Weak Interactions in Nuclei and Nuclear Matter

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Effect of isoscalar and isovector scalar fields on baryon semileptonic decays in nuclear matter

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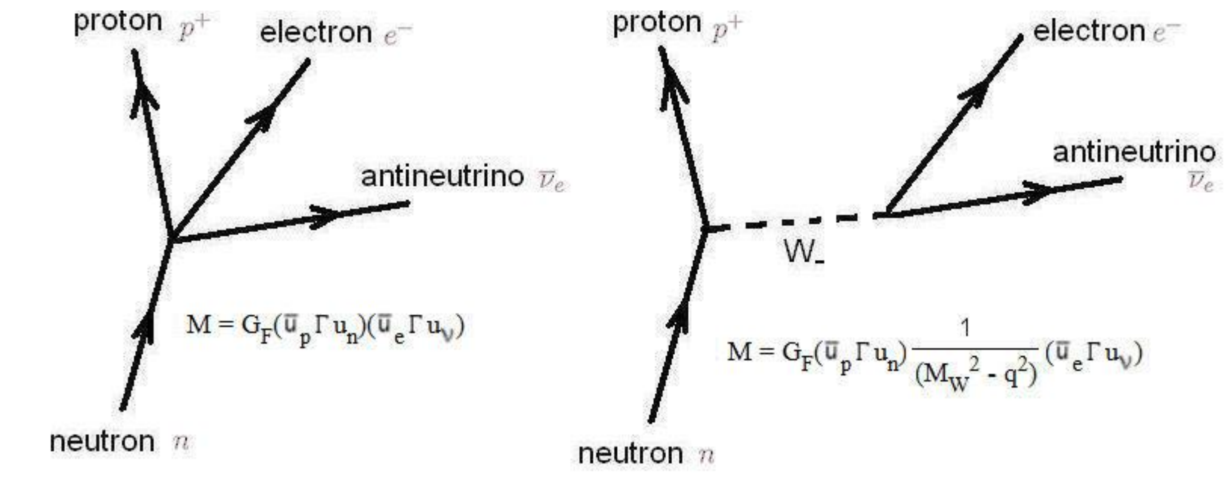
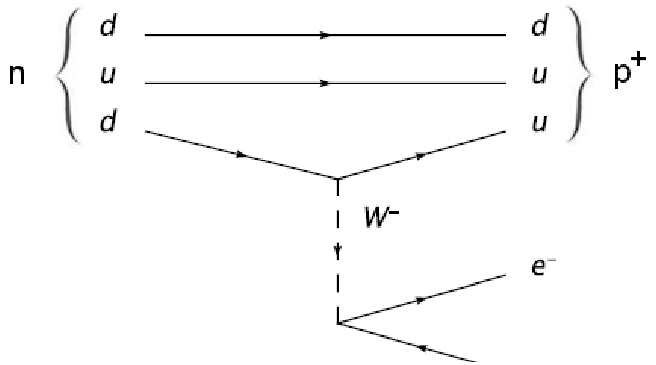


a. Fermi's 4-point Interaction, 1934 b. Weak Interaction mediated by boson, 1938



4. Quark-Level Decay Rate

The **beta decay** process ($n \to p + e^- + \bar{\nu}_e$) at the quark level involves the following transition:

$$d \to u + e^- + \bar{\nu}_e$$

The decay rate depends on both the **vector** and **axial-vector currents**, which involve G_F and g_A, respectively. The total decay rate Γ for beta decay can be written as:

$$\Gamma \propto G_F^2 \left(1 + g_A^2\right) Q^5$$

This reflects the fact that both G_F (from the **W boson exchange**) and g_A (from the **axial current**) contribute to the transition rate.

4. Effective Field Theory Connection:

In more advanced **effective field theory** (EFT) treatments of weak interactions, the connection between G_F and g_A may be more explicit through the **effective weak Hamiltonian** for low-energy processes. In this framework, the weak Hamiltonian includes terms that involve both the vector and axial currents:

$$\mathcal{H}_{\text{weak}} = \frac{G_F}{\sqrt{2}} \left(g_V \bar{\psi}_p \gamma^\mu \psi_n \, \bar{\psi}_e \gamma_\mu \psi_\nu + g_A \bar{\psi}_p \gamma^\mu \gamma_5 \psi_n \, \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu \right)$$

Here:

- g_V (vector coupling) and g_A (axial coupling) determine the relative contributions of the vector and axial currents to the transition rate in weak decays.

The **axial coupling constant** g_A is experimentally measured, while G_F is derived from the overall weak interaction strength. However, there isn't a simple **quantitative relation** between them in the standard model; they are determined by separate experimental observations but both contribute to weak decay processes.

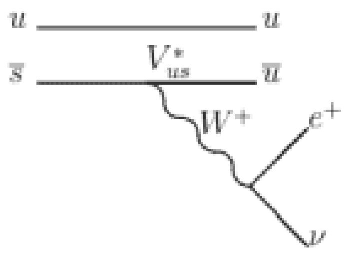
How to relate G_V and G_A?

pion decay constant

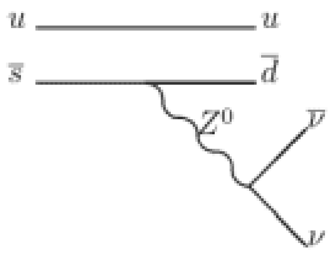
pion-nucleon coupling constant

$$g_A \equiv \frac{G_A}{G_V} = \frac{f_\pi g_{\pi n}}{M_N c^2}$$

Goldberger-Treiman relation



Ef



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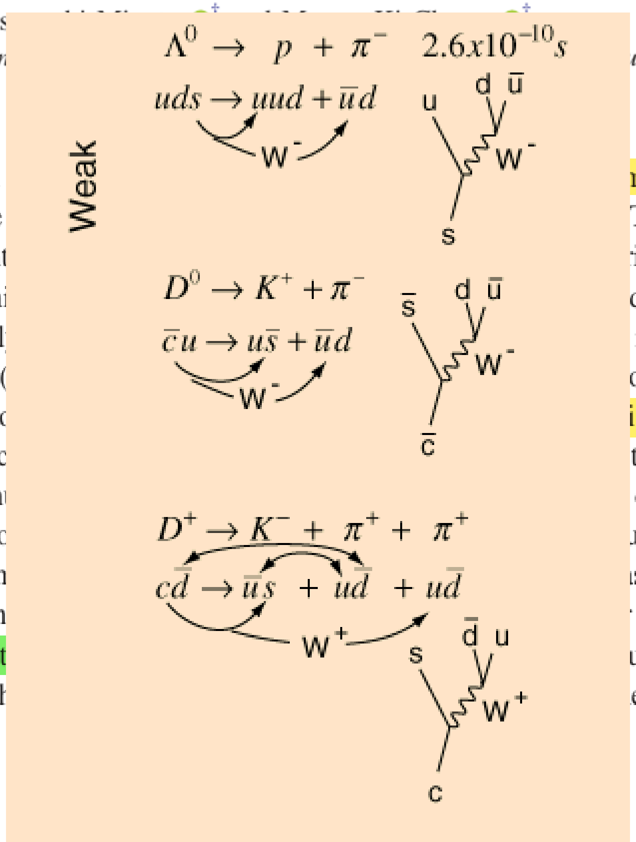


The precise determination ... matrix elements is very important because it could be ... This is particular true of V_{ud} because it is the main cont... ix elements. The level of accuracy for the test of the uni... der of 10^{-4}. Because the precise data for V_{ud} is usuall... it is quite significant to investigate the breaking of SU(... onstant in nuclear matter. The purpose of this paper is to ... isovector scalar (δ or a_0) mean-fields affect the weak vec... tonic baryon (neutron, Λ, or Ξ^-) decay in asymmetric nu... coupling (QMC) model, where nuclear matter consists o... und by the self-consistent exchange of scalar and vector m... ss correction to the quark currents in matter. We then fin... matter, the defect of the vector coupling constant due t... uclear saturation density, which is the same amount as th... ents. It is also interesting

Beta Decay

Microscopic picture

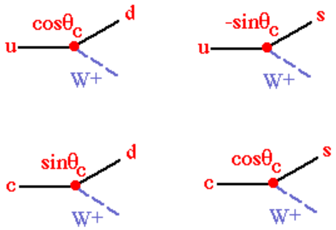
When a quark decays, the new quark does not have a definite flavor. For instance:

$$u \to d' = d\cos\theta_c + s\sin\theta_c$$

Cabibbo angle

However, the observed weak transitions are between quarks of definite flavor. The strong-interaction quark eigenstates

$$\begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix}\begin{pmatrix} t \\ b \end{pmatrix}$$



are different from weak interaction eigenstates).

$$\begin{pmatrix} u \\ d' \end{pmatrix}\begin{pmatrix} c \\ s' \end{pmatrix}\begin{pmatrix} t \\ b' \end{pmatrix}$$

This means that the observed beta-decay strength in reactions is modified by the mixing angle.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Cabibbo _Kobayashi-Maskawa (CKM) matrix

For nuclear beta-decay, we are mainly concerned with the transition between u- and d-quarks. As a result, only the product

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} \approx \begin{bmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{bmatrix}$$

Why nucleon mass becomes small than that in free space ?

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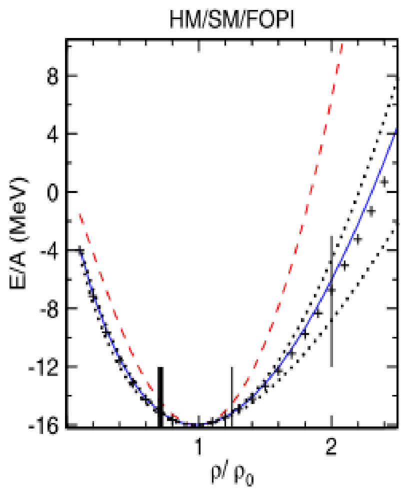

Fig. 4. Various symmetric nuclear matter EOS. Dashed (red) curve, HM; full (blue) curve, SM. The dotted curves and the thin vertical bars delimit the FOPI EOS (crosses) constraints. The thick vertical bar is the density relevant for the GMR according to [40]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Yes, the mass of a nucleon (such as a proton or neutron) can be smaller in certain environments compared to its mass in free space. This phenomenon is primarily observed under conditions like those inside atomic nuclei or in high-energy physics scenarios.

In a free space, the mass of a proton, for example, is about 938 MeV/c^2. However, inside a nucleus, the nucleon mass can be lower due to the binding energy of the nucleus. The binding energy is the energy required to break a nucleus into its constituent nucleons. This energy is released when the nucleons come together to form the nucleus, leading to a reduction in the effective mass of the nucleons in the bound state.

This decrease in mass is due to the strong nuclear force, which acts between nucleons and can bind them in a nucleus. The effective mass of the nucleons inside a nucleus is typically lower than their free-space mass because the system as a whole has a lower total energy than if the nucleons were free. This concept is connected to the idea of mass-energy equivalence, where energy contributes to the mass of a system.

In high-energy particle physics, under extreme conditions like those found in heavy-ion collisions or at the beginning of the universe, nucleons can also experience mass modifications due to factors such as the quark-gluon plasma, where quarks and gluons are no longer confined within individual nucleons.

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Quark mean field model for nuclear matter and finite nuclei

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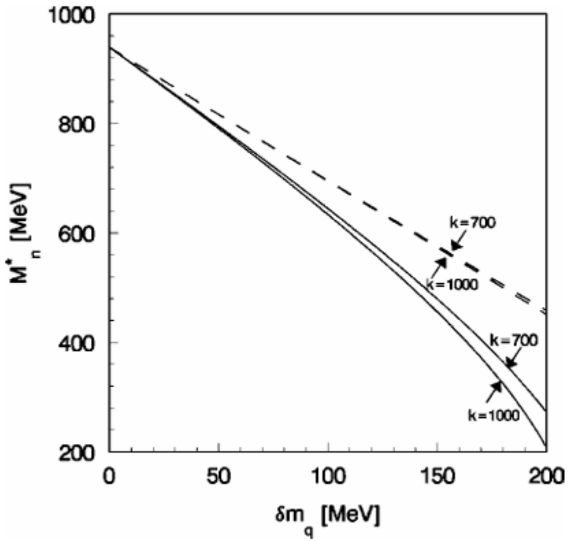

FIG. 1. The effective nucleon mass M_n^* as functions of the quark mass correction δm_q. The results in the QMF model with $\chi_c = \frac{1}{2}kr^2$ are shown by solid curves, while those with $\chi_c = \frac{1}{2}kr^2(1+\gamma^0)/2$ are shown by dashed curves. For each potential shown are the two results for two confining strengths.

The first step is to generate the nucleon system under the influence of the meson mean fields. In the constituent quark model, the quarks in a nucleon satisfy the following Dirac equation:

$$[i\gamma_\mu \partial^\mu - m_q - \chi_c - g_\sigma^q \sigma(r) - g_\omega^q \omega(r)\gamma^0 - g_\rho^q \rho(r)\tau_3\gamma^0]q(r) = 0, \tag{2}$$

where τ_3 is the isospin matrix in our nuclear physics convention. Assuming the meson mean fields are constant within the small nucleon volume, we can then write the Dirac equation as

$$[-i\vec{\alpha}\cdot\vec{\nabla} + \beta m_q^* + \beta\chi_c]q(r) = e^* q(r), \tag{3}$$

where $m_q^* = m_q + g_\sigma^q \sigma$ and $e^* = e - g_\omega^q \omega - g_\rho^q \rho\tau_3$, with σ, ω, and ρ being the mean fields at the middle of the nucleon. e is the energy of the quark under the influence of the σ, ω, and ρ mean fields. The quark mass is modified to m_q^* due to the presence of the σ mean field. Here, g_σ^q, g_ω^q, and g_ρ^q are the coupling constants of the σ, ω, and ρ mesons with quarks, respectively. We take into account the spin correlations, $E_{\rm spin}$, due to gluons and pions so that the mass difference between Δ and nucleon arises. Hence, the nucleon energy is expressed as $E_n^* = 3e^* + E_{\rm spin}$, where the vector contribution is removed here. There exists the spurious center of mass motion, which is removed in the standard method by $M_n^* = \sqrt{E_n^{*2} - \langle p_{\rm c.m.}^2\rangle}$, where $\langle p_{\rm c.m.}^2\rangle = \Sigma_{i=1}^3 \langle p_i^2\rangle$, since the three constituent quarks are moving in the confining potential independently.

We now move to the second step, in which the nuclear many body system will be solved with the change of the nucleon properties obtained in the first step. We assume the following QMF Lagrangian,

$$\mathcal{L}_{\rm QMF} = \bar{\psi}[i\gamma_\mu\partial^\mu - M_n^* - g_\omega \omega\gamma^0 - g_\rho \rho\tau_3\gamma^0]\psi + \mathcal{L}_M(\sigma,\omega,\rho). \tag{4}$$

TABLE I. The nuclear matter properties used to determine the five free parameters in the present model. The saturation density and the energy per particle are denoted by ρ_0 and E/A, and the incompressibility by k, the effective mass by M_n^*, and the symmetry energy by a_{sym}.

ρ_0 (fm^{-3})	E/A (MeV)	k (MeV)	M_n^*/M_n	$a_{\rm sym}$ (MeV)
0.145	−16.3	280	0.63	35

III. PROPERTIES OF NUCLEAR MATTER

We calculate first the change of the nucleon properties as a function of the quark mass correction, δm_q, which is defined as $\delta m_q = m_q - m_q^* = -g_\sigma^q \sigma$. Here, the constituent quark mass is taken to be one third of the nucleon mass; $m_q = M_n/3 = 313$ MeV. We take into account confinement in terms of the harmonic oscillator potential together with two Lorentz structures: (1) scalar potential $\chi_c = \frac{1}{2}kr^2$ and (2) scalar-vector potential $\chi_c = \frac{1}{2}kr^2(1+\gamma^0)/2$. As pointed out in Ref. [25], the quark cannot be confined when the vector potential is larger than the scalar one. Here, we just take two extreme types, since the Lorentz structure of the confinement is not established. As for the strength of the confining potential, we take $k = 700$ and 1000 MeV/fm^2, in order to see the results depending on this factor. The spin correlation, $E_{\rm spin}$, is fixed by the free nucleon mass as $M_n = \sqrt{(3e + E_{\rm spin})^2 - \langle p_{\rm c.m.}^2\rangle} = 939$ MeV. We assume further that the confining interaction and the spin correlations do not change in the nuclear medium.

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IV. QUARK-MESON COUPLING MODEL FOR ASYMMETRIC NUCLEAR MATTER

In this section, we introduce the quark-meson coupling (QMC) model [22–26], which starts with confined quarks as the degrees of freedom, with the relativistic confinement potential of the scalar-vector HO type [see Eq. (1)]. We assume that the strength parameter c in the potential does not change in matter.

We here consider the mean fields of σ, ω, ρ and δ mesons, which interact with the confined quarks, in uniformly distributed, asymmetric nuclear matter. Let the mean-field values for the σ, ω (the time component), ρ (the time component in the 3rd direction of isospin) and δ (in the 3rd direction of isospin) fields be $\bar{\sigma}$, $\bar{\omega}$, $\bar{\rho}$ and $\bar{\delta}$, respectively. The Dirac equation for the quark field ψ_i ($i = u$, d or s) is then given by [32]

$$\left[i\gamma\cdot\partial - (m_i - V_s) - \gamma_0 V_0 \boxed{- \frac{c}{2}(1+\gamma_0)r^2}\right]\psi_i(\vec{r},t) = 0, \tag{19}$$

where $V_s = g_\sigma^q\bar{\sigma} + \tau_3 g_\delta^q\bar{\delta}$ and $V_0 = g_\omega^q\bar{\omega} + \tau_3 g_\rho^q\bar{\rho}$ [$\tau_3 = \pm 1$ for $\binom{u}{d}$ quark] with the quark-meson coupling constants, g_σ^q, g_δ^q, g_ω^q and g_ρ^q. Motivated by the Okubo-Zweig-Iizuka (OZI) rule, we here assume that the σ, ω, ρ and δ mesons couple to the u and d quarks only, not to the s quark. This breaks SU(3) symmetry explicitly. Furthermore, the isoscalar σ meson couples to the u and d quarks *equally*, while the isovector δ meson couples to the u and d quarks *oppositely*. This breaks isospin symmetry. Now we respectively define the effective quark mass and the effective single-particle energy as $m_i^* \equiv m_i - V_s = m_i - g_\sigma^q\bar{\sigma} \mp g_\delta^q\bar{\delta}$ and $\epsilon_i^* \equiv \epsilon_i - V_0 = \epsilon_i - g_\omega^q\bar{\omega} \mp g_\rho^q\bar{\rho}$ for $\binom{u}{d}$ quark, where ϵ_i is the eigenenergy of Eq. (19). Note that $m_s^* = m_s$ and $\epsilon_s^* = \epsilon_s$.

The static, lowest-state wave function in matter is presented by

$$\psi_i(\vec{r},t) = \exp[-i\epsilon_i t]\psi_i(\vec{r}). \tag{20}$$

The wave function $\psi_i(\vec{r})$ is then given by Eqs. (2) and (3), in which ϵ_i, m_i, λ_i and a_i are replaced with ϵ_i^*, m_i^*, λ_i^* and a_i^*, and the effective energy ϵ_i^* is determined by $\sqrt{\epsilon_i^* + m_i^*}(\epsilon_i^* - m_i^*) = 3\sqrt{c}$ [see Eq. (4)].

The zeroth-order energy E_B^{0*} of the low-lying baryon is thus given by

$$E_B^{0*} = \sum_i \epsilon_i^*, \tag{21}$$

and we have the effective baryon mass in matter

$$M_B^* = E_B^{0*} + E_B^{\rm spin} - E_B^{c.m.*}. \tag{22}$$

In this model, we have eight parameters: c, m_u, m_d, m_s, $E_p^{\rm spin}$, $E_n^{\rm spin}$, $E_\Lambda^{\rm spin}$, $E_{\Xi^-}^{\rm spin}$. In order to reduce the number of parameters, we assume $E_N^{\rm spin} = E_p^{\rm spin} = E_n^{\rm spin}$, and consider the three cases:

(i) case 1; $m_u = 250$ MeV, $m_s = 450$ MeV,
(ii) case 2; $m_u = 300$ MeV, $m_s = 500$ MeV,
(iii) case 3; $m_u = 350$ MeV, $m_s = 550$ MeV.

The remaining five parameters, c, m_d, $E_N^{\rm spin}$, $E_\Lambda^{\rm spin}$, and $E_{\Xi^-}^{\rm spin}$, are determined so as to reproduce the proton charge radius $\langle r^2\rangle_p^{1/2} = 0.841$ fm [3] and the baryon masses: $M_p = 937.64$ MeV, $M_n = 939.70$ MeV, $M_\Lambda = 1115.68$ MeV, and $M_{\Xi^-} = 1321.71$ MeV. Here, the electromagnetic self-energy for proton or neutron is subtracted from the observed mass [32].

TABLE I. Parameters and quark energies in vacuum.

Case	m_u [a]	m_d	m_s [a]	ϵ_u	ϵ_d	ϵ_s	$E_N^{\rm spin}$	$E_\Lambda^{\rm spin}$	$E_{\Xi^-}^{\rm spin}$	c
	(MeV)						(MeV)			(fm^{-3})
1	250	252.63	450	493.1	495.0	649.8	−236.8	−227.7	−190.2	0.635
2	300	302.48	500	517.8	519.7	681.2	−334.5	−331.3	−299.8	0.561
3	350	352.38	550	546.4	548.3	715.3	−441.9	−443.3	−416.5	0.500

[a]Input.

Here, we assume that the spin correlation $E_B^{\rm spin}$ does not change in matter, and the c.m. correction $E_B^{c.m.*}$ is given by Eqs. (5)–(8) with ϵ_i^* and m_i^*, instead of ϵ_i and m_i.

For describing asymmetric nuclear matter, we now start from the following Lagrangian density in mean-field approximation

$$\mathcal{L} = \bar{\psi}_N[i\gamma\cdot\partial - M_N^*(\bar{\sigma},\bar{\delta}) - g_\omega\gamma_0\bar{\omega} - g_\rho\gamma_0\tau_3\bar{\rho}]\psi_N - \frac{m_\sigma^2}{2}\bar{\sigma}^2 - \frac{m_\delta^2}{2}\bar{\delta}^2 + \frac{m_\omega^2}{2}\bar{\omega}^2 + \frac{m_\rho^2}{2}\bar{\rho}^2 - \frac{g_2}{3}\bar{\sigma}^3 \tag{23}$$

with ψ_N the (isodoublet) nucleon field, τ_3 the 3rd component of Pauli matrix and $M_N^*(\bar{\sigma},\bar{\delta})$ the mass matrix whose component is given by Eq. (22). The nucleon-meson coupling constants, g_σ, g_ω, g_ρ, and g_δ, are respectively related to the quark-meson coupling constants as $g_\sigma = 3g_\sigma^q$, $g_\omega = 3g_\omega^q$, $g_\rho = g_\rho^q$, and $g_\delta = g_\delta^q$. The meson masses are taken to be $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV, $m_\rho = 770$ MeV, and $m_\delta = 983$ MeV. We add the last term to the Lagrangian, which is the nonlinear, self-coupling term of σ meson, in order to reproduce the properties of nuclear matter as discussed later. Here, we do not include the nonlinear term $\frac{1}{4}g_3\sigma^4$, because it plays similar roles as $\frac{1}{3}g_2\sigma^3$.

The total energy per nucleon of asymmetric nuclear matter is then obtained by

$$E = \sum_{j=p,n}\frac{1}{\pi^2\rho_N}\int_0^{k_{F_j}} dk\,k^2\sqrt{M_j^{*2}+k^2} + g_\omega\bar{\omega} + g_\rho\left(\frac{\rho_3}{\rho_N}\right)\bar{\rho} + \frac{1}{2\rho_N}\left(m_\sigma^2\bar{\sigma}^2 - m_\omega^2\bar{\omega}^2 + m_\delta^2\bar{\delta}^2 - m_\rho^2\bar{\rho}^2\right) + \frac{1}{3\rho_N}g_2\bar{\sigma}^3, \tag{24}$$

where $k_{F_{p(n)}}$ is the Fermi momentum for protons (neutrons), and this is related to the density of protons (neutrons), $\rho_{p(n)}$, through $\rho_{p(n)} = k_{F_{p(n)}}^3/(3\pi^2)$. The total nucleon density is given by $\rho_N = \rho_p + \rho_n$, and the difference in proton and neutron densities is defined by $\rho_3 \equiv \rho_p - \rho_n$. Using Eq. (24), we can calculate pressure by $P = \rho_N^2(\partial E/\partial\rho_N)$. Then, the binding energy per nucleon, E_b, is defined by

$$E_b(\rho_N,\alpha) \equiv E(\rho_N,\alpha) - \frac{1}{2}[(M_n+M_p)+(M_n-M_p)\alpha] \tag{25}$$

where $\alpha = (\rho_n-\rho_p)/\rho_N$ and $E(\rho_N,\alpha)$ is given by Eq. (24).

$$(m_\sigma^2 + g_2\bar{\sigma})\bar{\sigma} = -\sum_{j=p,n}\left(\frac{\partial M_j^*}{\partial\bar{\sigma}}\right)\rho_j^s \equiv g_\sigma\sum_j G_{\sigma j}(\bar{\sigma},\bar{\delta})\rho_j^s$$

$$m_\omega^2\bar{\omega} = g_\omega\rho_N, \tag{27}$$

$$m_\delta^2\bar{\delta} = -\sum_j\left(\frac{\partial M_j^*}{\partial\bar{\delta}}\right)\rho_j^s \equiv g_\delta\sum_j G_{\delta j}(\bar{\sigma},\bar{\delta})\rho_j^s, \tag{28}$$

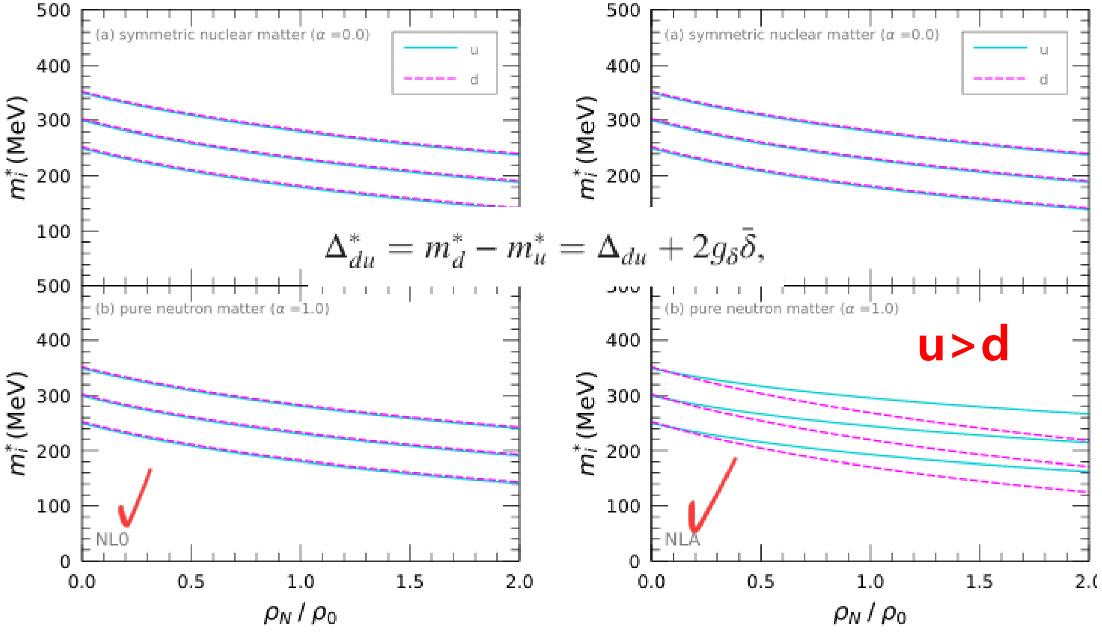
$$m_\rho^2\bar{\rho} = g_\rho\rho_3, \tag{29}$$

with $G_{\sigma j}$ ($G_{\delta j}$) the isoscalar (isovector) quark scalar density in j (= proton or neutron) (see Appendix C), and ρ_j^s the scalar density of j in matter

$$\rho_j^s = \frac{1}{\pi^2}\int_0^{k_{F_j}} dk\,k^2\frac{M_j^*}{\sqrt{M_j^{*2}+k^2}}. \tag{30}$$

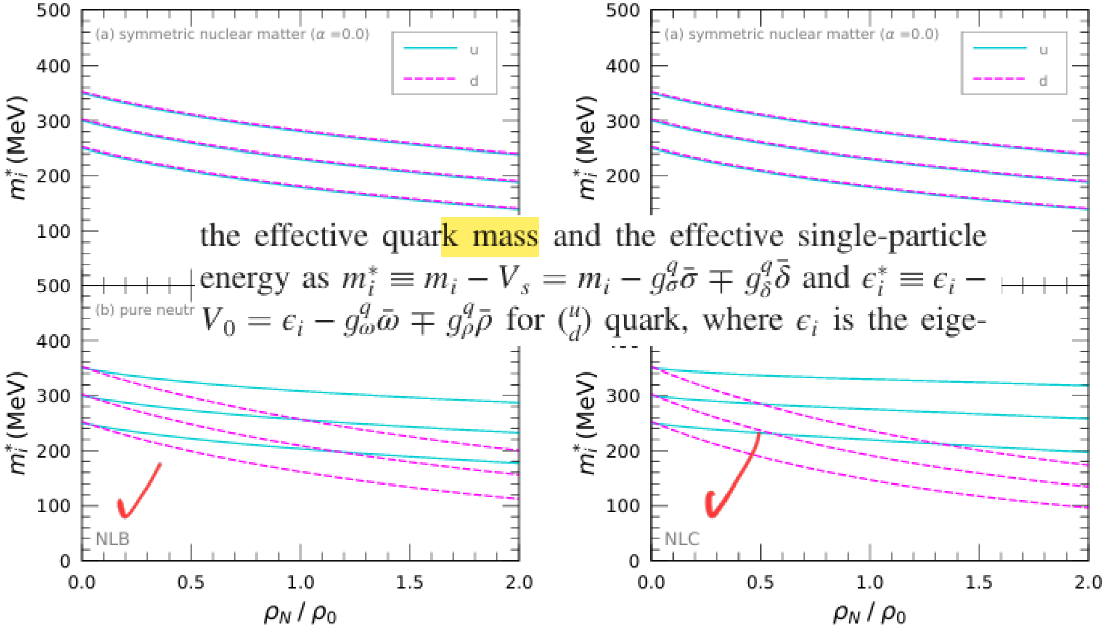
TABLE IV. Coupling constants. For detail, see the text.

Model	Case	g_σ	g_ω	g_δ	g_ρ	g_2 (fm^{-1})
NL0	1	10.34	7.34	...	4.37	23.61
	2	9.98	7.69	...	4.35	24.97
	3	9.72	7.95	...	4.33	25.67
NLA	1	10.34	7.34	4.04	4.83	23.61
	2	9.98	7.69	4.04	4.90	24.97
	3	9.72	7.95	4.04	4.96	25.67
NLB	1	10.34	7.34	5.59	5.16	23.61
	2	9.98	7.69	5.59	5.31	24.97
	3	9.72	7.95	5.59	5.43	25.67
NLC	1	10.34	7.34	7.70	5.65	23.61
	2	9.98	7.69	7.70	5.93	24.97
	3	9.72	7.95	7.70	6.16	25.67



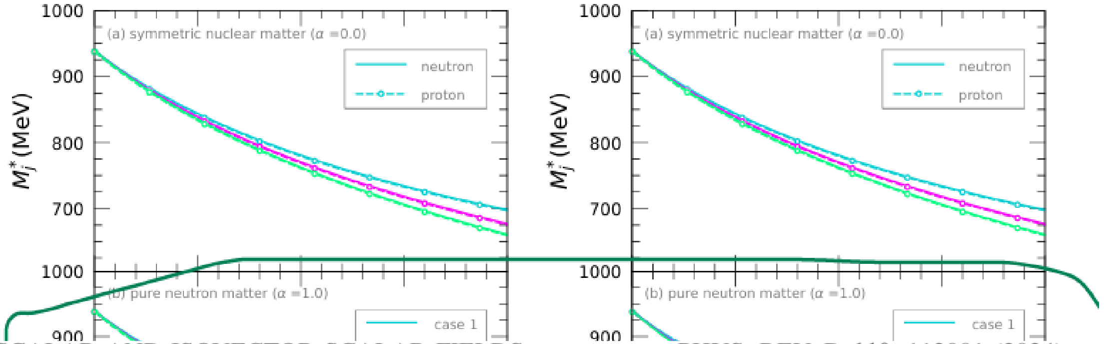
$$\Delta_{du}^* = m_d^* - m_u^* = \Delta_{du} + 2 g_\delta \bar{\delta},$$

u>d



the effective quark mass and the effective single-particle energy as $m_i^* \equiv m_i - V_s = m_i - g_\sigma^q \bar{\sigma} \mp g_\delta^q \bar{\delta}$ and $\epsilon_i^* \equiv \epsilon_i - V_0 = \epsilon_i - g_\omega^q \bar{\omega} \mp g_\rho^q \bar{\rho}$ for $\binom{u}{d}$ quark, where ϵ_i is the eige-

FIG. 2. Effective quark mass ($i = u$ or d) as a function of ρ_N/ρ_0.



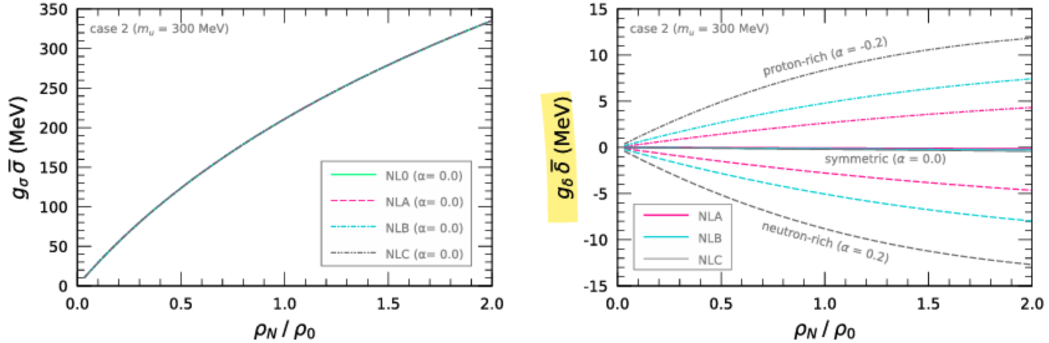


FIG. 2. Mean-field values of the meson fields as a function of ρ_N/ρ_0. We show the results in case 2 only in Table VI. The left panel is for the σ field in symmetric nuclear matter. The model dependence of the σ field is very small. The right panel is for the δ field in case of $\alpha = -0.2$ (the dot-dashed curves), 0 (the solid curves), or 0.2 (the dashed curves). Note that $\alpha = 0.2$ corresponds to $^{208}_{82}$Pb.

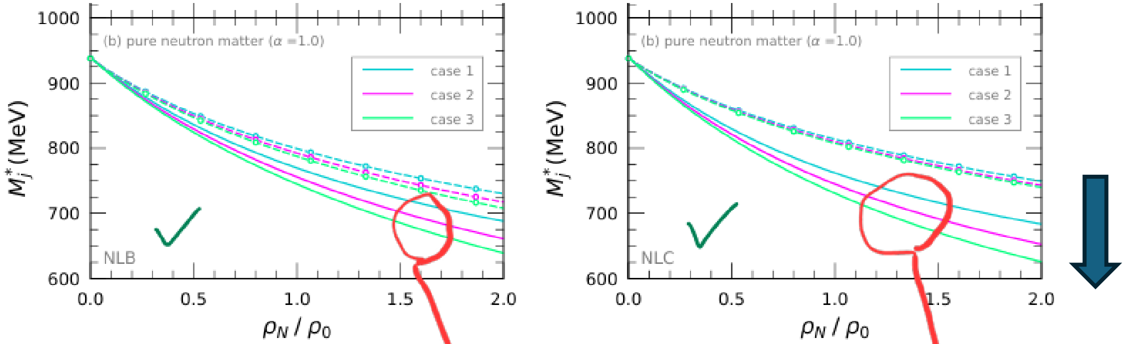


FIG. 3. Effective nucleon mass ($j = p$ or n) as a function of ρ_N/ρ_0.

Change of

III. WEAK COUPLING CONSTANTS IN VACUUM

We are interested in calculating the weak form factors. The transition matrix element for the decay of an initial baryon B_1 to a final baryon B_2 and lepton l with its antineutrino $\bar{\nu}_l$, that is $B_1 \to B_2 + l + \bar{\nu}_l$, is proportional to the matrix element of the baryon weak current $J^\mu(x)$, which consists of the vector and axial-vector currents, as

$$\langle B_2|J^\mu(x)|B_1\rangle = C\left[\langle B_2|V^\mu(x)|B_1\rangle + \langle B_2|A^\mu(x)|B_1\rangle\right], \tag{12}$$

and

$$\langle B_2|V^\mu(x)|B_1\rangle = \bar{u}_2(p_2)\left[f_1(q^2)\gamma^\mu + \cdots\right]u_1(p_1)e^{iq\cdot x}, \tag{13}$$

$$\langle B_2|A^\mu(x)|B_1\rangle = \bar{u}_2(p_2)\left[g_1(q^2)\gamma^\mu\gamma_5 + \cdots\right]u_1(p_1)e^{iq\cdot x}, \tag{14}$$

where $C = \cos\theta_c(\sin\theta_c)$ is the Cabibbo angle for the strangeness transition $\Delta S = 0\,(\Delta S = 1)$, and $p_{1(2)}$ is the four-momentum of baryon 1(2). Here, $q^2 = (p_1 - p_2)^2$ is the momentum transfer squared, which is generally quite small for any such decays. The ellipses in Eqs. (13) and (14) refer to additional vector and axial-vector currents, which contribute to the decay rates only in order q/M_B or higher. Therefore, in order to focus on the leading form factors $f_1(0)$ (vector coupling constant) and $g_1(0)$ (axial-vector coupling constant), we only consider the $\mu = 0$ component of V^μ and the $\mu = 3$ component of A^μ (all baryons are treated with spin up).

The β decay of the octet baryons, $B_1 \to B_2 + e^- + \bar{\nu}_e$, can be interpreted as the quark β decays like $q_i \to q_j + e^- + \bar{\nu}_e$ inside the baryon, where $j = u$ and i could be d or s quarks. The weak coupling constants without the cm correction, $f_1^{(0)}$ and $g_1^{(0)}$, are calculated by the quark model

$$f_1^{(0)} = \int d\vec{r}\,\langle B_2|V^0(x)|B_1\rangle\,, \quad g_1^{(0)} = \int d\vec{r}\,\langle B_2|A^3(x)|B_1\rangle\,. \tag{15}$$

Then, they are rewritten by

$$\begin{aligned} f_1^{(0)} &= f_1^{\mathrm{SU(3)}} \times \int d\vec{r}\,\bar{\psi}_j(\vec{r})\gamma_0\psi_i(\vec{r}) \\ &= f_1^{\mathrm{SU(3)}} \times 2^{5/2}\left(\frac{a_i a_j}{a_i^2 + a_j^2}\right)^{3/2}\frac{\lambda_i a_i \lambda_j a_j + 3a_i a_j/(a_i^2 + a_j^2)}{\sqrt{2\lambda_i^2 a_i^2 + 3}\sqrt{2\lambda_j^2 a_j^2 + 3}} \\ &\equiv f_1^{\mathrm{SU(3)}} \times (1 + \boxed{\delta f_1^{(0)}}) \end{aligned} \tag{16}$$

SU(3) sym. bre.

and

$$\begin{aligned} g_1^{(0)} &= g_1^{\mathrm{SU(3)}} \times \int d\vec{r}\,\bar{\psi}_j(\vec{r})\gamma^3\gamma_5\psi_i(\vec{r}), \\ &= g_1^{\mathrm{SU(3)}} \times 2^{5/2}\left(\frac{a_i a_j}{a_i^2 + a_j^2}\right)^{3/2}\frac{\lambda_i a_i \lambda_j a_j - a_i a_j/(a_i^2 + a_j^2)}{\sqrt{2\lambda_i^2 a_i^2 + 3}\sqrt{2\lambda_j^2 a_j^2 + 3}} \\ &\equiv g_1^{\mathrm{SU(3)}} \times (1 - \boxed{\delta g_1^{(0)}}) \end{aligned} \tag{17}$$

where in both cases the superscript $^{\mathrm{SU(3)}}$ indicates the usual value obtained in exact SU(3) [46] (for example, in case of neutron β decay, $f_1^{\mathrm{SU(3)}} = 1$ and $g_1^{\mathrm{SU(3)}} = 5/3$). Furthermore, using Eq. (4), we can easily verify that the vector coupling $f_1^{(0)}$ obeys the AG theorem, namely $\delta f_1^{(0)} = \mathcal{O}(\Delta_{ij}^2)$.

SU(3) sym. br. cm correci relativ. corr cm.

TABLE III. Weak coupling constants in vacuum.

case	$\delta f_1^{(0)} \times 10^6$	ρ_V	$\delta f_1 \times 10^6$	$\delta g_1^{(0)}$	ρ_A	δg_1
			$n \to p$			
1	3.12	0.999999789	2.91	0.187	0.9177	0.114
2	2.28	0.999999810	2.08	0.156	0.9195	0.082
3	1.71	0.999999825	1.54	0.131	0.9209	0.057
			$\Lambda \to p$			
case	$\delta f_1^{(0)}$	ρ_V	δf_1	$\delta g_1^{(0)}$	ρ_A	δg_1
1	0.012	0.9988	0.011	0.154	0.9254	0.086
2	0.010	0.9989	0.009	0.131	0.9271	0.062
3	0.008	0.9990	0.007	0.111	0.9285	0.043
			$\Xi^- \to \Lambda$			
case	$\delta f_1^{(0)}$	ρ_V	δf_1	$\delta g_1^{(0)}$	ρ_A	δg_1
1	0.012	0.9973	0.009	0.154	0.9404	0.101
2	0.010	0.9975	0.007	0.131	0.9420	0.077
3	0.008	0.9976	0.006	0.111	0.9433	0.058

in the case 2, we find that $\delta f_1^{(0)} \simeq 2.3 \times 10^{-6}$ and $\delta g_1^{(0)} \simeq 0.16$ in the neutron β decay. It is not surprising that $\delta f_1^{(0)}$ is of the order of 10^{-6}, because the present calculation does not involve the breaking of SU(3) symmetry due to gluons and pions [21]. In contrast, we find that $\delta f_1^{(0)} \simeq 0.01$ and $\delta g_1^{(0)} \simeq 0.13$ in the Λ or Ξ^- decay. Here, $\delta g_1^{(0)}$ in the hyperon decay is smaller than that in the neutron decay, which is caused by the fact that the s quark is heavier than the d quark and thus the lower component of the s-quark wave function in hyperon is smaller than that of the d-quark wave function in neutron.

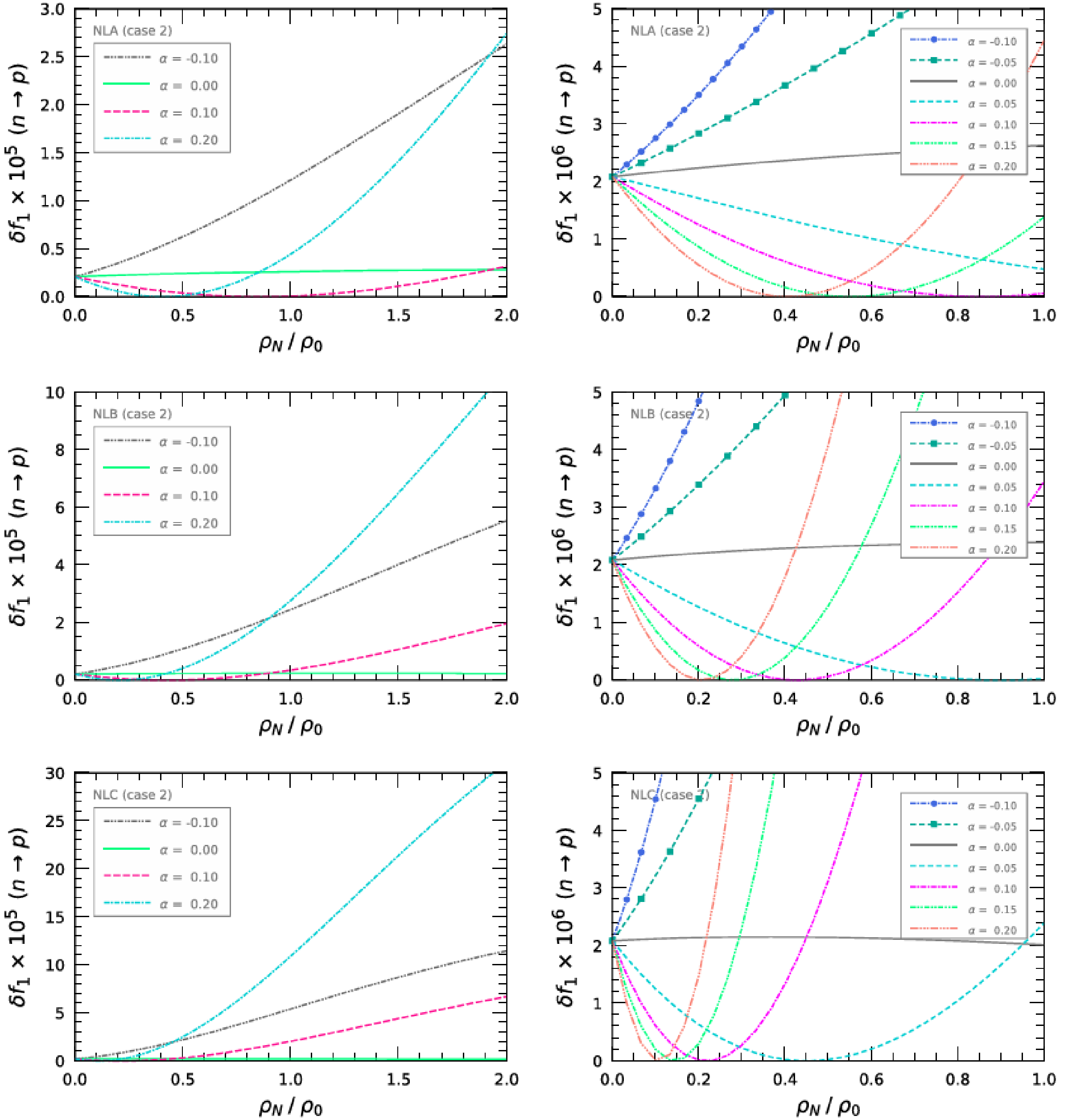


FIG. 9. Deviation, δf_1, for the neutron β decay in the NLA, NLB, or NLC model. We show the results in case 2. In the left panels, the nuclear density varies up to $\rho_N/\rho_0 = 2.0$. In the right panels, δf_1 at low density is magnified.

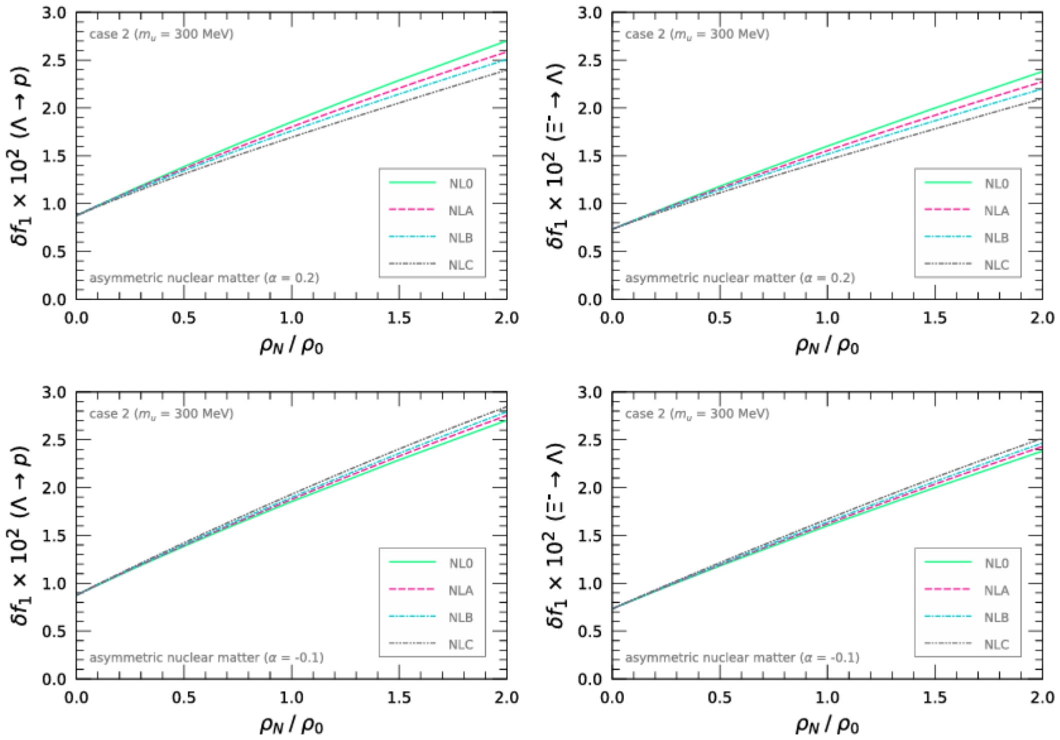


FIG. 10. Deviation, δf_1, for the hyperon β decay in the NL0, NLA, NLB, or NLC model. We show the results in case 2 only. The left (right) two panels are for the Λ (Ξ^-) decay. The top (bottom) two panels are for $\alpha = 0.2$ (-0.1).

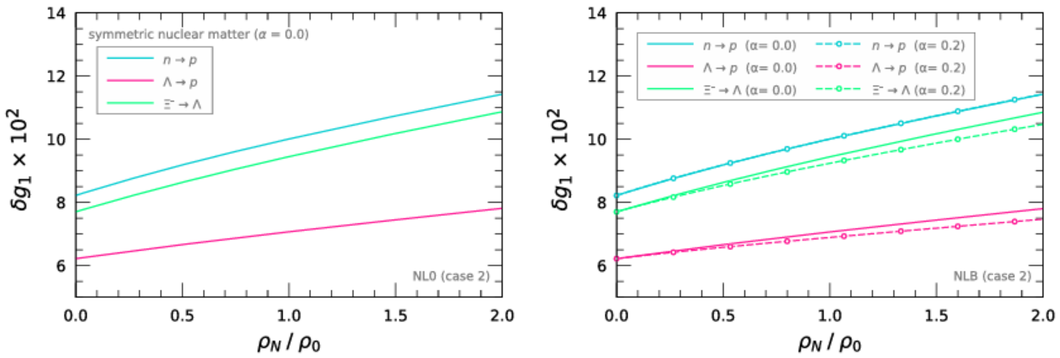


FIG. 11. Deviation, δg_1, in the axial-vector coupling constant as a function of ρ_N/ρ_0. We show the results in case 2. In the left panel, δg_1 in the NL0 model is displayed, in which the δ field is not included. Because the dependence of δg_1 on α is quite small in this model, we show the result with $\alpha = 0$ only. In the right panel, δg_1 in the NLB model is presented. In the neutron β decay, the results with $\alpha = 0$ and 0.2 are very close to each other.

Summary

1. We investigated effects of the isoscalar scalar (sigma) and isovector scalar (delta) fields on the semi-leptonic decay in free space and nuclear matter.
2. The evolution of u and d quark masses (m_d>m_u) in nuclear matter are evaluated. The smaller d quark mass than u quark mass are found with the increase of density.

$$\Delta^*_{du} = m^*_d - m^*_u = \Delta_{du} + 2g_\delta\bar{\delta},$$
$$\Delta^*_{su} = m^*_s - m^*_u = \Delta_{su} + \frac{g_\sigma}{3}\bar{\sigma} + g_\delta\bar{\delta},$$

where Δ_{du} and Δ_{du} are the original breaking in vacuum, and the meson-field terms are the additional ones in matter. It should be noticed that the mean field $\bar{\delta}$ is negative (positive) in neutron-rich (proton-rich) matter. Because the

3. We found that the deviation of vector CC from the SU symmetry amounts to O(10^{-6}) for SU(2), but it is **O(10^{-2}) for SU(3).** For axial CC, they are up to O(10^{-1}) for SU(2), but it is **O(10^{-2}) for SU(3).**

4. Main reason of the deviation for the vector CC comes from SU(3) symmetry breaking, but the axial CC stems from the relativistic effect due to the lower component of the wave function.

Electromagnetic |GE/GM| ratios of hyperons at large timelike q2

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Weak interaction axial form factors of the octet baryons in nuclear medium

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e-Print: 2406.07958 [hep-ph]

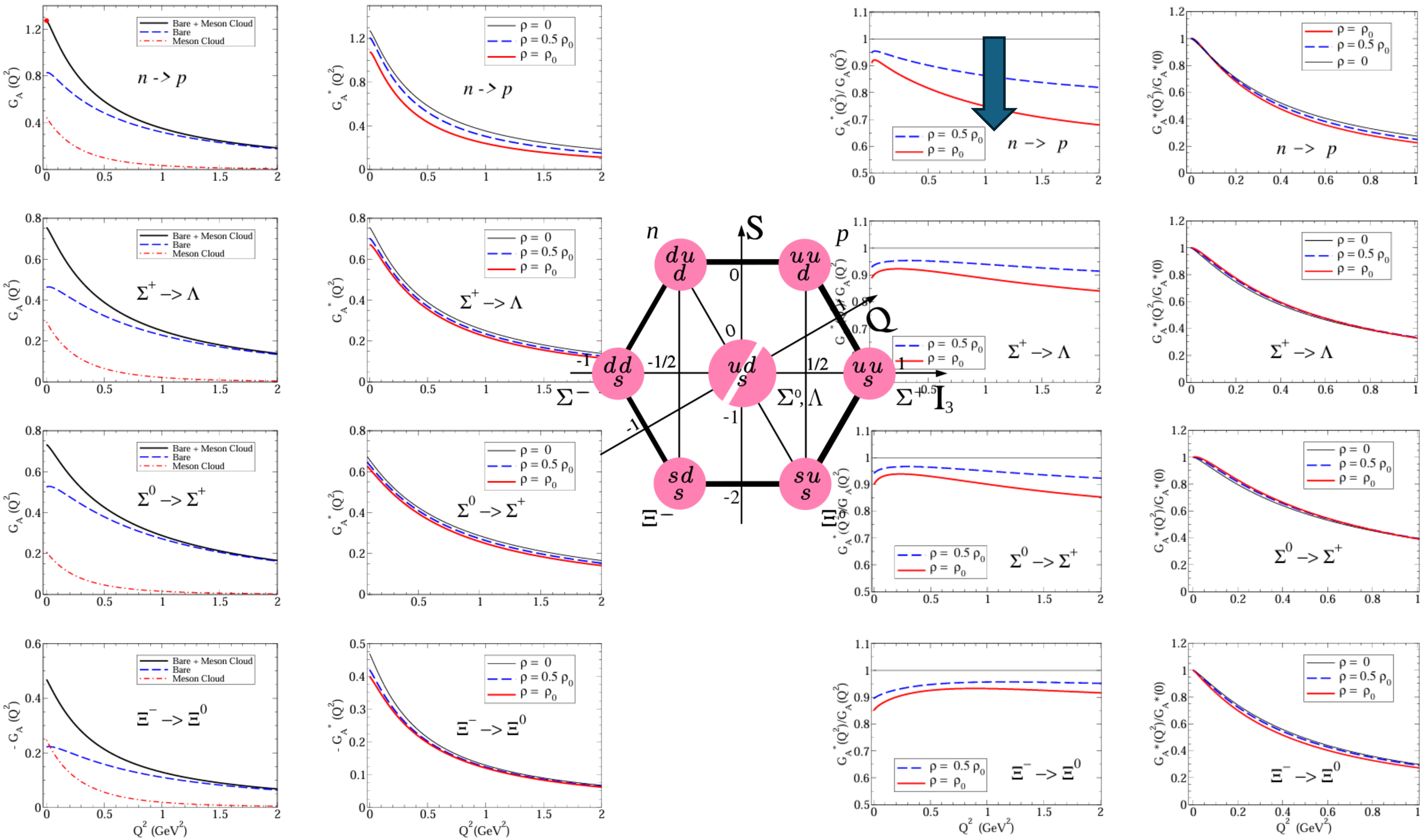
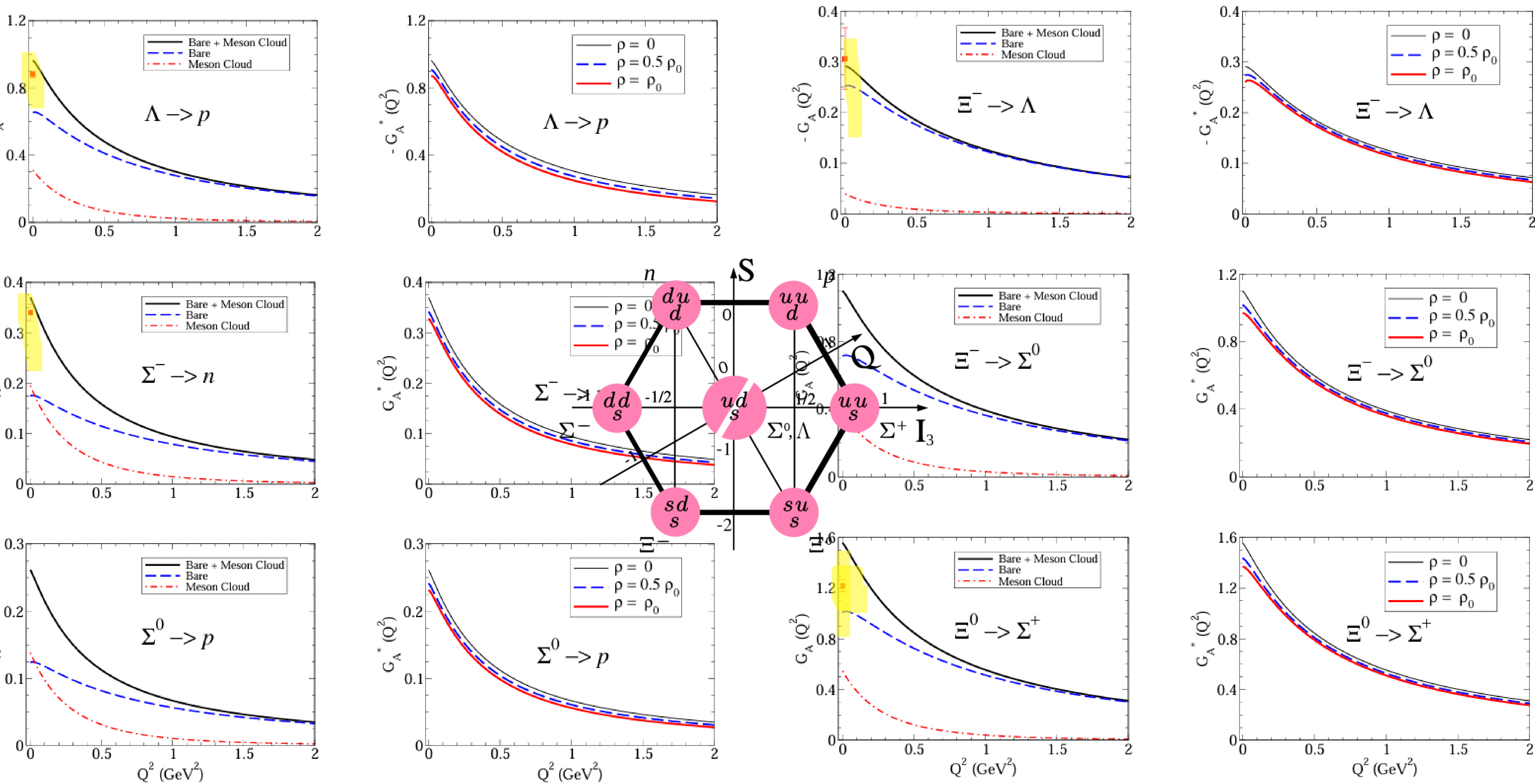


FIG. 3. Axial-vector form factor G_A for $\Delta I = 1$ transitions. **Left panel:** results in vacuum (bare, meson cloud and total). **Right panel:** total results for the medium $\rho = 0.5\rho_0$ and ρ_0 compared with vacuum ($\rho = 0$).

FIG. 9. Axial-vector form factor G_A for $\Delta I = 1$ transitions. Ratios G_A^*/G_A and $G_A^*(Q^2)/G_A^*(0)$.



xial-vector form factor G_A for $\Delta S = 1$ transitions (part 1). **Left panel:** results in vacuum (bare, meson cloud and total).
el: total results for the medium $\rho = 0.5\rho_0$ and ρ_0 compared with vacuum ($\rho = 0$).

. Axial-vector form factor G_A for $\Delta S = 1$ transitions (part 2). **Left panel:** results in vacuum (bare, meson cloud and
panel: total results for the medium $\rho = 0.5\rho_0$ and ρ_0 compared with vacuum ($\rho = 0$).

Summary

1. We investigated effects of the isoscalar scalar (sigma) and isovector scalar (delta) fields on the semi-leptonic decay in free space and nuclear matter.
2. The evolution of u and d quark masses (m_d>m_u) in nuclear matter are evaluated. The smaller d quark mass than u quark mass are found with the increase of density.

$$\Delta^*_{du} = m^*_d - m^*_u = \Delta_{du} + 2g_\delta\bar{\delta},$$
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4. Main reason of the deviation for the vector CC comes from SU(3) symmetry breaking, but the axial CC stems from the relativistic effect due to the lower component of the wave function.
This was also confirmed by using a different quark model, covariant spectator quark model.

5. In nuclear matter, the deviation becomes larger with increase of the density.

6. Now we are applying this evolution of axial and vectorial CC to the supernovae evolution and BBC.

GR1D : Relativistic Boltzmann equation

$$p^{\alpha}\left[\frac{\partial f_{\nu_i}}{\partial x^{\alpha}} - \Gamma^{\beta}_{\alpha\gamma}\, p^{\gamma}\, \frac{\partial f_{\nu}}{\partial p^{\beta}}\right] = \left[\frac{d f_{\nu}}{d\tau}\right]_{coll}$$

$\Gamma^{\beta}_{\alpha\gamma}$: connection coefficients-Christoffel symbols

f_{ν} : distribution function

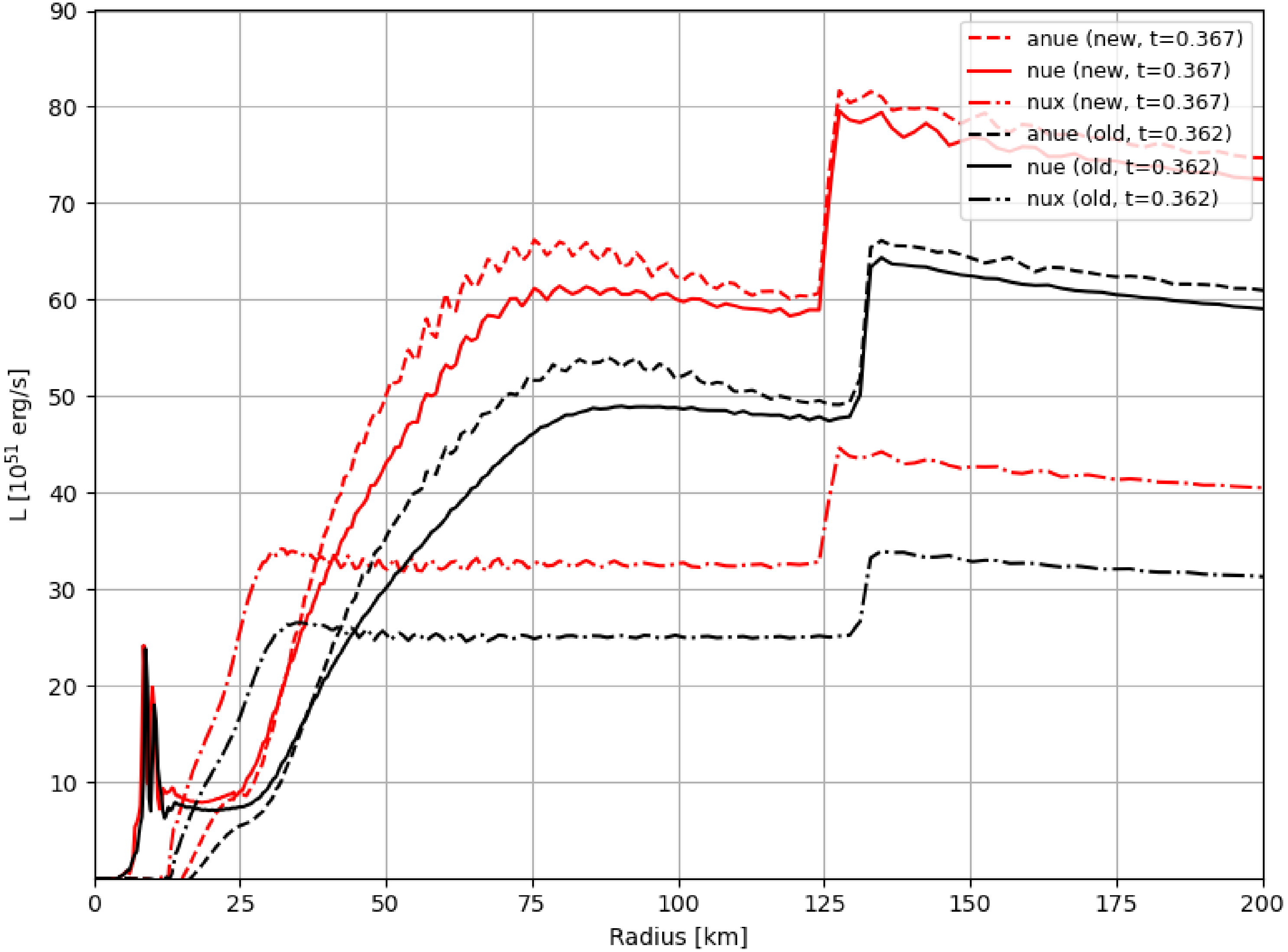
$\tau \rightarrow \frac{dx^{\alpha}}{d\tau} = p^{\alpha}$

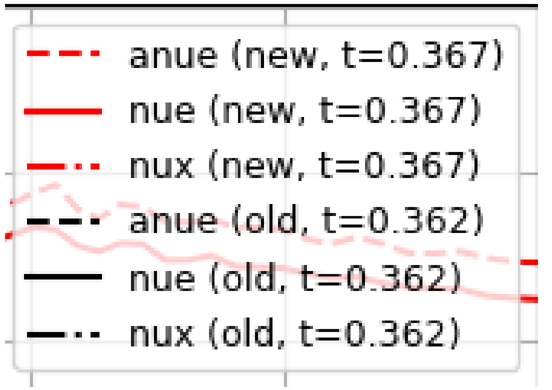
$\left[\frac{d f_{\nu}}{d\tau}\right]_{coll}$ = collision term

Neutrinos in supernovae is **High energy + not equilibrium**

→ use relativistic Boltzmann equation

Preliminary data : (g_A =1) vs (g_A =1.27) data





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Thanks for your attention !!