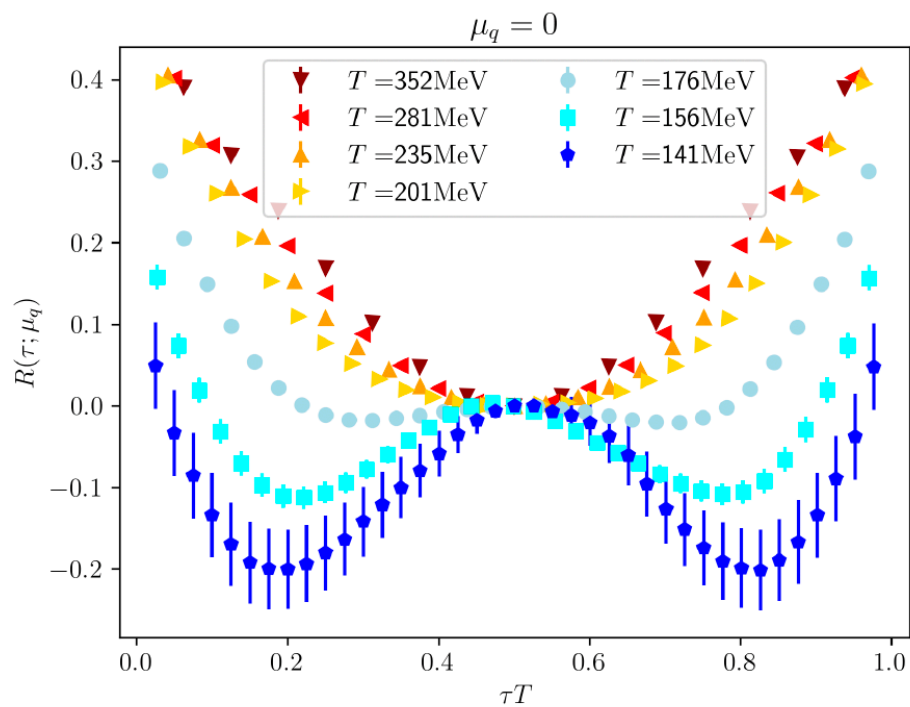


Indication of partial restoration of S/QCS

[Numerical results: LQCD data analyses]

A ratio is defined for phase line:

$$R(\tau; \mu_q) = \frac{\tilde{G}_V(\tau; \mu_q) - \tilde{G}_A(\tau; \mu_q)}{\tilde{G}_V(\tau; \mu_q) + \tilde{G}_A(\tau; \mu_q)},$$

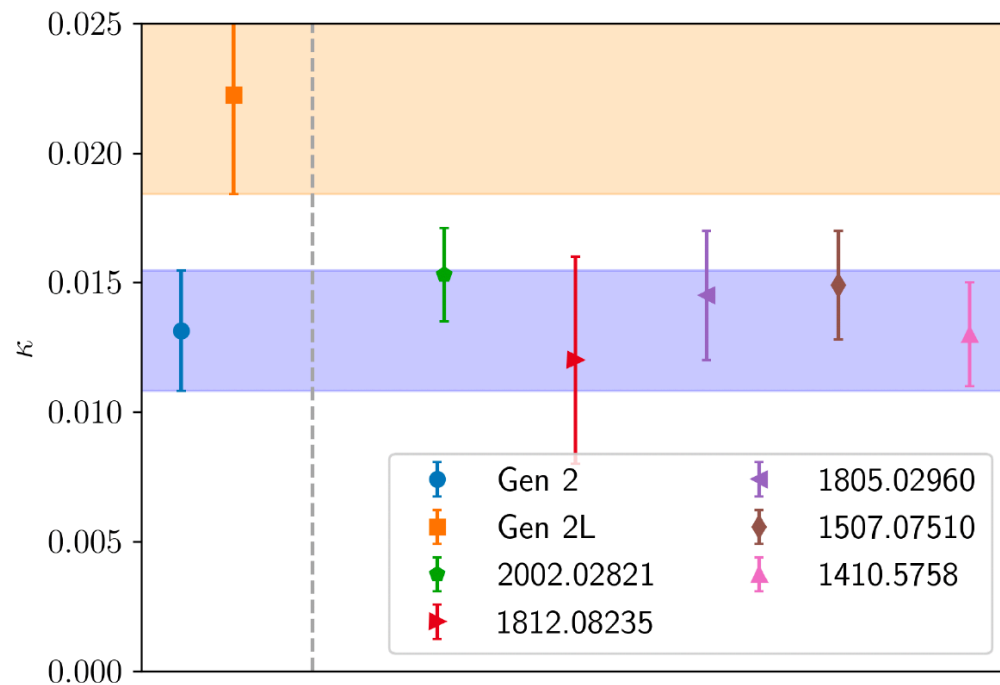
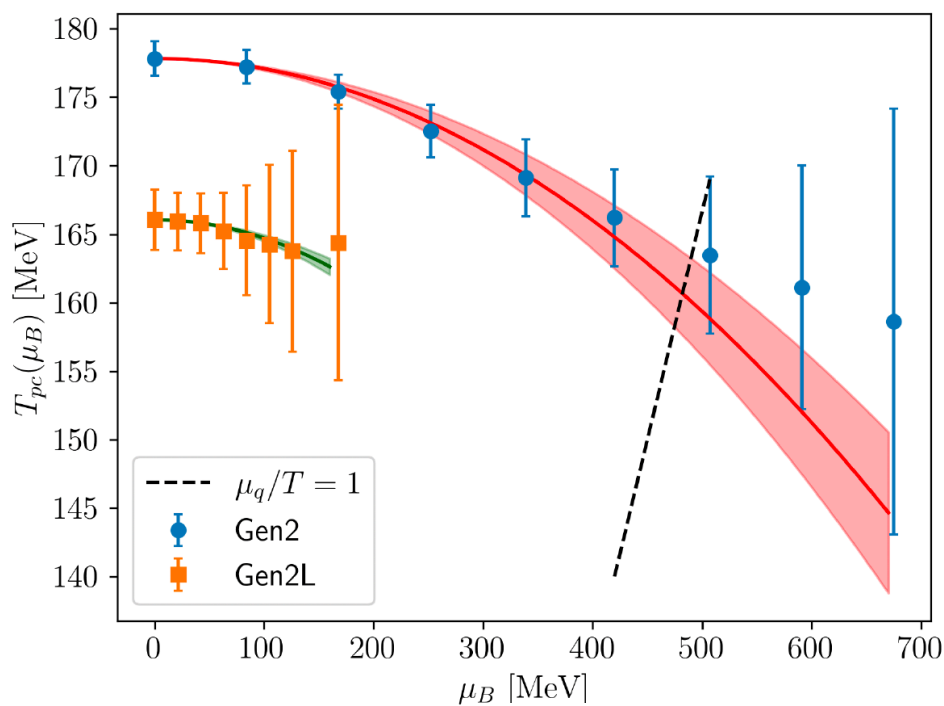


[Numerical results: LQCD data analyses]

If A-V degenerated at T_c at certain μ , one can trace chiral boundary of QCD phase.

$$R(T, \mu) = \frac{G_V - V_A}{G_V + V_A}$$

Picking up (T, μ) for $R=0$, chiral boundary can be obtained.



$$\frac{T_c(\mu_B)}{T_c(\mu_B = 0)} = 1 - \boxed{\kappa} \left(\frac{\mu_B}{T_c(\mu_B = 0)} \right)^2 + \mathcal{O}(\mu_B^4),$$

[Summary]

Connections between hadron decays and A-V degeneracy in heat bath.

A and V meson correlators for light-flavor sector studied via LQCD.

Improved Wilson-clover fermion in anisotropic lattice with FASTSUM ensemble.

Finite μ included by Taylor-expansion method at finite T.

A and V correlators computed as functions of Euclidean time.

Partial restoration of SBCS shown from A-V degeneracy at finite μ and T.

A-V degeneracy from LQCD data analyses confined QCD phase boundary

More works in progress: Spectral function, Dalitz analyses, strangeness, T_c traceback, etc.



Thank you for your attention!!

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